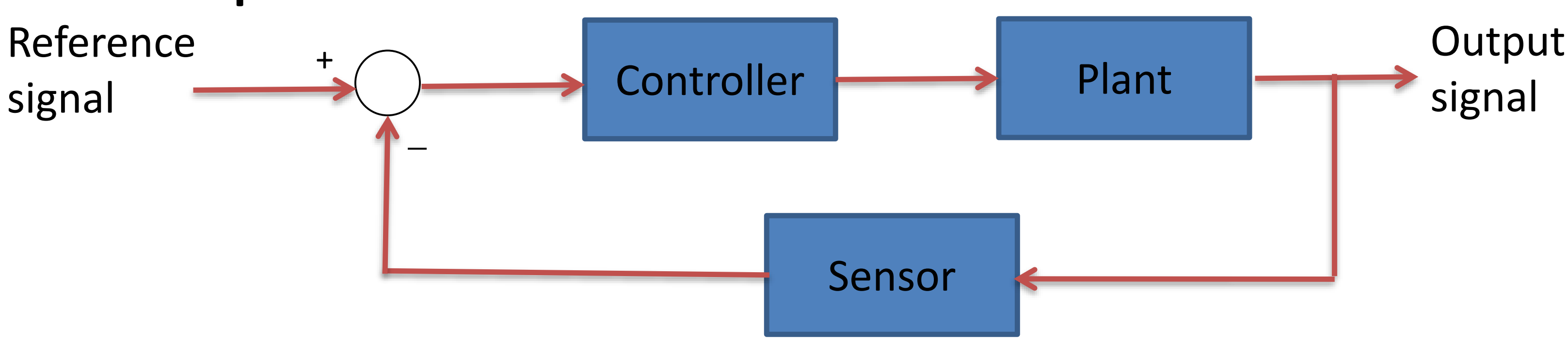
Outline

- Closed Loop Control of DC Drives
- Closed-loop Control with Controlled Rectifier – Two-quadrant
  - Transfer Functions of Subsystems
  - Design of Controllers
- Closed-loop Control with Field Weakening – Two-quadrant
- Closed-loop Control with Controlled Rectifier – Four-quadrant
- References

Closed Loop Control of DC Drives

- **Closed loop control** is when the firing angle is varied automatically by a controller to achieve a reference speed or torque
- This requires the use of sensors to feed back the actual motor speed and torque to be compared with the reference values

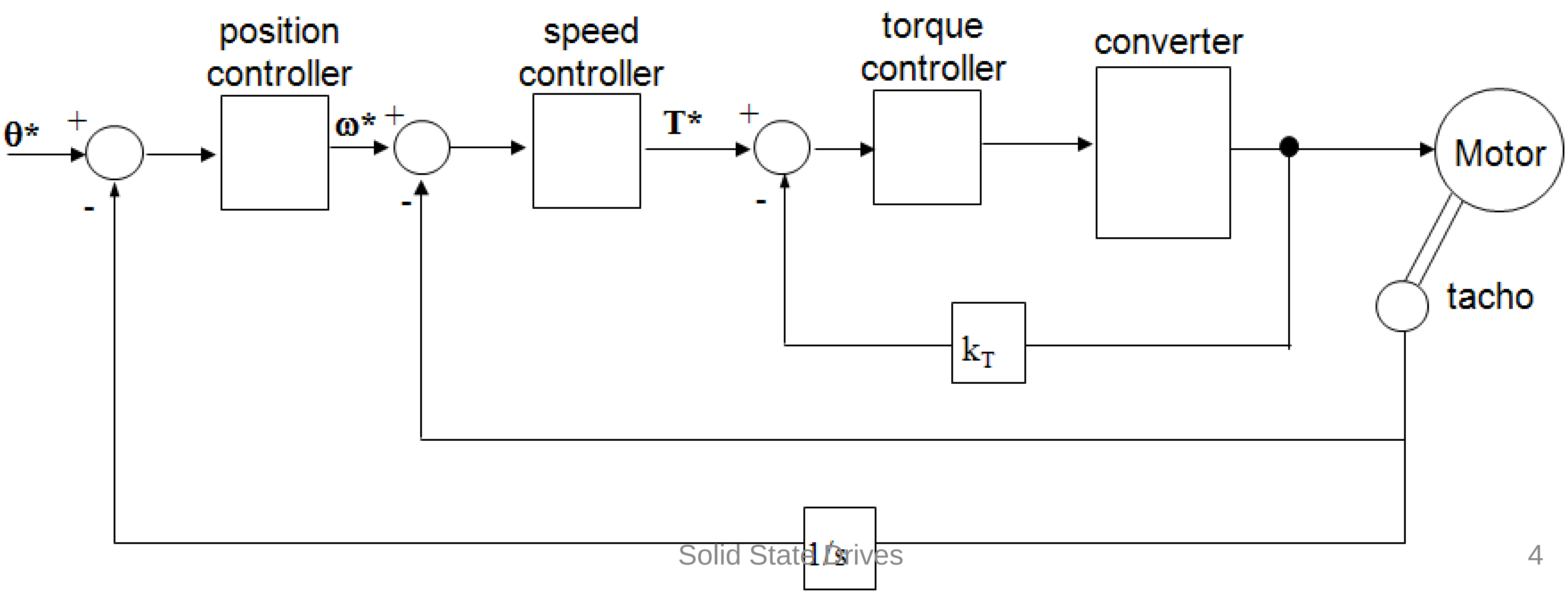


Closed Loop Control of DC Drives

- Feedback loops may be provided to satisfy one or more of the following:
  - Protection
  - Enhancement of response – fast response with small overshoot
  - Improve steady-state accuracy
- Variables to be controlled in drives:
  - Torque – achieved by controlling current
  - Speed
  - Position

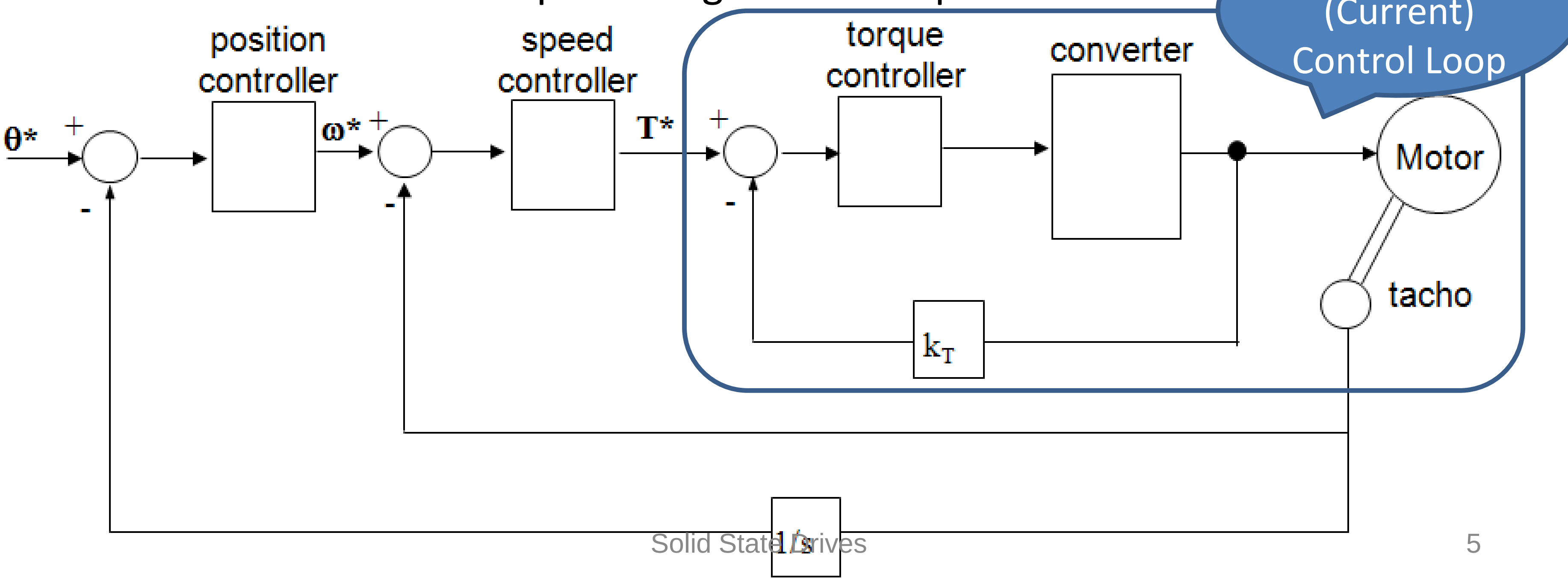
Closed Loop Control of DC Drives

- Cascade control structure
  - Flexible – outer loops can be added/removed depending on control requirements.
  - Control variable of inner loop (eg: speed, torque) can be limited by limiting its reference value
  - Torque loop is fastest, speed loop – slower and position loop - slowest



