Jaipur Engineering College & Research Centre, Jaipur

Session 2020-21

Notes – Unit I

Power System-II

Name of Faculty Ritu Soni
Designation Asst. Professor
Department Electrical Engineering

Scanned with CamScanner

Syllabus of Power System-II

1 Introduction: Objective, scope and outcome of the course.

2 Power Flow Analyses

Review of the structure of a Power System and its components. Analysis of Power Flows: Formation of Bus Admittance Matrix. Real and reactive power balance equations at a node. Load and Generator Specifications. Application of numerical methods for solution of nonlinear algebraic equations – Gauss Seidel and Newton-Raphson methods for the solution of the power flow equations. Computational Issues in Large-scale Power Systems.

3 Stability Constraints in synchronous grids

Swing Equations of a synchronous machine connected to an infinite bus. Power angle curve. Description of the phenomena of loss of synchronism in a single-machine infinite bus system following a disturbance like a three--phase fault. Analysis using numerical integration of swing equations (using methods like Forward Euler, Runge-Kutta 4th order methods), as well as the Equal Area Criterion. Impact of stability constraints on Power System Operation. Effect of generation rescheduling and series compensation of transmission lines on stability.

4 Control of Frequency and Voltage

Turbines and Speed-Governors. Frequency dependence of loads, Droop Control and Power Sharing. Automatic Generation Control. Generation and absorption of reactive power by various components of a Power System. Excitation System Control in synchronous generators, Automatic Voltage Regulators. Shunt Compensators, Static VAR compensators and STATCOMs. Tap Changing Transformers. Power flow control using embedded dc links, phase shifters

5 Monitoring and Control

Overview of Energy Control Centre Functions: SCADA systems. Phasor Measurement Units and Wide-Area Measurement Systems. State-estimation. System Security Assessment. Normal, Alert, Emergency, Extremis states of a Power System. Contingency Analysis. Preventive Control and Emergency Control

6 Power System Economics and Management

Basic Pricing Principles: Generator Cost Curves, Utility Functions, Power Exchanges, Spot Pricing. Electricity Market Models (Vertically Integrated, Purchasing Agency, Whole-sale competition, Retail Competition), Demand Side-management, Transmission and Distributions charges, Ancillary Services. Regulatory framework.

Scanned with CamScanner

Course outcomes for Power system-II

CO1- Able to study about load flow and stability analysis of power system using different computational methods.

CO2- Able to understand the frequency, speed, voltage and power flow control and monitoring of power system components using different methods.

CO3- Able to understand power system economics and management using different methods.

Scanned with CamScanner

→ Symmetrical steady state is the most important mode of operation of a power system.
Three major problems encountered in this mode of operation are listed below

1. Load flow problem
2. Optimal load ~~sh~~ scheduling problem
3. System control problem.

Load flow solution is a solution of the network under steady state condition subject to certain inequality constraints under which system operates. These constraints can be in the form of load nodal voltages, reactive power generation of the generators, the tap settings of a tap changing under load transformer etc.

The load flow solution gives the nodal voltages and phase angles and hence the power injection at all buses and power flows through interconnecting power channels (transmission lines)
Load flow solution is essential for designing a new power system and for planning extension of the existing one for increased load demand.
These analysis require the calculation of numerous load flows under both normal and abnormal operating conditions. it also gives the initial conditions of the system when the transient behaviour of the system is to be studied.

Scanned with CamScanner

Load flow solution for Power n/w can be worked out both ways according as it is operating under (i) balanced or (ii) unbalanced conditions.

The following treatment will be for a system operating under balanced condition only. for such a system a single phase representation is adequate. A load flow solution of the power system requires mainly the following steps.

(i) Formulation or the n/w equations.

(ii) Suitable mathematical technique for solution of the equation.

→ Since we are studying the system under steady state conditions the network equations will be in the form of simple algebric equations.

Scanned with CamScanner

Bus Classification ⇒

In a Power System each bus or node is associated with four quantities, real and reactive Powers, bus voltage magnitude and it's Phase angle.
In a load flow solution two out of four quantities are specified and the remaining two are required to be obtain through the solution of the equations.
Depending upon which quantities have been specified, the buses are classified in the following three categories:

① Load bus or PQ Bus ⇒

At this type of bus, the net Powers P_i and Q_i are known (P_{Di} and Q_{Di} are known from load forecasting and P_{Gi} and Q_{Gi} are specified).
The unknowns are $|V_i|$ and δ_i through load flow solution.
A Pure load bus (no generating facility at the bus, i.e $P_{Gi} = Q_{Gi} = 0$) is a PQ bus.

② Generator bus or Voltage controlled bus or PV Bus ⇒

At this type of bus P_{Di} and Q_{Di} are known a priori and $|V_i|$ and P_i (hence P_{Gi}) are specified.
The unknowns are Q_i (hence Q_{Gi}) and δ_i.

③ Slack bus/ Swing bus / Reference bus ⇒

This is distinguish from the other two types by the fact that real and reactive Powers at this bus are not specified. Instead, Voltage magnitude and Phase angle (normally set equal to zero) are specified.

→ normally there is only one bus of this type in a given Power system.

→ In a load flow study real and reactive Powers (Complex Power) Can not be fixed a Periori at all the buses as the net Complex Power flow into the n/w is not Known in advance, the system Power loss being unknown till the load flow study is complete. It is therefore necessary to have one bus (slack bus) at which complex Power is unspecified so that is supplies the difference in the total system load Plus losses and the sum of the complex Power specified at the remaining buses.

By the same reasoning slack bus must be a generator bus.

Bus type	Quantities specified	Quantities to be obtain
Load bus	P, Q	|V|, δ
Generator bus	P, |V|	Q, δ
slack bus	|V|, δ	P, Q

Scanned with CamScanner

Bus Admittance matrix ⇒

A Power system may comprise several buses interconnected through transmission lines. Power is injected into a bus from generators, while the loads are tapped from it. Of course there may be buses with only generators, and there may be others with only loads. Some buses may have both generators and loads while some other may ~~be~~ have static capacitors (or synchronous condensers) for reactive power compensation or voltage control. The surplus power at some of the buses is transported through transmission lines to the buses deficient in power.

Single line dia of a simple 4-bus system with generators and loads at each bus is shown in fig 1. To arrive at network model of a power system,-

Short line $\xrightarrow{\text{represented by}}$ series impedance

long line $\xrightarrow{\text{by}}$ nominal π model

Very long line $\longrightarrow$ equivalent π

line resistance is usually neglected with a ~~slow~~ small loss in accuracy ~~by~~ but a great deal of saving in computation time.

It is convenient to consider loads as negative generators and lump together the generator and load powers at the buses.

Thus at the ith bus, the net complex power injected into the bus is given by -

$$S_i = P_i + jQ_i = (P_{Gi} - P_{Di}) + j(Q_{Gi} - Q_{Di})$$

where the complex power supplied by the generator is

$$S_{Gi} = P_{Gi} + jQ_{Gi}$$

and complex power drawn by the load is

$$S_{Di} = P_{Di} + jQ_{Di}$$

The real and reactive powers injected into the ith bus are then

$$P_i = P_{Gi} - P_{Di} \quad ; \quad i = 1, 2, \ldots n \quad \text{— (1)}$$
$$Q_i = Q_{Gi} - Q_{Di}$$

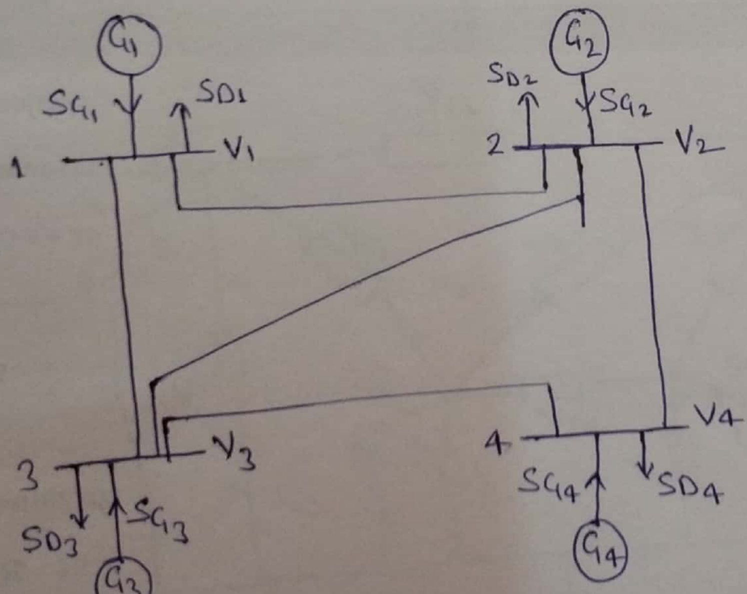


fig 1. one line dia. of 4-Bus system.

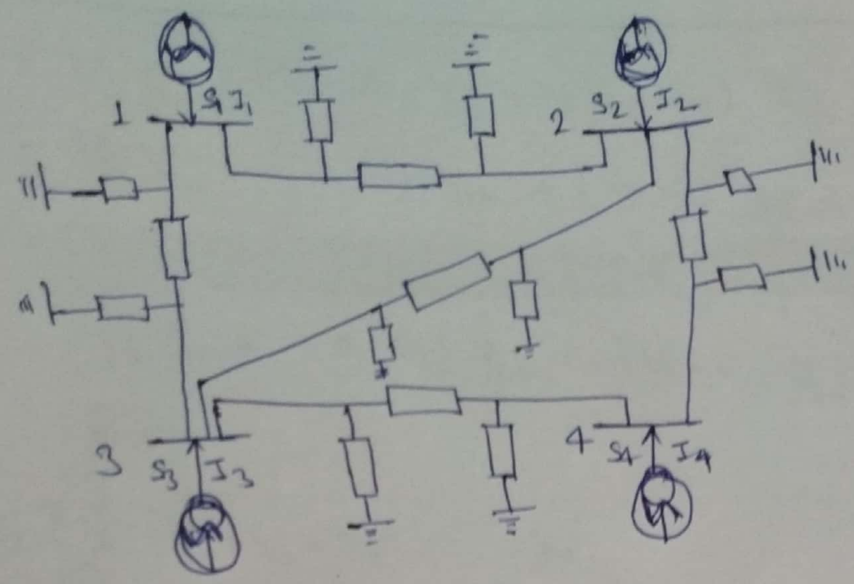


fig 2 (a) Equivalent circuit

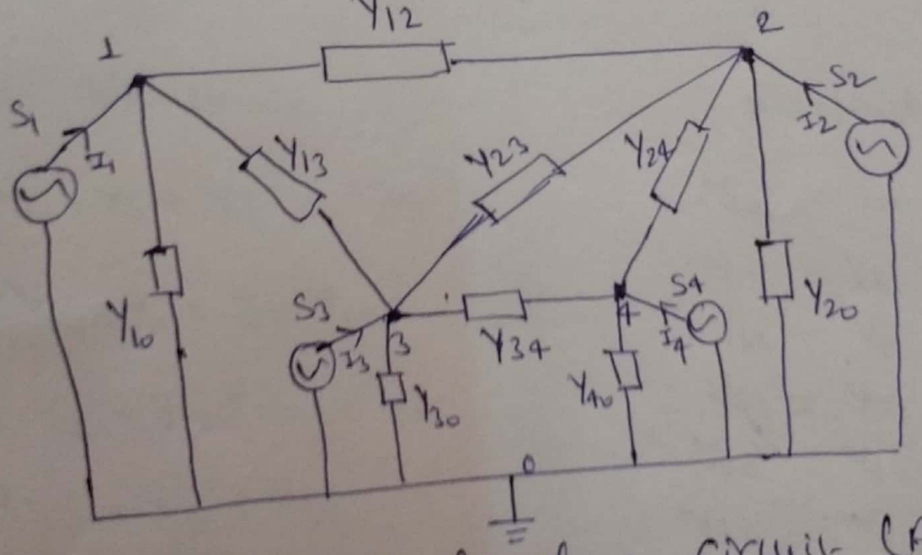


fig 2 (b) modified equivalent circuit (Admittance network) of four bus system shown in fig 1.

