Unit -2

Page No. ______
Date

Stability Constraints in synchronous grids

The Stability of an interconnected Power System is it's ability to return to normal or stable Operation after having been subjected to some form of disturbance.

Conversely, instability means a condition denoting loss of synchronism or falling out of step.

With interconnected system continually growing in size and extending over vast geographical regions, it is becoming increasingly more difficult to maintain synchronism b/w various Parts of Power system.

The dynamics of a Power system are characterised by it's basic features given below:

① Synchrous tie exhibits the typical behaviour that as Power transfer is gradually increased as max. limit is reached beyond which the system can not stay in synchronism i.e, it falls out of step.

② The system is basically a spring - inertia Oscillatory system with inertia on the mechanical side and spring action provided by the synchronous tie wherein Power transfer is Proportional to $\sin\delta$ or δ (for small δ, δ being the relative internal angle of machines)

Page No.
Date

(3). Because of Power transfer being proportional to Sin δ, the equation determining system dynamics is non linear for disturbances causing large variation in angle δ. Stability phenomenon peculiar to non-linear system as distinguished from linear system is therefore exhibited by power system.

(Stable up to certain magnitude of disturbance and unstable for larger disturbances).

According Power system stability problems are classified into three basic types.

(1) Steady state stability

(2) Dynamic stability

(3) Transient stability.

Steady state stability ⇒

The study of steady state stability is basically concerned with the determination of upper limit of machine loading before losing synchronism, provided the loading is increased gradually.

Page No. ______
Date

Dynamic Stability =>

Dynamic instability is more Probable than steady state instability. Small disturbances are Continually occurring in a Power system (Variations in disturbances are continually occurring in a Power system (Variations in loadings, Changes in turbine speed etc) which are small enough not to cause the system to lose synchronism but do excite the system into the state of natural oscillations.

"The system is said to be dynamically stable if the oscillations do not acquire more than certain amplitude and die out quickly (i.e. system is well damped)."

In dynamically unstable system, the oscillation amplitude is large and these Persist for a long time (i.e, the system is underdamped). This kind of instability behaviour Constitutes a serious threat to system security and creates very difficult operating Conditions.

→ Dynamic stability Can be significantly improved through the use of Power system Stabilizers. Dynamic system study has to be carried out for 5-10 s and sometimes upto 30s. Computer simulation is the only effective means of studying dynamic stability problems.

→ The same simulation Programs are, of course, applicable to transient stability studies.

Page No. ______
Date

Transient Stability =>

Following a sudden disturbance on a Power system rotor speeds, rotor angular differences and Power transfer undergoes ~~fast~~ fast changes whose magnitudes are dependent upon the severity of disturbance. "For a large disturbance, changes in angular differences may be so large as to cause the machines to fall out of step. This type of unstability is known as transient instability" and is a fast Phenomenon usually occurring with in 1 s for a generator close to the ~~close~~ cause of disturbance.

There is a large range of disturbances which may occur on a Power system, but a fault on a heavily loaded line which requires opening the line to clear the fault is usually of greatest concern. The tripping of a loaded generator or the abrupt dropping of a large load may also cause instability.

Page No.
Date

Dynamics of a synchronous machine ⇒

The kinetic energy of the rotor at synchronous machine is

$$KE = \frac{1}{2} J \omega_{sm}^2 \times 10^{-6} \text{ MJ}$$

where, J = rotor moment of inertia in kg-m²

ω_{sm} = Synchronous speed in rad (mech)/s

But $\omega_s = \left(\frac{P}{2}\right) \omega_{sm}$ = rotor speed in rad (elect)/s

where P = number of machine Poles

$$KE = \frac{1}{2}\left[J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}\right) \omega_s$$

$$= \frac{1}{2} M \omega_s$$

Where $M = J\left(\frac{2}{P}\right)^2 \omega_s \times 10^{-6}$
= moment of inertia in MJ-sec/elec rad

we shall define the inertia constant H such that

$$GH = KE = \frac{1}{2} M\omega_s \text{ MJ}$$

where G = machine rating (base) in MVA (3-Phase)
H = inertia constant in MJ/MVA or MW-s/MVA

it follows that

$$\boxed{M = \frac{2GH}{\omega_s} = \frac{GH}{\pi f} \text{ MJ-s/elec rad}} \quad -(1)$$

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Page No. ____
Date

$$M = \frac{GH}{180f} \text{ MJ-s/elect degree}$$

M is also called the inertia constant

taking G as base, the inertia constant in pu is

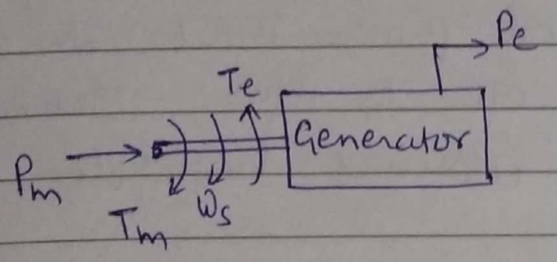
$$M(pu) = \frac{H}{\pi f} \text{ s}^2\text{/elect } \sout{\text{degree}} \text{ rad} \quad \text{— (2)}$$

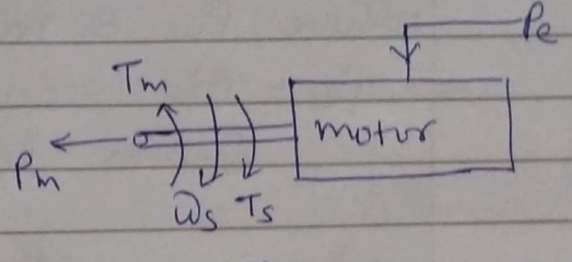
$$= \frac{H}{180f} \text{ s}^2\text{/ elect degree}$$

Page No.
Date

The Swing equation ⇒

Fig shows the torque, speed and flow of mechanical and electrical powers in a synchronous machine. It is assumed that the windage, friction and iron-loss torque is negligible.





The differential equation governing the rotor dynamics can then be written as

$$J \frac{d^2 \theta_m}{dt^2} = T_m - T_e \quad Nm \qquad — (3)$$

where θ_m = angle in rad (mech)

T_m = turbine torque in Nm ; it acquires a negative value for a motoring machine.

T_e = electromagnetic torque developed in Nm ; it acquires negative value for motoring machine.

While the rotor undergoes dynamics as per eq. (3), the rotor speed changes by insignificant magnitude for the time period of interest (1s). eq. (3) can therefore be converted into it's more convenient power form by assuming the

Date

the rotor speed to remain constant at the synchronous speed (ω_{sm}).
multiplying both side of eq. ③ by ω_{sm} we can write -

$$J\omega_{sm}\frac{d^2\theta_m}{dt^2}\times 10^{-6} = P_m - P_e \text{ MW} \quad —④$$

where P_m = mechanical power input in MW
P_e = electrical power O/P in MW; stator copper loss is assumed negligible.

rewriting eq ④

$$\left(J\left(\frac{2}{P}\right)^2\omega_s \times 10^{-6}\right)\frac{d^2\theta_e}{dt^2} = P_m - P_e \text{ MW}$$

where θ_e = angle in rad (elect)

or

$$M\frac{d^2\theta_e}{dt^2} = P_m - P_e \quad —⑤$$

it is more convenient to measure the angular position of the rotor with respect to a synchronously rotating frame of reference.
Let

$\delta = \theta_e - \omega_s t$; rotor angular displacement from synchronously rotating reference frame. (called torque angle or power angle).

—⑥

Page No.
Date

from eq. (6)

$$\frac{d^2\theta_e}{dt^2} = \frac{d^2\delta}{dt^2} \quad -(7)$$

hence eq (5) can be written in terms of δ as

$$M\frac{d^2\delta}{dt^2} = P_m - P_e \quad MW \quad -(8)$$

with M as defined in eq. (1) we can write

$$\frac{GH}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad MW \quad -(9)$$

Dividing throughout by G, the MVA rating of the machine

$$M(pu)\frac{d^2\delta}{dt^2} = P_m - P_e \quad -(10)$$

; in PU of machine rating as base

where $M(pu) = \frac{H}{\pi f}$

or $$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad PU \quad -(11)$$

This eq. (10/11), is called "Swing equation" and it describes the rotor dynamics for a synchronous m/c (generating/motoring)
it is a second order diff. eq. where the damping term (proportional to $d\delta/dt$) is absent because of assumption of a lossless m/c and the fact that the torque of damper winding has been ignored. This assumption leads to pessimistic results in transient stability analysis.

Page No.
Date

damping helps to stabilize the system. Damping must of course be considered in a dynamic stability study.

Multimachine system —

In a multimachine system a common system base must be chosen.

Let G_{mach} = machine rating base
G_{system} = System base

Eq. (11) can be rewritten as

$$\frac{G_{mach}}{G_{system}}\left(\frac{H_{mach}}{\pi f}\frac{d^2\delta}{dt^2}\right) = (P_m - P_e)\frac{G_{mach}}{G_{system}}$$

or

$$\frac{H_{system}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \quad \text{PU in system base} \qquad — (12)$$

where

$$H_{system} = H_{mach}\left(\frac{G_{mach}}{G_{system}}\right) \qquad — (13)$$

= machine inertia constant in system base.

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Page No. ____
Date

Machines swinging coherently –

Consider the swing equation of two machines on a common system base.

$$\frac{H_1}{\pi f} \frac{d^2 \delta_1}{dt^2} = P_{m_1} - P_{e_1} \text{ pu} \quad — (14)$$

$$\frac{H_2}{\pi f} \frac{d^2 \delta_2}{dt^2} = P_{m_2} - P_{e_2} \text{ pu} \quad — (15)$$

Since the machine rotors swings together (coherently or in unison)

$$\delta_1 = \delta_2 = \delta$$

adding eq. (14) and (15)

$$\frac{H_{eq}}{\pi f} \frac{d^2 \delta}{dt^2} = P_m - P_e \quad — (16)$$

where
$$P_m = P_{m_1} + P_{m_2}$$
$$P_e = P_{e_1} + P_{e_2} \quad — (17)$$
$$H_{eq} = H_1 + H_2$$

Two machines swinging coherently are thus reduces to a single machine as in eq (16)

$$H_{eq} = H_{1\,mach}\, G_{1\,mach} / G_{system} + H_{2\,mach}\, G_{2\,mach} / G_{system} \quad — (18)$$

The above results are easily extendable to any no. of machines swinging coherently.

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✡ The Power Flow ⇒

Consider fig for the calculation of Power flow. All the quantities have been expressed in Polar form.

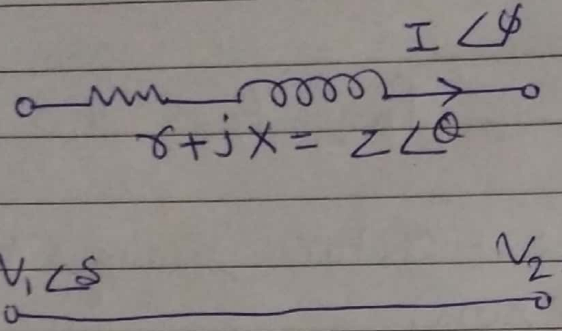

fig:- Power flow in a 1-φ line

$$I = \frac{V_1 \angle \delta - V_2 \angle 0}{Z \angle \theta} = \frac{V_1}{Z} \angle \delta - \theta - \frac{V_2}{Z} \angle -\theta$$

Power recieved is given by

$$P_2 = Re\left[V_2 I^*\right]$$

$$P_2 = Re\left[V_2 \left\{\frac{V_1}{Z} \angle(\theta - \delta) - \frac{V_2}{Z} \angle \theta\right\}\right]$$

$$= \frac{V_1 V_2}{Z} \cos(\theta - \delta) - \frac{V_2^2}{Z} \cos\theta \quad -① $$

Let $\theta = 90 - \alpha$

$$P_2 = \frac{V_1 V_2}{Z} \cos(90 - \alpha - \delta) - \frac{V_2^2}{Z} \cos(90 - \alpha)$$

$$P_2 = \frac{V_1 V_2}{Z} \sin(\alpha + \delta) - \frac{V_2^2}{Z} \sin\alpha \quad -②$$

Now α is a function of the impedance of the line, therefore, the Power P_2 recieved is maximum when $\alpha + \delta = 90°$ or $\delta = (90 - \alpha)$

$$P_{2\,max} = \frac{V_1 V_2}{Z} - \frac{V_2^2}{Z} \sin\alpha$$

also $\sin\alpha = \frac{r}{Z}$

$$\therefore P_{2\,max} = \frac{V_1 V_2}{\sqrt{r^2+x^2}} - \frac{V_2^2}{\sqrt{r^2+x^2}} \cdot \frac{r}{\sqrt{r^2+x^2}}$$

and when $V_1 = V_2$,

$$P_{2\,max} = V_2^2 \left[\frac{1}{\sqrt{r^2+x^2}} - \frac{r}{r^2+x^2} \right] \quad \text{--- (3)}$$

for $P_{2\,max}$ to be maximum

$$\frac{dP_{2\,max}}{dx} = 0 = V_2^2 \left[\frac{x}{(r^2+x^2)^{3/2}} - \frac{2xr}{(r^2+x^2)^2} \right]$$

or
$$\frac{V_2^2\, x}{(r^2+x^2)^2} \left[\sqrt{r^2+x^2} - 2r \right] = 0$$

or
$$\sqrt{r^2+x^2} = 2r$$

$$r^2 + x^2 = 4r^2$$

$$x = \sqrt{3}\, r \quad \text{--- (4)}$$

This shows that the max. Power can be transferred from end 1 to end 2 when the reactance of line is $\sqrt{3}$ times it resistance.
Normally, the reactance is quite large as

Page No. ____
Date

compared to the resistance.
The eq. shows that it is not necessarily desirable to compensate the series capacitance for all the reactance. Also it is clear that the power can be transferred only if reactance is present. In case reactance is zero power can not be transmitted.

for a lossless line $r = 0$ and transmitted power

$$P_2 = \frac{V_1 V_2}{x} \sin \delta \quad — (5)$$

This eq. shows that the power transmitted depends upon the system reactance and angle b/w two rotors.
The curve drawn b/w P_2 and δ is known as the power angle curve and is shown in fig below.

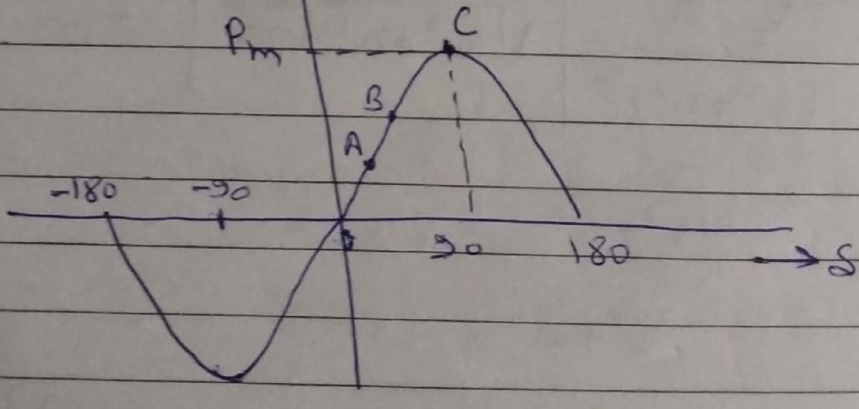

fig. Power angle curve

The max. power transmitted is given by

$$P_m = \frac{V_1 V_2}{x}$$

Page No.
Date

→ For a given V_1, V_2 and x and occurs at one angle of 90° the torque angle δ is Positive for generator action and negative for motor action.

→ In case of generator action the rotor advances in the direction of rotation whereas for motor action, the rotor retards or falls back opposite to the direction of rotation.

→ max. value of Power transmitted can be varied by varying V_1, V_2 and x (the circuit reactance)

→ The system is stable if and only if for an increase in rotor angle δ, the transmitted Power also increases i.e, $dP/d\delta$ should be Positive.

→ it can be seen from fig that the range where $dP/d\delta$ is Positive lies b/w +90 and -90°

→ when tie-line impedance is Purely Capacitive (negative reactance), the range of angle for delivery Power to the system is from 180° to 270° in-stead of from 0 to 90°.

→ At 0° with inductive reactance the Power transmitted is zero whereas at 180° with Capacitive reactance even though the Power to be transmitted is zero but a large wattless current will flow which is not desirable, and therefore, normally over compensation of lines (by using series

Date

Capacitor is never done.

→ We study the Power angle curve in detail -
let P be the mech. I/P to the generator and
the mech. O/P from the motor assuming
negligible friction and transmission losses.
Say initially this Power Corresponds to Point A
on the Power angle curve.
if a small increment of shaft load is
added to the motor, The O/P Power of motor is
increases as the speed does not change moment-
-arily where as the I/P to the motor remains
unchanged. Therefore, there is a net torque
on the motor tending to retard it and it's
speed decreases temporarily. As a result of
reduction in motor speed, the rotor angle δ
increases and Cosequently the Power input to the
motor increases until finally the I/P and O/P
are again in equilibrium and steady operation
takes Place at a new Point B higher than A on
the Power angle curve.
The gradual addition of load on the motor shaft
is Possible till the Point C is reached on the
Power angle curve when $P = P_{max}$. and any
further addition of load will result in increase
in angle δ but reduction in I/P Power to motor
and, Therefore, the motor will decelerate further
and it will Pull out of step and will Probably
stall unless it has damper winding which may
keep it running as an induction motor.

Page No.
Date

→ Pm is known as steady state stability limit of the system which means that it is the max. Power that can be transmitted and synchronism will be lost if an attempt is made to transmit Power more than this limit.

→ The steady state stability limit can be increased by

(i) increasing the excitation of the motor or generator or both so that the internal emfs are increased, and

(ii) reducing the reactance. This is done either by running parallel lines or by using the series capacitors.

Page No. ____
Date

Steady State Stability =>

The study of steady state stability of Power system involves the study of dyna-mics of the system when the rate of appli-cation of load is quite slow as compared with the natural freq. of oscillation of the system. The dynamics of the system can be described by the swing equation, which is non linear.