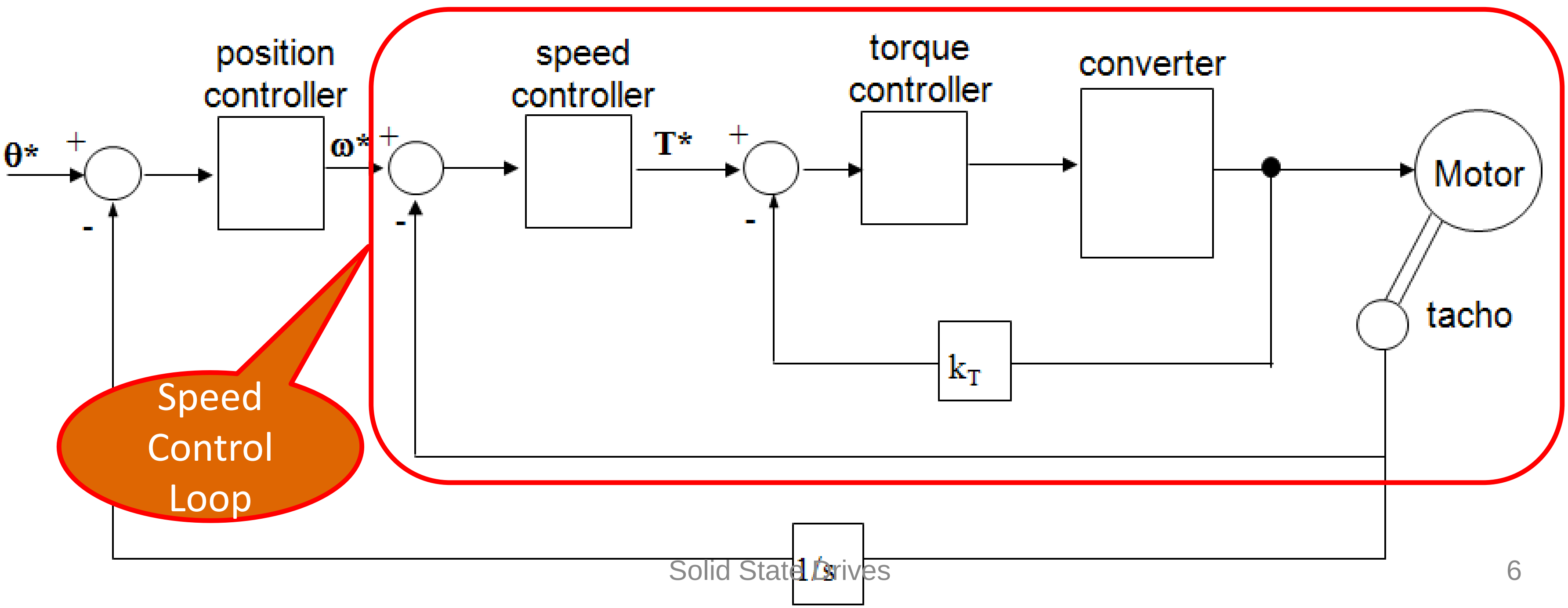
Closed Loop Control of DC Drives

- Cascade control structure:
  - <u>Inner Torque (Current) Control Loop:</u>
    - **Current control loop** is used to **control torque via armature current (i_a)** and **maintains current within a safe limit**
    - Accelerates and decelerates the drive at maximum permissible current and torque during transient operations



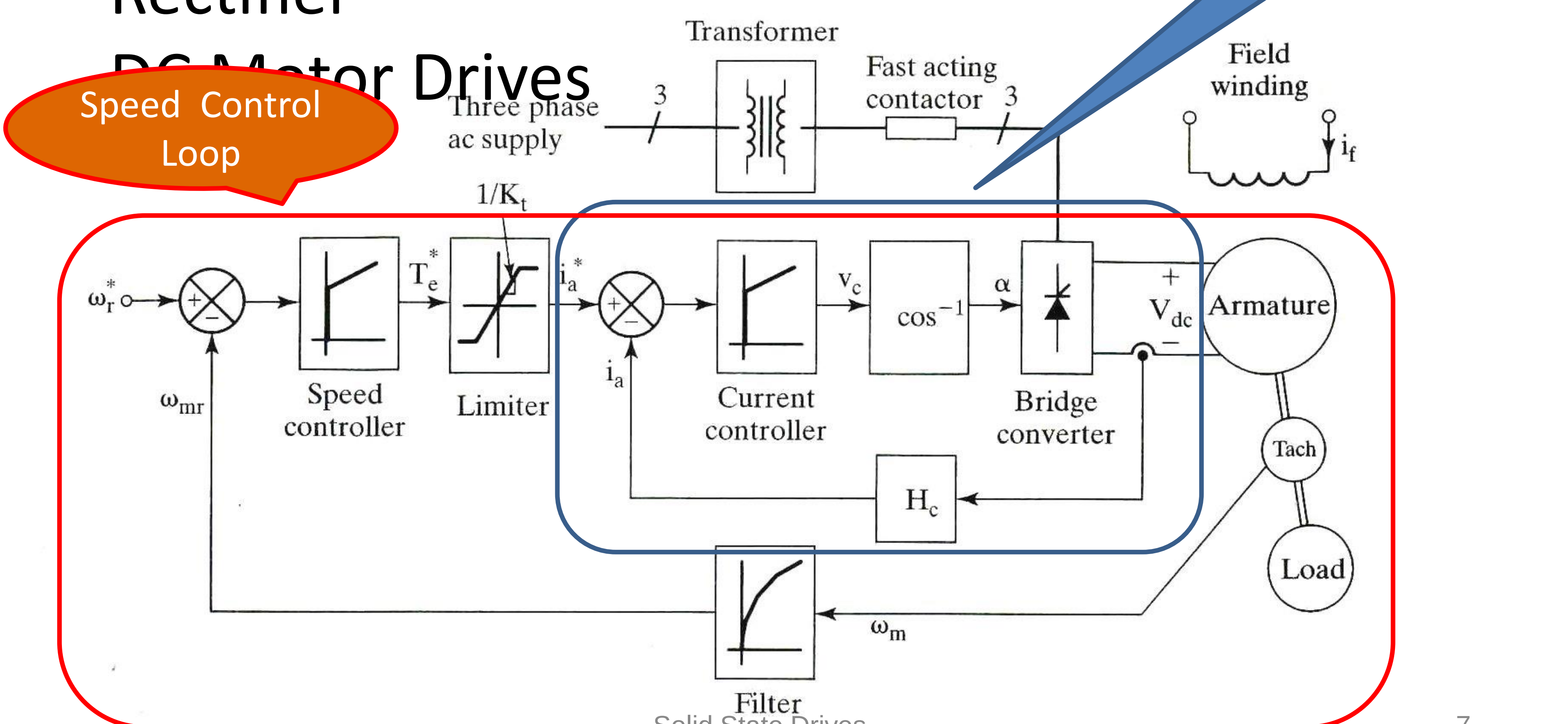
Closed Loop Control of DC Drives

- Cascade control structure
  - <u>Speed Control Loop:</u>
    - **Ensures that the actual speed is always equal to reference speed ω***
    - Provides fast response to changes in ω*, T_L and supply voltage (i.e. any transients are overcome within the shortest feasible time) without exceeding motor and converter capability



Closed Loop Control with Controlled Rectifiers – Two-quadrant

- **Two-quadrant** Three-phase Controlled Rectifier DC Motor Drives



Closed Loop Control with Controlled Rectifiers – Two-quadrant

- Actual motor speed ω_m measured using the tachogenerator (Tach) is filtered to produce feedback signal ω_{mr}
- The reference speed ω_r^* is compared to ω_{mr} to obtain a speed error signal
- The **speed (PI) controller** processes the speed error and **produces the torque command T_e^***
- **T_e^* is limited** by the limiter to keep within the safe current limits and the **armature current command i_a^* is produced**
- i_a^* is compared to actual current i_a to obtain a current error signal
- The **current (PI) controller** processes the error to **alter the control signal v_c**
- **v_c modifies the firing angle α** to be sent to the converter to obtained the motor armature voltage for the desired motor operation speed

Closed Loop Control with Controlled Rectifiers – Two-quadrant

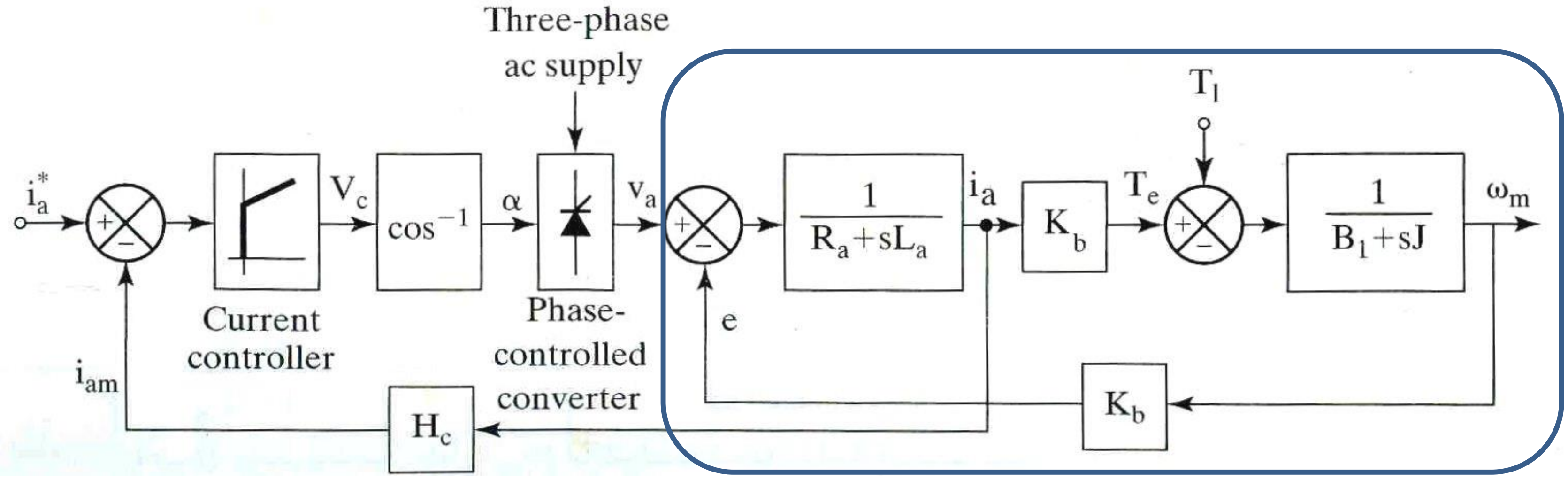
- **Design of speed and current controller (gain and time constants) is crucial in meeting the dynamic specifications of the drive system**
- Controller design procedure:
  1. Obtain the transfer function of all drive subsystems
     a) DC Motor & Load
     b) Current feedback loop sensor
     c) Speed feedback loop sensor
  2. Design current (torque) control loop first
  3. Then design the speed control loop

Transfer Function of Subsystems – DC Motor and Load

- Assume load is proportional to speed

$$T_L = B_L \omega_m$$

- DC motor has inner loop due to induced emf magnetic coupling, which is not physically seen
- This creates complexity in current control loop design



Transfer Function of Subsystems – DC Motor and Load

- Need to split the DC motor transfer function between ω_m and V_a

$$\frac{\omega_m(s)}{V_a(s)} = \frac{\omega_m(s)}{I_a(s)} \cdot \frac{I_a(s)}{V_a(s)} \qquad (1)$$

- where

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b}{B_t(1+sT_m)} \qquad (2)$$

$$\frac{I_a(s)}{V_a(s)} = K_1 \frac{(1+sT_m)}{(1+sT_1)(1+sT_2)} \qquad (3)$$

- This is achieved through redrawing of the DC motor and load block diagram.