The line admittance b/w the nodes i and k is depicted by $y_{ik} = y_{ki}$. Further, the mutual admittance b/w lines is assumed to be zero.

Scanned with CamScanner

$$I_1 = V_1 y_{10} + (V_1 - V_2) y_{12} + (V_1 - V_3) y_{13}$$
$$I_2 = V_2 y_{20} + (V_2 - V_1) y_{12} + (V_2 - V_3) y_{23} + (V_2 - V_4) y_{24}$$
$$I_3 = V_3 y_{30} + (V_3 - V_1) y_{13} + (V_3 - V_2) y_{23} + (V_3 - V_4) y_{34}$$
$$I_4 = V_4 y_{40} + (V_4 - V_2) y_{24} + (V_4 - V_3) y_{34} \quad \text{---- (2)}$$

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} y_{10}+y_{12}+y_{13} & -y_{12} & -y_{13} & 0 \\ -y_{12} & y_{20}+y_{12}+y_{23}+y_{24} & -y_{23} & -y_{24} \\ -y_{13} & -y_{23} & y_{30}+y_{13}+y_{23}+y_{34} & -y_{34} \\ 0 & -y_{24} & -y_{34} & y_{40}+y_{24}+y_{34} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \text{--- (3)}$$

Self admittance

$$Y_{11} = y_{10} + y_{12} + y_{13}$$
$$Y_{22} = y_{20} + y_{12} + y_{23} + y_{24}$$
$$Y_{33} = y_{30} + y_{13} + y_{23} + y_{34}$$
$$Y_{44} = y_{40} + y_{24} + y_{34}$$

Y_{ii} = Self admittance / driving point admittance

Y_{ii} = Sum of admittances connected ith node

mutual admittance

$$Y_{12} = Y_{21} = -y_{12}$$
$$Y_{13} = Y_{31} = -y_{13}$$
$$Y_{14} = Y_{41} = -y_{14}$$
$$Y_{23} = Y_{32} = -y_{23}$$
$$Y_{24} = Y_{42} = -y_{24}$$
$$Y_{34} = Y_{43} = -y_{34}$$
$$Y_{ik} = Y_{ki}$$

Each off diagonal term Y_{ik} is known as mutual admittance (or transfer admittance) b/w ith and kth node and is equal to the negative of the sum of all the admittances connected directly b/w ith and kth nodes.

Scanned with CamScanner

$$\begin{bmatrix} I_1 \\ I_2 \\ I_3 \\ I_4 \end{bmatrix} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \\ V_3 \\ V_4 \end{bmatrix} \quad \text{— (4)}$$

$$[I_{bus}] = [Y_{bus}][V_{bus}] \quad \text{— (5)}$$

where $[I]$ = node Current matrix

$[V]$ = node Voltage matrix

$[Y_{Bus}]$ = Bus admittance matrix

General eq. for n-bus N/w based on KCL and admittance form is

$$[I] = [Y_{Bus}][V] \quad \text{or} \quad I = Y_{Bus} V \quad \text{— (6)}$$

where
$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & \cdots & Y_{1n} \\ Y_{21} & Y_{22} & \cdots & Y_{2n} \\ \cdots & \cdots & \cdots & \cdots \\ Y_{n1} & Y_{n2} & \cdots & Y_{nn} \end{bmatrix} \quad \text{— (7)}$$

→ Y_{Bus} matrix for n-bus N/w has n rows and n columns. each of Y term has two subscripts:

→ The first subscript refers to the bus number on which the current is expressed

→ The second subscript refers to the bus no. whose voltage has caused that current component

→ The term on diagonal are self-admittances..

→ All the non diagonal terms are ~~too~~ mutual admittances

$$I_i = \sum_{k=1}^{n} Y_{ik} V_k \quad ; \quad i = 1, 2, \cdots n \quad \text{— (8)}$$

Scanned with CamScanner

→ Nodal admittance matrix is a sparse matrix (a few no. of elements are non-zero) for an actual power system.

In a large system of 100 nodes, these non-zero elements may be as small as 2% of the total elements. Computer memory requirement for storing the nodal admittance matrix is very long.

→ Nodal admittance matrix is a symmetric matrix along the leading diagonal, the computer need store the upper or lower triangular nodal admittance matrix only.

Thus computer memory requirement for storing the nodal admittance is all the more reduced and numerical computation is also reduced due to sparsity of bus admittance matrix.

→

Scanned with CamScanner

Formulation of load flow equations and methods of Solution =>

The complex Power injected by the generating source into the ith bus of a Power system is given as

$$S_i = P_i + jQ_i = V_i I_i^* \quad -① \quad i = 1, 2, \ldots n$$

where V_i is the Voltage at the ith bus with respect to ground and I_i^* is the complex Conjugate of source current I_i injected to the bus.

It is convenient to handle load flow Problems by using I_i rather than I_i^*. So taking the complex of eq. -①, we have

$$S_i^* = P_i - jQ_i = V_i^* I_i \quad ; \quad i = 1, 2, 3, \ldots n \quad -(1(a))$$

Substituting $I_i = \sum_{K=1}^{n} Y_{iK} V_K$ in above eq., we have

$$S_i^* = P_i - jQ_i = V_i^* \sum_{K=1}^{n} Y_{iK} V_K \quad ; \quad i = 1, 2, 3, \ldots n \quad -(1(b))$$

equating real and imaginary Parts, we have

Real Power $P = Re\left\{V_i^* \sum_{K=1}^{n} Y_{iK} V_K\right\}$ — ②(a)

and Reactive Power $Q = -Im\left\{V_i^* \sum_{K=1}^{n} Y_{iK} V_K\right\}$ — ②(b)

in Polar form $V_i = V_i \angle \delta_i$, $V_i^* = V_i \angle -\delta_i$

and $Y_{iK} = Y_{iK} \angle \theta_{iK}$,

So real and reactive Powers can now be expressed as -

· Real Power, $P_i = V_i \sum_{K=1}^{n} V_K Y_{iK} \cos(\theta_{iK} + \delta_K - \delta_i)$ — ③

Reactive Power, $Q_i = -V_i \sum_{K=1}^{n} V_K Y_{iK} \sin(\theta_{iK} + \delta_K - \delta_i)$ — ④

Scanned with CamScanner

Above equations (3) and (4) are known as static load flow equations (SLFE).

These equations are non-linear equations and, therefore only a numerical solution is possible.

→ For each of the n system buses we have two such equations giving a total of 2n equations (n real flow power equations and n reactive power flow equations).

→ Each bus is characterized by four variables P_i, Q_i, V_i and δ_i giving a total 4n variables.

→ To obtain a solution it necessary to ~~obtain~~ specify two variables at each bus so that no. of unknown is reduced to 2n.

→ Evidently we should specify the variables over which we have physical control. The choice is influenced somewhat by the devices which are connected to a particular bus.

→ Depending upon the quantities specified, the buses can be classified in to three types, as - ~~already~~

Type of Bus	Known or Specified Quantities	Unknown Quantities ~~of~~
(1) Generation or P-V Bus	P, |V|	Q, δ
(2) Load or P-Q Bus	P, Q	|V|, δ
(3) Slack or reference Bus	|V|, δ	P, Q.

→ Phase angle of the voltage at the slack bus is usually taken as reference or zero.

Scanned with CamScanner

→ The load buses are most common in Power system. at these buses P_i and Q_i are known because P_{Di}, Q_{Di} are known from the load forecast data and P_{Gi} and Q_{Gi} are either zero (no generation at these buses) or specified.

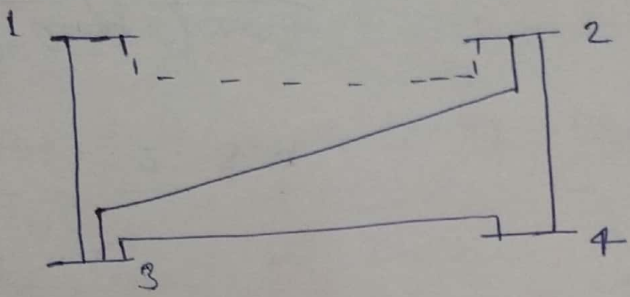
→ Power system can be analysed using either mesh current method or nodal voltage method. but nodal voltage method is usually employed from the computer time and memory point of view.

Scanned with CamScanner

Q. fig shows the one-line dia of a simple four-bus system. The shunt admittances at all the buses is assumed negligible.

(a) Find Y_{Bus} assuming that the line shown dotted is not connected.

(b) What modifications need to be carried out in Y_{Bus} if the line shown dotted is connected.



Line, bus to bus	R, pu	X, pu
1-2	0.05	0.15
1-3	0.10	0.30
2-3	0.15	0.45
2-4	0.10	0.30
3-4	0.05	0.15

Solution –

Line	G, pu	B, pu
1-2	2.0	-6.0
1-3	1.0	-3.0
2-3	0.666	-2.0
2-4	1.0	-3.0
3-4	2.0	-6.0

$$Y_{Bus} = \begin{bmatrix} Y_{11} & Y_{12} & Y_{13} & Y_{14} \\ Y_{21} & Y_{22} & Y_{23} & Y_{24} \\ Y_{31} & Y_{32} & Y_{33} & Y_{34} \\ Y_{41} & Y_{42} & Y_{43} & Y_{44} \end{bmatrix} = \begin{bmatrix} Y_{13} & 0 & -Y_{13} & 0 \\ 0 & Y_{23}+Y_{24} & -Y_{23} & -Y_{24} \\ -Y_{13} & -Y_{23} & Y_{13}+Y_{23}+Y_{34} & -Y_{34} \\ 0 & -Y_{24} & -Y_{34} & Y_{24}+Y_{34} \end{bmatrix}$$

Scanned with CamScanner

$$Y_{Bus} = \begin{bmatrix} 1-j3 & 0 & -1+j3 & 0 \\ 0 & 1.666-j5 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

(b) The following elements of Y_{Bus} of Part (a) are modified when a line is added b/w buses 1 & 2.

$$Y_{12new} = Y_{12old} - (2-j6) = 0-(2-j6) = -2+j6$$

$$Y_{11new} = Y_{11old} + (2-j6) = 1-j3+2-j6 = 3-j9$$

$$Y_{22new} = Y_{22old} + (2-j6) = 1.666-j5+2-j6 = 3.666-j11$$

Modified Y_{Bus} is written as

$$Y_{Bus} = \begin{bmatrix} 3-j9 & -2+j6 & -1+j3 & 0 \\ -2+j6 & 3.666-j11 & -0.666+j2 & -1+j3 \\ -1+j3 & -0.666+j2 & 3.666-j11 & -2+j6 \\ 0 & -1+j3 & -2+j6 & 3-j9 \end{bmatrix}$$

Scanned with CamScanner

Formation of Y_{Bus} by Singular Transformation ⇒

The matrix Pair Y_{Bus} and Z_{Bus} form the n/w models for load flow studies. Y_{Bus} can alternatively assembled by use of singular transformations given by a graph theoretical approach. This alternative approach is of great theoretical and practical significance. ~~and~~ ~~is~~

Graph –
To describe the geometrical features of a n/w, it is replaced by single line segments called elements whose terminals are called nodes.