If the changes are small, these eq. can be linearized around the initial operating point.

consider a system consisting of a genera-tor connected to an infinite busbar through a lossless network with initial operating point on power angle curve as (P_0, δ_0)

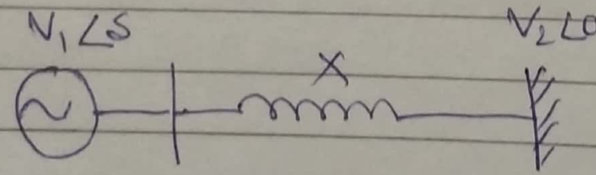

fig- finite machine connected to an infinite bus

Say the load is changed (increased) by ΔP, as a result of which the load angle changes by $\Delta\delta$ and the swing equation can be linearized around the point (P_0, δ_0) and is given as

$$M \frac{d^2 \Delta\delta}{dt^2} = P_i - P_e$$

Page No. ____
Date

$$= P_i - (P_o + \Delta P) = -\Delta P$$

$$= -\left(\frac{\partial P}{\partial \delta}\right)\Bigg|_{\delta_0} \cdot \Delta\delta \quad — (1)$$

Now let $\frac{d}{dt} = p$, eq. (1) reduces to

$$m p^2 \Delta\delta + \frac{\partial P}{\partial \delta}\Big|_{\delta_0} \cdot \Delta\delta = 0$$

or $$\left(m p^2 + \frac{\partial P}{\partial \delta}\Big|_{\delta_0}\right)\Delta\delta = 0 \quad — (2)$$

eq. (2) is the characteristic eq. with the two roots.

$$p = \pm\left(\frac{-\frac{\partial P}{\partial \delta}\Big|_{\delta_0}}{m}\right)^{1/2} \quad — (3)$$

When $\frac{\partial P}{\partial \delta}$ is positive $\delta < \frac{\pi}{2}$, the two roots are pure imaginary and conjugate, the rotor motion is oscillatory and undamped around δ_0. Line resistance and damper winding of m/c which have been ignored in above modelling, cause the system oscillation to decay and system is therefore stable for small increment in power.

When $\frac{\partial P}{\partial \delta}$ is negative $(\delta > 90^\circ)$, both the roots are real, one positive and the other negative respectively and hence the system is unstable.

When $\frac{\partial P}{\partial \delta} = 0$, $\delta = 90^\circ$ the system is critically stable. The freq. of oscillation of the system is given by the roots of

Page No.
Date

Characteristics equation.

Let us now study the steady state stability of the system when series resistance and shunt capacitance are included. The power transmitted b/w bus 1 and 2 is given by

$$P = \frac{V_1 V_2}{|B|} \cos(\beta - \delta) - \frac{A V_2^2}{|B|} \cos(\beta - \alpha) \quad — (4)$$

for system to be stable under steady-state operation

$$\frac{\partial P}{\partial \delta} > 0$$

$$\frac{V_1 V_2}{|B|} \sin(\beta - \delta) > 0$$

or $\beta > \delta$

And the system is critically stable

$$\frac{\partial P}{\partial \delta} = 0$$

$$\beta = \delta$$

And the max. power transfer is given as

$$P_m = \frac{V_1 V_2}{|B|} - \frac{A V_2^2}{|B|} \cos(\beta - \alpha) \quad — (5)$$

which is an indication of steady state

Page No. ______
Date

Stability limit:

If resistance is considered but shunt ~~cha~~ capacitance neglected, we have
$|B| > X$, $\beta < 90^\circ$ and $A = 1.0 \angle 0^\circ$

i.e $\alpha = 0^\circ$
So that from eq. (5)

$$P_m = \frac{V_1 V_2}{|B|} - \frac{A V_2^2}{|B|} \cos\beta \quad -(6)$$

Comparing eq. (6) with $P_2 = \frac{V_1 V_2}{X} \sin\delta$,

since $|B| > X$ and eq. (6) contains a negative term also, the steady state stability limit is, therefore, lowered and hence eq. $P = \frac{V_1 V_2}{X} \sin\delta$ will gives optimistic results. it is to be noted, however, that transient stability ~~analy~~ results are optimistic when resistence is included.
If capacitance is also considered when $|A| < 1$ and $\alpha > 0$ it can be seen that P_m will now have a slightly higher value as compared to when capacitance is neglected but this value is still lower than the one given by eq. $P_2 = \frac{V_1 V_2}{X} \sin\delta$.

While derive eq. $P_2 = \frac{V_1 V_2}{X} \sin\delta$ we assumed that the emf induced ~~the~~ behind the reactance of the generator is constant which means the excitation of the generator is

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Kept constant. But in actual Practice, it is the terminal voltage which is kept constant and hence with change in load the internal emf of generator will change.

Date

Q. A 50 Hz four pole turbogenerator rated 20 MVA 13.2 kV has an inertia constant of H = 9.0 kW-sec/kVA. Determine the KE stored in the rotor at synchronous speed. Determine the acceleration if the input less the rotational losses is 25000 HP and the electric power developed is 15000 kW. If the acceleration computed for the generator is constant for a period of 15 cycles, determine the change in torque angle in that period and the rpm at the end of 15 cycles. Assume the generator is synchronized with a large system and has no accelerating torque before the 15 cycle period begins.

Solution – (a) Since $H = \dfrac{\text{stored energy in MJ}}{\text{machine rating in MVA}}$

∴ KE stored in rotor in MJ = H × G

$$KE = 9 \times 20 = 180 \text{ MJ}$$

(b) The accelerating power = $P_m - P_e$

$$P_a = 25000 \times 0.735 - 15000$$
$$= 18375 - 15000 = 3.375 \text{ kW}$$

Now the acceleration is

$$\frac{d^2\delta}{dt^2} = \frac{P_a}{M} = \frac{3.375}{M}$$

$$M = \frac{GH}{\pi f} = \frac{20 \times 9}{\pi \times 50} = 1.146 \text{ MJ sec/radian}$$

$$\frac{d^2\delta}{dt^2} = \frac{3.375}{1.146} = 2.945 \text{ rad/sec}^2$$

Page No.
Date

(c) Again using swing equation

$$\frac{d^2\delta}{dt^2} = \frac{Pa}{m} = \text{Constant} = 2.945$$

$$2\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2}\,dt = 2\times 2.945\,\frac{d\delta}{dt}\,dt$$

$$\left(\frac{d\delta}{dt}\right)^2 = 5.89\,\delta + A$$

Since at $t=0$, $\frac{d\delta}{dt} = 0$ $\therefore A = 0$

$$\frac{d\delta}{dt} = \sqrt{5.89}\cdot\sqrt{\delta}$$

$$\frac{d\delta}{\sqrt{\delta}} = \sqrt{5.89}\,dt$$

$$\delta^{-1/2}\,d\delta = 2.427\,dt$$

$$2\delta^{1/2} = 2.427\,t$$

$$\delta^{1/2} = 1.2135t$$

$$\delta = 1.47258\times t^2$$

Now $t = \frac{15}{50} = \frac{3}{10} = 0.3$ sec.

$\delta = 1.47258\times 0.09 = 0.1325$ rad

$= 7.5955$ electrical rad

Now $\frac{d\delta}{dt} = 2.425\sqrt{\delta} = 2.425\sqrt{0.1325}$

$= 0.8827$ rad/sec

$= \frac{0.8827}{4\pi}$ rps

or $\frac{0.8827}{4\pi}\times 60$ rpm $= 4.2$ rpm

$\therefore$ Rotor speed at the end of 15 cycles $= 1504.2$ rpm

Q. A synchronous generator of reactance 1.20 pu is connected to an infinite bus (|V| = 1.0 pu) through transformers and a line of total reactance of 0.60 pu. The generator no load voltage is 1.20 pu and it's inertia constant is H = 4 MW-s/MVA. The resistance and machine damping may be assumed ~~to~~ negligible. The system frequency is 50 Hz. Calculate the frequency of natural oscillations if the generator is loaded to

(i) 50% and (ii) 80% of it's maximum power limit.

Solution:- Since the system is operating initially under steady state condition, a small perturbation in power will make the rotor oscillate. The natural freq of oscillation is given by

$$f_n = \left\{ \left(\frac{\partial P_e}{\partial \delta}\right)_{\delta_0} / M \right\}^{1/2}$$

(i) for 50% loading → $\sin\delta_0 = \frac{P_e}{P_{max}} = 0.5$ or $\delta_0 = 30°$

$$\frac{\partial P_e}{\partial \delta} = \frac{V_1 V_2}{X} \cos\delta$$

Page No.
Date

$$\left[\frac{\partial P_e}{\partial \delta}\right]_{30°} = \frac{1.2 \times 1}{(1.2+0.6)} \cos 30° = \frac{1.2 \times 1}{1.8} \cos 30°$$

$= 0.577$ MW (PU/elect rad)

$$M(PU) = \frac{H}{\pi f} = \frac{4}{\pi \times 50} \; s^2/\text{elect rad}$$

from characteristics eq.

$$P = \pm j \left[\left(\frac{\partial P_e}{\partial \delta}\right)_{30°} / M \right]^{1/2}$$

$$= \pm j \left[\frac{0.577 \times 50\pi}{4} \right]^{1/2} = \pm j 4.76$$

frequency of oscillation $= 4.76$ rad/sec.

$$= \frac{4.76}{2\pi} = 0.758 \text{ HZ.}$$

for (ii) 80% loading

$$\sin\delta_0 = \frac{P_e}{P_{max}} = 0.8 \quad \text{or} \quad \delta_0 = 53.1°$$

$$\left(\frac{\partial P_e}{\partial \delta}\right)_{53.1°} = \frac{1.2 \times 1}{1.8} \cos 53.1° = 0.4 \text{ MW (PU/ele rad)}$$

$$P = \pm j \left(\frac{0.4 \times 50\pi}{4}\right)^{1/2} = \pm j 3.96$$

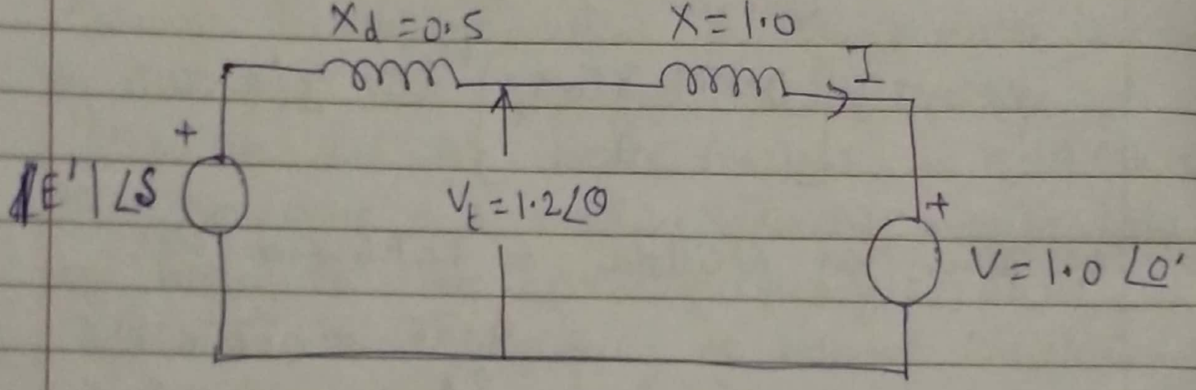
frequency of oscillations $= 3.96$ rad/sec

$$= \frac{3.96}{2\pi} = 0.63 \text{ HZ}$$

Page No. ______
Date

Q: Find the steady state power limit of a system consisting of a generator equivalent reactance 0.50 pu connected to an infinite bus through a series reactance of 1.0 pu. The terminal voltage of the generator is held at 1.20 pu and the voltage of the infinite bus is 1.0 p.u

Solution:- The system is shown in fig. Let the voltage of infinite bus be taken as reference



then $V = 1.0 \angle 0'$, $V_t = 1.2 \angle \theta$

Now $I = \dfrac{V_t - V}{jX} = \dfrac{1.2\angle\theta - 1.0}{j1}$

$E = V_t + jX_d I = 1.2\angle\theta + j0.5\left[\dfrac{1.2\angle\theta - 1.0}{j1}\right]$

or

$E = 1.8\angle\theta - 0.5 = (1.8\cos\theta - 0.5) + j1.8\sin\theta$

Steady state power limit is reached when E has an angle of $\delta = 90'$ i.e it's real part is zero

$1.8\cos\theta - 0.5 = 0$

or $\theta = 73.87°$

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Page No. ______
Date

Now $V_t = 1.2\angle 73.87° = 0.332 + j1.152$

$$I = \frac{0.332 + j1.152 - 1}{j1} = 1.152 + j0.668$$

$E = 0.332 + j1.152 + j0.5(1.152 + j0.668)$

$= -0.002 + j1.728 \doteq 1.728\angle 90°$

Steady state Power limit is given by

$$P_{max} = \frac{|E||V|}{X_d + X} = \frac{1.728 \times 1}{1.5} = 1.152 \text{ PU}$$

if instead, the generator emf is held fixed at a value of 1.2 PU, the steady state Power limit would be

$$P_{max.} = \frac{1.2 \times 1}{1.5} = 0.8 \text{ PU}$$

→ it is observed here that regulating the generator emf to hold the terminal generator Voltage at 1.2 PU raises the Power limit from 0.8 PU to 1.152 PU; this is how the voltage regulating loop helps in Power system stability.

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Page No.
Date

Equal area criterion =>

The equal area criterion is derived using the swing eq. for a machine connected to an infinite bus.
The swing eq. is given as

$$M \frac{d^2\delta}{dt^2} = P_a = P_s - P_e$$

Multiplying both the sides of the eq. by $2\,d\delta/dt$, and integrating with respect to time we get

$$\int 2M \frac{d^2\delta}{dt^2} \cdot \frac{d\delta}{dt}\, dt = \int 2(P_s - P_e) \frac{d\delta}{dt}\, dt$$

$$M \left(\frac{d\delta}{dt}\right)^2 = 2\int_{\delta_0}^{\delta} (P_s - P_e)\, d\delta$$

or $$\frac{d\delta}{dt} = \sqrt{\frac{2}{M}\int_{\delta}^{\delta} (P_s - P_e)\, d\delta + c} \quad \text{—(1)}$$

where δ_0 is the initial torque angle before any disturbance occurs and at this time $d\delta/dt = 0$. The angle δ will stop changing and m/c will again be operating at synchronous speed after a disturbance when $d\delta/dt = 0$ or

when $$\int_{\delta_0}^{\delta} (P_s - P_e)\, d\delta = 0 = \int_{\delta_0}^{\delta} P_a\, d\delta \quad \text{—(2)}$$

This means that the area under the curve Pa

Should be zero which is Possible only when Pa is both accelerating and deaccelerating Powers, i.e, for a Part of the graph $P_s > P_e$ and for other $P_e > P_s$.
for a generator action $P_s > P_e$ for Positive area A_1 and $P_e > P_s$ for negative area A_2 for stable operation hence the name equal area criterion.

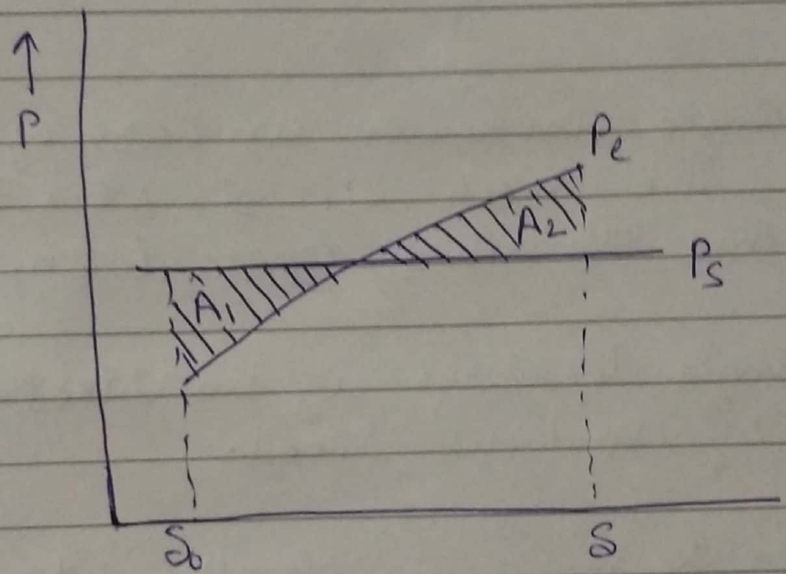

fig 1 equal area criterion.

The area A_1 represents the Kinetic energy stored by rotor during acceleration and the area A_2 represents the Kinetic energy given up by the rotor to the system and when it is all give up, the m/c has returned to its original speed.

Page No.
Date

⇒ Operation of synchronous motor using equal area criterion when sudden increase in mechanical load on that motor

The following points are to be noted with regard to the change in torque-angle whenever a disturbance occurs.

① There is no change in torque angle when the speed of the rotor is the synchronous speed.

② The angle increases in case of a motor if $P_s > P_e$ i.e mech. O/P is more then elect. input and the speed goes down.

③ The angle decreases if the speed is more than synchronous speed.

initially the motor is operating when torque angle is δ_0 with mech. O/P P_0. Let this load be increased to P_s. Momentarily there is no change in angle δ corresponding to electrical i/p to motor $P_e < P_s$. Therefor motor deaccelerates (K.E supplied by the motor) as a result torque angle increases and P_e starts increasing.
At point b, $P_e = P_s$ and therefore deaccelerating force is zero. but due to inertia of the rotor, torque angle goes on increasing. The speed of the roltor beyond b, starts increasing. The speed is minimum at b. when the speed equals the synchronous speed beyond point b, say

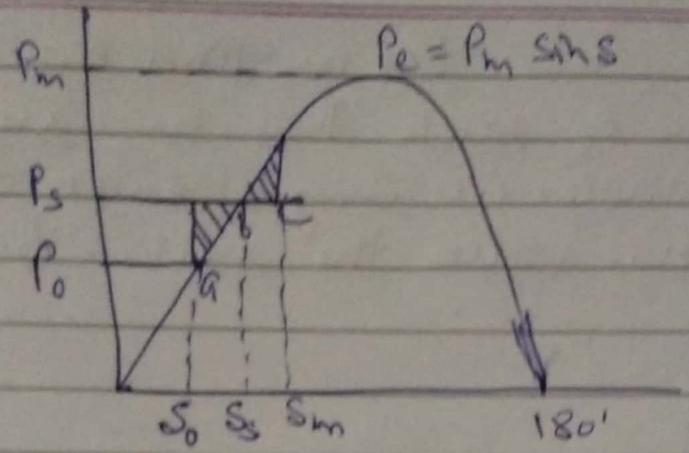

fig.2 sudden change of load - equal area criterion

at point c, the rotor angle stops increasing. but b/w c & b $P_e > P_s$; therefore motors accelerates and torque angle starts decreasing. The speed goes on increasing till it reaches the point b where this time the speed is max. and is more than synchronous speed. Between b and a, $P_s > P_e$ therefore, the rotor starts deaccelerating but the speed is more than synchronous speed till it reaches the point a where once again the speed equals to synchronous speed. and angle stops decreasing.