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Transfer Function of Subsystems – DC Motor and Load

- In (2),

  - mechanical motor time constant: $$T_m = \frac{J}{B_t} \qquad (4)$$

  - motor and load friction coefficient: $B_t = B_1 + B_L$ (5)

- In (3),

$$K_1 = \frac{B_t}{K_b{}^2 + R_a B_t} \qquad (6)$$

$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left(\frac{R_a}{L_a} + \frac{B_t}{J}\right) \pm \sqrt{\frac{1}{4}\left(\frac{R_a}{L_a} + \frac{B_t}{J}\right)^2 - \left(\frac{R_a B_t}{J L_a} + \frac{K_b{}^2}{J L_a}\right)} \qquad (7)$$

Note: J = motor inertia, B_1 = motor friction coefficient,
B_L = load friction coefficient

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Transfer Function of Subsystems – Three-phase Converter

- Need to obtain linear relationship between control signal v_c and delay angle α (i.e. using 'cosine wave crossing' method)

$$\alpha = \cos^{-1}\left(\frac{v_c}{V_{cm}}\right) \qquad (8)$$

where v_c = control signal (output of current controller)

V_{cm} = maximum value of the control voltage

- Thus, dc output voltage of the three-phase converter

$$V_{dc} = \frac{3}{\pi} V_{\mathrm{L-L},m} \cos\alpha = \frac{3}{\pi} V_{\mathrm{L-L},m} \cos\left(\cos^{-1}\frac{v_c}{V_{cm}}\right) = \frac{3}{\pi}\frac{V_{\mathrm{L-L},m}}{V_{cm}} v_c = K_r v_c \qquad (9)$$

Transfer Function of Subsystems – Three-phase Converter

- Gain of the converter

$$K_r = \frac{3}{\pi}\frac{V_{L-L,m}}{V_{cm}} = \frac{3}{\pi}\frac{\sqrt{2}V}{V_{cm}} = 1.35\frac{V}{V_{cm}} \qquad (10)$$

where V = rms line-to-line voltage of 3-phase supply

- Converter also has a delay

$$T_r = \frac{1}{2}\times\frac{60}{360}\times\frac{1}{f_s} = \frac{1}{12}\times\frac{1}{f_s} \qquad (11)$$

where f_s = supply voltage frequency

- Hence, the **converter transfer function**

$$G_r(s) = \frac{K_r}{(1+sT_r)} \qquad (12)$$

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Transfer Function of Subsystems – Current and Speed Feedback

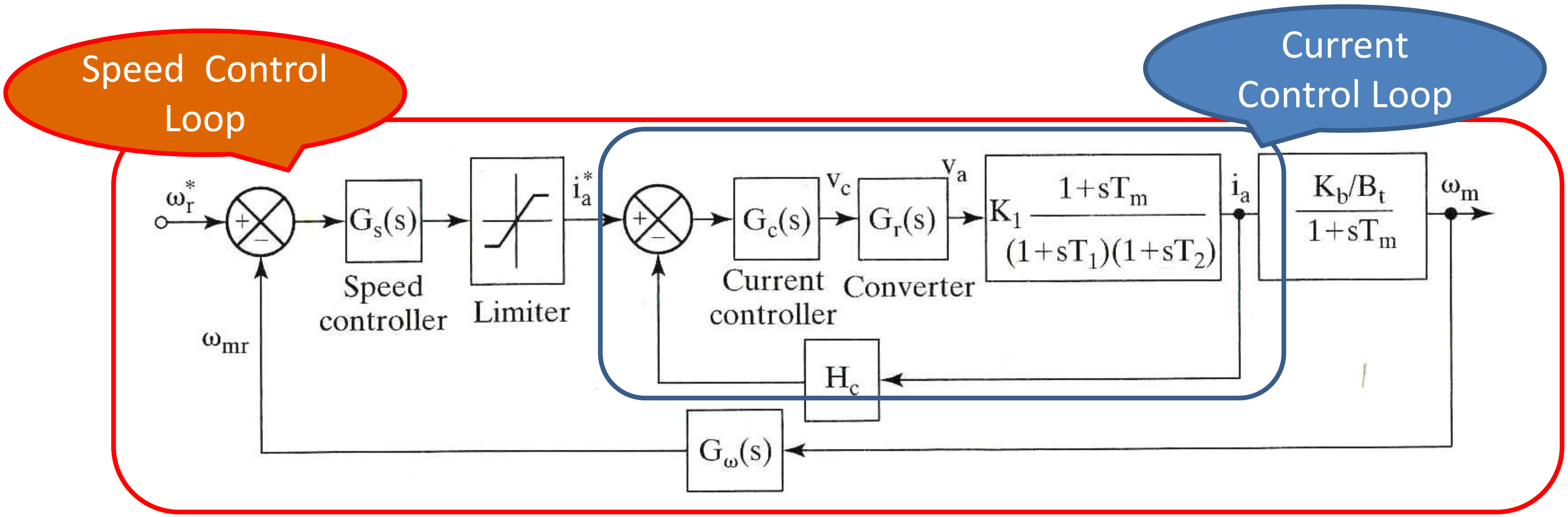
- **Current Feedback**
  - Transfer function: H_c
  - No filtering is required in most cases
  - If filtering is required, a low pass-filter can be included (time constant < 1ms).
- **Speed Feedback**
  - Transfer function:

$$G_\omega(s) = \frac{K_\omega}{(1 + sT_\omega)} \qquad (13)$$

  where K_ω = gain, T_ω = time constant

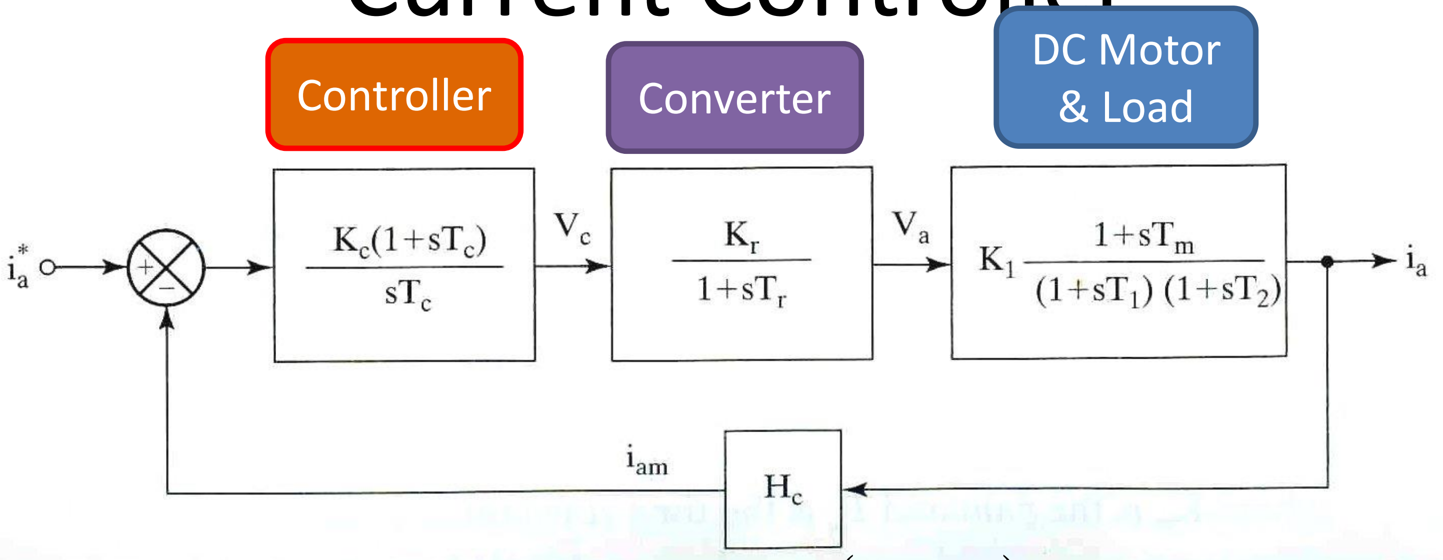
  - Most high performance systems use dc tacho generator and low-pass filter
  - Filter time constant < 10 ms

Design of Controllers – Block Diagram of Motor Drive



- **Control loop design starts from inner (fastest) loop to outer(slowest) loop**
  - Only have to solve for one controller at a time
  - Not all drive applications require speed control (outer loop)
  - Performance of outer loop depends on inner loop

Design of Controllers– Current Controller



- PI type current controller: $G_c(s) = \dfrac{K_c(1+sT_c)}{sT_c}$ (14)
- Open loop gain function:

$$GH_{ol}(s) = \left\{\frac{K_1 K_c K_r H_c}{T_c}\right\} \frac{(1+sT_c)(1+sT_m)}{s(1+sT_1)(1+sT_2)(1+sT_r)} \quad (15)$$

- From the open loop gain, the system is of 4^{th} order (due to 4 poles of system)

Design of Controllers– Current Controller

- If designing without computers, **simplification is needed**.
- **Simplification 1:** T_m is in order of 1 second. Hence,

$$\boxed{(1+sT_m) \cong sT_m} \qquad (16)$$

Hence, the open loop gain function becomes:

$$\mathrm{GH}_{\mathrm{ol}}(\mathrm{s}) = \left\{\frac{K_1 K_c K_r H_c}{T_c}\right\} \frac{(1+sT_c)(1+sT_m)}{s(1+sT_1)(1+sT_2)(1+sT_r)}$$

$$\cong \left\{\frac{K_1 K_c K_r H_c}{T_c}\right\} \frac{(1+sT_c)(sT_m)}{s(1+sT_1)(1+sT_2)(1+sT_r)}$$

$$\mathrm{GH}_{\mathrm{ol}}(\mathrm{s}) \cong K \frac{(1+sT_c)}{(1+sT_1)(1+sT_2)(1+sT_r)} \quad \text{where } K = \frac{K_1 K_c K_r H_c T_m}{T_c} \qquad (17)$$

i.e. system zero cancels the controller pole at origin.