Linear graph – it depicts the geometrical interconnection of the elements of n/w.

Connected graph – if and only if there is a path b/w every pair of nodes.

Oriented graph – if each element of a connected graph is assigned a direction.

Power system are so structured that
Total nodes $= m$
1 node is always at ground Potential
remaining, $n = m-1$, nodes are buses at which the source Power is injected.

Scanned with CamScanner

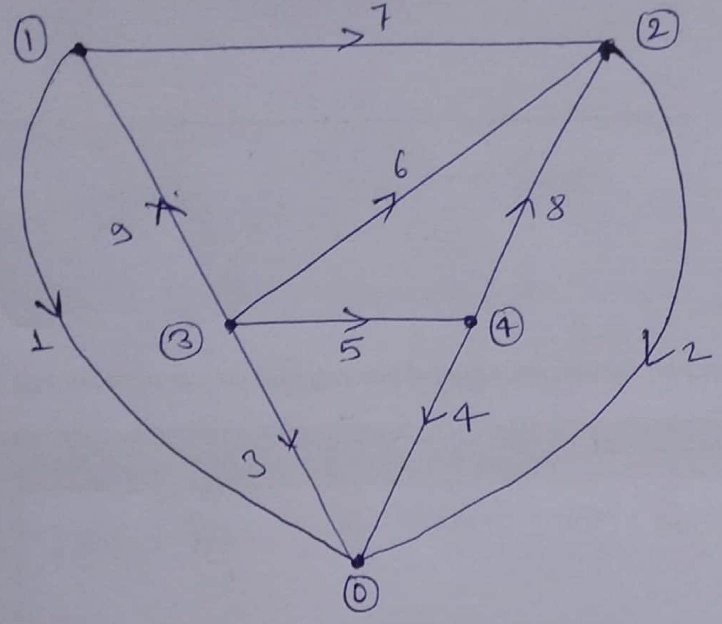


Fig 1 Linear graph.

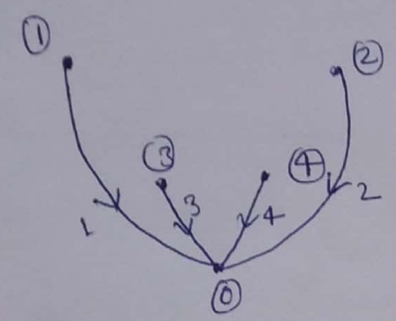
→ Each source and the shunt admittance connected across it are represented by a single element. In fact, this combination represents the most general n/w element is described under sub heading "Primitive network".

→ Tree – A connected subgraph containing all the nodes of a graph but having no closed path is called a tree. The elements of a tree are called branches or tree branches.
no. of branches b that form a tree are given by

$$b = m - 1 = n \text{ (number of buses)}$$

→ Links – Those elements of of the graph that are not included in the tree are called links. and they form a subgraph, not necessarily connected, called Cotree.
The no. of links l of a connected graph with e elements is

$$l = e - b = e - m + 1$$



— Branch
--- Link
$e = 9$
$n = 5$
$b = n - 1 = 5 - 1 = 4 = n$
$l = e - b = 5$

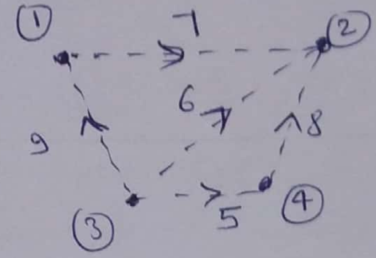


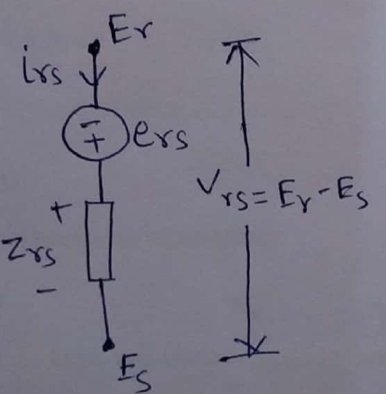
fig 2 (a) tree (b) Co-tree

→ note that a tree of a graph is not unique.

→ If a link is added to the tree, the corresponding graph contains one closed path called a loop. Thus a graph has as many loops as the no. of links.

Primitive n/w ⇒

A n/w element may in general contain active and passive components. Fig. (a) and (b) shows respectively, the alternative impedance and admittance form representation of general n/w element.



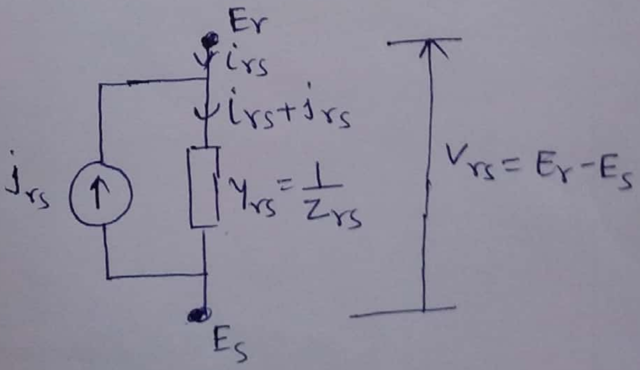


fig 3 fig (a) impedance form fig (b) Admittance form

Representation of a n/w element

Scanned with CamScanner

The impedance form is a voltage source e_{rs} in series with an impedance Z_{rs}; while the admittance form is a current source j_{rs} in parallel with an admittance Y_{rs}. The element current is i_{rs} and the element voltage is $V_{rs} = E_r - E_s$

The voltage relation for fig (a) can be written as

$$V_{rs} + e_{rs} = Z_{rs}\, i_{rs} \qquad — (1)$$

and for fig (2) is

$$i_{rs} + j_{rs} = Y_{rs} V_{rs} \qquad — (2)$$

The forms of fig (a) and (b) are equivalent wherein the parallel source current in admittance form is related to the series voltage in impedance form by

$$j_{rs} = -Y_{rs}\, e_{rs}$$

Also $$Y_{rs} = \frac{1}{Z_{rs}}$$

A set of unconnected element is defined as a primitive n/w.

The performance eq. of a primitive n/w are given below:

in impedance form

$$V+E = ZI \qquad — (3)$$

$$I+J = YV \qquad — (4)$$

here V & I are element current and vectors respectively J and E are source vectors. Z and Y are reffered to as

Scanned with CamScanner

Primitive impedance and admittance matrices, respectively.

and $z = y^{-1}$

Bus incidence matrix $\Rightarrow$

for fig 2 (a), we obtain following relation b/w nine element voltages and 4 bus voltages

$$\begin{aligned} V_{b1} &= V_1 & V_{15} &= V_3 - V_4 \\ V_{b2} &= V_2 & V_{16} &= V_3 - V_2 \\ V_{b3} &= V_3 & V_{17} &= V_1 - V_2 \\ V_{b4} &= V_4 & V_{18} &= V_4 - V_2 \\ & & V_{19} &= V_3 - V_1 \end{aligned} \quad -(5)$$

or in matrix form

$$V = A\, V_{bus} \quad -(6)$$

where Bus incidence matrix A is

$$\begin{array}{c} \text{bus} \\ e \\ 1 \\ 2 \\ 3 \\ 4 \\ 5 \\ 6 \\ 7 \\ 8 \\ 9 \end{array} \left[\begin{array}{cccc} 1 & 2 & 3 & 4 \\ \hline 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ \hline 0 & 0 & 1 & -1 \\ 0 & -1 & 1 & 0 \\ 1 & -1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ -1 & 0 & 1 & 0 \end{array}\right] \quad -(7)$$

This matrix is rectangular and therefore singular.

it element a_{ik} are :

a_{ik} = 1 if ith element is incident to and oriented away from the kth node

= -1 if ith element is incident to but oriented towards the kth node

= 0 if the ith element is not incident to the kth node.

Substituting eq. (6) into (4) we get

$$I + J = Y A V_{bus} \quad -(8)$$

Premultiplying by A^T

$$A^T I + A^T J = A^T Y A V_{bus} \quad -(9)$$

each component of n-dimentional vector $A^T I$ is algebric sum of the element currents leaving nodes 1, 2, ---n.

therefore, the application of KCL must results in

$$A^T I = 0 \quad -(10)$$

Similarly each component of vector $A^T J$ can be recognized as algebric sum of all source currents injected into the nodes 1, 2, -- n

$$A^T J = J_{bus} \quad -(11)$$

eq. (9) then is simplified to $J_{bus} = A^T Y A V_{bus}$ -(12)

Scanned with CamScanner

The bus admittance matrix can then be obtained from singular transformation of the Primitive Y

ie $$Y_{Bus} = A^T Y A \quad \text{—} \quad (13)$$

Scanned with CamScanner

☆ Gauss - Seidel method ⇒

The Gauss - Seidel method is an iterative algorithm for solving a set of non-linear algebraic equations. To start with, a solution vector is ~~required~~ assumed, based on guidance from practical experience in physical situation. One of the equations is then used to obtain the revised value of a particular variable by substituting in it the present values of the remaining variables.
The solution vector is immediately updated in respect of this variable. The process is then repeated for all the variables thereby completing one iteration. The iterative process is then repeated till the solution vector converges with in prescribed accuracy.

Let it be assumed that all buses other than the slack bus are PQ buses.
The slack bus voltage being specified, there are (n-1) bus voltages starting values of whose magnitudes and angles are assumed. These values are then updated through an iterative process.

from eq. $P_i - jQ_i = V_i^* I_i$

$$I_i = \frac{P_i - jQ_i}{V_i^*} \quad —①$$

and from eq. $I_i = \sum_{k=1}^{n} Y_{ik} V_k$

$$V_i = \frac{1}{Y_{ii}} \left[I_i - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k \right] \quad —②$$

Scanned with CamScanner

Substituting ~~I_i~~ I_i from eq. (1) into (2)

$$V_i = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{V_i^*} - \sum_{\substack{k=1 \\ k \neq i}}^{n} Y_{ik} V_k\right] ; i = 2, 3, \ldots n \qquad -(3)$$

The Voltage substituted in the right hand side of eq. (3) are the most recently calculated (updated values for the corresponding buses.