The speed at a and c is the synchronous speed where as at b, the speed is below synchronous speed when the rotor oscillates from a to b and is above synchronous speed when it oscillates b/w c and b.

we have seen that with power angle curve shown in fig 2. if the load on the motor shaft is increased suddenly from P_0 to P_s, the system is stable.

Page No.
Date

For this system let us find out the max value of the P_s such that the system is critically stable. i.e any attempt to increase P_s beyond this value the system becomes unstable.

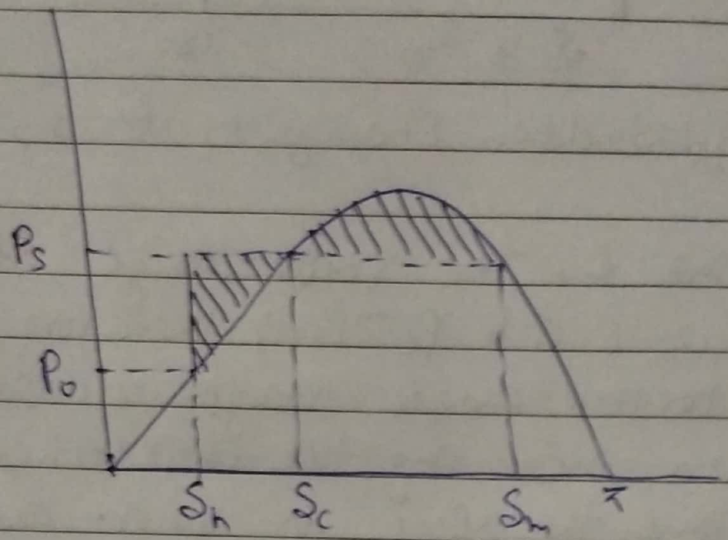

fig3: Transient stability limit

Reffering to fig 3.

$$P_s = P_m \sin\delta_c = P_m \sin\delta_m$$

$$\therefore \quad \delta_m = (\pi - \delta_c)$$

Here δ_c is known as critical torque angle. for the two shaded areas to be equal, the following condition should be satisfied:

$$P_s(\delta_m - \delta_o) = \int_{\delta_o}^{\delta_m} P_m \sin\delta \, d\delta \quad \text{—(3)}$$

also $P_s = P_m \sin\delta_m$

$$P_m \sin\delta_m (\delta_m - \delta_o) = \int_{\delta_o}^{\delta_m} P_m \sin\delta \, d\delta$$

$$P_m \sin\delta_m (\delta_m - \delta_o) = P_m (\cos\delta_o - \cos\delta_m) \quad \text{—(4)}$$

Here δ_m is the only unknown which can be obtained and hence P_s can be calculated.

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Some Comments on Steady State Stability

A knowledge of steady state stability limit is important for various reasons. A system can be operated above its transient stability limit but not above its steady state limit. Now, with increased fault clearing speeds, it is possible to make the transient limit closely approach the steady state limit.

As is clear from Eq. (12.51), the methods of improving steady state stability limit of a system are to reduce X and increase either or both $|E|$ and $|V|$. If the transmission lines are of sufficiently high reactance, the stability limit can be raised by using two parallel lines which incidently also increases the reliability of the system. Series capacitors are sometimes employed in lines to get better voltage regulation and to raise the stability limit by decreasing the line reactance. Higher excitation voltages and quick excitation system are also employed to improve the stability limit.

12.7 TRANSIENT STABILITY

It has been shown in Sec. 12.4 that the dynamics of a single synchronous machine connected to infinite bus bars is governed by the nonlinear differential equation

$$M \frac{d^2\delta}{dt^2} = P_m - P_e$$

where $\quad P_e = P_{max} \sin \delta$

or
$$M \frac{d^2\delta}{dt^2} = P_m - P_{max} \sin \delta \tag{12.52}$$

As said earlier, this equation is known as the *swing equation*. No closed form solution exists for swing equation except for the simple case $P_m = 0$ (not a practical case) which involves elliptical integrals. For small disturbance (say, gradual loading), the equation can be linearized (see Sec. 12.6) leading to the concept of steady state stability where a unique criterion of stability ($\partial P_e/\partial \delta > 0$) could be established. No generalized criteria are available* for determining system stability with large disturbances (called transient stability). The practical approach to the transient stability problem is therefore to list all important severe disturbances along with their possible locations to which the system is likely to be subjected according to the experience and judgement of the power system analyst. Numerical solution of the swing equation (or equations for a multimachine case) is then obtained in the presence of such disturbances giving a plot of δ vs. t called the *swing curve*. If δ starts to decrease after reaching a maximum value, it is normally assumed that the system is stable and the oscillation of δ around the equilibrium point will decay

*Recent literature gives methods of determining transient stability through Liapunov and Popov's stability criteria, but these have not been of partical use so far.

and finally die out. As already pointed out in the introduction, important severe disturbances are a short circuit or a sudden loss of load.

For ease of analysis certain assumptions and simplifications are always made (some of these have already been made in arriving at the swing equation (Eq. (12.52)). All the assumptions are listed, below along with their justification and consequences upon accuracy of results.

1. Transmission line as well as synchronous machine resistance are ignored. This leads to pessimistic result as resistance introduces damping term in the swing equation which helps stability. In Example 12.11, line resistance has been taken into account.
2. Damping term contributed by synchronous machine damper windings is ignored. This also leads to pessimistic results for the transient stability limit.
3. Rotor speed is assumed to be synchronous. In fact it varies insignificantly during the course of the stability transient.
4. Mechanical input to machine is assumed to remain constant during the transient, i.e., regulating action of the generator loop is ignored. This leads to pessimistic results.
5. Voltage behind transient reactance is assumed to remain constant, i.e., action of voltage regulating loop is ignored. It also leads to pessimistic results.
6. Shunt capacitances are not difficult to account for in a stability study. Where ignored, no greatly significant error is caused.
7. Loads are modelled as constant admittances. This is a reasonably accurate representation.

Note: Since rotor speed and hence frequency vary insignificantly, the network parameters remain fixed during a stability study.

A digital computer programme to compute the transient following sudden disturbance can be suitably modified to include the effect of governor action and excitation control.

Present day power systems are so large that even after lumping of machines (Eq. (12.17)), the system remains a multimachine one. Even then, a simple two-machine system greatly aids the understanding of the transient stability problem. It has been shown in Section 12.4 that an equivalent single-machine infinite bus system can be found for a two-machine system (Eqs. (12.41) to (12.43)).

Upon occurrence of a severe disturbance, say a short circuit, the power transfer between machines is greatly reduced, causing the machine torque angles to swing relatively. The circuit breakers near the fault disconnect the unhealthy part of the system so that power transfer can be partially restored, improving the chances of the system remaining stable. The shorter the time to breaker operating, called *clearing time*, the higher is the probability of the system being stable. Most of the line faults are transient in nature and get cleared on opening the line. Therefore, it is common practice now to employ *autoreclose breakers* which automatically close rapidly after each of the two sequential openings. If the fault still persists, the circuit breakers open and lock permanently till cleared manually. Since in the majority of faults the first reclosure will be successful, the chances of system stability are greatly enhanced by using autoreclose breakers.

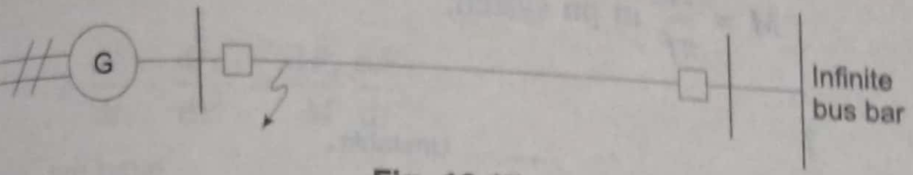


Fig. 12.17

The procedure of determining the stability of a system upon occurrence of a disturbance followed by various switching off and switching on actions is called a *stability study*. Steps to be followed in a stability study are outlined below for a single-machine infinite bus bar system shown in Fig. 12.17. The fault is assumed to be a transient one which is cleared by the time of first reclosure. In the case of a permanent fault, this system completely falls apart. This will not be the case in a multimachine system. The steps listed, in fact, apply to a system of any size.

1. From prefault loading, determine the voltage behind transient reactance and the torque angle δ_0 of the machine with reference to the infinite bus.
2. For the specified fault, determine the power transfer equation $P_e(\delta)$ during fault. In this system $P_e = 0$ for a three-phase fault.
3. From the swing equation starting with δ_0 as obtained in step 1, calculate δ as a function of time using a numerical technique of solving the non-linear differential equation.
4. After clearance of the fault, once again determine $P_e(\delta)$ and solve further for $\delta(t)$. In this case, $P_e(\delta) = 0$ as when the fault is cleared, the system gets disconnected.
5. After the transmission line is switched on, again find $P_e(\delta)$ and continue to calculate $\delta(t)$.
6. If $\delta(t)$ goes through a maximum value and starts to reduce, the system is regarded as stable. It is unstable if $\delta(t)$ continues to increase. Calculation is ceased after a suitable length of time.

An important numerical method of calculating $\delta(t)$ from the swing equation will be given in Section 12.9. For the single machine infinite bus bar system, stability can be conveniently determined by the equal area criterion presented in the following section.

12.8 EQUAL AREA CRITERION

In a system where one machine is swinging with respect to an infinite bus, it is possible to study transient stability by means of a simple criterion, without resorting to the numerical solution of a swing equation.

Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_e) = \frac{P_a}{M};\ P_a = \text{accelerating power}$$

$$M = \frac{H}{\pi f} \text{ in pu system} \tag{12.53}$$

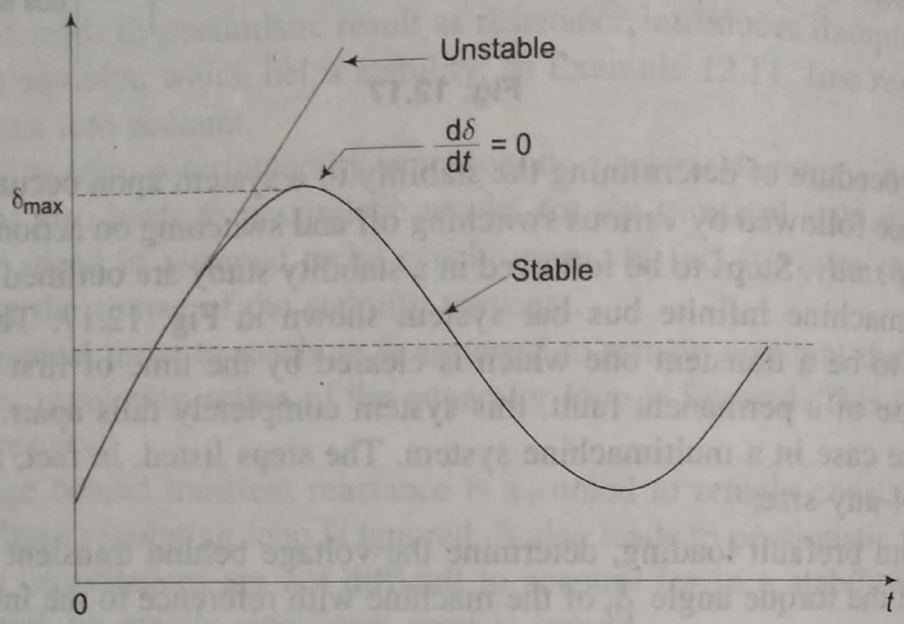


Fig. 12.18 Plot of δ *vs* t for stable and unstable systems

If the system is unstable δ continues to increase indefinitely with time and the machine loses synchronism. On the other hand, if the system is stable, $\delta(t)$ performs oscillations (nonsinusoidal) whose amplitude decreases in actual practice because of damping terms (not included in the swing equation). These two situations are shown in Fig. 12.18. Since the system is non-linear, the nature of its response [$\delta(t)$] is not unique and it may exhibit instability in a fashion different from that indicated in Fig. 12.18, depending upon the nature and severity of disturbance. However, experience indicates that the response $\delta(t)$ in a power system generally falls in the two broad categories as shown in the figure. It can easily be visualized now (this has also been stated earlier) that for a stable system, indication of stability will be given by observation of the first swing where δ will go to a maximum and will start to reduce. This fact can be stated as a stability criterion, that the system is stable if at some time

$$\frac{d\delta}{dt} = 0 \tag{12.54}$$

and is unstable, if

$$\frac{d\delta}{dt} > 0 \tag{12.55}$$

for a sufficiently long time (more than 1 s will generally do).

The stability criterion for power systems stated above can be converted into a simple and easily applicable form for a single machine infinite bus system.

Multiplying both sides of the swing equation by $\left(2\frac{d\delta}{dt}\right)$, we get

$$2\frac{d\delta}{dt}\cdot\frac{d^2\delta}{dt^2}=\frac{2P_a}{M}\frac{d\delta}{dt}$$

Integrating, we have

$$\left(\frac{d\delta}{dt}\right)^2=\frac{2}{M}\int_{\delta_0}^{\delta}P_a\,d\delta$$

or

$$\frac{d\delta}{dt}=\left(\frac{2}{M}\int_{\delta_0}^{\delta}P_a\,d\delta\right)^{\frac{1}{2}} \tag{12.56}$$

where δ_0 is the initial rotor angle before it begins to swing due to disturbance.

From Eqs. (12.55) and (12.56), the condition for stability can be written as

$$\left(\frac{2}{M}\int_{\delta_0}^{\delta}P_a d\,\delta\right)^{\frac{1}{2}}=0$$

or

$$\int_{\delta_0}^{\delta}P_a\,d\delta=0 \tag{12.57}$$

The condition of stability can therefore be stated as: the system is stable if the area under P_a (accelerating power) – δ curve reduces to zero at some value of δ. In other words, the positive (accelerating) area under $P_a - \delta$ curve must equal the negative (decelerating) area and hence the name 'equal area' criterion of stability.

To illustrate the equal area criterion of stability, we now consider several types of disturbances that may occur in a single machine infinite bus bar system.

Sudden Change in Mechanical Input

Figure 12.19 shows the transient model of a single machine tied to infinite bus bar. The electrical power transmitted is given by

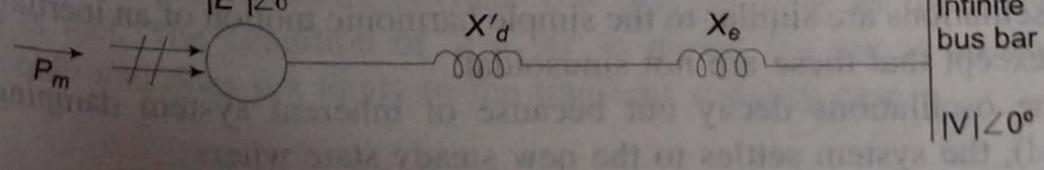


Fig. 12.19

$$P_e = \frac{|E'||V|}{X'_d + X_e} \sin \delta = P_{max} \sin \delta$$

Under steady operating condition

$$P_{m0} = P_{e0} = P_{max} \sin \delta_0$$

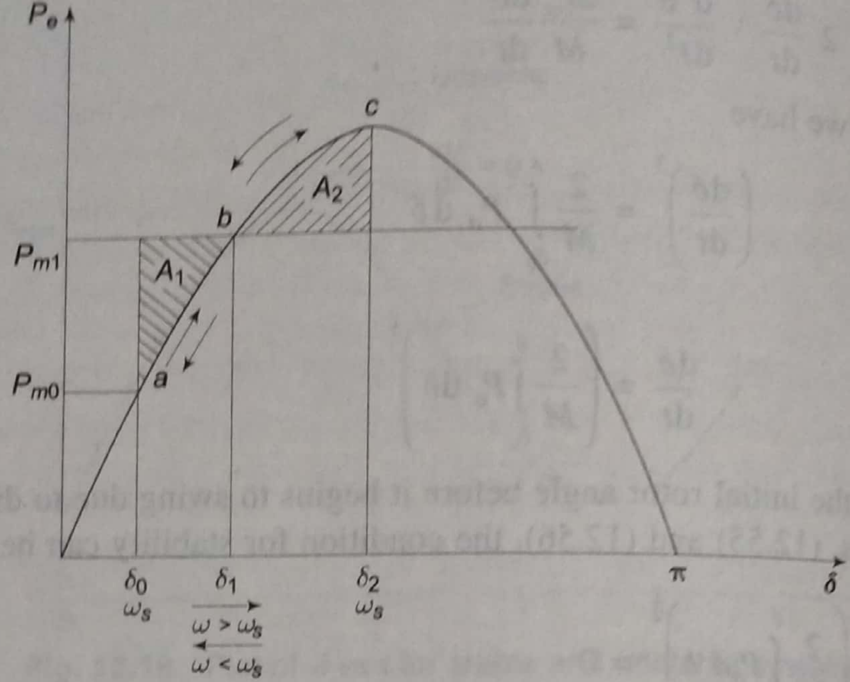


Fig. 12.20 $P_e - \delta$ diagram for sudden increase in mechanical input to generator of Fig. 12.19

This is indicated by the point a in the $P_e - \delta$ diagram of Fig. 12.20.