Design of Controllers– Current Controller

- Relationship between the denominator time constants in (17):

$$\boxed{T_r < T_2 < T_1}$$

- **Simplification 2:** Make controller time constant equal to T_2

$$\boxed{T_c = T_2} \qquad (18)$$

Hence, the open loop gain function becomes:

$$\mathrm{GH}_{\mathrm{ol}}(\mathrm{s}) \cong K \frac{(1+sT_c)}{(1+sT_1)(1+sT_2)(1+sT_r)}$$

$$\cong K \frac{(1+sT_2)}{(1+sT_1)(1+sT_2)(1+sT_r)}$$

$$\mathrm{GH}_{\mathrm{ol}}(\mathrm{s}) \cong \frac{K}{(1+sT_1)(1+sT_r)} \text{ where } K = \frac{K_1 K_c K_r H_c T_m}{T_c}$$

i.e. controller zero cancels one of the system poles.

Design of Controllers– Current Controller

- **After simplification**, the final open loop gain function:

$$\mathrm{GH_{ol}}(\mathrm{s}) \cong \frac{K}{(1+sT_1)(1+sT_r)} \qquad (19)$$

where

$$\boxed{K = \frac{K_1 K_c K_r H_c T_m}{T_c}} \qquad (20)$$

- The system is now of 2nd order.
- From the closed loop transfer function: $\mathrm{G_{cl}}(\mathrm{s}) = \frac{\mathrm{GH_{ol}}(\mathrm{s})}{1+\mathrm{GH_{ol}}(\mathrm{s})}$,

the closed loop characteristic equation is:

$$\boxed{(1+sT_1)(1+sT_r)+K}$$

or when expanded becomes:

$$\boxed{T_1 T_r \left\{ s^2 + s\left(\frac{T_1+T_r}{T_1 T_r}\right) + \frac{K+1}{T_1 T_r} \right\}} \qquad (21)$$

Design of Controllers– Current Controller

- Design the controller by comparing system characteristic equation (eq. 21) with the standard 2nd order system equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

- Hence,

$$\omega_n{}^2 = \frac{K+1}{T_1 T_r} \qquad (22)$$

$$\zeta = \frac{\left(\dfrac{T_1 + T_r}{T_1 T_r}\right)}{2\sqrt{\dfrac{K+1}{T_1 T_r}}} \qquad (23)$$

- So, **for good dynamic performance *ζ=0.707***
  - **Hence equating the damping ratio to 0.707 in (23) we get**

$$0.707 = \frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)}{2\sqrt{\frac{K+1}{T_1 T_r}}}$$

Squaring the equation on both sides

$$0.5 = \left(\frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)}{2\sqrt{\frac{K+1}{T_1 T_r}}}\right)^2 \Rightarrow 0.5 = \frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)^2}{2 \text{ x } 2 \text{ x } \frac{\mathrm{K}+1}{\mathrm{T_1 T_r}}} \Rightarrow 1 = \frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)^2}{2 \text{ x } \frac{\mathrm{K}+1}{\mathrm{T_1 T_r}}}$$

$$K + 1 = \frac{\left(\frac{T_1 + T_r}{T_1 T_r}\right)^2}{\frac{2}{T_1 T_r}} \Rightarrow \mathrm{K} + 1 = \left(\frac{T_1 + T_r}{T_1 T_r}\right)^2 X \frac{T_1 T_r}{2} \Rightarrow K + 1 = \frac{(T_1 + T_r)^2}{2 T_1 T_r}$$

$$K+1=\frac{\left(\frac{T_1+T_r}{T_1T_r}\right)^2}{\frac{2}{T_1T_r}} \Rightarrow K+1=\left(\frac{T_1+T_r}{T_1T_r}\right)^2 X\,\frac{T_1T_r}{2} \Rightarrow K+1=\frac{(T_1+T_r)^2}{2T_1T_r}$$

An approximation K >> 1 & $T_1 >> T_r$ **Which leads to**

$$K \cong \frac{T_1^{\,2}}{2T_1T_r}=\frac{T_1}{2T_r}$$

Equating above expression with (20) we get the gain of current controller

$$\frac{K_1K_cK_rH_cT_m}{T_c}=\frac{T_1}{2T_r}$$

$$K_c=\frac{T_1T_c}{2T_r}\left[\frac{1}{K_1K_rH_cT_m}\right]$$

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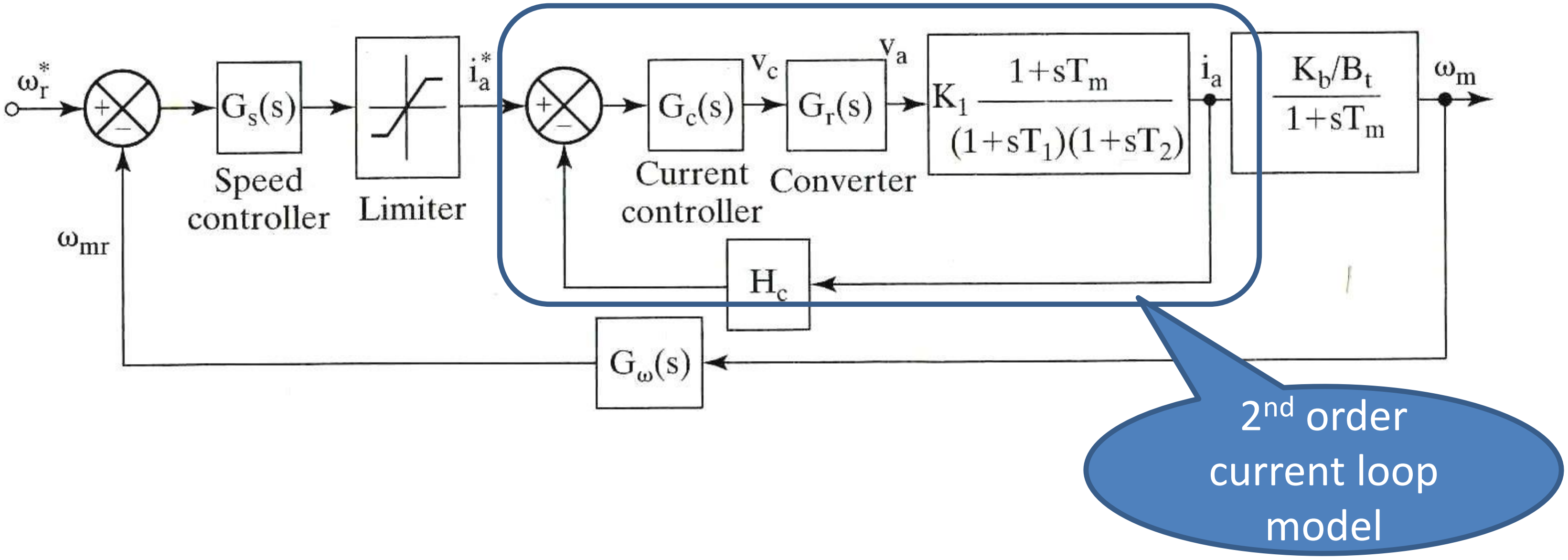
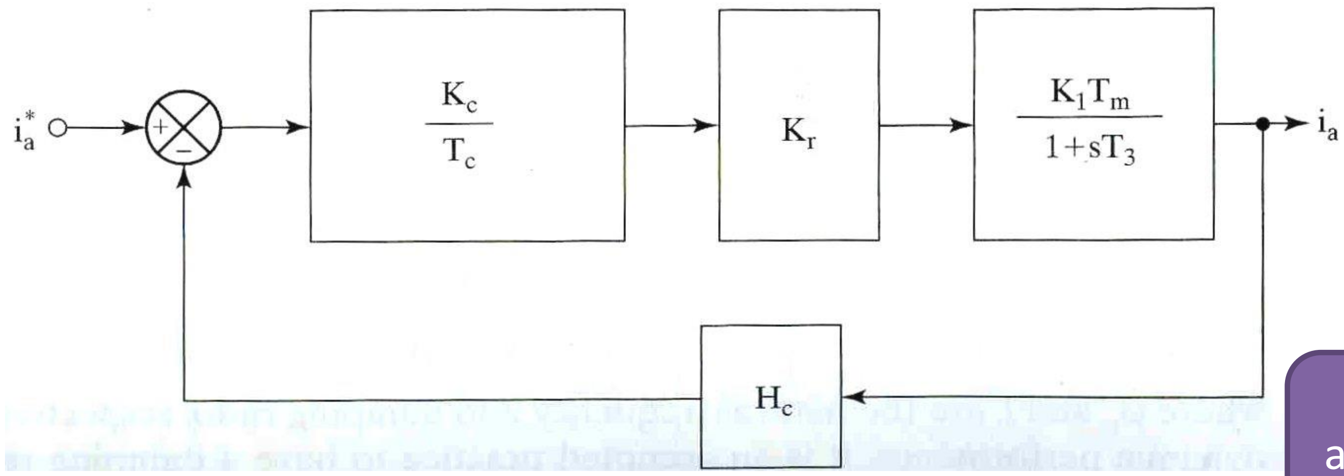
Design of Controllers– Current loop 1st order approximation