During each iteration voltages at buses $i = 2, 3, 4 \ldots n$ are sequentially updated through use of eq. (3). V_1, the slack bus voltage being fixed is not required to be updated.

iterations are repeated till no bus voltage magnitude changes by more than a Prescribed value during an iteration. The Computation Process is then said to converge to a solution.

Algorithm for load flow solution =>

Consider the case where all buses other than the slack bus are PQ buses.

The steps of a computational algorithm are given below:

(1) With the load Profile known at each bus (i.e P_{Di} and Q_{Di} known), P_{Gi} and Q_{Gi} to all generating stations. While active and reactive allocated to the slack bus, these are permitted to vary during iterative computation. This is necessary as voltage magnitude and angle are specified at this bus (only two variables can be specified at any bus).

Scanned with CamScanner

~~with~~

with this step, bus injections $(P_i + jQ_i)$ are known at all buses other than the slack bus.

(2) Assembly of Bus admittance matrix Y_{Bus}:

with the line and shunt admittance data stored in the computer, Y_{Bus} is assembled by using the rule for self and mutual admittances.

(3) Iterative computation of bus voltages (V_i, $i = 2, 3, \dots n$):

To start the iterations a set of initial voltage values is assumed. {Since in power system the voltage spread is not too wide, it is normal practice to use a flat voltage start, i.e initially all voltages are set equal to $(1 + j0)$ except the voltage of the slack bus which is fixed.}

→ $(n-1)$ equations — (3) in complex numbers are to be solved iteratively for finding $(n-1)$ complex voltages $V_2, V_3, \dots V_n$.

Define

$$A_i = \frac{P_i - jQ_i}{Y_{ii}}, \quad i = 2, 3, \dots n \quad — (4)$$

$$B_{ik} = \frac{Y_{ik}}{Y_{ii}}, \quad i = 2, 3, \dots n \quad — (5)$$

$$k = 1, 2, \dots n$$
$$k \neq i$$

Now for the $(r+1)^{th}$ iteration, the ~~at~~ Voltage eq. (3) becomes

$$V_i^{(r+1)} = \frac{A_i}{(V_i^r)^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} , i=2,3,--n \quad — (6)$$

The iterative Process is continued till the change in magnitude of bus voltage, $|\Delta V_i^{(r+1)}|$, b/w two consecutive iterations is less than a certain tolerance for all bus voltages i.e.

$$|\Delta V_i^{(r+1)}| = |V_i^{(r+1)} - V_i^{(r)}| < \epsilon \quad ; \quad i=2,3,---n \quad — (7)$$

(4) Computation of Slack bus Power:

Substitution of all bus voltages computed in step 3 along with V_1 in eq. $S_1^* = P_1 - jQ_1$

(5) Computation of line flows:

This is the last step in the load flow analysis wherein the Power flows on various lines of N/w are computed.

Consider the line connecting buses i and k.

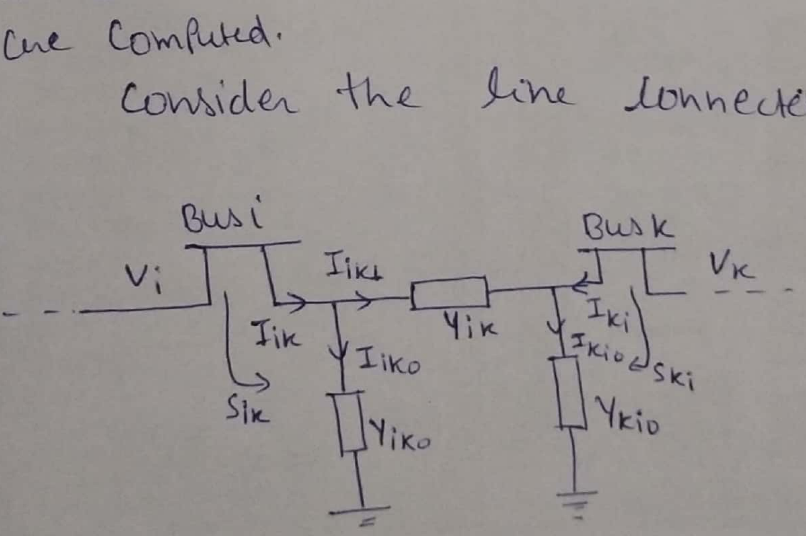


fig. π - representation of a line and transformer connected b/w two buses.

The current fed by bus i into line

$$I_{ik} = I_{ik1} + I_{ik0} = (V_i - V_k) Y_{ik} + V_i Y_{ik0} \quad — (8)$$

The Power fed into the line from bus i is

$$S_{ik} = P_{ik} + jQ_{ik} = V_i I_{ik}^*$$

$$= V_i (V_i^* - V_k^*) Y_{ik}^* + V_i V_i^* Y_{ik0}^* \quad — (9)$$

Similarly Power fed into the line from bus k is

$$S_{ki} = V_k (V_k^* - V_i^*) Y_{ik}^* + V_k V_k^* Y_{k0}^* \quad — (10)$$

The Power loss in the $(i-k)$th line is the sum of the Power flows determined from eq. (9) and (10).

Total transmission losses can be computed by summing all line flows i.e ($S_{ik} + S_{ki}$ for all i, k)

It may be noted the slack bus Power can also be found by assuming the flows on the lines terminating at the slack bus.

Acceleration of Convergence —

Convergence in the GS method can sometimes be speeded up by the use of the acceleration factor. For ith bus, the accelerated value of voltage at the $(r+1)^{th}$ iteration is given by

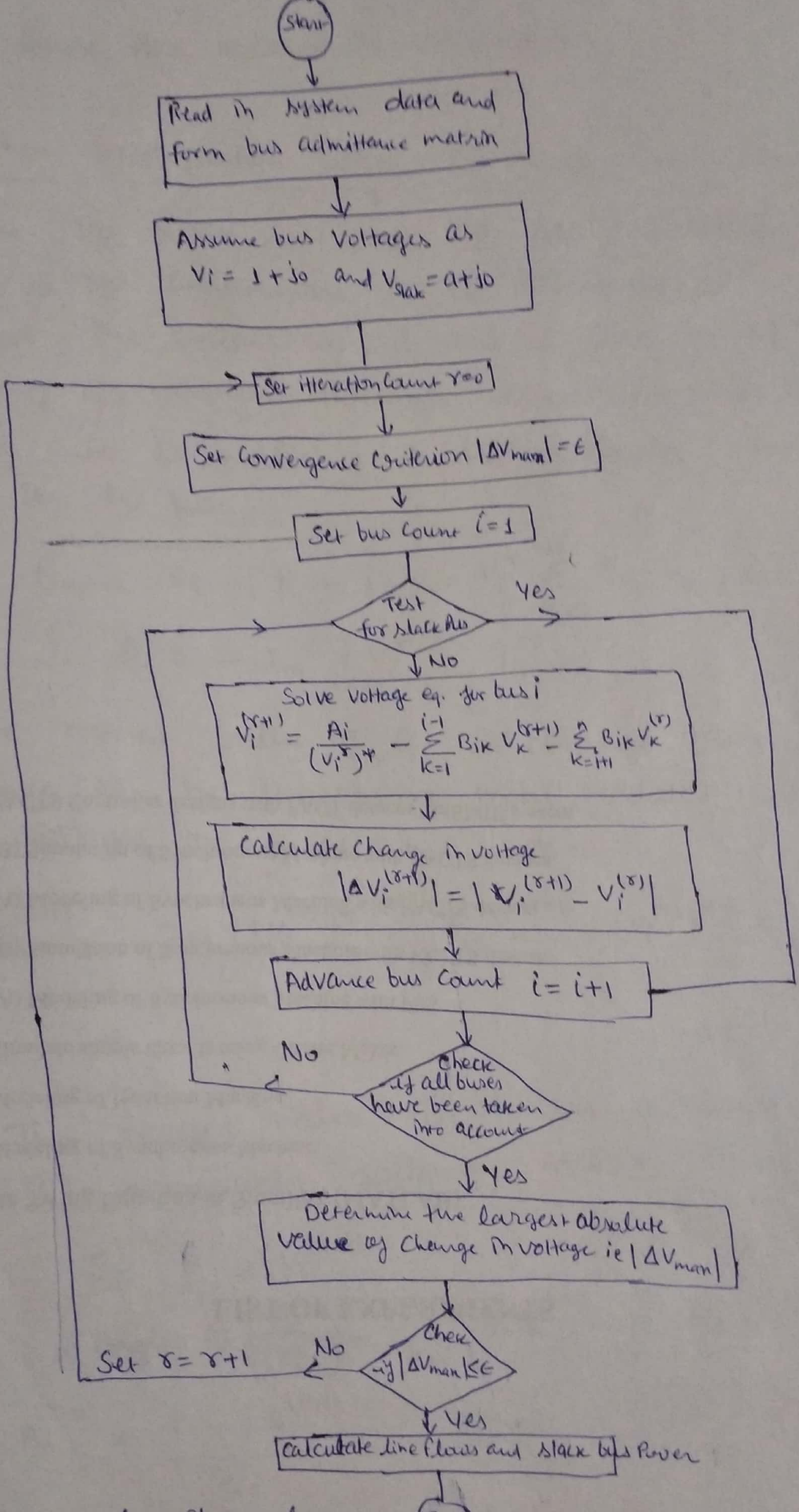
$$V_i^{(r+1)} (\text{accelerated}) = V_i^{(r)} + \alpha \left(V_i^{(r+1)} - V_i^{(r)}\right)$$

where α = real number = acceleration factor — (11)

→ Suitable value of α for any system can be obtained by trail load flow studies.

→ Generally recommended value of $\alpha = 1.6$

Scanned with CamScanner



flow chart for load flow solution using Gauss method (Load bus system only)

Scanned with CamScanner

→ A wrong choice of α may indeed slow down convergence or even cause the method to diverge.

✱ Algorithm modification when PV buses are also present

At the PV buses, P and $|V|$ are specified and Q and δ are unknowns to be determined.
Therefore the values of Q and δ are to be updated in every GS iteration through appropriate bus eq.

This is accomplished in the following steps for the ith PV bus.

(1) from eq. $P_i - jQ_i = V_i^* \sum_{k=1}^{n} Y_{ik} V_k$, $i = 1, 2, \ldots n$

$$Q_i = -\operatorname{Im}\left\{ V_i^* \sum_{k=1}^{n} Y_{ik} V_k \right\}$$

The revised value of Q_i is obtained from the above eq. by substituting most updated values of voltages on right hand side.

$$Q_i^{(r+1)} = -\operatorname{Im}\left\{ (V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik} V_k^{(r+1)} + (V_i^{(r)})^* \sum_{i=1}^{n} Y_{ik} V_k^{(r)} \right\} \quad \text{— (12)}$$

(2) The revised value of δ_i is obtained from eq. (6) immediately following step 1. Thus

$$\delta_i^{(r+1)} = \angle V_i^{(r+1)}$$

$$= \text{Angle of} \left[\frac{A_i}{(V_i^{(r)})^*} - \sum_{k=1}^{i-1} B_{ik} V_k^{(r+1)} - \sum_{k=i+1}^{n} B_{ik} V_k^{(r)} \right] \quad \text{— (13)}$$

where $A_i^{(r+1)} = \dfrac{P_i - jQ_i^{(r+1)}}{Y_{ii}}$ — (14)

The algorithm for PQ buses remains unchanged.