Let the mechanical input to the rotor be suddenly increased to P_{m1} (by opening the steam valve). The accelerating power $P_a = P_{m1} - P_e$ causes the rotor speed to increase ($\omega > \omega_s$) and so does the rotor angle. At angle δ_1, $P_a = P_{m1} - P_e (= P_{max} \sin \delta_1) = 0$ (state point at b) but the rotor angle continues to increase as $\omega > \omega_s$. P_a now becomes negative (decelerating), the rotor speed begins to reduce but the angle continues to increase till at angle δ_2, $\omega = \omega_s$ once again (state point at c. At c), the decelerating area A_2 equals the accelerating area A_1 (areas are shaded), i.e., $\int_{\delta_0}^{\delta_2} P_a \, d\delta = 0$. Since the rotor is decelerating, the speed reduces below ω_s and the rotor angle begins to reduce. The state point now traverses the $P_e - \delta$ curve in the opposite direction as indicated by arrows in Fig. 12.20. It is easily seen that the system oscillates about the new steady state point b ($\delta = \delta_1$) with anglc excursion up to δ_0 and δ_2 on the two sides. These oscillations are similar to the simple harmonic motion of an inertia-spring system except that these are not sinusoidal.

As the oscillations decay out because of inherent system damping (not modelled), the system settles to the new steady state where

$$P_{m1} = P_e = P_{max} \sin \delta_1$$

From Fig. 12.20, areas A_1 and A_2 are given by

$$A_1 = \int_{\delta_0}^{\delta_1} (P_{m1} - P_e)\, d\delta$$

$$A_2 = \int_{\delta_1}^{\delta_2} (P_e - P_{m1})\, d\delta$$

For the system to be stable, it should be possible to find angle δ_2 such that $A_1 = A_2$. As P_{m1} is increased, a limiting condition is finally reached when A_1 equals the area above the P_{m1} line as shown in Fig. 12.21. Under this condition, δ_2 acquires the maximum value such that

$$\delta_2 = \delta_{max} = \pi - \delta_1 = \pi - \sin^{-1} \frac{P_{m1}}{P_{max}} \quad (12.58)$$

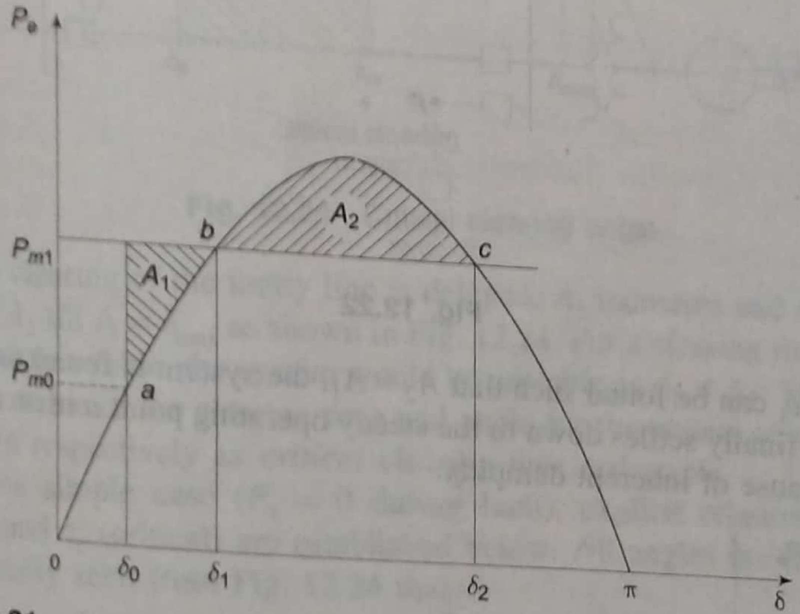


Fig. 12.21 Limiting case of transient stability with mechanical input suddenly increased

Any further increase in P_{m1} means that the area available for A_2 is less than A_1, so that the excess kinetic energy causes δ to increase beyond point c and the decelerating power changes over to accelerating power, with the system consequently becoming unstable. It has thus been shown by use of the equal area criterion that there is an upper limit to sudden increase in mechanical input $(P_{m1} - P_{m0})$, for the system in question to remain stable.

It may also be noted from Fig. 12.21 that the system will remain stable even though the rotor may oscillate beyond $\delta = 90°$, so long as the equal area criterion is met. The condition of $\delta = 90°$ is meant for use in steady state stability only and does not apply to the transient stability case.

Effect of Clearing Time on Stability

Let the system of Fig. 12.22 be operating with mechanical input P_m at a steady angle of δ_0 ($P_m = P_e$) as shown by the point a on the $P_e - \delta$ diagram of Fig. 12.23. If a 3-phase fault occurs at the point P of the outgoing radial line, the electrical output of the generator instantly reduces to zero, i.e., $P_e = 0$ and the state point drops to b. The acceleration area A_1 begins to increase and so does the rotor angle while the state point moves along bc. At time t_c corresponding to angle δ_c the faulted line is cleared by the opening of the line circuit breaker. The values of t_c and δ_c are respectively known as *clearing time* and *clearing angle*. The system once again becomes healthy and transmits $P_e = P_{max} \sin \delta$ i.e. the state point shifts to d on the original $P_e - \delta$ curve. The rotor now decelerates and the decelerating area A_2 begins while the state point moves along de.

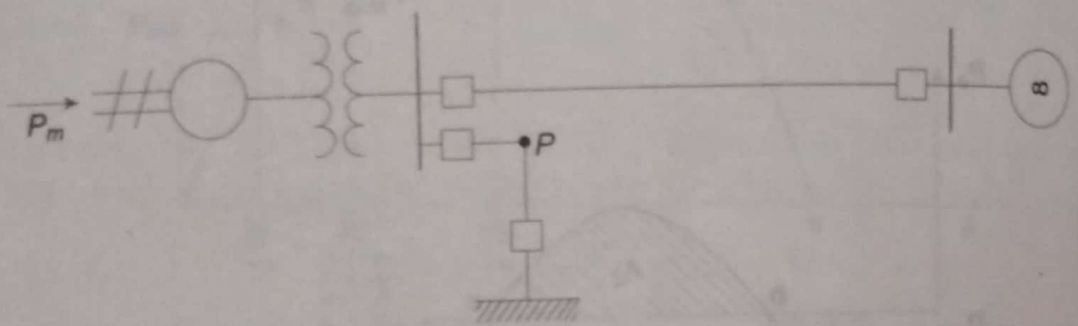


Fig. 12.22

If an angle δ_1 can be found such that $A_2 = A_1$, the system is found to be stable. The system finally settles down to the steady operating point a in an oscillatory manner because of inherent damping.

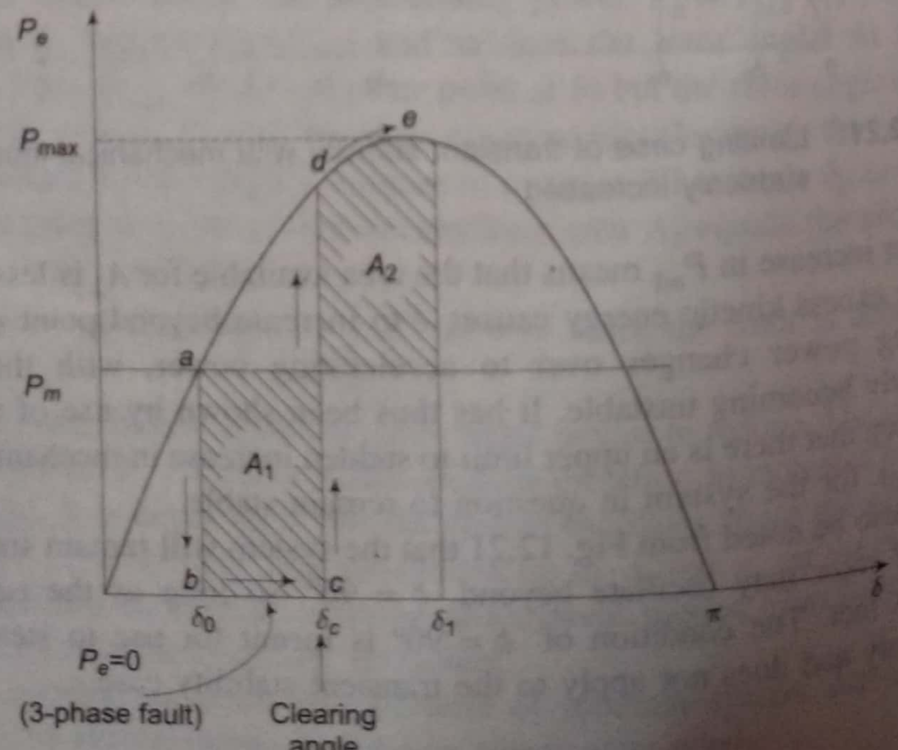


Fig. 12.23

The value of clearing time corresponding to a clearing angle can be established only by numerical integration except in this simple case. The equal area criterion therefore gives only qualitative answer to system stability as the time when the breaker should be opened is hard to establish.

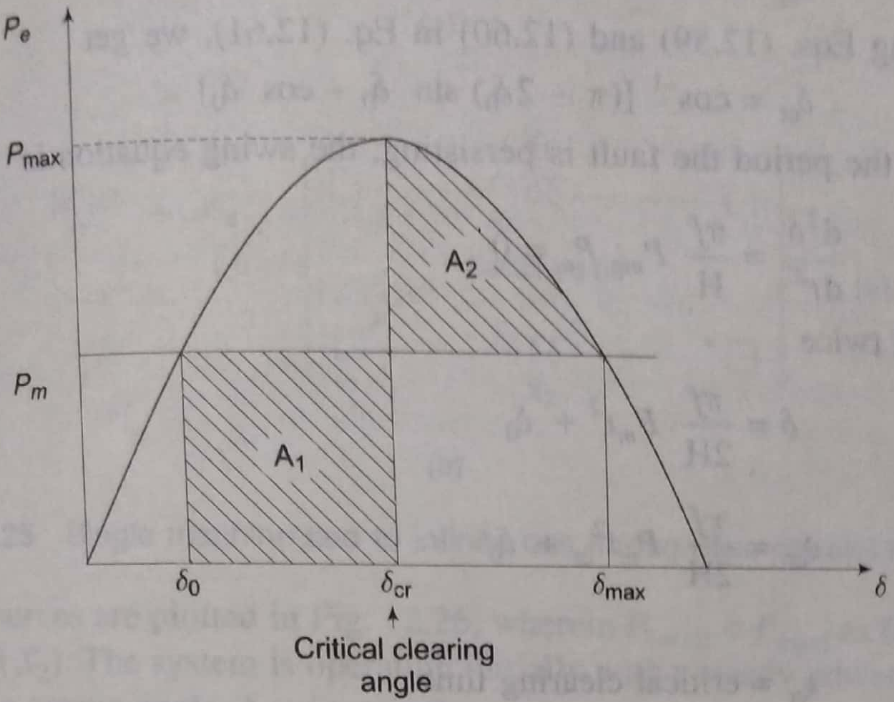


Fig. 12.24 Critical clearing angle

As the clearing of the faulty line is delayed, A_1 increases and so does δ_1 to find $A_2 = A_1$ till $\delta_1 = \delta_{max}$ as shown in Fig. 12.24. For a clearing time (or angle) larger than this value, the system would be unstable as $A_2 < A_1$. The maximum allowable value of the clearing time and angle for the system to remain stable are known respectively as *critical clearing time* and *angle*.

For this simple case ($P_e = 0$ during fault), explicit relationships for δ_c (critical) and t_c (critical) are established below. All angles are in *radians*.

It is easily seen from Fig. 12.24 that

$$\delta_{max} = \pi - \delta_0 \tag{12.59}$$

and

$$P_m = P_{max} \sin \delta_0 \tag{12.60}$$

Now

$$A_1 = \int_{\delta_0}^{\delta_{cr}} (P_m - 0)\, d\delta = P_m (\delta_{cr} - \delta_0)$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{max} \sin \delta - P_m)\, d\delta$$

$$= P_{max} (\cos \delta_{cr} - \cos \delta_{max}) - P_m (\delta_{max} - \delta_{cr})$$

For the system to be stable, $A_2 = A_1$, which yields

$$\cos\delta_{cr} = \frac{P_m}{P_{max}}(\delta_{max} - \delta_0) + \cos\delta_{max} \quad (12.61)$$

where δ_{cr} = critical clearing angle

Substituting Eqs. (12.59) and (12.60) in Eq. (12.61), we get

$$\delta_{cr} = \cos^{-1}[(\pi - 2\delta_0)\sin\delta_0 - \cos\delta_0] \quad (12.62)$$

During the period the fault is persisting, the swing equation is

$$\frac{d^2\delta}{dt^2} = \frac{\pi f}{H}P_m;\ P_e = 0 \quad (12.63)$$

Integrating twice

$$\delta = \frac{\pi f}{2H}P_m t^2 + \delta_0$$

or

$$\delta_{cr} = \frac{\pi f}{2H}P_m t^2_{cr} + \delta_0 \quad (12.64)$$

where

t_{cr} = critical clearing time

δ_{cr} = critical clearing angle

From Eq. (12.64)

$$t_{cr} = \sqrt{\frac{2H(\delta_{cr} - \delta_0)}{\pi f P_m}} \quad (12.65)$$

where δ_{cr} is given by the expression of Eq. (12.62)

An explicit relationship for determining t_{cr} is possible in this case as during the faulted condition $P_e = 0$ and so the swing equation can be integrated in closed form. This will not be the case in most other situations.

Sudden Loss of One of Parallel Lines

Consider now a single machine tied to infinite bus through two parallel lines as in Fig. 12.25a. Circuit model of the system is given in Fig. 12.25b.

Let us study the transient stability of the system when one of the lines is suddenly switched off with the system operating at a steady load. Before switching off, power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X_d + X_1 \| X_2}\sin\delta = P_{maxI}\sin\delta$$

Immediately on switching off line 2, power angle curve is given by

$$P_{eII} = \frac{|E'||V|}{X'_d + X_1}\sin\delta = P_{maxII}\sin\delta$$

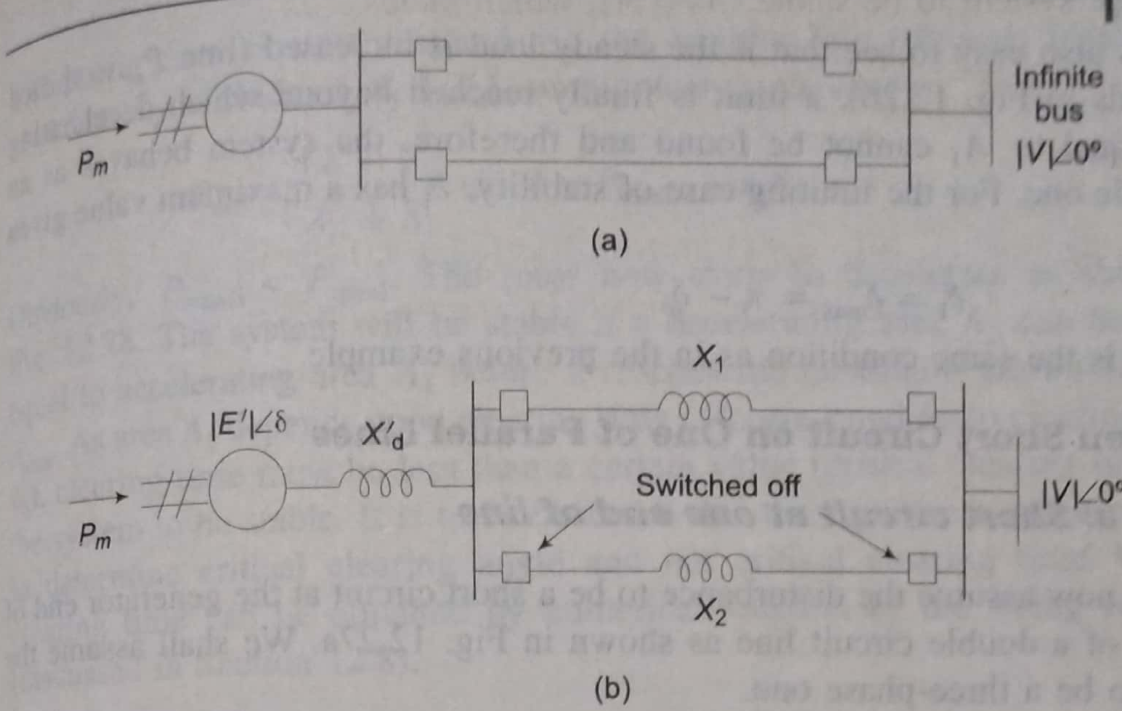


Fig. 12.25 Single machine tied to infinite bus through two parallel lines

Both these curves are plotted in Fig. 12.26, wherein $P_{\max II} < P_{\max I}$ as $(X'_d + X_1) > (X'_d + X_1 \parallel X_2)$. The system is operating initially with a steady power transfer $P_e = P_m$ at a torque angle δ_0 on curve I.

Immediately on switching off line 2, the electrical operating point shifts to curve II (point b). Accelerating energy corresponding to area A_1 is put into rotor followed by decelerating energy for $\delta > \delta_1$. Assuming that an area A_2 corresponding to decelerating energy (energy out of rotor) can be found such that $A_1 = A_2$, the system will be stable and will finally operate at c corresponding to a new rotor angle $\delta_1 > \delta_0$. This is so because a single line offers larger reactance and larger rotor angle is needed to transfer the same steady power.