- To design the speed loop, the 2nd order model of current loop must be replaced with an approximate 1st order model
- Why?
- To reduce the order of the overall speed loop gain function



Design of Controllers–
Current loop 1st order approximation

- Approximated by adding T_r to T_1 $\Rightarrow$ $\boxed{T_3 = T_1 + T_r}$



- Hence, current model transfer function is given by:

$$\frac{I_a(s)}{I_a^*(s)} = \frac{\frac{K_c K_r K_1 T_m}{T_c}\frac{1}{(1+sT_3)}}{1+\frac{K_c K_r K_1 H_c T_m}{T_c}\frac{1}{(1+sT_3)}} = \frac{K_i}{(1+sT_i)} \qquad (24)$$

1st order approximation of current loop

[Full derivation available here.](#)

Design of Controllers– Current Controller

- **After simplification**, the final open loop gain function:

$$\mathrm{GH_{ol}}(\mathrm{s}) \cong \frac{K}{(1+sT_1)(1+sT_r)} = \frac{K}{1+s(T_1+T_r)+s^2T_1T_r}$$

Where $\boxed{K = \dfrac{K_1K_cK_rT_m}{T_c}}$

$$\mathrm{GH_{ol}}(\mathrm{s}) \cong \frac{K}{1+s(T_3)+s^2T_1T_r}$$

Since $\boxed{T_1 + T_r = T_3}$

and since $\boxed{T_1 >> T_r}$ **Therefore** $\mathrm{GH_{ol}}(\mathrm{s}) \cong \dfrac{K}{1+sT_3}$

Design of Controllers–
Current loop 1st order approximation

where

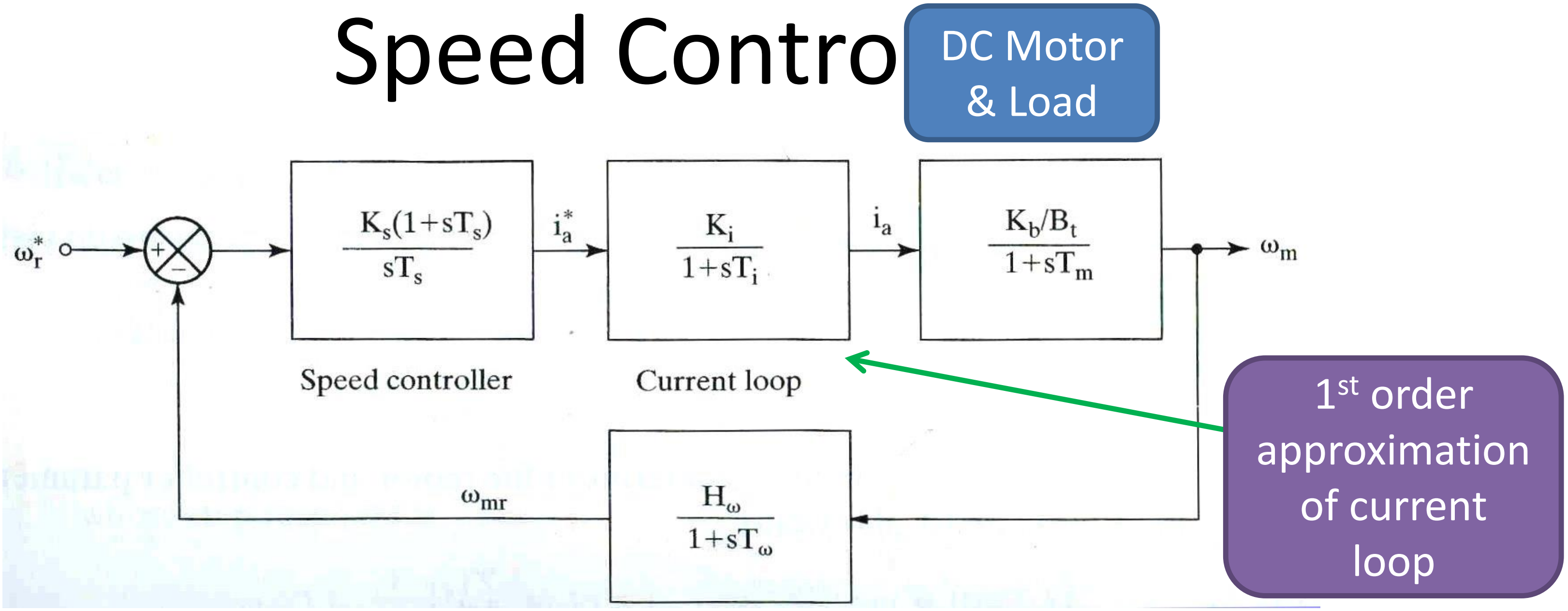
$$T_i = \frac{T_3}{1+K_{fi}} \quad (26)$$

$$K_i = \frac{K_{fi}}{H_c}\frac{1}{(1+K_{fi})} \quad (27)$$

$$K_{fi} = \frac{K_1 K_c K_r H_c T_m}{T_c} \quad (28)$$

- 1st order approximation of current loop **used in speed loop design**.
- If more accurate speed controller design is required, values of K_i and T_i should be obtained experimentally.

Design of Controllers– Speed Contro



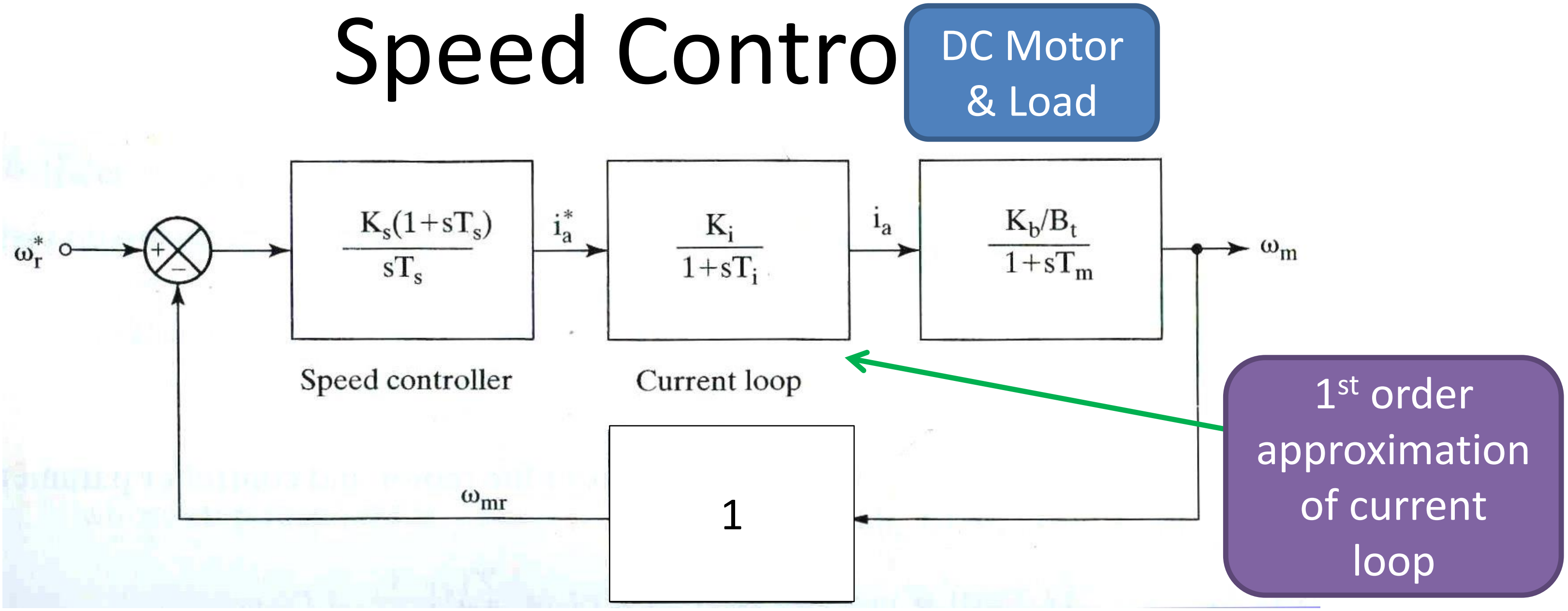
- PI type speed controller: (29)

$$G_s(s) = \frac{K_s(1 + sT_s)}{sT_s}$$

- Assume there is **unity speed feedback**:

$$G_\omega(s) = \frac{H_\omega}{(1 + sT_\omega)} = 1 \qquad (30)$$

Design of Controllers– Speed Control



- Open loop gain function:

$$GH(s) = \left\{ \frac{K_B K_s K_i}{B_t T_s} \right\} \frac{(1+sT_s)}{s(1+sT_i)(1+sT_m)} \quad (31)$$

- From the loop gain, the system is of 3rd order.
- If designing without computers, **simplification is needed**.