Scanned with CamScanner

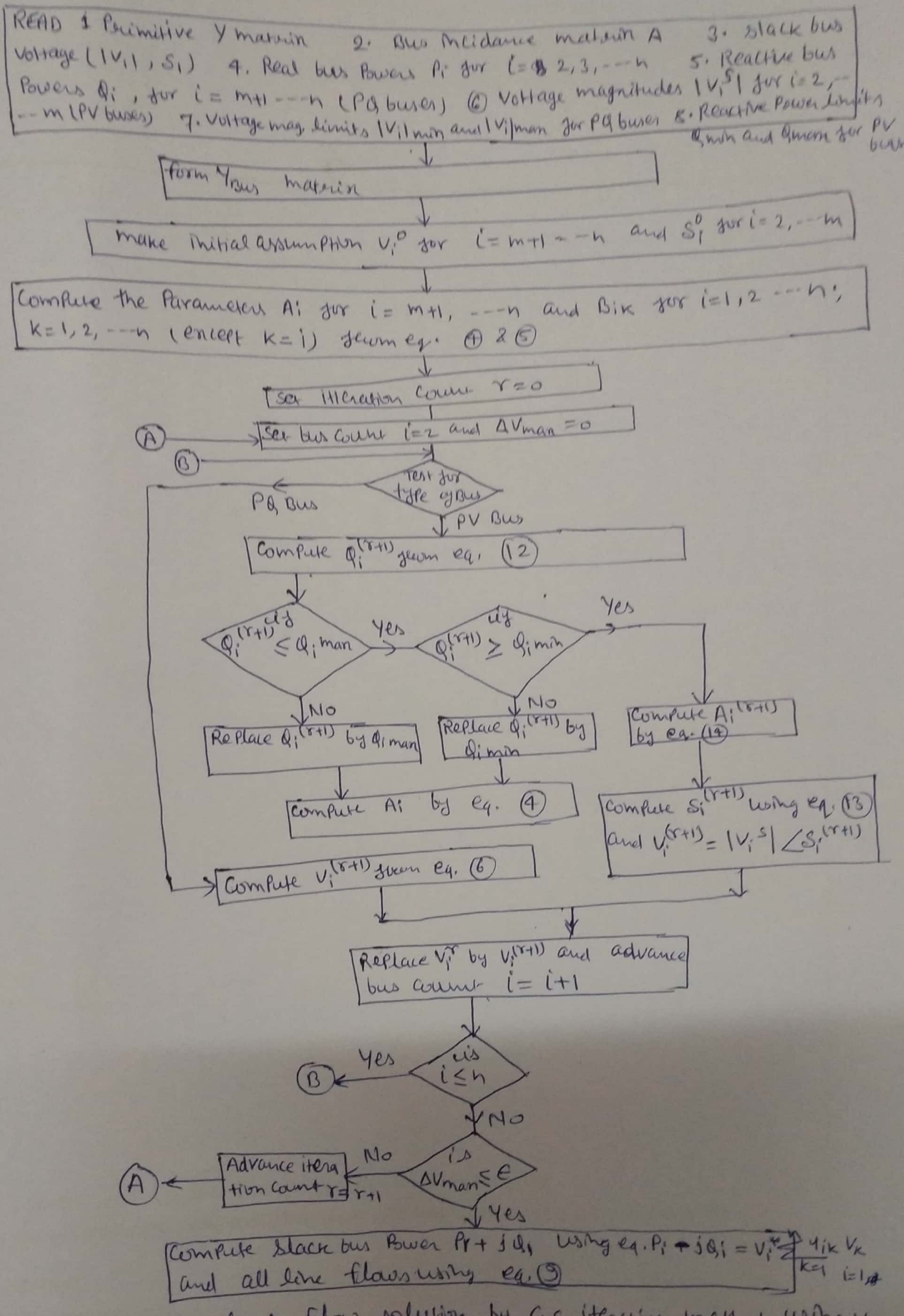
→ Q must be in the range $Q_{min} \rightarrow Q_{max}$.

③ if $Q_i^{(r+1)} < Q_{i,min}$, set $Q_i^{(r+1)} = Q_{i,min}$ and treat bus i as PQ bus. Compute $A_i^{(r+1)}$ and $V_i^{(r+1)}$ from eq. (14) and (6) respectively.

if $Q_i^{(r+1)} > Q_{i,max}$, set $Q_i^{(r+1)} = Q_{i\,max}$ and treat bus i as a PQ bus. Compute $A_i^{(r+1)}$ and $V_i^{(r+1)}$ from eq. (14) and (6) respectively.

→ All the computational steps are summerized in the detailed flow chart

it is assumed that out of n buses, the first bus is slack bus as usual, then 2, 3, --- m are PV buses and the remaining m+1, --- --- n are PQ buses.



fig= Flow chart for load flow solution by GS iterative method using Y_{Bus}

Q.1 The following is the system data for a load flow solution:

The line admittances:

Bus Code	Admittance
1-2	2 - j8.0
1-3	1 - j4.0
2-3	0.666 - j 2.664
2-4	1 - j4.0
3-4	2 - j8.0

The Schedule of active and reactive Powers:

Bus Code	P	Q	V	Remarks
1	—	—	1.06	Slack
2	0.5	0.2	1 + j0.0	PQ
3	0.4	0.3	1 + j0.0	PQ
4	0.3	0.1	1 + j0.0	PQ

Determine the Voltage at the end of first iteration using Gauss-Seidal method. Take $\alpha = 1.6$.

Solution - The admittance matrix will be given below:

$$Y = \begin{bmatrix} 3 - j12 & -2 + j8 & -1 + j4 & 0 \\ -2 + j8 & 3.666 - j14.664 & -0.666 + j2.664 & -1 + j4 \\ -1 + j4 & -0.666 + j2.664 & 3.666 - j14.664 & -2 + j8 \\ 0.0 & -1 + j4 & -2 + j8 & 3 - j12 \end{bmatrix}$$

Scanned with CamScanner

The Powers for load buses are to be taken as negative and that for generator buses are Positive.

$$V_i^{(r+1)} = \frac{1}{Y_{ii}}\left[\frac{P_i - jQ_i}{(V_i^*)^r} - \sum_{\substack{k=1 \\ k\neq i}}^{n} Y_{ik} V_k^r\right] ; \; i = 2, 3, \cdots n$$

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{(V_2^*)^0} - Y_{21}V_1^0 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{(3.666 - j14.664)}\left[\frac{0.5 + j0.2}{1 - j0} - (-2+j8)1.06 - 1.0(-0.666 + j2.664) - 1.0(-1+j4)\right]$$

$$= (1.01187 - j0.02888)$$

$$V_i^{(r+1)} \text{(accelerated)} = V_i^{(r)} + \alpha(V_i^{(r+1)} - V_i^{(r)})$$

$$V_{2\,acc}^1 = (1+j0) + 1.6(1.01187 - j0.02888 - 1 - j0)$$

$$= 1.01899 - j0.046208$$

Ans

Similarly,

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^*} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{(3.666 - j14.664)}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1+j4.0)1.06 - (-0.666 + j2.664)(1.01899 - j0.046208) - (-2+j8)(1+j0)\right]$$

$$= 0.994119 - j0.029248$$

$$V_{3\,acc}^1 = (1+j0.0) + 1.6\left[0.994119 - j0.029248 - 1 - j0.0\right]$$

$$= 0.99059 - j0.0467968 \text{ Ans}$$

Scanned with CamScanner

$$V_4^1 = \frac{1}{Y_{44}}\left[\frac{P_4 - jQ_4}{V_4^{0*}} - Y_{42}V_2^1 - Y_{43}V_3^1\right]$$

$$= \frac{1}{(3-j12)}\left[\frac{-0.3+j0.1}{1-j0.0} - (-1+j4.0)(1.01835 - j0.046208) - (-2+j8)(0.99059 - j0.0467968)\right]$$

$$= 0.9716032 - j0.064684$$

$$V_{4\,acc}^1 = 1.0 + j0.0 + 1.6\left[0.9716032 - j0.064684 - 1 - j0.0\right]$$

$$= 0.954565 - j0.1034944 \;\; \underline{\text{Ans}}$$

__Q.2__ if in Q.1, bus 2 is taken as a generator bus with $|V_2| = 1.04$ and reactive power constraints is

$$0.1 \leq Q_2 \leq 1.0$$

Determine the voltages starting with a flat-voltage profile and assuming accelating factor as 1.0.

__Solution__ ⇒ Since bus 2 is taken as a generator bus Q_2 is not specified and $P_2 = 0.5$

To find V_2^1 we first find Q_2 with $V_2 = 1.04 + j0$ as the phase angle of the voltage is 0.0 to begin with

$$Q_i^{(r+1)} = -\text{Im}\left\{(V_i^{(r)})^* \sum_{k=1}^{i-1} Y_{ik}V_k^{(r+1)} + (V_i^{(r)})^* \sum_{k=i}^{n} Y_{ik}V_k^{(r)}\right\}$$

Scanned with CamScanner

$$Q_2^1 = -I_m\left[V_2^{0*}\left[Y_{21}V_1 + Y_{22}V_2 + Y_{23}V_3 + Y_{24}V_4\right]\right]$$

$$= -I_m\Big[(1.04 - j0)\big[(-2+j8)(1.06) + (3.666 - j14.664)(1.04)$$

$$+ (-0.666 + j2.664)(1 + j0) + (-1+j4)(1+j0)\big]\Big]$$

$$= 0.1108$$

Since Q_2 lies within the limits, therefore V_2 will taken as $|V_2|_{spec}$ and phase angle as in this iteration,

$$V_2 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

bus 2 being a generator bus, P_2 and Q_2 are to be taken as Positive and value of P_2 as the specified and Q_2 as the once calculated above, i.e $Q_2 = 0.1108$

$$V_2^1 = \frac{1}{(3.666 - j14.664)}\left[\frac{0.5 - j0.1108}{1.04 - j0.0} - (-2+j8)1.06 - (-0.666 + j2.664)1.0 - (-1+j4)1.0\right]$$

$$V_2^1 = 1.0472846 + j0.0291476$$

$$\delta = 1.59^\circ$$

and $V_2^1 = 1.04\angle 1.59^\circ = 1.0395985 + j0.0288158$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^{0*}} - Y_{31}V_1 - Y_{32}V_2^1 - Y_{34}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1+j4)1.06 - (-0.666 + j2.664)(1.0395985 + j0.0288158) - (-2+j8)(1+j0)\right]$$

Scanned with CamScanner

$= 0.9978866 - j0.015607057$

Similarly V_4^1 can be obtained

$$V_4^1 = 0.998065 - j0.022336$$

Ans

Q.3 if the reactive Power Constraint on generator 2 (in Q.2) is $-0.2 \leq Q_2 \leq 1.0$
Solve the Previous example for voltage at the end of first iteration.