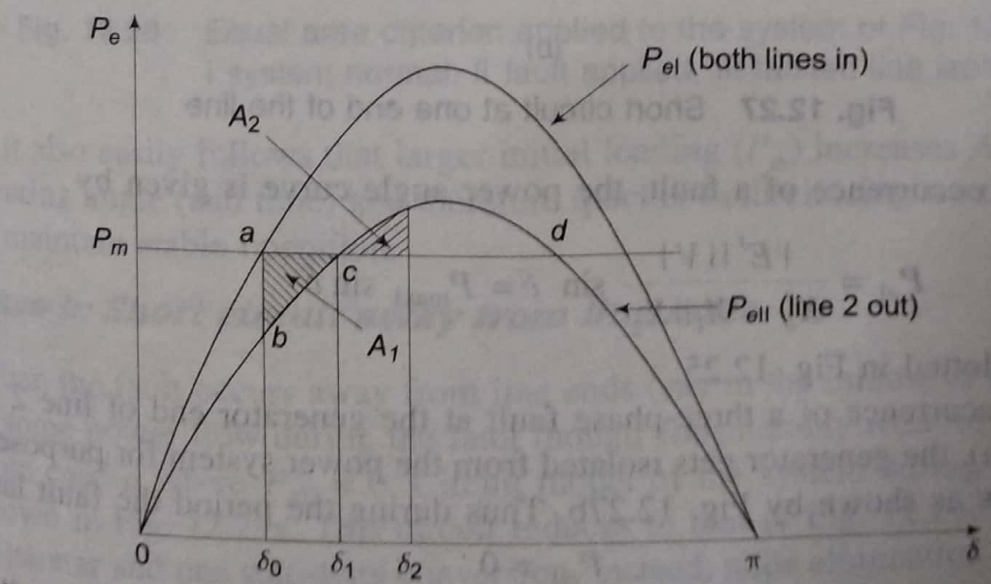


Fig. 12.26 Equal area criterion applied to the opening of one of the two lines in parallel

It is also easy to see that if the steady load is increased (line P_m is shifted upwards in Fig. 12.26), a limit is finally reached beyond which decelerating area equal to A_1 cannot be found and therefore, the system behaves as an unstable one. For the limiting case of stability, δ_1 has a maximum value given by

$$\delta_1 = \delta_{max} = \pi - \delta_c$$

which is the same condition as in the previous example.

Sudden Short Circuit on One of Parallel Lines

Case a: Short circuit at one end of line

Let us now assume the disturbance to be a short circuit at the generator end of line 2 of a double circuit line as shown in Fig. 12.27a. We shall assume the fault to be a three-phase one.

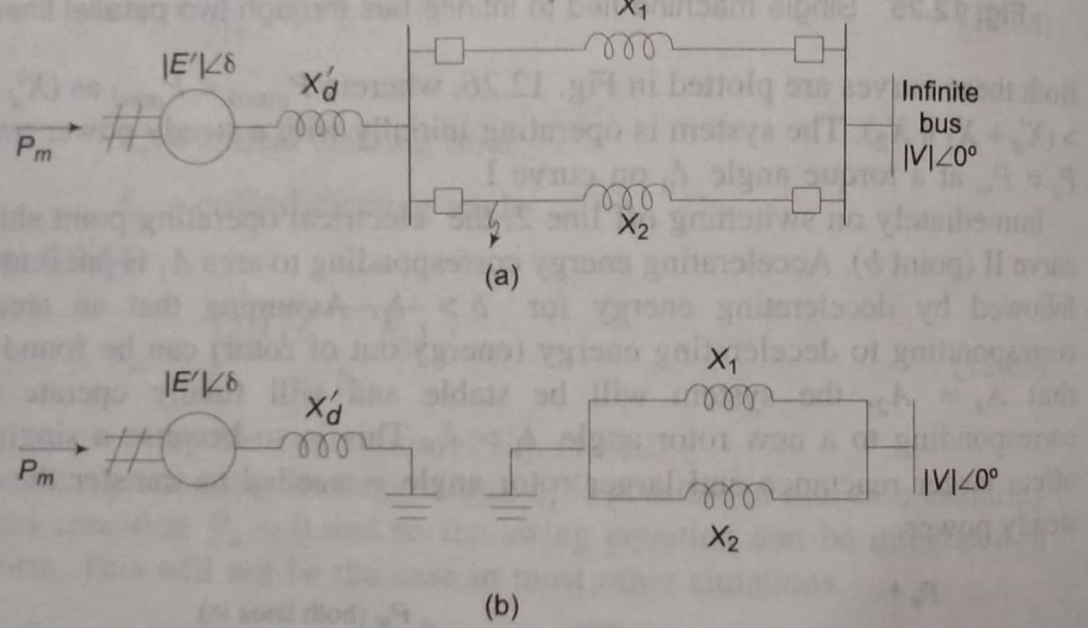


Fig. 12.27 Short circuit at one end of the line

Before the occurrence of a fault, the power angle curve is given by

$$P_{eI} = \frac{|E'||V|}{X'_d + X_1 \| X_2} \sin \delta = P_{maxI} \sin \delta$$

which is plotted in Fig. 12.25.

Upon occurrence of a three-phase fault at the generator end of line 2 (see Fig. 12.24a), the generator gets isolated from the power system for purposes of power flow as shown by Fig. 12.27b. Thus during the period the fault lasts,

$$P_{eII} = 0$$

The rotor therefore accelerates and angles δ increases. Synchronism will be lost unless the fault is cleared in time.

The circuit breakers at the two ends of the faulted line open at time t_c (corresponding to angle δ_c), the clearing time, disconnecting the faulted line.

The power flow is now restored via the healthy line (through higher line reactance X_2 in place of $X_1 \parallel X_2$), with power angle curve

$$P_{eIII} = \frac{|E'||V|}{X'_d + X_1} \sin\delta = P_{\max II} \sin\delta$$

Obviously, $P_{\max II} < P_{\max I}$. The rotor now starts to decelerate as shown in Fig. 12.28. The system will be stable if a decelerating area A_2 can be found equal to accelerating area A_1 before δ reaches the maximum allowable value δ_{max}. As area A_1 depends upon clearing time t_c (corresponding to clearing angle δ_c), clearing time must be less than a certain value (critical clearing time) for the system to be stable. It is to be observed that the equal area criterion helps to determine critical clearing angle and not critical clearing time. Critical clearing time can be obtained by numerical solution of the swing equation (discussed in Section 12.8).

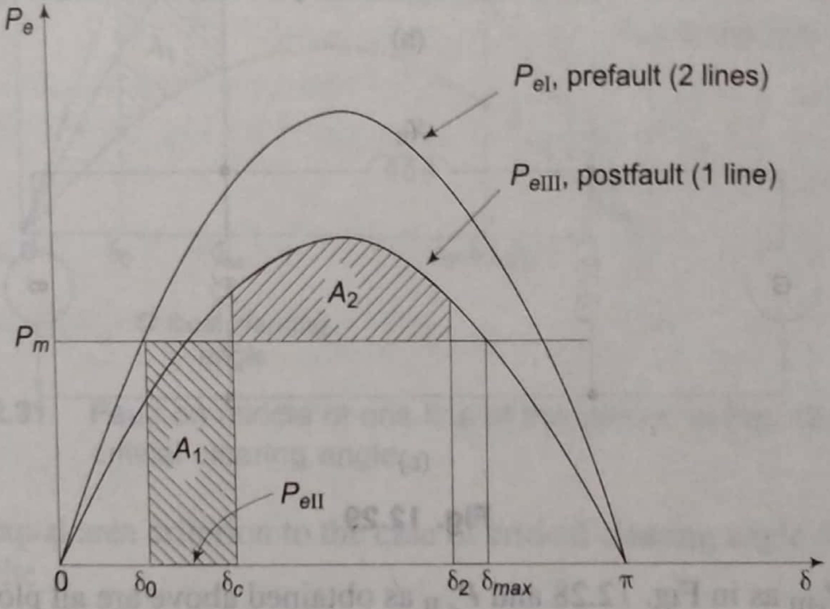


Fig. 12.28 Equal area criterion applied to the system of Fig. 12.24a, I system normal, II fault applied, III faulted line isolated.

It also easily follows that larger initial loading (P_m) increases A_1 for a given clearing angle (and time) and therefore quicker fault clearing would be needed to maintain stable operation.

Case b: Short circuit away from line ends

When the fault occurs away from line ends (say in the middle of a line), there is some power flow during the fault though considerably reduced, as different from case *a* where $P_{eII} = 0$. Circuit model of the system during fault is now shown in Fig. 12.29a. This circuit reduces to that of Fig. 12.29c through one delta-star and one star-delta conversion. Instead, node elimination technique of Section 12.3 could be employed profitably. The power angle curve during fault is therefore given by

$$P_{eII} = \frac{|E'||V|}{X_{II}} \sin\delta = P_{\max II} \sin\delta$$

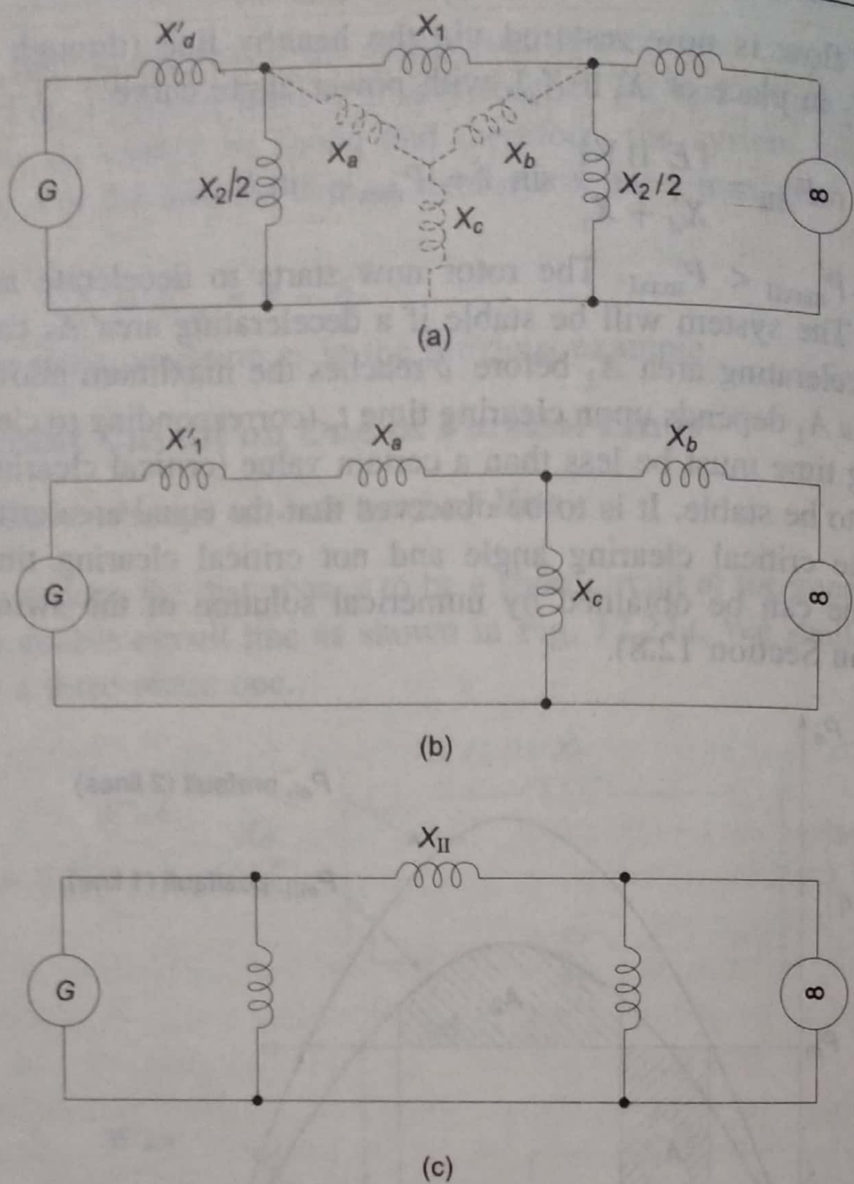


Fig. 12.29

$P_{e\text{I}}$ and $P_{e\text{III}}$ as in Fig. 12.28 and $P_{e\,\text{II}}$ as obtained above are all plotted in Fig. 12.30. Accelerating area A_1 corresponding to a given clearing angle δ_c is less

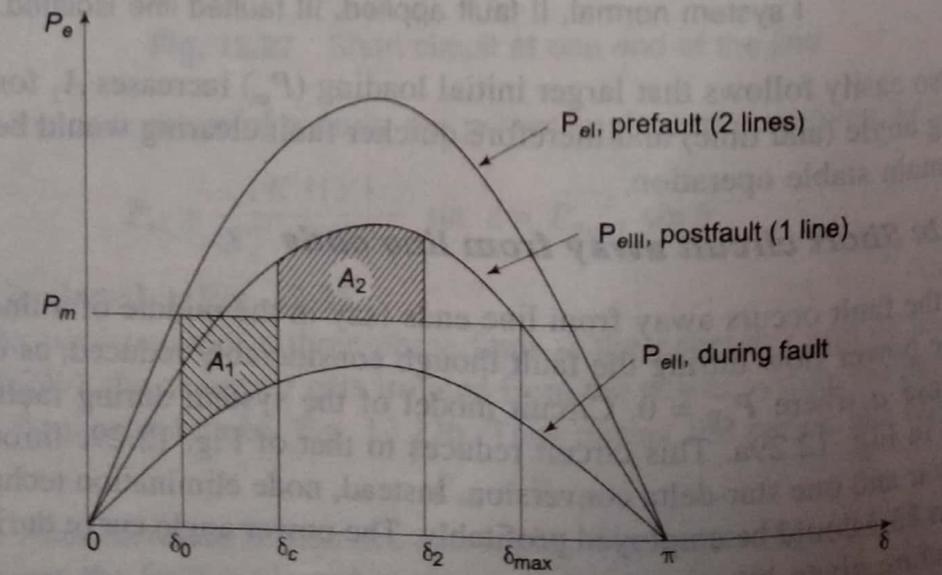


Fig. 12.30 Fault on middle of one line of the system of Fig. 12.24a with $\delta_c < \delta_{cr}$

in this case, than in case *a*, giving a better chance for stable operation. Stable system operation is shown in Fig. 12.30, wherein it is possible to find an area A_2 equal to A_1 for $\delta_2 < \delta_{max}$. As the clearing angle δ_c is increased, area A_1 increases and to find $A_2 = A_1$, δ_2 increases till it has a value δ_{max}, the maximum allowable for stability. This case of critical clearing angle is shown in Fig. 12.31.

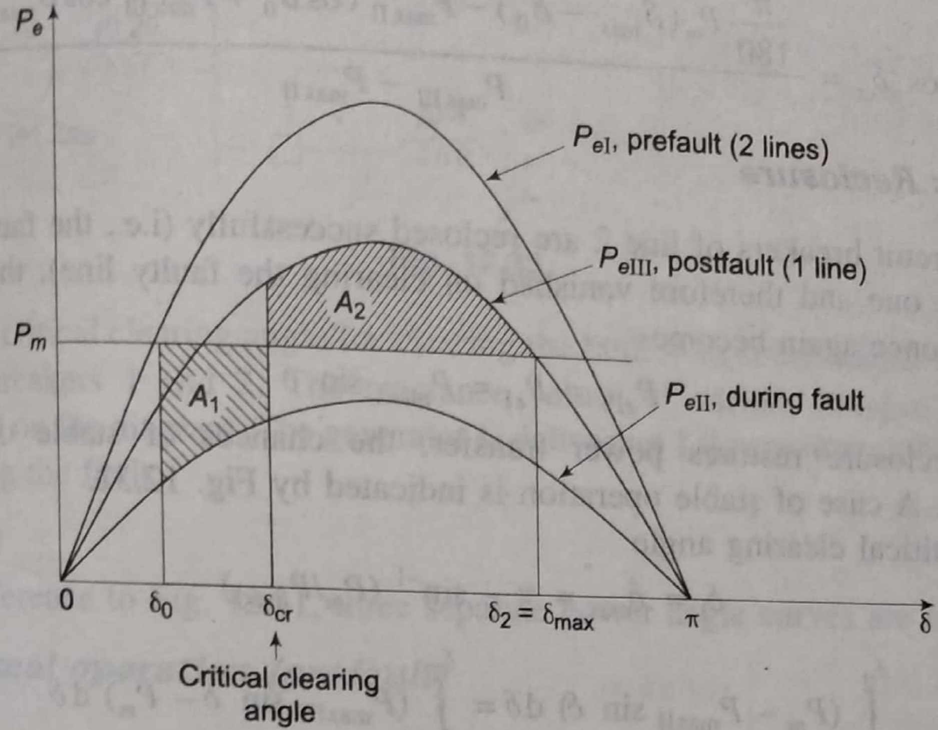


Fig. 12.31 Fault on middle of one line of the system of Fig. 12.24a, case of critical clearing angle

Applying equal area criterion to the case of critical clearing angle of Fig. 12.31, we can write

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{maxII} \sin \delta) \, d\delta = \int_{\delta_{cr}}^{\delta_{max}} (P_{maxIII} \sin \delta - P_m) \, d\delta$$

where

$$\delta_{max} = \pi - \sin^{-1}\left(\frac{P_m}{P_{max\,III}}\right) \tag{12.66}$$

Integrating, we get

$$(P_{m\delta} + P_{maxII} \cos \delta)\Big|_{\delta_o}^{\delta_{cr}} + (P_{maxIII} \cos \delta + P_m \delta)\Big|_{\delta_{cr}}^{\delta_{max}} = 0$$

or

$$P_m (\delta_{cr} - \delta_0) + P_{maxII} (\cos \delta_{cr} - \cos \delta_0) + P_m (\delta_{max} - \delta_{cr}) + P_{maxIII} (\cos \delta_{max} - \cos \delta_{cr}) = 0$$

or

$$\cos \delta_{cr} = \frac{P_m(\delta_{max} - \delta_0) - P_{max\,II} \cos \delta_0 + P_{max\,III} \cos \delta_{max}}{P_{max\,III} - P_{max\,II}} \quad (12.67)$$

Critical clearing angle can be calculated from Eq. (12.67) above. The angles in this equation are in radians. The equation modifies as below if the angles are in degrees.

$$\cos \delta_{cr} = \frac{\frac{\pi}{180} P_m(\delta_{max} - \delta_0) - P_{max\,II} \cos \delta_0 + P_{max\,III} \cos \delta_{max}}{P_{max\,III} - P_{max\,II}}$$

Case c: Reclosure

If the circuit breakers of line 2 are reclosed successfully (i.e., the fault was a transient one and therefore vanished on clearing the faulty line), the power transfer once again becomes

$$P_{eIV} = P_{eI} = P_{maxI} \sin \delta$$

Since reclosure restores power transfer, the chances of stable operation improve. A case of stable operation is indicated by Fig. 12.32.