Design of Controllers– Speed Controller

- Relationship between the denominator time constants in (31):

$$T_i < T_m \quad (32)$$

- Hence, design the speed controller such that:

$$\boxed{T_s = T_m} \quad (33)$$

The open loop gain function becomes:

$$\mathrm{GH}(s) = \left\{\frac{K_B K_s K_i}{B_t T_s}\right\} \frac{(1 + sT_s)}{s(1 + sT_i)(1 + sT_m)}$$

$$\cong \left\{\frac{K_B K_s K_i}{B_t T_s}\right\} \frac{(1 + sT_m)}{s(1 + sT_i)(1 + sT_m)}$$

$$\mathrm{GH}(s) \cong \frac{K_\omega}{s(1 + sT_i)} \text{ where } K_\omega = \frac{K_B K_s K_i}{B_t T_s}$$

i.e. controller zero cancels one of the system poles.

Design of Controllers– Speed Controller

- **After simplification**, loop gain function:

$$\mathrm{GH(s)} \cong \frac{K_\omega}{s(1+sT_i)} \qquad (34)$$

where

$$\boxed{K_\omega = \frac{K_B K_s K_i}{B_t T_s}} \qquad (35)$$

- The controller is now of 2nd order.
- From the closed loop transfer function: $\mathrm{G_{cl}(s)} = \dfrac{\mathrm{GH(s)}}{1+\mathrm{GH(s)}}$, the closed loop characteristic equation is:

$$\boxed{s(1+sT_i)+K_\omega}$$

or when expanded becomes:

$$\boxed{T_i\left\{s^2 + s\left(\frac{1}{T_i}\right) + \frac{K_\omega}{T_i}\right\}} \qquad (36)$$

Design of Controllers– Speed Controller

- Design the controller by comparing system characteristic equation with the standard equation:

$$s^2 + 2\zeta\omega_n s + \omega_n^2$$

- Hence:

$$2\zeta\omega_n = \qquad (37)$$

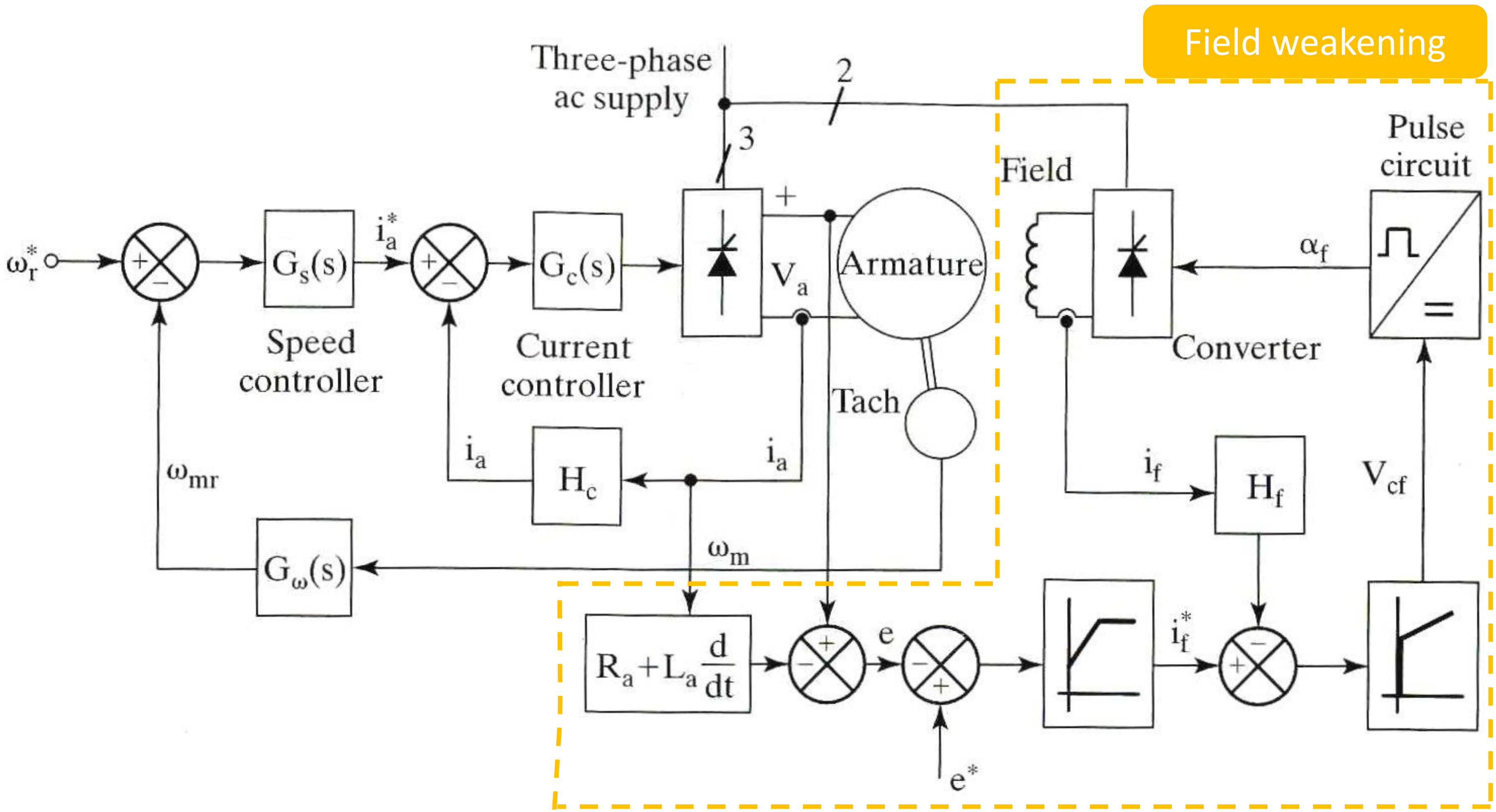
$$\omega_n^2 = \qquad (38)$$

- So, **for a given value of ζ:**
  - **use (37) to calculate ω_n**
  - **Then use (38) to calculate the controller gain K_S**

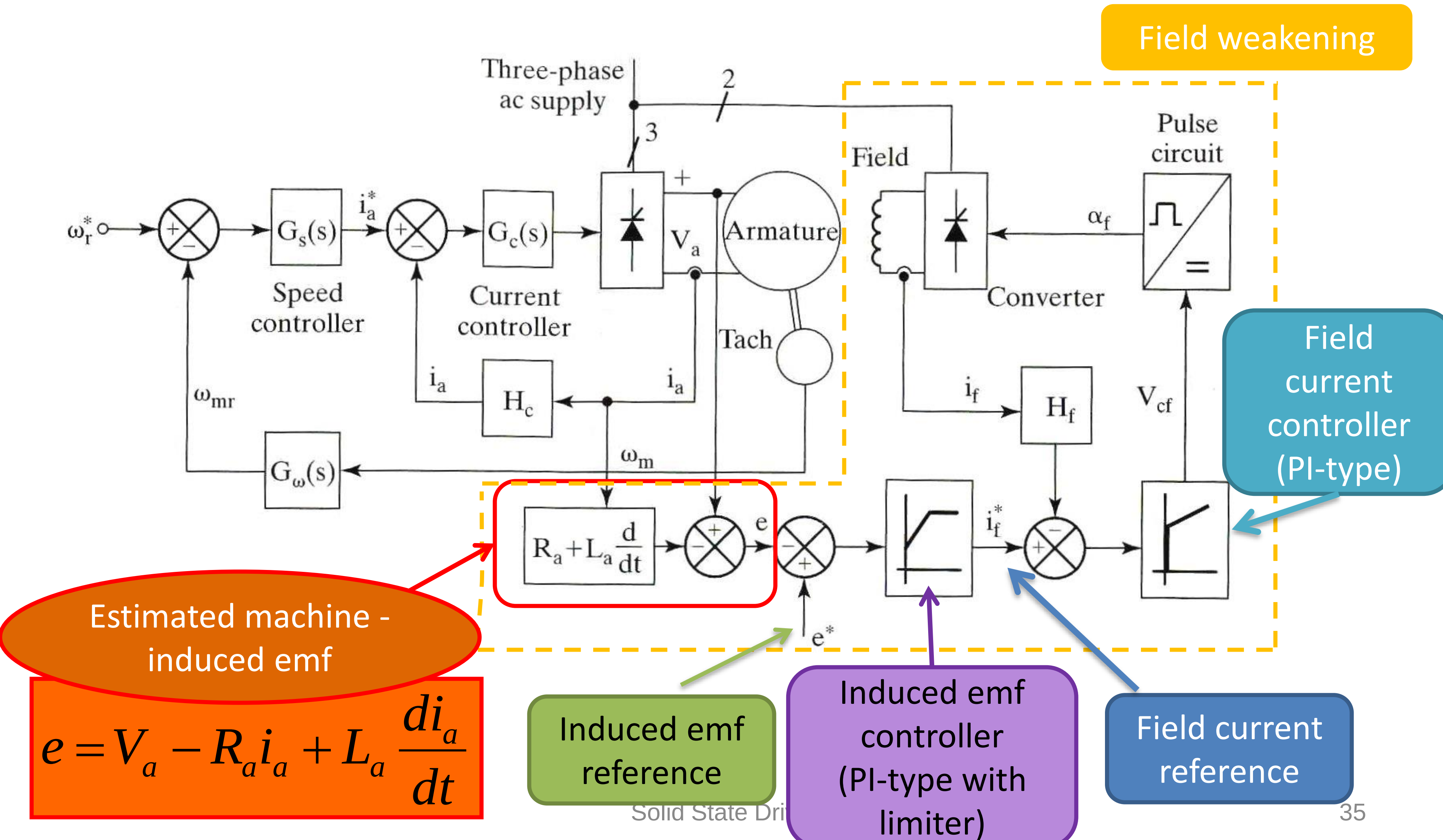
Closed Loop Control with Field Weakening – Two-quadrant

- Motor operation above base speed requires field weakening
- Field weakening obtained by varying field winding voltage using controlled rectifier in:
  - single-phase or
  - three-phase
- Field current has no ripple – due to large L_f
- Converter time lag negligible compared to field time constant
- Consists of two additional control loops on field circuit:
  - Field current control loop (inner)
  - Induced emf control loop (outer)