Solution ⇒ Since Q_2 calculated corresponding to initial guess $V_2 = 1.04 + j0.0$ is 0.1108 PU. which is less than the min. specified, the reactive Power generation for bus 2 is fixed at 0.2, the lower limit.
then The bus is treated as the load bus for this iteration., Voltage V_2 will be $V_2^0 = 1 + j0$ as for all other load buses for this iteration.
it is noted that the generator bus to be treated as load bus means - specified quantities are P and Q and unknown are $|V|$ and δ

$$V_2^1 = \frac{1}{Y_{22}}\left[\frac{P_2 - jQ_2}{V_2^*} - Y_{21}V_1 - Y_{23}V_3^0 - Y_{24}V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{0.5 + j0.2}{1 - j0} - (-2 + j8)1.06 - (-0.666 + j0.2664)1.0 - (-1 + j4)1.0\right]$$

Scanned with CamScanner

$$= 1.098221 + j0.030105662$$

$$V_3^1 = \frac{1}{Y_{33}}\left[\frac{P_3 - jQ_3}{V_3^0} - Y_{31}V_1 - Y_{32}V_2^1 - 34\,V_4^0\right]$$

$$= \frac{1}{3.666 - j14.664}\left[\frac{-0.4 + j0.3}{1 - j0.0} - (-1 + j4)1.06 - (-0.666 + j2.664)(1.098221 + j0.030105662) - (-2 + j8)(1 + j0.0)\right]$$

Similarly V_4^1 can be evaluated.

Scanned with CamScanner

Newton-Raphson (NR) method ⇒

The Newton-Raphson method is a Powerful method of solving non-linear algebric equation. It works faster and is sure to converge in most cases as compared to the GS method. It is indeed the Practical method of load flow solution of large Power networks.

Its only drawback is the large requirement of computer memory which has been overcome through a compact storage scheme.

Convergence can be considerably speeded up by performing the first iteration through the GS method and using the values so obtained for starting the NR iteration.

First review the method in it's general form and then it will applied to solve the load flow solution Problem,

Consider a set of n non-linear algebric equations -

$$f_i(x_1, x_2, ----, x_n) = 0 \; ; \quad i = 1, 2, 3, ---n. \quad \text{--- (1)}$$

Assume initial values of unknowns as $x_1^0, x_2^0, --- x_n^0$.

Let $\Delta x_1^0, \Delta x_2^0, --- \Delta x_n^0$ be the corrections, which on being added to the initial guess, give the actual solution.

Therefore -

$$f_i(x_1^0 + \Delta x_1^0, x_2^0 + \Delta x_2^0, ----, x_n^0 + \Delta x_n^0) = 0 \; ; \quad i = 1, 2, --- n \quad \text{--- (2)}$$

expanding these equations in taylor series around the initial guess, we have -

$$f_i(x_1^0, x_2^0, --- x_n^0) + \left[\left(\frac{\partial f_i}{\partial x_1}\right)^0 \Delta x_1^0 + \left(\frac{\partial f_i}{\partial x_2}\right)^0 \Delta x_2^0 + --- + \left(\frac{\partial f_i}{\partial x_n}\right)^0 \Delta x_n^0 \right] + \text{higher order terms} = 0 \quad — (3)$$

where $\left(\frac{\partial f_i}{\partial x_1}\right)^0, \left(\frac{\partial f_i}{\partial x_2}\right)^0, ----, \left(\frac{\partial f_i}{\partial x_n}\right)^0$ are the deriva-tives of f_i with respect to $x_1, x_2, --- x_n$ evaluated at $(x_1^0, x_2^0, --- x_n^0)$.

Neglecting higher order terms we can write the above eq. in matrix form as

$$\begin{bmatrix} f_1^0 \\ f_2^0 \\ \vdots \\ f_n^0 \end{bmatrix} + \begin{bmatrix} \left(\frac{\partial f_1}{\partial x_1}\right)^0 & \left(\frac{\partial f_1}{\partial x_2}\right)^0 & --- & \left(\frac{\partial f_1}{\partial x_n}\right)^0 \\ \left(\frac{\partial f_2}{\partial x_1}\right)^0 & \left(\frac{\partial f_2}{\partial x_2}\right)^0 & --- & \left(\frac{\partial f_2}{\partial x_n}\right)^0 \\ \vdots & \vdots & \vdots & \vdots \\ \left(\frac{\partial f_n}{\partial x_1}\right)^0 & \left(\frac{\partial f_n}{\partial x_2}\right)^0 & --- & \left(\frac{\partial f_n}{\partial x_n}\right)^0 \end{bmatrix} \begin{bmatrix} \Delta x_1^0 \\ \Delta x_2^0 \\ \vdots \\ \Delta x_n^0 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ \vdots \\ 0 \end{bmatrix} \quad — (4.a)$$

or in vector matrix form

$$f^0 + J^0 \Delta x^0 \simeq 0 \quad — (4.b)$$

J^0 is known as Jacobian matrix (obtained by differentiating the function vector f with respect to x and evaluating it at x^0)

$$f^0 = [-J^0] \Delta x^0 \quad — (5)$$

approximate values of Δx can be obtained from eq. (5) updated values of x are then

$$x^1 = \cancel{x_0}\, x^0 + \Delta x^0$$

or in general, for the $(r+1)^{th}$ iteration

$$x^{(r+1)} = x^{(r)} + \Delta x^{(r)} \quad — (6)$$

Scanned with CamScanner

iterations are continued till eq. (1) is satisfied to any desired accuracy, i.e

$$|f_i(x^r)| < \varepsilon \text{ (a specified value)}; \; i = 1, 2, \ldots n \quad \text{— (7)}$$

~~NR algro~~

NR Algorithm for load flow solution:-

First assume that all buses are PQ buse. At any PQ bus the load flow solution must satisfy the following non-linear algebric eq.

$$f_{iP}(|V|, \delta) = P_i \text{(specified)} - P_i = 0 \quad \text{— (8)(a)}$$

$$f_{iQ}(|V|, \delta) = Q_i \text{(specified)} - Q_i = 0 \quad \text{— (8)(b)}$$

for a trail set of variables $|V_i|$, δ_i the vector residuals f^0 of eq. (6) corresponds to

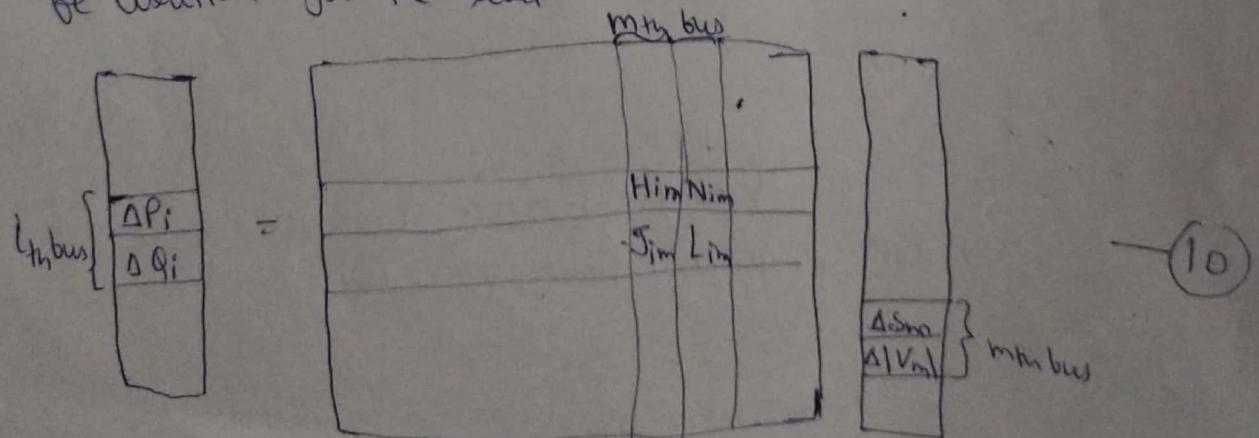
$$f_{iP} = P_i \text{(specified)} - P_i \text{(calculated)} = \Delta P_i \quad \text{— (9)(a)}$$

$$f_{iQ} = Q_i \text{(specified)} - Q_i \text{(calculated)} = \Delta Q_i \quad \text{— (9)(b)}$$

While the vector of correction Δx^0 corresponds to

$\Delta|V_i|$, $\Delta \delta_i$

eq. (6) for obtaining the approximate corrections vector can be written for the load flow case as-



— (10)

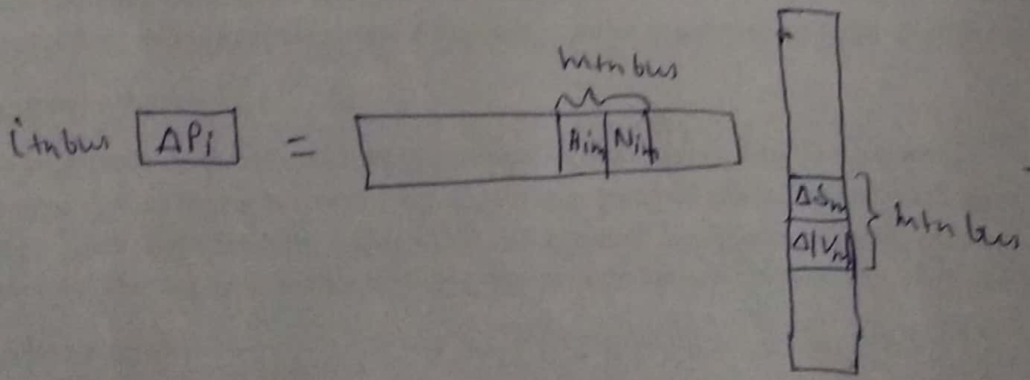
where $H_{im} = \frac{\partial P_i}{\partial \delta_m}$; $N_{im} = \frac{\partial P_i}{\partial |V_m|}$

$J_{im} = \frac{\partial Q_i}{\partial \delta_m}$; $L_{im} = \frac{\partial Q_i}{\partial |V_m|}$ — (11)

it is to be immediately observed that the Jacobian elements corresPonding to the ith bus residuals and mth bus corrections are a 2x2 matrin enclosed in box in eq. (10) where i and m are both PQ buses.