For critical clearing angle

$$\delta_1 = \delta_{max} = \pi - \sin^{-1} (P_m/P_{maxI})$$

$$\int_{\delta_0}^{\delta_{cr}} (P_m - P_{maxII} \sin \delta)\, d\delta = \int_{\delta_{cr}}^{\delta_{rc}} (P_{maxIII} \sin \delta - P_m)\, d\delta + \int_{\delta_{rc}}^{\delta_{max}} (P_{maxI} \sin \delta - P_m)\, d\delta$$

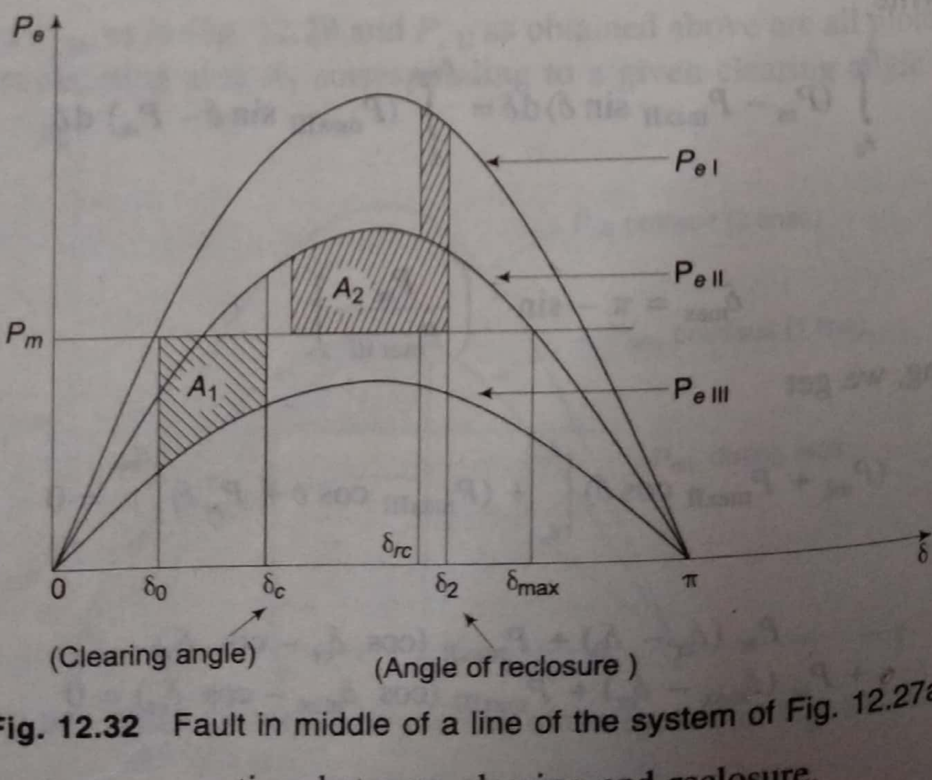


Fig. 12.32 Fault in middle of a line of the system of Fig. 12.27a

where $t_{rc} = t_{cr} + \tau$; τ = time between clearing and reclosure.

Example 12.7

Give the system of Fig. 12.33 where a three-phase fault is applied at the point *P* as shown.

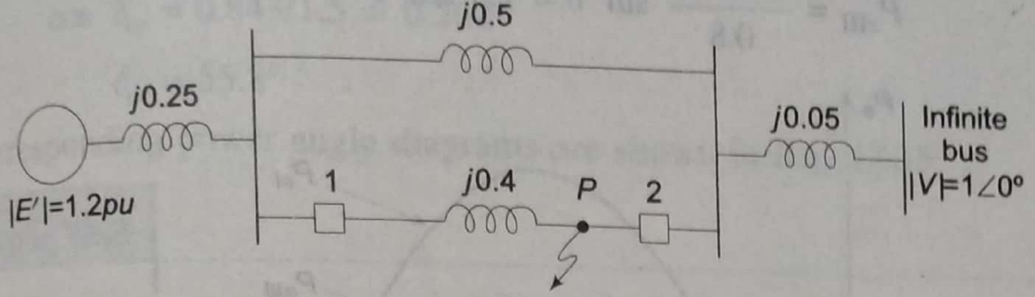


Fig. 12.33

Find the critical clearing angle for clearing the fault with simultaneous opening of the breakers 1 and 2. The reactance values of various components are indicated on the diagram. The generator is delivering 1.0 pu power at the instant preceding the fault.

Solution

With reference to Fig. 12.31, three separate power angle curves are involved.

I. Normal operation (prefault)

$$X_I = 0.25 + \frac{0.5 \times 0.4}{0.5 + 0.4} + 0.05$$

$$= 0.522 \text{ pu}$$

$$P_{eI} = \frac{|E'||V|}{X_I} \sin\delta = \frac{1.2 \times 1}{0.522} \sin\delta$$

$$= 2.3 \sin\delta \qquad \text{(i)}$$

Prefault operating power angle is given by

$$1.0 = 2.3 \sin\delta_0$$

or

$$\delta_0 = 25.8° = 0.45 \text{ radians}$$

II. During fault

It is clear from Fig. 12.31 that no power is transferred during fault, i.e.,

$$P_{eII} = 0$$

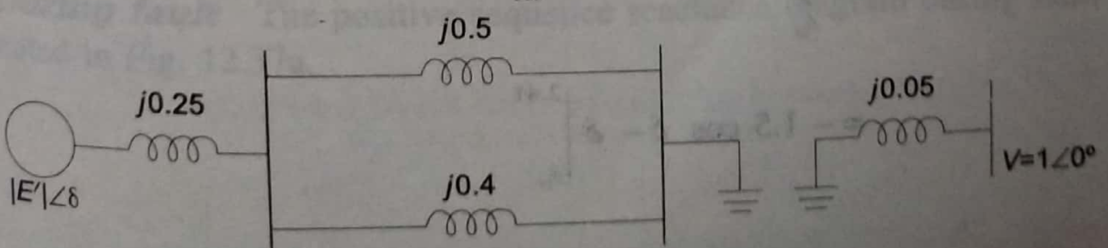


Fig. 12.34

III. Post fault operation (fault cleared by opening the faulted line)

$$X_{III} = 0.25 + 0.5 + 0.05 = 0.8$$

$$P_{eIII} = \frac{1.2 \times 1.0}{0.8} \sin \delta = 1.5 \sin \delta \qquad \text{(iii)}$$

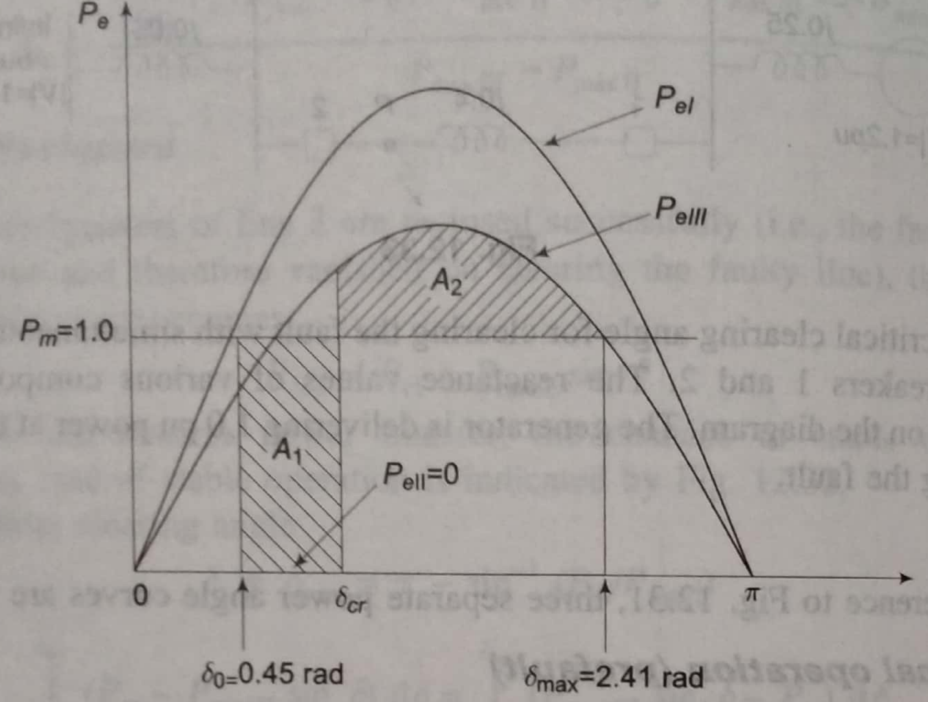


Fig. 12.35

The maximum permissible angle δ_{max} for area $A_1 = A_2$ (see Fig. 12.35) is given by

$$\delta_{max} = \pi - \sin^{-1} \frac{1}{1.5} = 2.41 \text{ radians}$$

Applying equal area criterion for critical clearing angle δ_c

$$A_1 = P_m (\delta_{cr} - \delta_0)$$

$$= 1.0 (\delta_{cr} - 0.45) = \delta_{cr} - 0.45$$

$$A_2 = \int_{\delta_{cr}}^{\delta_{max}} (P_{eIII} - P_m) \, d\delta$$

$$= \int_{\delta_{cr}}^{2.41} (1.5 \sin \delta - 1) \, d\delta$$

$$= -1.5 \cos \delta - \delta \Big|_{\delta_{cr}}^{2.41}$$

$$= -1.5(\cos 2.41 - \cos \delta_{cr}) - (2.41 - \delta_{cr})$$

$$= 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

Setting $A_1 = A_2$ and solving

$$\delta_{cr} - 0.45 = 1.5 \cos \delta_{cr} + \delta_{cr} - 1.293$$

or

$$\cos \delta_{cr} = 0.843/1.5 = 0.562$$

or

$$\delta_{cr} = 55.8^\circ$$

The corresponding power angle diagrams are shown in Fig. 12.35.

Example 12.8

Find the critical clearing angle for the system shown in Fig. 12.36 for a three-phase fault at the point P. The generator is delivering 1.0 pu power under prefault conditions.

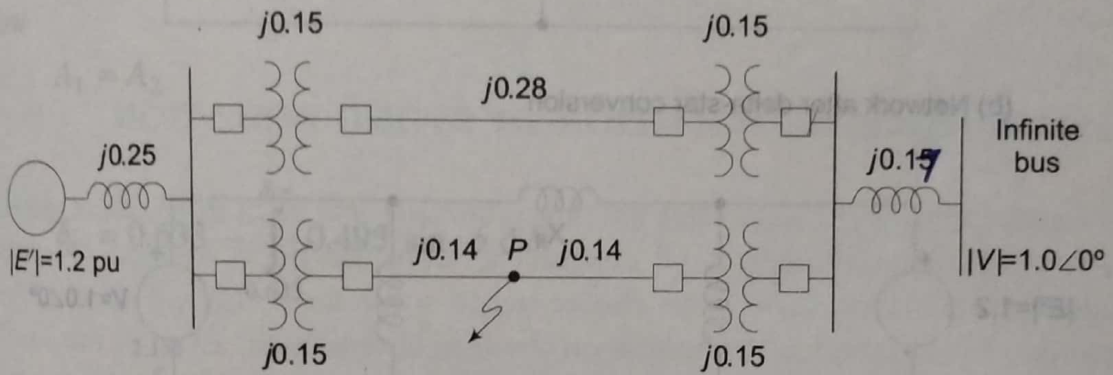


Fig. 12.36

Solution

I. Prefault operation Transfer reactance between generator and infinite bus is

$$X_I = 0.25 + 0.17 + \frac{0.15 + 0.28 + 0.15}{2} = 0.71$$

$$\therefore \quad P_{el} = \frac{1.2 \times 1}{0.71} \sin \delta = 1.69 \sin \delta \qquad \text{(i)}$$

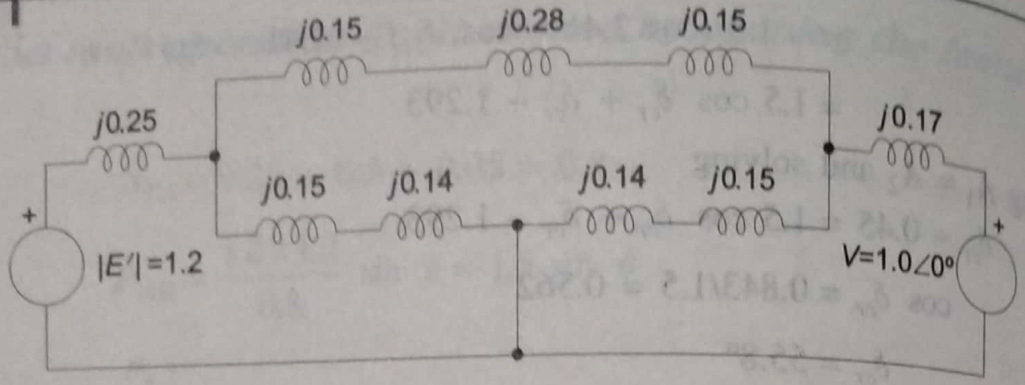
The operating power angle is given by

$$1.0 = 1.69 \sin \delta_0$$

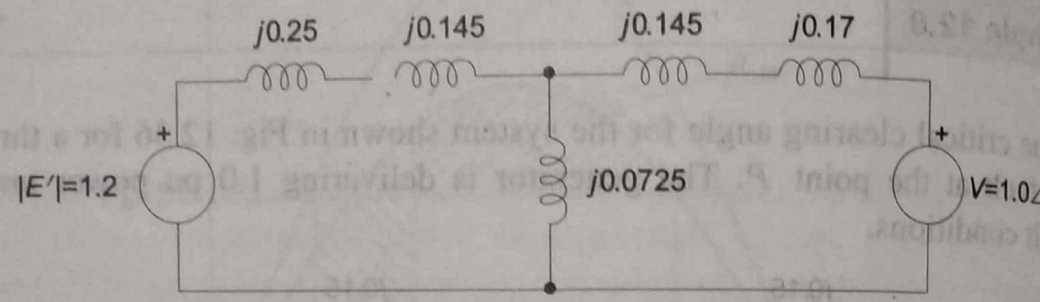
or

$$\delta_0 = 0.633 \text{ rad}$$

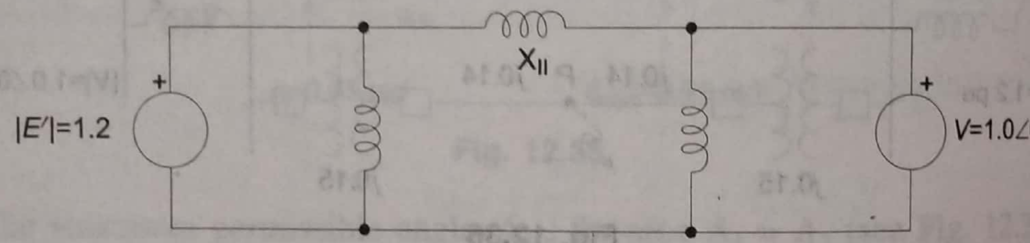
II. During fault The positive sequence reactance diagram during fault is presented in Fig. 12.37a.



(a) Positive sequence reactance diagram during fault



(b) Network after delta-star conversion



(c) Network after star-delta conversion

Fig. 12.37

Converting delta to star*, the reactance network is changed to that of Fig. 12.37(b). Further, upon converting star to delta, we obtain the reactance network of Fig. 12.37(c). The transfer reactance is given by

$$X_{\text{II}} = \frac{\dfrac{(0.25+0.145)\,0.0725+(0.145+0.17)\,0.0725+(0.25+0.145)}{(0.145+0.17)}}{0.075}$$

$$= 2.424$$

$$\therefore \quad P_{e\text{II}} = \frac{1.2\times 1}{2.424}\sin\,\delta = 0.495\sin\,\delta \qquad (ii)$$

III. Postfault operation (faulty line switched off)

$$X_{\text{III}} = 0.25 + 0.15 + 0.28 + 0.15 + 0.17 = 1.0$$

*Node elimination technique would be used for complex network.

$$P_{eIII} = \frac{1.2 \times 1}{1} \sin \delta = 1.2 \sin \delta \qquad \text{(iii)}$$

With reference to Fig. 12.30 and Eq. (12.66), we have

$$\delta_{max} = \pi - \sin^{-1}\frac{1}{1.2} = 2.155 \text{ rad}$$

To find the critical clearing angle, areas A_1 and A_2 are to be equated.

$$A_1 = 1.0\,(\delta_{cr} - 0.633) - \int_{\delta_0}^{\delta_{cr}} 0.495 \sin \delta \, d\delta$$

and

$$A_2 = \int_{\delta_{cr}}^{\delta max} 1.2 \sin \delta \, d\delta - 1.0\,(2.155 - \delta_c)$$

Now

$$A_1 = A_2$$

or

$$\delta_{cr} = 0.633 - \int_{0.633}^{\delta cr} 0.495 \sin \delta \, d\delta$$

$$= \int_{\delta cr}^{2.155} 1.2 \sin \delta \, d\delta - 2.155 + \delta_{cr}$$

or $-0.633 + 0.495 \cos \delta \Big|_{0.633}^{\delta_{cr}} = -1.2 \cos \delta \Big|_{\delta_{cr}}^{2.155} - 2.155$

or $-0.633 + 0.495 \cos \delta_{cr} - 0.399 = 0.661 + 1.2 \cos \delta_{cr} - 2.155$

or $\cos \delta_{cr} = 0.655$

or $\delta_{cr} = 49.1°$

Example 12.9

A generator operating at 50 Hz delivers 1 pu power to an infinite bus through a transmission circuit in which resistance is ignored. A fault takes place reducing the maximum power transferable to 0.5 pu whereas before the fault, this power was 2.0 pu and after the clearance of the fault, it is 1.5 pu. By the use of equal area criterion, determine the critical clearing angle.

Solution

All the three power angle curves are shown in Fig. 12.30.