Closed Loop Control with Field Weakening – Two-quadrant



Closed Loop Control with Field Weakening – Two-quadrant



$$e = V_a - R_a i_a + L_a \frac{di_a}{dt}$$

Closed Loop Control with Field Weakening – Two-quadrant

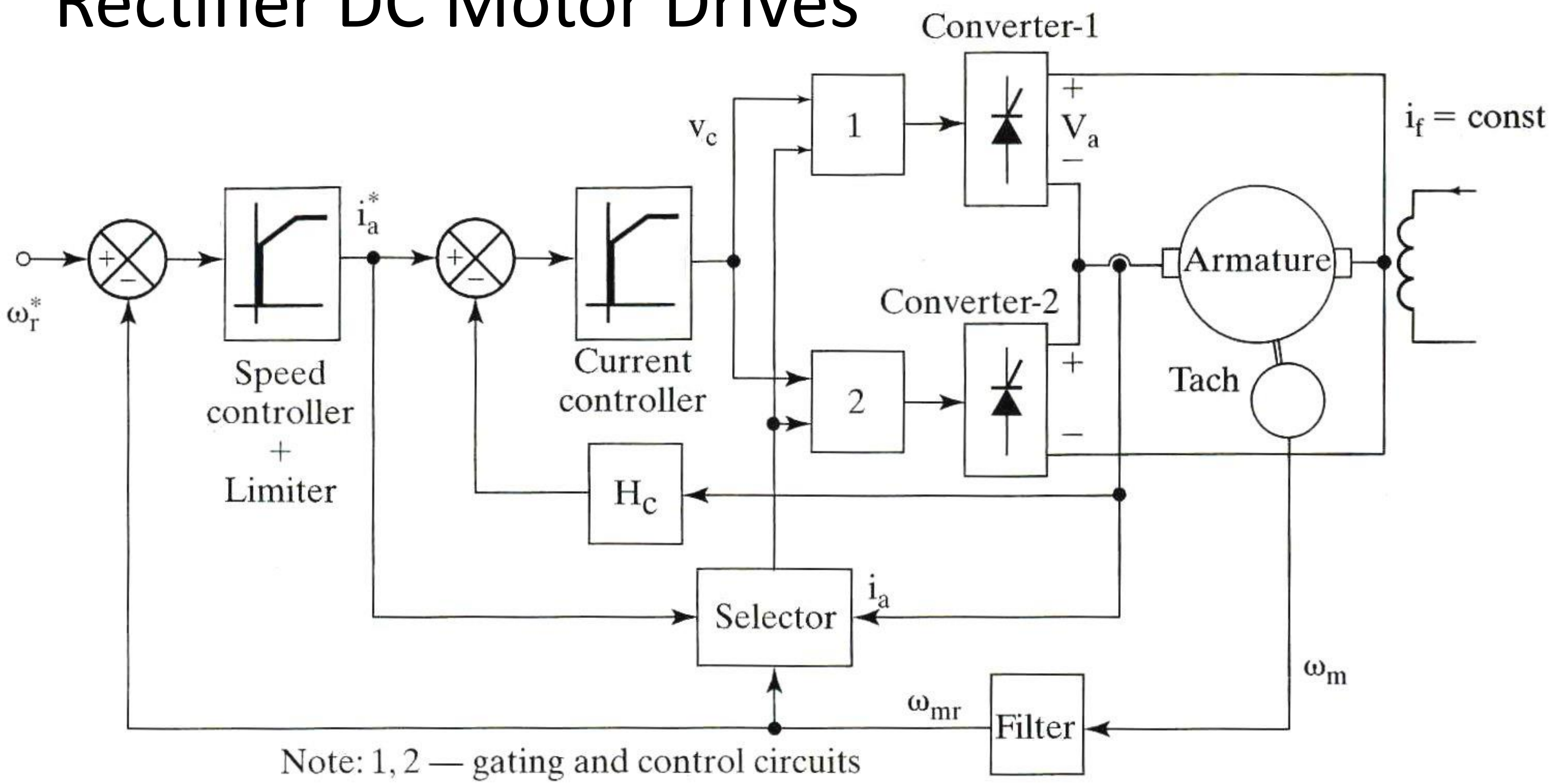
- The estimated machine-induced emf is obtained from:

$$e = V_a - R_a i_a + L_a \frac{di_a}{dt}$$

  (the estimated emf is machine-parameter sensitive and must be adaptive)
- The reference induced emf e^* is compared to e to obtain the induced emf error signal (for speed above base speed, e^* kept constant at rated emf value so that $\phi \propto 1/\omega$)
- The induced emf (PI) controller processes the error and produces the field current reference i_f^*
- i_f^* is limited by the limiter to keep within the safe field current limits
- i_f^* is compared to actual field current i_f to obtain a current error signal
- The field current (PI) controller processes the error to alter the control signal v_{cf} (similar to armature current i_a control loop)
- v_{cf} modifies the firing angle α_f to be sent to the converter to obtained the motor field voltage for the desired motor field flux

Closed Loop Control with Controlled Rectifiers – Four-quadrant

- **Four-quadrant** Three-phase Controlled Rectifier DC Motor Drives



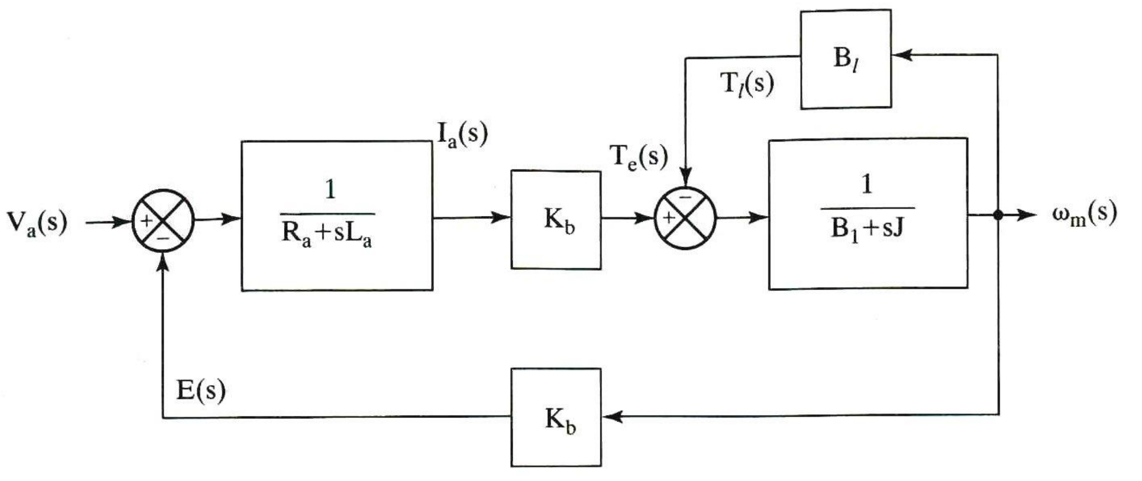
Closed Loop Control with Controlled Rectifiers – Four-quadrant

- Control very similar to the two-quadrant dc motor drive.
- Each converter must be energized depending on quadrant of operation:
  - Converter 1 – for forward direction / rotation
  - Converter 2 – for reverse direction / rotation
- Changeover between Converters 1 & 2 handled by monitoring
  - Speed
  - Current-command
  - Zero-crossing current signals

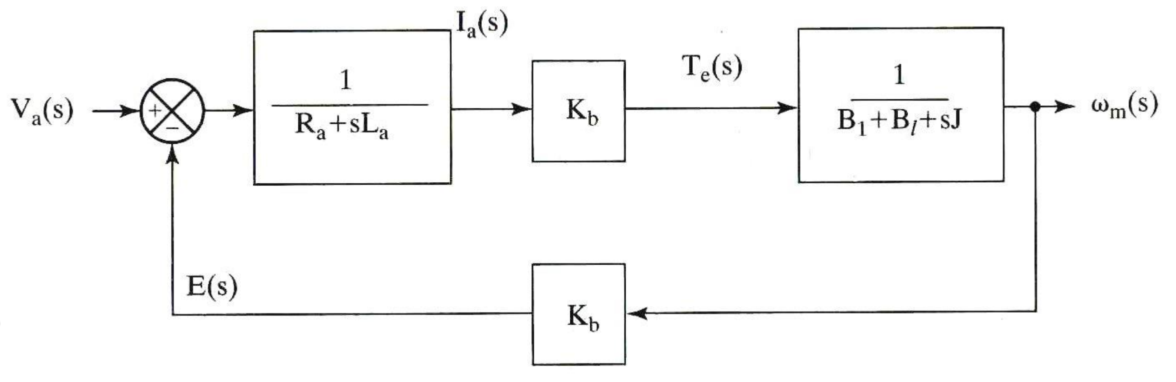
  Inputs to 'Selector' block
- 'Selector' block determines which converter has to operate by assigning pulse-control signals
- Speed and current loops shared by both converters
- Converters switched only when current in outgoing converter is zero (i.e. does not allow circulating current. One converter is on at a time.)