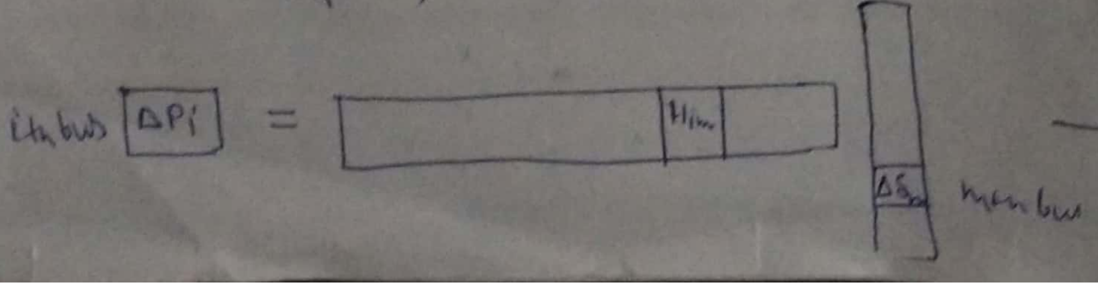
→ Since at slack bus (bus no.1), P_1 and Q_1 are unspecified and $|V_1|$, δ_1 are fined, there are no eq. corresPonding to eq. (8) at this bus. hence slack bus does not enter the Jacobian in eq. (10)

Consider now the Presence of PV buses. if the ith bus is a PV bus, Q_i is unspecified so that there is no eq. corresponds to eq. (b) for this bus therefore, The Jacobian element of the ith bus become a single row Pertaining to ΔP_i, i.e



— (10)(b)

if the mth bus is also a PV bus, $|V_m|$ becomes fined so that $\Delta|V_m| = 0$, we can now write



— (10)(c)

Scanned with CamScanner

Also if the ith bus is a PQ bus while the mth bus is a PV bus, we can then write

$$\begin{array}{c} \\ \text{ith bus} \end{array} \begin{bmatrix} \Delta P_i \\ \Delta Q_i \end{bmatrix} = \begin{array}{c} \text{mth bus} \\ \begin{bmatrix} \cdots & H_{im} & \cdots \\ \cdots & J_{im} & \cdots \end{bmatrix} \end{array} \begin{bmatrix} \vdots \\ \Delta\delta_m \\ \vdots \end{bmatrix} \begin{array}{c} \\ \text{mth bus} \end{array} \quad \text{— (10)(d)}$$

it is convenient for numerical solution to normalize the voltage corrections as

$$\frac{\Delta|V_m|}{|V_m|}$$

as a consequence of which, the corresponding Jacobian elements become

$$N_{im} = \frac{\partial P_i}{\partial |V_m|}|V_m| \qquad \text{— (11)(b)}$$

$$L_{im} = \frac{\partial Q_i}{\partial |V_m|}|V_m|$$

Expression for elements of the Jacobian (in normalized form) of the load flow equation (8)(1) & (b) are derived below,-

$$P_i - jQ_i = V_i^* \sum_{K=1}^{n} Y_{ik} V_K = |V_i| \exp(-j\delta_i) \sum_{K=1}^{n} |Y_{ik}| \exp(j\theta_{ik}) V_K \exp(j\delta_K) \quad (D_1)$$

differentiating partially with respect to δ_m $(m \neq i)$

$$\frac{\partial P_i}{\partial \delta_m} - j\frac{\partial Q_i}{\partial \delta_m} = j|V_i| \exp(-j\delta_i)\left(|Y_{im}| \exp(j\theta_{im})|V_m| \exp(j\delta_m)\right)$$

$$= j(e_i - jf_i)(a_m + jb_m) \quad \text{— (12)}$$

where $Y_{im} = G_{im} + jB_{im}$

$V_i = e_i + jf_i$

$(a_m + jb_m) = (G_{im} + jB_{im})(e_m + jf_m)$

$$\frac{\partial P_i}{\partial \delta_m} = (a_m f_i - b_m e_i) = H_{im} \qquad \frac{\partial Q_i}{\partial \delta_m} = -(a_m e_i + b_m f_i) = J_{im}$$

for case of $m = i$ we ~~can~~ have

$$\frac{\partial P_i}{\partial \delta_i} - j\frac{\partial Q_i}{\partial \delta_i} = -j|V_i|\exp(-j\delta_i)\sum_{k=1}^{n}|Y_{ik}|\exp(j\theta_{ik})|V_k|\exp(j\delta_k) + j|V_i|\exp(-j\delta_i)\left(|Y_{ii}|\exp(j\theta_{ii})|V_i|\exp(j\delta_i)\right)$$

$$= -j(P_i - jQ_i) + j|V_i|^2(G_{ii} + jB_{ii}) \quad - D_3$$

from D_3,

$$\frac{\partial P_i}{\partial \delta_i} = -Q_i - B_{ii}|V_i|^2 = H_{ii}$$

$$\frac{\partial Q_i}{\partial \delta_i} = P_i - G_{ii}|V_i|^2 = J_{ii}$$

differentiating D_1 with respect to $|V_m|$ $(m \neq i)$ we have

$$\frac{\partial P_i}{\partial |V_m|} - j\frac{\partial Q_i}{\partial |V_m|} = |V_i|\exp(-j\delta_i)\left(|Y_{im}|\exp(j\theta_{im})\exp(j\delta_m)\right)$$

multiplying $|V_m|$ on both sides

$$\frac{\partial P_i}{\partial |V_m|}|V_m| - j\frac{\partial Q_i}{\partial |V_m|}|V_m| = |V_i|\exp(-j\delta_i)|Y_{im}|\exp(j\theta_{im})|V_m|\exp(j\delta_m)$$

$$= (e_i - jf_i)(a_m + jb_m) \quad - D_4$$

it follows from eq. D_4 that

$$\frac{\partial P_i}{\partial |V_m|}|V_m| = a_m e_i + b_m f_i = N_{im}$$

$$\frac{\partial Q_i}{\partial |V_m|}|V_m| = a_m f_i - b_m e_i = L_{im}$$

Now for case $m = i$,

$$\frac{\partial P_i}{\partial |V_i|} - j\frac{\partial Q_i}{\partial |V_i|} = \exp(-j\delta_i)\sum_{k=1}^{n}|Y_{ik}|\exp(j\theta_{ik})|V_k|\exp(j\delta_k) + |V_i|\exp(-j\delta_i)|Y_{ii}|\exp(j\theta_{ii})\exp(j\delta_i)$$

multiplying both side by $|V_i|$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| - j\frac{\partial Q_i}{\partial |V_i|}|V_i| = |V_i|\exp(-j\delta_i)\sum_{k=1}^{n}|Y_{ik}|\exp(j\theta_{ik})|V_k|\exp(j\delta_k) + \cancel{2}|V_i|^2|Y_{ii}|\exp(j\theta_{ii})$$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| - j\frac{\partial Q_i}{\partial |V_i|}|V_i| = (P_i - jQ_i) + |V_i|^2(G_{ii} + jB_{ii}) \quad - D\text{-}5$$

$$\frac{\partial P_i}{\partial |V_i|}|V_i| = P_i + G_{ii}|V_i|^2 = N_{ii}$$

$$\frac{\partial Q_i}{\partial |V_i|}|V_i| = Q_i - jB_{ii}|V_i|^2 = L_{ii}$$

The above results are summerised below:-

Case1- $m \neq i$

$$H_{im} = L_{im} = a_m f_i - b_m e_i$$
$$N_{im} = -J_{im} = a_m e_i + b_m f_i \quad - D_6$$

Case 2- $m = i$

$$H_{ii} = -Q_i - B_{ii}|V_i|^2$$
$$N_{ii} = P_i + G_{ii}|V_i|^2 \quad - D_8$$
$$J_{ii} = P_i - G_{ii}|V_i|^2$$
$$L_{ii} = Q_i - B_{ii}|V_i|^2$$

Scanned with CamScanner

Case 1 - $m \neq i$

$$H_{im} = L_{im} = a_m f_i - b_m e_i$$
$$N_{im} = -J_{im} = a_m e_i + b_m f_i \qquad \text{—(12)}$$

where $Y_{im} = G_{im} + jB_{im}$ and $V_i = e_i + jf_i$

$$(a_m + jb_m) = (G_{im} + jB_{im})(e_m + jf_m)$$

Case 2 - $m = i$

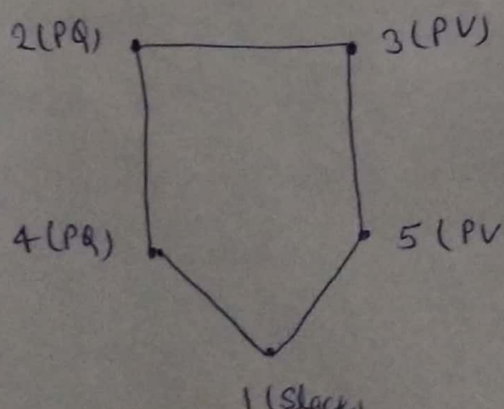
$$H_{ii} = -Q_i - B_{ii}|V_i|^2$$
$$N_{ii} = P_i + G_{ii}|V_i|^2 \qquad \text{—(13)}$$
$$J_{ii} = P_i - G_{ii}|V_i|^2$$
$$L_{ii} = Q_i - B_{ii}|V_i|^2$$

An important observation can be made in respect of the Jacobian by examination of the Ybus matrix. if bus i and m are not connected, $Y_{im} = 0$ ($G_{im} = B_{im} = 0$) hence from eq. (11) and (12), we can write

$$H_{im} = H_{mi} = 0$$
$$N_{im} = N_{mi} = 0$$
$$J_{im} = J_{mi} = 0$$
$$L_{im} = L_{mi} = 0$$

Thus Jacobian is as sparse as the Ybus matrix.

For example -



Sample 5 bus N/w

$$\begin{array}{c} 2 \\ \\ 3 \\ 4 \\ \\ \end{array}\begin{bmatrix} \cancel{\Delta P_2} \\ \cancel{\Delta Q_2} \\ \cancel{\Delta P_3} \\ \cancel{\Delta P_4} \\ \cancel{\Delta Q_4} \end{bmatrix} \qquad \begin{array}{c} 2 \\ \\ 3 \\ 4 \\ \\ 5 \end{array}\begin{bmatrix} \Delta P_2 \\ \Delta Q_2 \\ \Delta P_3 \\ \Delta P_4 \\ \Delta Q_4 \\ \Delta P_5 \end{bmatrix} = \begin{array}{c} \begin{array}{cccc} 2 & 3 & 4 & 5 \end{array} \\ \begin{bmatrix} H_{22} & N_{22} & H_{23} & H_{24} & N_{24} & \\ J_{22} & L_{22} & J_{23} & J_{24} & L_{24} & \\ H_{32} & N_{32} & H_{33} & & & H_{35} \\ H_{42} & N_{42} & & H_{44} & N_{44} & H_{45} \\ J_{42} & L_{42} & & J_{44} & L_{44} & J_{45} \\ & & H_{53} & H_{54} & N_{54} & H_{55} \end{bmatrix} \end{array} \begin{bmatrix} \Delta\delta_2 \\ \frac{\Delta |V_2|}{|V_2|} \\ \Delta\delta_3 \\ \Delta\delta_4 \\ \frac{\Delta |V_4|}{|V_4|} \\ \Delta\delta_5 \end{bmatrix} \quad -(14)$$