Here $P_{\text{maxI}} = 2.0$ pu, $P_{\text{maxII}} = 0.5$ pu and $P_{\text{maxIII}} = 1.5$ pu

Initial loading $P_m = 1.0$ pu

$$\delta_0 = \sin^{-1}\left(\frac{P_m}{P_{\max I}}\right) = \sin^{-1}\frac{1}{2} = 0.523 \text{ rad}$$

$$\delta_{\max} = \pi \sin^{-1}\left(\frac{P_m}{P_{\max \text{III}}}\right)$$

$$= \pi - \sin^{-1}\frac{1}{1.5} = 2.41 \text{ rad}$$

Applying Eq. (12.67)

$$\cos \delta_{cr} = \frac{1.0(2.41 - 0.523) - 0.5 \cos 0.523 + 1.5 \cos 2.41}{1.5 - 0.5} = 0.337$$

or $\delta_{cr} = 70.3°$

12.5 SIMPLE SYSTEMS

Machine Connected to Infinite Bus

Figure 12.14 is the circuit model of a single machine connected to infinite bus through a line of reactance X_e. In this simple case

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$$X_{\text{transfer}} = X'_d + X_e$$

From Eq. (12.29b)

$$P_e = \frac{|E'||V|}{X_{\text{transfer}}} \sin\delta = P_{\max} \sin\delta \tag{12.36}$$

The dynamics of this system are described in Eq. (12.11) as

$$\frac{H}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \text{ pu} \tag{12.37}$$

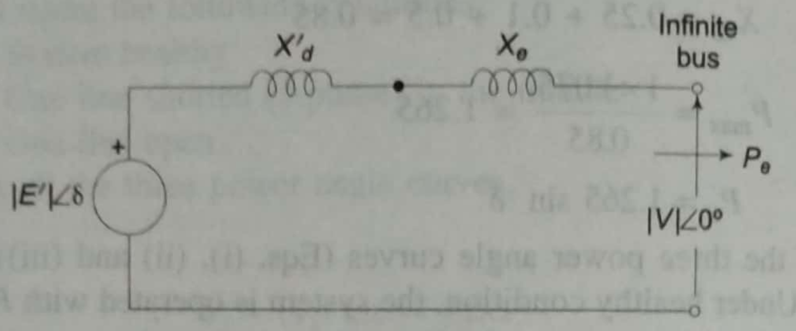

Fig. 12.14 Machine connected to infinite bus

Two Machine System

The case of two finite machines connected through a line (X_e) is illustrated in Fig. 12.15 where one of the machines must be generating and the other must be motoring. Under steady condition, before the system goes into dynamics and

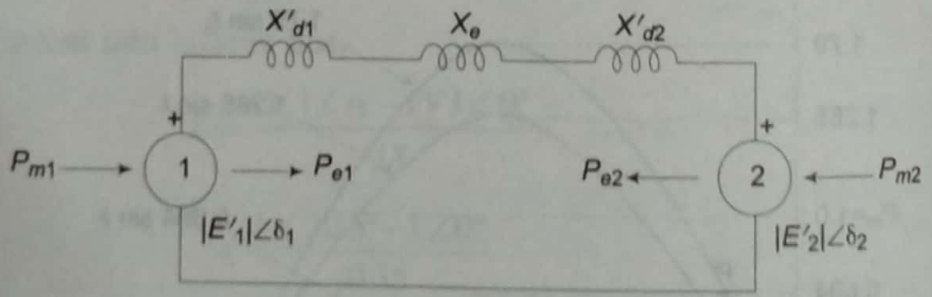

Fig. 12.15 Two-machine system

$$P_{m1} = -P_{m2} = P_m \tag{12.38a}$$

the mechanical input/output of the two machines is assumed to remain constant at these values throughout the dynamics (governor action assumed slow). During steady state or in dynamic condition, the electrical power output of the generator must be absorbed by the motor (network being lossless). Thus at all time

$$P_{e1} = -P_{e2} = P_e \tag{12.38b}$$

The swing equations for the two machines can now be written as

$$\frac{d^2\delta_1}{dt^2} = \pi f\left(\frac{P_{m1} - P_{e1}}{H_1}\right) = \pi f\left(\frac{P_m - P_e}{H_1}\right) \tag{12.39a}$$

and
$$\frac{d^2\delta_2}{dt^2} = \pi f\left(\frac{P_{m2}-P_{e2}}{H_2}\right) = \pi f\left(\frac{P_e - P_m}{H_2}\right) \tag{12.39b}$$

Subtracting Eq. (12.39b) from Eq. (12.39a)

$$\frac{d^2(\delta_1-\delta_2)}{dt^2} = \pi f\left(\frac{H_1+H_2}{H_1H_2}\right)(P_m - P_e) \tag{12.40}$$

or
$$\frac{H_{eq}}{\pi f}\frac{d^2\delta}{dt^2} = P_m - P_e \tag{12.41}$$

where
$$\delta = \delta_1 - \delta_2 \tag{12.42}$$

$$H_{eq} = \frac{H_1H_2}{H_1+H_2} \tag{12.43}$$

The electrical power interchange is given by expression

$$P_e = \frac{|E_1'||E_2'|}{X_{d1}' + X_e + X_{d2}'}\sin\delta \tag{12.44}$$

The swing equation Eq. (12.41) and the power angle equation Eq. (12.44) have the same form as for a single machine connected to infinite bus. Thus a two-machine system is equivalent to a single machine connected to infinite bus. Because of this, the single-machine (connected to infinite bus) system would be studied extensively in this chapter.

Example 12.4

In the system of Example 12.3, the generator has an inertia constant of 4 MJ/MVA, write the swing equation upon occurrence of the fault. What is the initial angular acceleration? If this acceleration can be assumed to remain constant for $\Delta t = 0.05$s, find the rotor angle at the end of this time interval and the new acceleration.

Solution

Swing equation upon occurrence of fault

$$\frac{H}{180f}\frac{d^2\delta}{dt^2} = P_m - P_e$$

$$\frac{4}{180\times 50}\frac{d^2\delta}{dt^2} = 1 - 0.694\sin\delta$$

or
$$\frac{d^2\delta}{dt^2} = 2250\,(1 - 0.694\sin\delta).$$

Initial rotor angle $\delta_0 = 33.9°$ (calculated in Example 12.3)

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0^+} = 2250\ (1 - 0.694\ \sin 33.9°)$$

$$= 1379 \text{ elect deg/s}^2$$

$$\left.\frac{d\delta}{dt}\right|_{t=0^+} = 0; \text{ rotor speed cannot change suddenly}$$

$$\Delta\delta \text{ (in } \Delta t = 0.05\text{s)} = \frac{1}{2} \times 1379 \times (0.05)^2$$

$$= 1.7°$$

$$\delta_1 = \delta_0 + \Delta\delta = 33.9 + 1.7° = 35.6°$$

$$\left.\frac{d^2\delta}{dt^2}\right|_{t=0.05s} = 2250\ (1 - 0.694\ \sin 35.6°)$$

$$= 1341 \text{ elect deg/s}^2$$

Observe that as the rotor angle increases, the electrical power output of the generator increases and so the acceleration of the rotor reduces.

or $\delta_{cr} = 70.5$

12.9 NUMERICAL SOLUTION OF SWING EQUATION

In most practical systems, after machine lumping has been done, there are still more than two machines to be considered from the point of view of system stability. Therefore, there is no choice but to solve the swing equation of each machine by a numerical technique on the digital computer. Even in the case of a single machine tied to infinite bus bar, the critical clearing time cannot be obtained from equal area criterion and we have to make this calculation numerically through swing equation. There are several sophisticated methods now available for the solution of the swing equation including the powerful Runge-Kutta method. Here we shall treat the point-by-point method of solution which is a conventional, approximate method like all numerical methods but a well tried and proven one. We shall illustrate the point-by-point method for one machine tied to infinite bus bar. The procedure is, however, general and can be applied to every machine of a multimachine system.

Consider the swing equation

$$\frac{d^2\delta}{dt^2} = \frac{1}{M}(P_m - P_{max}\sin\delta) = P_a/M;$$

$$\left(M = \frac{GH}{\pi} \text{ or in pu system } M = \frac{H}{\pi f}\right)$$

The solution $\delta(t)$ is obtained at discrete intervals of time with interval spread of Δt uniform throughout. Accelerating power and change in speed which are continuous functions of time are discretized as below:

1. The accelerating power P_a computed at the beginning of an interval is assumed to remain constant from the middle of the preceding interval to the middle of the interval being considered as shown in Fig. 12.38.

2. The angular rotor velocity $\omega = d\delta/dt$ (over and above synchronous velocity ω_s) is assumed constant throughout any interval, at the value computed for the middle of the interval as shown in Fig. 12.38.

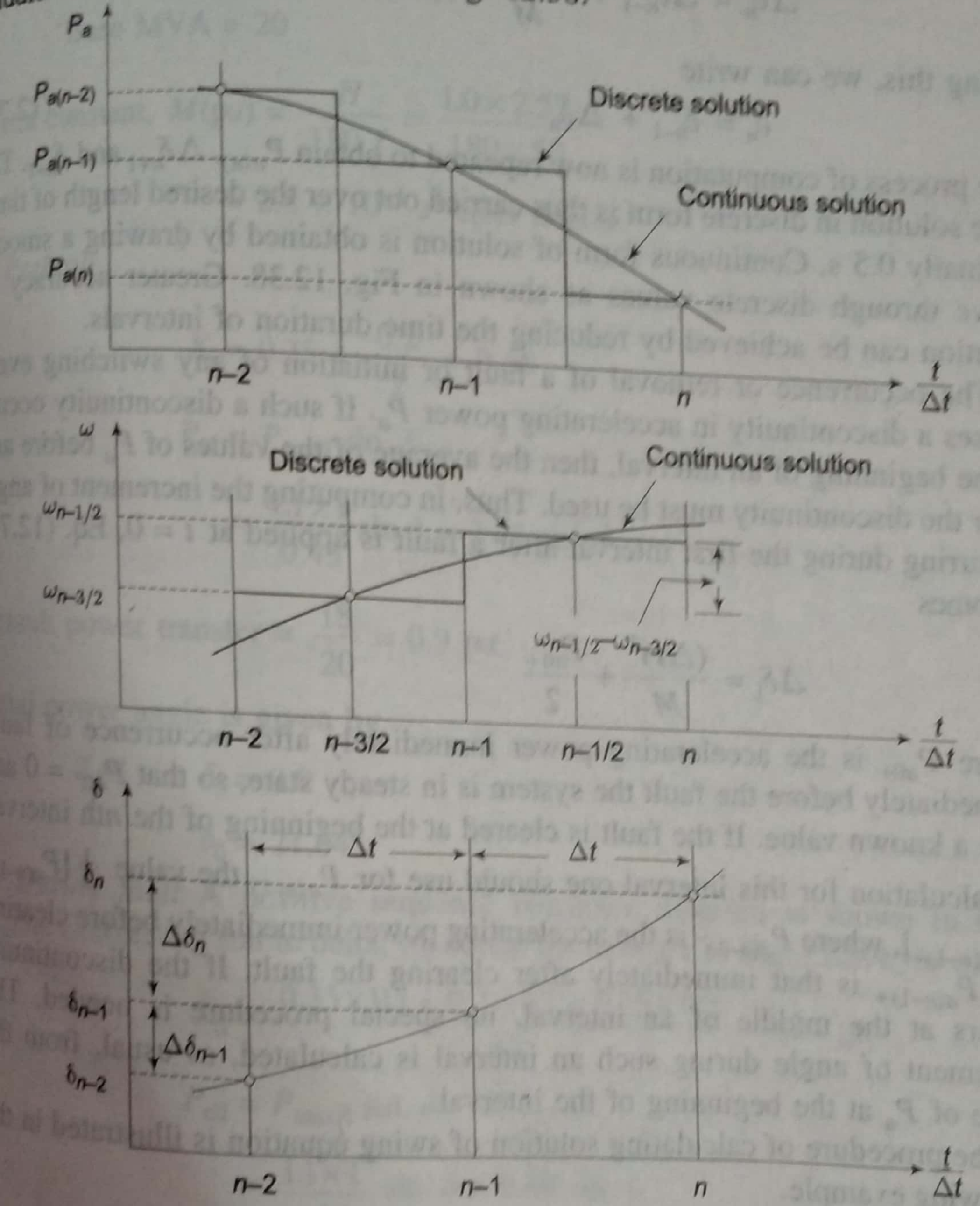

Fig. 12.38 Point-by-point solution of swing equation

In Fig. 12.38, the numbering on $t/\Delta t$ axis pertains to the end of intervals. At the end of the $(n-1)$th interval, the acceleration power is

$$P_{a\,(n-1)} = P_m - P_{max} \sin \delta_{n-1} \tag{12.68}$$

where δ_{n-1} has been previously calculated. The change in velocity ($\omega = d\delta/dt$), caused by the $P_{a(n-1)}$, assumed constant over Δt from $(n-3/2)$ to $(n-1/2)$ is

$$w_{n-1/2} - w_{n-3/2} = (\Delta t/M)\ P_{a\,(n-1)} \tag{12.69}$$

The change in δ during the $(n-1)$th interval is

$$\Delta\delta_{n-1} = \delta_{n-1} - \delta_{n-2} = \Delta t\omega_{n-3/2} \tag{12.70a}$$

and during the nth interval

$$\Delta\delta_n = \delta_n - \delta_{n-1} = \Delta t\omega_{n-1/2} \tag{12.70b}$$

Subtracting Eq. (12.70a) from Eq. (12.70b) and using Eq. (12.69), we get

$$\Delta\delta_n = \Delta\delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \tag{12.71}$$

Using this, we can write

$$\delta_n = \delta_{n-1} + \Delta\delta_n \tag{12.72}$$

The process of computation is now repeated to obtain $P_{a(n)}$, $\Delta\delta_{n+1}$ and δ_{n+1}. The time solution in discrete form is thus carried out over the desired length of time, normally 0.5 s. Continuous form of solution is obtained by drawing a smooth curve through discrete values as shown in Fig. 12.38. Greater accuracy of solution can be achieved by reducing the time duration of intervals.

The occurrence or removal of a fault or initiation of any switching event causes a discontinuity in accelerating power P_a. If such a discontinuity occurs at the beginning of an interval, then the average of the values of P_a before and after the discontinuity must be used. Thus, in computing the increment of angle occurring during the first interval after a fault is applied at $t = 0$, Eq. (12.71) becomes

$$\Delta\delta_1 = \frac{(\Delta t)^2}{M} + \frac{P_{a0+}}{2}$$

where P_{a0+} is the accelerating power immediately after occurrence of fault. Immediately before the fault the system is in steady state, so that $P_{a0^-} = 0$ and δ_0 is a known value. If the fault is cleared at the beginning of the nth interval, in calculation for this interval one should use for $P_{a(n-1)}$ the value $\frac{1}{2}[P_{a(n-1)^-} + P_{a(n-1)+}]$, where $P_{a(n-1)^-}$ is the accelerating power immediately before clearing and $P_{a(n-1)+}$ is that immediately after clearing the fault. If the discontinuity occurs at the middle of an interval, no special procedure is needed. The increment of angle during such an interval is calculated, as usual, from the value of P_a at the beginning of the interval.

The procedure of calculating solution of swing equation is illustrated in the following example.

Example 12.10

A 20 MVA, 50 Hz generator delivers 18 MW over a double circuit line to an infinite bus. The generator has kinetic energy of 2.52 MJ/MVA at rated speed. The generator transient reactance is $X'_d = 0.35$ pu. Each transmission circuit has $R = 0$ and a reactance of 0.2 pu on a 20 MVA base. $|E'| = 1.1$ pu and infinite bus voltage $V = 1.0 \angle 0°$. A three-phase short circuit occurs at the mid point of one of the transmission lines. Plot swing curves with fault cleared by simultaneous opening of breakers at both ends of the line at 2.5 cycles and 6.25 cycles after the occurrence of fault. Also plot the swing curve over the period of 0.5 s if the fault is sustained.

Solution Before we can apply the step-by-step method, we need to calculate the inertia constant M and the power angle equations under prefault and postfault conditions.

Base MVA = 20

$$\text{Inertia constant, } M(\text{pu}) = \frac{H}{180\,f} = \frac{1.0 \times 2.52}{180 \times 50}$$

$$= 2.8 \times 10^{-4}\ s^2/\text{elect degree}$$

I *Prefault*

$$X_I = 0.35 + \frac{0.2}{2} = 0.45$$

$$\therefore \quad P_{eI} = P_{\max I} \sin \delta$$

$$= \frac{1.1 \times 1}{0.45} \sin \delta = 2.44 \sin \delta \qquad \text{(i)}$$

$$\text{Prefault power transfer} = \frac{18}{20} = 0.9 \text{ pu}$$

Initial power angle is given by

$$2.44 \sin \delta_0 = 0.9$$

or

$$\delta_0 = 21.64°$$

II *During fault* A positive sequence reactance diagram is shown in Fig. 12.39a. Converting star to delta, we obtain the network of Fig. 12.39b, in which

$$X_{II} = \frac{0.35 \times 0.1 + 0.2 \times 0.1 + 0.35 \times 0.2}{0.1} = 1.25 \text{ pu}$$

$$\therefore \quad P_{eII} = P_{\max II} \sin \delta$$

$$= \frac{1.1 \times 1}{1.25} \sin \delta = 0.88 \sin \delta \qquad \text{(ii)}$$

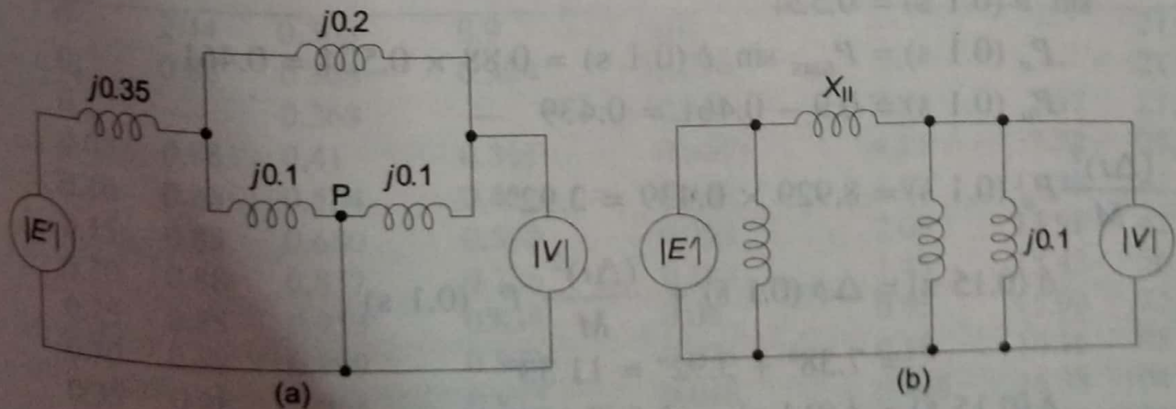

Fig. 12.39

III *Postfault* With the faulted line switched off,

$$X_{III} = 0.35 + 0.2 = 0.55$$

$$\therefore \quad P_{eIII} = P_{maxIII} \sin \delta$$

$$= \frac{1.1 \times 1}{0.55} \sin \delta = 2.0 \sin \delta \quad \text{(iii)}$$

Let us choose $\Delta t = 0.05$ s

The recursive relationships for step-by-step swing curve calculation are reproduced below.

$$P_{a(n-1)} = P_m - P_{max} \sin \delta_{n-1} \quad \text{(iv)}$$

$$\Delta \delta_n = \Delta \delta_{n-1} + \frac{(\Delta t)^2}{M} P_{a(n-1)} \quad \text{(v)}$$

$$\delta_n = \delta_{n-1} + \Delta \delta_n \quad \text{(vi)}$$

Since there is a discontinuity in P_e and hence in P_a, the average value of P_a must be used for the first interval.

$$P_a(0_-) = 0 \text{ pu and } P_a(0_+) = 0.9 - 0.88 \sin 21.64° = 0.576 \text{ pu}$$

$$P_a(0_{average}) = \frac{0 + 0.576}{2} = 0.288 \text{ pu}$$

Sustained Fault

Calculations are carried out in Table 12.2 in accordance with the recursive relationship (iv), (v) and (vi) above. The second column of the table shows P_{max} the maximum power that can be transferred at time t given in the first column. P_{max} in the case of a sustained fault undergoes a sudden change at $t = 0_+$ and remains constant thereafter. The procedure of calculations is illustrated below by calculating the row corresponding to $t = 0.15$ s.

$$(0.1 \text{ sec}) = 31.59°$$

$$P_{max} = 0.88$$

$$\sin \delta (0.1 \text{ s}) = 0.524$$

$$P_e (0.1 \text{ s}) = P_{max} \sin \delta (0.1 \text{ s}) = 0.88 \times 0.524 = 0.461$$

$$P_a (0.1 \text{ s}) = 0.9 - 0.461 = 0.439$$

$$\frac{(\Delta t)^2}{M} P_a (0.1 \text{ s}) = 8.929 \times 0.439 = 3.92°$$

$$\delta (0.15 \text{ s}) = \Delta\delta (0.1 \text{ s}) + \frac{(\Delta t)^2}{M} P_a (0.1 \text{ s})$$

$$= 7.38° + 3.92° = 11.33°$$

$$\delta (0.15 \text{ s}) = \delta (0.1 \text{ s}) + \Delta\delta (0.15 \text{ s})$$

$$= 31.59° + 11.30° = 42.89°$$

$\delta(t)$ for sustained fault as calculated in Table 12.2 is plotted in Fig. 12.40 from which it is obvious that the system is unstable.

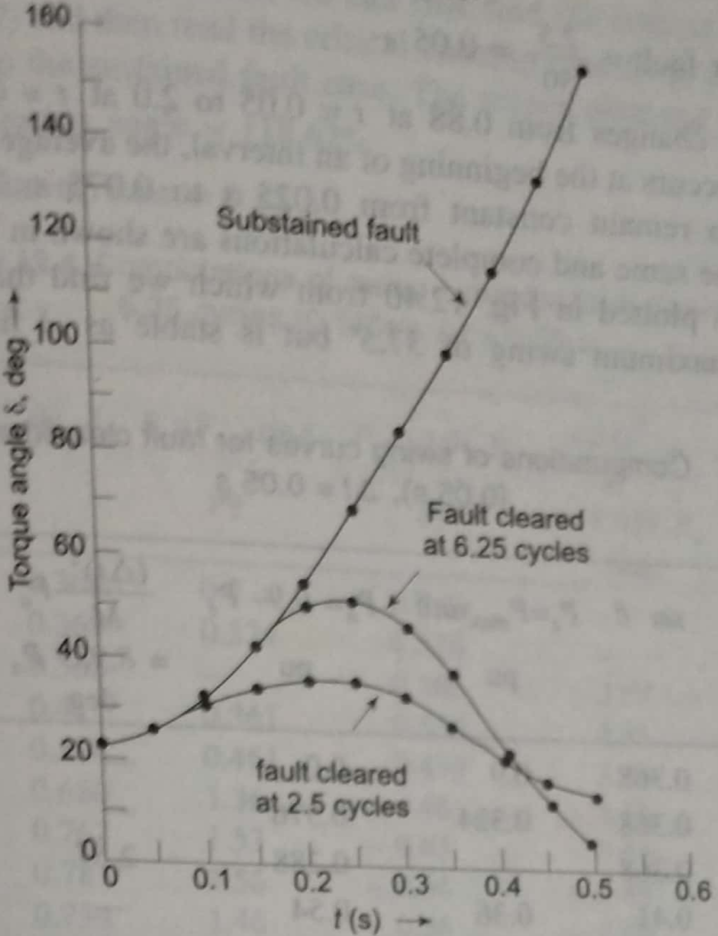

Fig. 12.40 Swing curves for Example 12.10 for a sustained fault and for clearing in 2.5 and 6.25 cycles

Table 12.2 Point-by-point computations of swing curve for sustained fault, $\Delta t = 0.05$ s

t sec	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929\, P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	0.88	0.680	0.598	0.301	2.68	13.98	42.89
0.20	0.88	0.837	0.736	0.163	1.45	15.43	56.87
0.25	0.88	0.953	0.838	0.06	0.55	15.98	72.30
0.30	0.88	0.999	0.879	0.021	0.18	16.16	88.28
0.35	0.88	0.968	0.852	0.048	0.426	16.58	104.44
0.40	0.88	0.856	0.754	0.145	1.30	17.88	121.02
0.45	0.88	0.657	0.578	0.321	2.87	20.75	138.90
0.50	0.88	—	—	—	—	—	159.65

Fault Cleared in 2.5 Cycles

Time to clear fault = $\frac{2.5}{50}$ = 0.05 s

P_{max} suddenly changes from 0.88 at t = 0.05 to 2.0 at $t = 0.05_+$. Since the discontinuity occurs at the beginning of an interval, the average value of P_a will be assumed to remain constant from 0.025 s to 0.075 s. The rest of the procedure is the same and complete calculations are shown in Table 12.3. The swing curve is plotted in Fig. 12.40 from which we find that the generator undergoes a maximum swing of 37.5° but is stable as δ finally begins to decrease.

Table 12.3 Computations of swing curves for fault cleared at 2.5 cycles (0.05 s), Δt = 0.05 s

t sec	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929\,P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	—	—	21.64
0_+	0.88	0.368	0.324	0.576	—	—	21.64
0_{avg}	—	0.368	—	0.288	2.57	2.57	21.64
0.05_-	0.88	0.41	0.36	0.54	—	—	24.21
0.05_+	2.00	0.41	0.82	0.08	—	—	24.21
0.05_{avg}				0.31	2.767	5.33	24.21
0.10	2.00	0.493	0.986	– 0.086	– 0.767	4.56	29.54
0.15	2.00	0.56	1.12	– 0.22	– 1.96	2.60	34.10
0.20	2.00	0.597	1.19	– 0.29	– 2.58	0.02	36.70
0.25	2.00	0.597	1.19	– 0.29	– 2.58	– 2.56	37.72
0.30	2.00	0.561	1.12	– 0.22	– 1.96	– 4.52	34.16
0.35	2.00	0.494	0.989	– 0.089	– 0.79	– 5.31	29.64
0.40	2.00	0.41	0.82	0.08	0.71	– 4.60	24.33
0.45	2.00	0.337	0.675	0.225	2.0	– 2.6	19.73
0.50							17.13

Fault Cleared in 6.25 Cycles

Time to clear fault = $\frac{6.25}{50}$ = 0.125 s

Since the discontinuity now lies in the middle of an interval, no special procedure is necessary, as in deriving Eqs. (iv) – (vi) discontinuity is assumed to occur in the middle of the time interval. The swing curve as calculated in Table 12.4 is also plotted in Fig. 12.40. It is observed that the system is stable with a maximum swing of 52.5° which is much larger than that in the case of 2.5 cycle clearing time.

To find the critical clearing time, swing curves can be obtained, similarly, for progressively greater clearing time till the torque angle δ increases without bound. In this example, however, we can first find the critical clearing angle using Eq. (12.67) and then read the critical clearing time from the swing curve corresponding to the sustained fault case. The values obtained are:

Critical clearing angle = 118.62°

Critical clearing time = 0.38 s

Table 12.4 Computations of swing curve for fault cleared at 6.25 cycles (0.125s), $\Delta t = 0.05$ s

t sec	P_{max} pu	$\sin\delta$	$P_e = P_{max}\sin\delta$ pu	$P_a = 0.9 - P_e$ pu	$\frac{(\Delta t)^2}{M} P_a = 8.929\, P_a$ deg	$\Delta\delta$ deg	δ deg
0_-	2.44	0.368	0.9	0.0	–	–	21.64
0_+	0.88	0.368	0.324	0.576	–	–	21.64
0_{avg}	–	0.368	–	0.288	2.57	2.57	21.64
0.05	0.88	0.41	0.361	0.539	4.81	7.38	24.21
0.10	0.88	0.524	0.461	0.439	3.92	11.30	31.59
0.15	2.00	0.680	1.36	– 4.46	– 4.10	7.20	42.89
0.20	2.00	0.767	1.53	– 0.63	– 5.66	1.54	50.09
0.25	2.00	0.78	1.56	– 0.66	– 5.89	– 4.35	51.63
0.30	2.00	0.734	1.46	– 0.56	– 5.08	– 9.43	47.28
0.35	2.00	0.613	1.22	– 0.327	– 2.92	– 12.35	37.85
0.40	2.00	0.430	0.86	0.04	0.35	– 12.00	25.50
0.45	2.00	0.233	0.466	0.434	3.87	– 8.13	13.50
0.50	2.00						5.37

Stability Study of Large Systems

To limit the computer memory and the time requirements and for the sake of computational efficiency, a large multi-machine system is divided into a study subsystem and an external system. The study subsystem is modelled in detail whereas approximate modelling is carried out for the external subsystem. The total study is rendered by the modern technique of dynamic equivalencing. In the external subsystem, number of machines is drastically reduced using various methods—coherency based methods being most popular and widely used by various power utilities in the world.

12.11 SOME FACTORS AFFECTING TRANSIENT STABILITY

We have seen in this chapter that the two-machine system can be equivalently reduced to a single machine connected to infinite bus bar. The qualitative conclusions regarding system stability drawn from a two-machine or an equivalent one-machine infinite bus system can be easily extended to a multimachine system. In the last article we have studied the algorithm for determining the stability of a multimachine system.

It has been seen that transient stability is greatly affected by the type and location of a fault, so that a power system analyst must at the very outset of a stability study decide on these two factors. In our examples we have selected a 3–phase fault which is generally more severe from point of view of power transfer. Given the type of fault and its location let us now consider other factors which affect transient stability and therefrom draw the conclusions, regarding methods of improving the transient stability limit of a system and making it as close to the steady state limit as possible.

For the case of one machine connected to infinite bus, it is easily seen from Eq. (12.71) that an increase in the inertia constant M of the machine reduces the angle through which it swings in a given time interval offering thereby a method of improving stability but this cannot be employed in practice because of economic reasons and for the reason of slowing down the response of the speed governor loop (which can even become oscillatory) apart from an excessive rotor weight.

With reference to Fig. 12.30, it is easily seen that for a given clearing angle, the accelerating area decreases but the decelerating area increases as the maximum power limit of the various power angle curves is raised, thereby adding to the transient stability limit of the system. The maximum steady power of a system can be increased by raising the voltage profile of the system and by reducing the transfer reactance. These conclusions along with the various transient stability cases studied, suggest the following method of improving the transient stability limit of a power system.

1. Increase of system voltages, use of AVR.
2. Use of high speed excitation systems.
3. Reduction in system transfer reactance.
4. Use of high speed reclosing breakers (see Fig. 12.32). Modern tendency is to employ single-pole operation of reclosing circuit breakers.

When a fault takes place on a system, the voltages at all buses are reduced. At generator terminals, these are sensed by the automatic voltage regulators which help restore generator terminal voltages by acting within the excitation system. Modern exciter systems having solid state controls quickly respond to bus voltage reduction and can achieve from one-half to one and one-half cycles ($1/2–1\frac{1}{2}$) gain in critical clearing times for three-phase faults on the HT bus of the generator transformer.

Reducing transfer reactance is another important practical method of increasing stability limit. Incidentally this also raises system voltage profile. The reactance of a transmission line can be decreased (i) by reducing the conductor spacing, and (ii) by increasing conductor diameter (see Eq. (2.37)). Usually, however, the conductor spacing is controlled by other features such as lightning protection and minimum clearance to prevent the arc from one phase moving to another phase. The conductor diameter can be increased by using material of low conductivity or by hollow cores. However, normally, the conductor configuration is fixed by economic considerations quite apart from stability. The use of bundled conductors is, of course, an effective means of reducing series reactance.

Compensation for line reactance by series capacitors is an effective and economical method of increasing stability limit specially for transmission distances of more than 350 km. The degree of series compensation, however, accentuates the problems of protective relaying, normal voltage profiles, and overvoltages during line-to-ground faults. Series compensation becomes more effective and economical if part of it is switched on so as to increase the degree of compensation upon the occurrence of a disturbance likely to cause instability. Switched series capacitors simultaneously decrease fluctuation of load voltages and raise the transient stability limit to a value almost equal to the steady state limit. Switching shunt capacitors on or switching shunt reactors off also raises stability limits (see Example 12.2) but the MVA rating of shunt capacitors required is three to six times the rating of switched series capacitors for the same increase in stability limit. Thus series capacitors are preferred unless shunt elements are required for other purposes, say, control of voltage profile.

Increasing the number of parallel lines between transmission points is quite often used to reduce transfer reactance. It adds at the same time to reliability of the transmission system. Additional line circuits are not likely to prove economical unit *l* after all feasible improvements have been carried out in the first two circuits.

As the majority of faults are transient in nature, rapid switching and isolation of unhealthy lines followed by reclosing has been shown earlier to be a great help in improving the stability margins. The modern circuit breaker technology has now made it possible for line clearing to be done as fast as in two cycles. Further, a great majority of transient faults are line-to-ground in nature. It is natural that methods have been developed for selective single pole opening and reclosing which further aid the stability limits. With reference to Fig. 12.17, if a transient LG fault is assumed to occur on the generator bus, it is immediately seen that during the fault there will now be a definite amount of power transfer, as different from zero power transfer for the case of a three-phase fault. Also when the circuit breaker pole corresponding to the faulty line is opened, the other two lines (healthy ones) remain intact so that considerable power transfer continues to take place via these lines in comparison to the case of three-pole switching when the power transfer on fault clearing will be reduced to zero. It is, therefore, easy to see why the single pole switching and reclosing aids in stability problem and is widely adopted. These facts are illustrated by means of Example 12.12. Even when the stability margins are sufficient, single pole switching is adopted to prevent large swings and consequent voltage dips. Single pole switching and reclosing is, of course, expensive in terms of relaying and introduces the associated problems of overvoltages caused by single pole opening owing to line capacitances. Methods are available to nullify these capacitive coupling effects.

Recent Trends

Recent trends in design of large alternators tend towards lower short circuit ratio (SCR = $1/X_d$), which is achieved by reducing machine air gap with consequent savings in machine mmf, size, weight and cost. Reduction in the size of rotor reduces inertia constant, lowering thereby the stability margin. The loss in stability margin is made up by such features as lower reactance lines, faster circuit breakers and faster excitation systems as discussed already, and a faster system valving to be discussed later in this article.

A stage has now been reached in technology whereby the methods of improving stability, discussed above, have been pushed to their limits, e.g., clearing times of circuit breakers have been brought down to virtually irreducible values of the order of two cycles. With the trend to reduce machine inertias there is a constant need to determine availability, feasibility and applicability of new methods for maintaining and/or improving system stability. A brief account of some of the recent methods of maintaining stability is given below:

HVDC Links

Increased use of HVDC links employing thyristors would alleviate stability problems. A dc link is asynchronous, i.e., the two ac system at either end do not have to be controlled in phase or even be at exactly the same frequency as they do for an ac link, and the power transmitted can be readily controlled. There is no risk of a fault in one system causing loss of stability in the other system.

Breaking Resistors

For improving stability where clearing is delayed or a large load is suddenly lost, a resistive load called a breaking resistor is connected at or near the generator bus. This load compensates for at least some of the reduction of load on the generators and so reduces the acceleration. During a fault, the resistors are applied to the terminals of the generators through circuit breakers by means of an elaborate control scheme. The control scheme determines the amount of resistance to be applied and its duration. The breaking resistors remain on for a matter of cycles both during fault clearing and after system voltage is restored.

Short Circuit Current Limiters

These are generally used to limit the short circuit duty of distribution lines. These may also be used in long transmission lines to modify favourably the transfer impedance during fault conditions so that the voltage profile of the system is somewhat improved, thereby raising the system load level during the fault.

Turbine Fast Valving or Bypass Valving

The two methods just discussed above are an attempt at replacing the system load so as to increase the electrical output of the generator during fault conditions. Another recent method of improving the stability of a unit is to decrease the mechanical input power to the turbine. This can be accomplished by means of fast valving, where the difference between mechanical input and reduced electrical output of a generator under a fault, as sensed by a control scheme, initiates the closing of a turbine valve to reduce the power input. Briefly, during a fast valving operation, the interceptor valves are rapidly shut (in 0.1 to 0.2 sec) and immediately reopened. This procedure increases the critical switching time long enough so that in most cases, the unit will remain stable for faults with stuck-breaker clearing times. The scheme has been put to use in some stations in the USA.

Full Load Rejection Technique

Fast valving combined with high-speed clearing time will suffice to maintain stability in most of the cases. However, there are still situations where stability is difficult to maintain. In such cases, the normal procedure is to automatically trip the unit off the line. This, however, causes several hours of delay before the unit can be put back into operation. The loss of a major unit for this length of time can seriously jeopardize the remaining system.

To remedy these situations, a full load rejection scheme could be utilized after the unit is separated from the system. To do this, the unit has to be equipped with a large steam bypass system. After the system has recovered from the shock caused by the fault, the unit could be resynchronized and reloaded. The main disadvantage of this method is the extra cost of a large bypass system.