↑ residuals ↑ Jacobian ↑ correction in variables

Iteration Algorithm =>

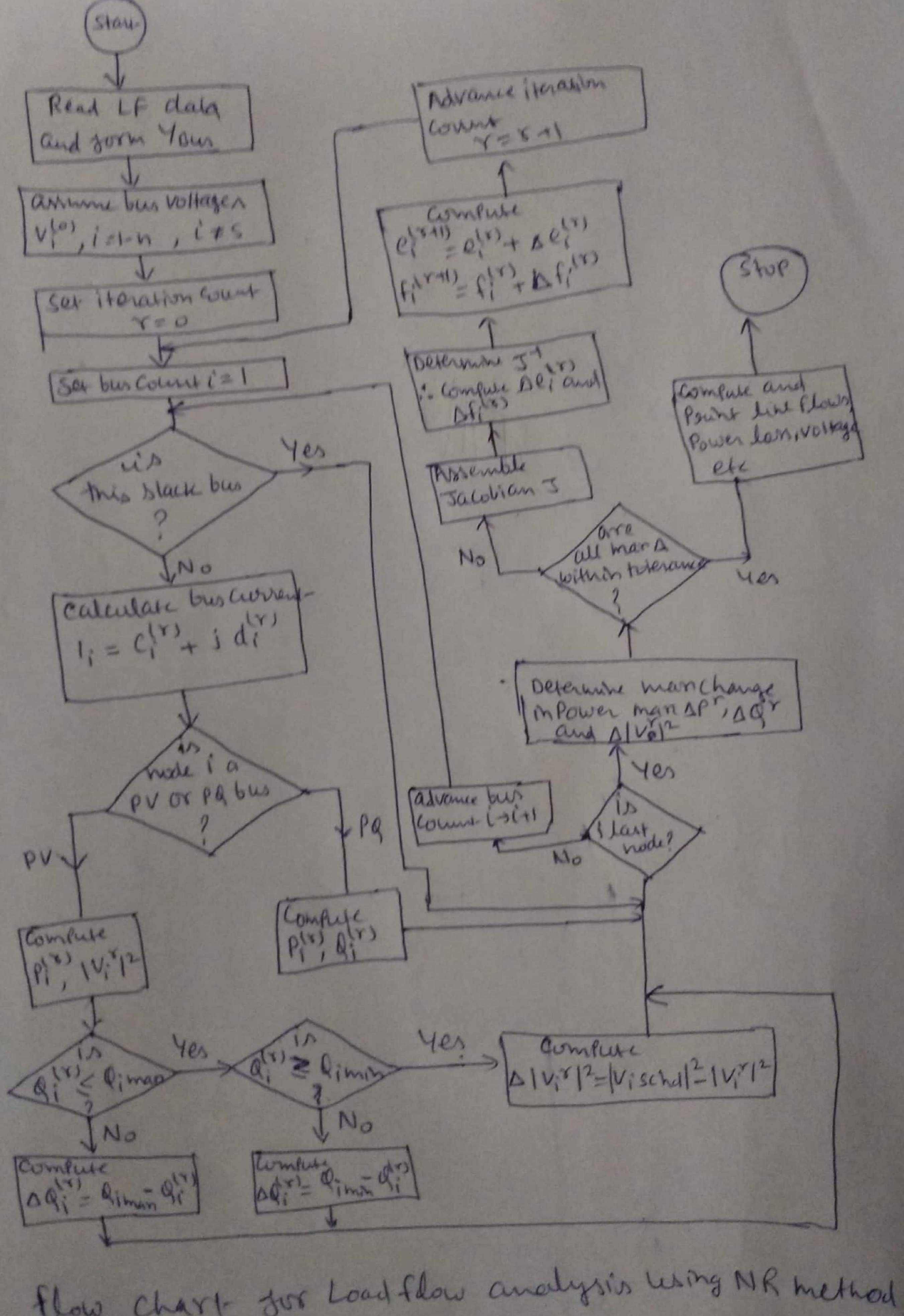
① With voltage and angle (usually $\delta = 0$) at slack bus fixed, assume $|V|$, δ at all PQ buses and δ at all PV buses in the absence of any other information flat voltage start is recommended.

② Compute ΔP_i (for PV and PQ buses) and ΔQ_i (for all PQ buses). if all the values are less than the prescribed tolerance, stop the iteration, calculate P_1 and Q_1 and print the entire solution including line flows.

③ if the convergence criteria is not satisfied, evaluate elements of the Jacobian using eq. (12) and (13)

④ Solve eq. (14) for corrections of voltage angles and magnitudes.

⑤ update voltage angles and magnitude by adding the corresponding changes to the previous values and return to step 2.



flow chart for Load flow analysis using NR method

Scanned with CamScanner

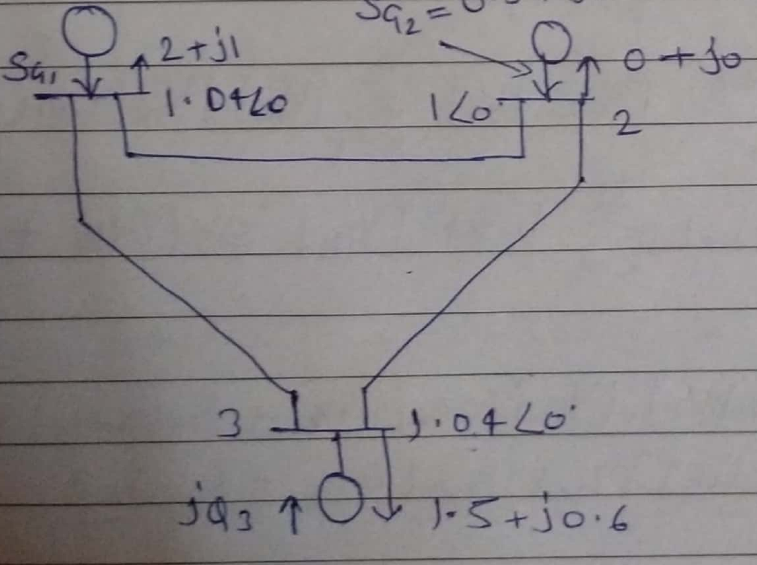
Page No.
Date

Q. Consider the 3 bus system in fig. each of the three lines has a series impedance of 0.02 + j0.08 Pu and a total shunt admittance of j0.02 Pu. The specified quantities at the buses are tabulated below:

Bus	Real load demand P_D	Reactive load demand Q_D	Real Power generation P_G	Reactive Power gen. Q_G	Voltage specification
1	2.0	1.0	unspecified	unspecified	1.04 + j0 (slack bus)
2	0.0	0.0	0.5	1.0	unspecified (PQ bus)
3	1.5	0.6	0.0	$Q_{G3} = ?$	$\lvert V \rvert = 1.04$ (PV bus)

Controllable reactive Power source is available at bus 3 with in constraint $0 \le Q_{G_3} \le 1.5$ PU

Find load flow solution using NR method. use a tolerance of 0.01 for Power mismatch.



Scanned with CamScanner

Page No.
Date

Solution:- Using nominal - π model for transmission lines, Y_{Bus} for the given system is obtained as follows

for each line

$$Y_{series} = \frac{1}{0.02 + j0.08} = 2.941 - j11.764$$
$$= 12.13 \angle -75.96^\circ$$

each off diagonal term $= -2.941 + j11.764$

each self term $= 2\left[(2.941 - j11.764) + j0.01\right]$
$$= 5.882 - j23.528 = 24.23\angle -75.9$$

$$Y_{Bus} = \begin{bmatrix} 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ & 12.13\angle 104.04^\circ \\ 12.13\angle 104.04^\circ & 24.23\angle -75.95^\circ & 12.13\angle 104.04^\circ \\ 12.19\angle 104.04^\circ & 12.13\angle 104.04^\circ & 24.23\angle -75.95 \end{bmatrix}$$

To start iteration Choose $V_2^\circ = 1 + j0$ and $\delta_3^\circ = 0$

from eq. $P_i = |V_i| \sum_{K=1}^{n} |V_K| |Y_{iK}| \cos(\theta_{iK} + \delta_K - \delta_i)$.

and $Q_i = -|V_i| \sum_{K=1}^{n} |V_K| |Y_{iK}| \sin(\theta_{iK} + \delta_K - \delta_i)$.

we get -

$P_2 = |V_2| |V_1| |Y_{21}| \cos(\theta_{21} + \delta_1 - \delta_2) + |V_2|^2 |Y_{22}| \cos\theta_{22} +$
$|V_2| |V_3| |Y_{23}| \cos(\theta_{23} + \delta_3 - \delta_2)$

Scanned with CamScanner

$$P_3 = |V_3||V_1||Y_{31}|\cos(\theta_{31} + \delta_1 - \delta_3) + |V_3||V_2||Y_{32}|\cos(\theta_{32} + \delta_2 - \delta_3) + |V_3|^2|Y_{33}|\cos\theta_{33}$$

$$Q_2 = -|V_2||V_1||Y_{21}|\sin(\theta_{21} + \delta_1 - \delta_2) - |V_2|^2|Y_{22}|\sin(\theta_{22}) - |V_2||V_3||Y_{23}|\sin(\theta_{23} + \delta_2 - \delta_3)$$

Substituting given and assumed values of different quantities we get-

$P_2^0 = -0.23$ PU

$P_3^0 = 0.12$ PU

$Q_2^0 = -0.96$ PU

Power residuals

$\Delta P_2^0 = P_2$ (specified) $- P_2^0$ (Calculated)

$= 0.5 - (-0.23) = 0.73$

$\Delta P_3^0 = -1.5 - (0.12) = -1.62$

$\Delta Q_2^0 = 1 - (-0.96) = 1.96$ PU

$$\begin{bmatrix} \Delta P_2 \\ \Delta P_3 \\ \Delta Q_2 \end{bmatrix} = \begin{bmatrix} \partial P_2/\partial\delta_2 & \partial P_2/\partial\delta_3 & \frac{\partial P_2}{\partial|V_2|} \\ \frac{\partial P_3}{\partial\delta_2} & \frac{\partial P_3}{\partial\delta_3} & \frac{\partial P_3}{\partial|V_2|} \\ \frac{\partial Q_2}{\partial\delta_2} & \frac{\partial Q_2}{\partial\delta_3} & \frac{\partial Q_2}{\partial|V_2|} \end{bmatrix} \begin{bmatrix} \Delta\delta_2 \\ \Delta\delta_3 \\ \Delta|V_2| \end{bmatrix}$$

$$\begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2^1| \end{bmatrix} = \begin{bmatrix} 24.47 & -12.23 & 5.64 \\ -12.23 & 24.95 & -3.05 \\ -6.11 & 3.05 & 22.54 \end{bmatrix}^{-1} \begin{bmatrix} 0.73 \\ -1.62 \\ 1.96 \end{bmatrix}$$

Scanned with CamScanner

Page No. ______

Date

$$\begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix}$$

$$\begin{bmatrix} \delta_2^1 \\ \delta_3^1 \\ |V_2|^1 \end{bmatrix} = \begin{bmatrix} \delta_2^0 \\ \delta_3^0 \\ |V_2|^0 \end{bmatrix} + \begin{bmatrix} \Delta\delta_2^1 \\ \Delta\delta_3^1 \\ \Delta|V_2|^1 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 1 \end{bmatrix} + \begin{bmatrix} -0.023 \\ -0.0654 \\ 0.089 \end{bmatrix} = \begin{bmatrix} -0.023 \\ -0.0654 \\ 1.089 \end{bmatrix}$$

$$Q_3^1 = 0.4677$$

$$Q_{G_3}^1 = Q_3^1 + Q_{D3} = 0.4677 + 0.6 = 1.0677$$

which is within limit

Scanned with CamScanner