DC Motor and Load Transfer Function - Decoupling of Induced EMF Loop

- Step 1:

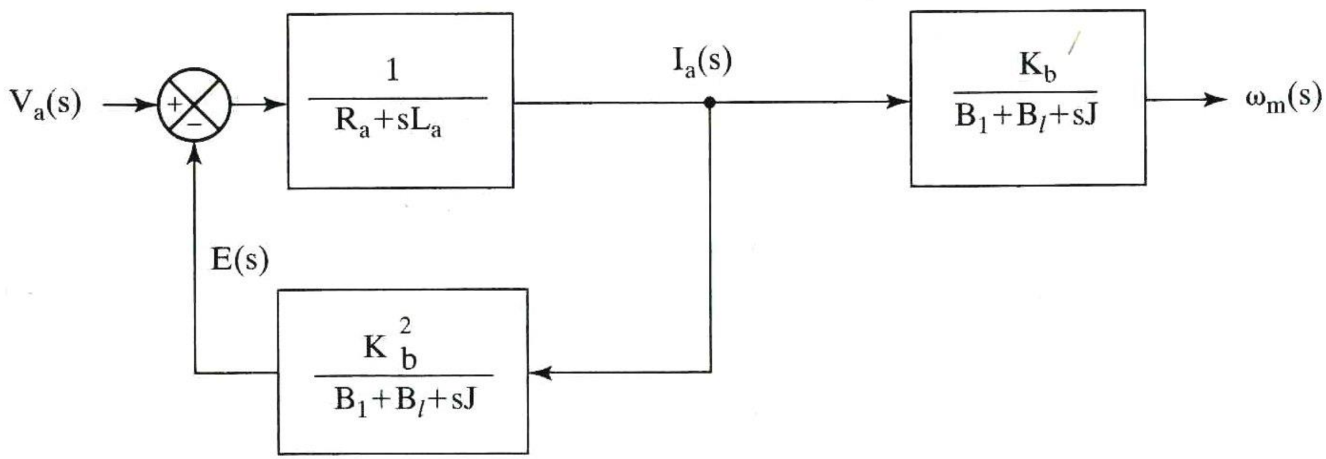


- Step 2:

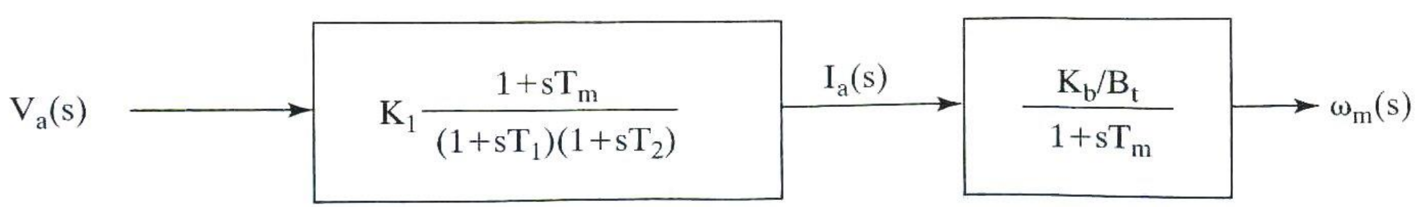


DC Motor and Load Transfer Function - Decoupling of Induced EMF Loop

- Step 3:



- Step 4:



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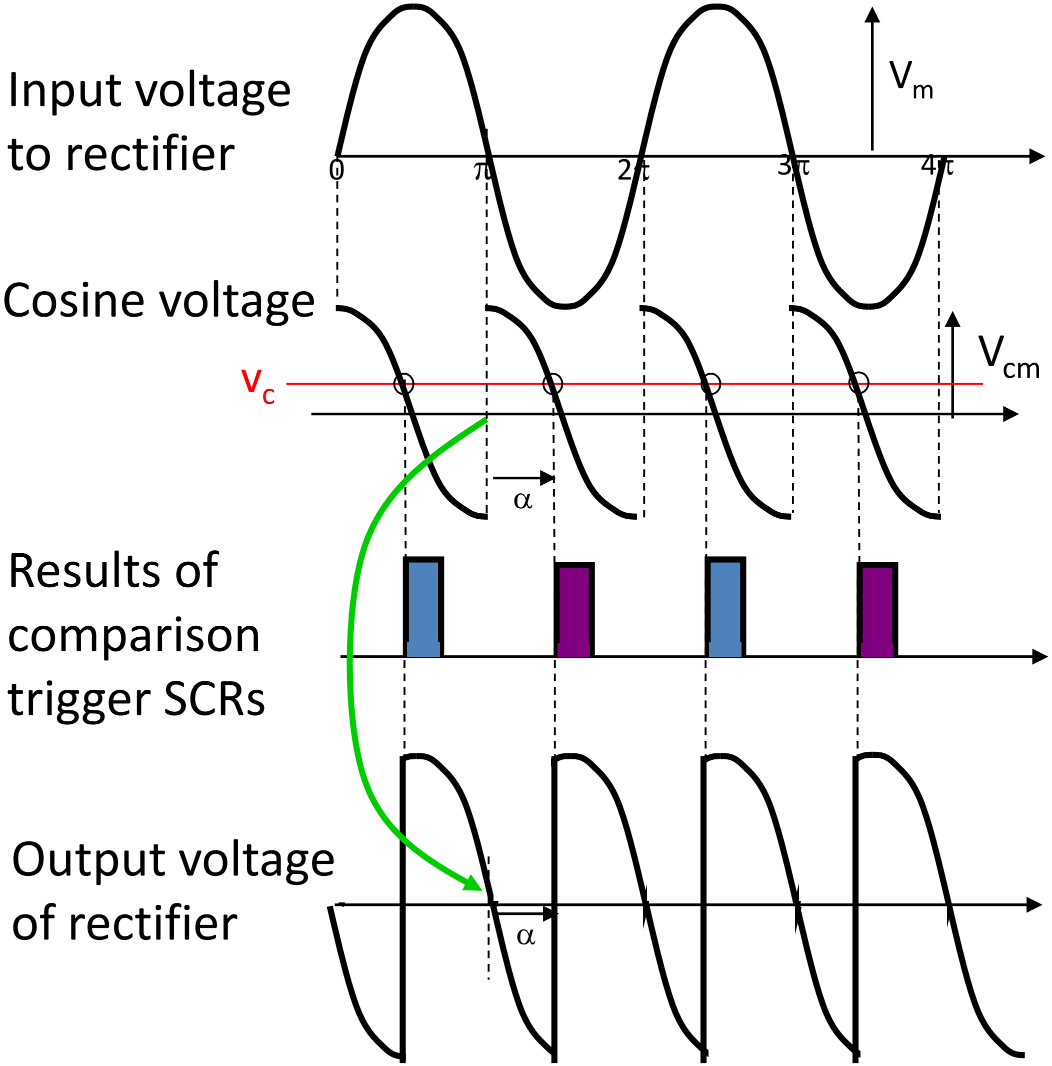
Cosine-wave Crossing Control for Controlled Rectifiers



Cosine wave compared with control voltage v_c

$$V_{cm}\cos(\alpha) = v_c$$

$$\alpha = \cos^{-1}\left(\frac{v_c}{V_{cm}}\right)$$

Back

Design of Controllers–
Current loop 1st order approximation

$$\frac{I_a(s)}{I_a^*(s)} = \frac{\frac{K_c K_r K_1 T_m}{T_c}\frac{1}{(1+sT_3)}}{1+\frac{K_c K_r K_1 H_c T_m}{T_c}\frac{1}{(1+sT_3)}} = \frac{\frac{K_{fi}}{H_c}\frac{1}{(1+sT_3)}}{1+K_{fi}\frac{1}{(1+sT_3)}}$$

$$= \frac{\frac{K_{fi}}{H_c}}{(1+sT_3)+K_{fi}} = \frac{\frac{K_{fi}}{H_c}\frac{1}{(1+K_{fi})}}{1+s\left(\frac{T_3}{1+K_{fi}}\right)} = \frac{K_i}{(1+sT_i)}$$

Back

Design a speed-controlled dc motor drive maintaining the field flux constant. The motor parameters and ratings are as follows:

220 V, 8.3 A, 1470 rpm, $R_a = 4\ \Omega$, $J = 0.0607$ kg − m_2, $L_a = 0.072$ H, $B_t = 0.0869$ N·m / rad/sec, $K_b = 1.26$ V/rad/sec.

The converter is supplied from 230V, 3-phase ac at 60 Hz. The converter is linear, and its maximum control input voltage is ±10 V. The tachogenerator has the transfer function $G_\omega(s) = \frac{0.065}{(1 + 0.002s)}$. The speed reference voltage has a maximum of 10V. The maximum current permitted in the motor is 20 A.

Solution (i) Converter transfer function:

Go to

$$K_r = \frac{1.35\ V}{V_{cm}} = \frac{1.35 \times 230}{10} = 31.05\ V/V \qquad \frac{220}{310.5} x10 = 7.09$$

$$V_{dc}(\max) = 310.5\ V$$

The rated dc voltage required is 220 V, which corresponds to a control voltage of 7.09 V. The transfer function of the converter is

$$G_r(s) = \frac{31.05}{(1 + 0.00138s)}\ V/V$$

(ii) Current transducer gain: The maximum safe control voltage is 7.09 V, and this has to correspond to the maximum current error:

$$i_{max} = 20 \text{ A}$$

$$H_c = \frac{7.09}{I_{max}} = \frac{7.09}{20} = 0.355 \text{ V/A}$$

(iii) Motor transfer function:

$$K_1 = \frac{B_t}{K_b^2 + R_a B_t} = \frac{0.0869}{1.26^2 + 4 \times 0.0869} = 0.0449$$

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$$-\frac{1}{T_1}, -\frac{1}{T_2} = -\frac{1}{2}\left[\frac{B_t}{J} + \frac{R_a}{L_a}\right] \pm \sqrt{\frac{1}{4}\left(\frac{B_t}{J} + \frac{R_a}{L_a}\right)^2 - \left(\frac{K_b^2 + R_a B_t}{J L_a}\right)}$$

$$T_1 = 0.1077 \text{ sec}$$

$$T_2 = 0.0208 \text{ sec}$$

$$T_m = \frac{J}{B_t} = 0.7 \text{ sec}$$

The subsystem transfer functions are

Go to

$$\frac{I_a(s)}{V_a(s)} = K_1\frac{(1 + sT_m)}{(1 + sT_1)(1 + sT_2)} = \frac{0.0449(1 + 0.7s)}{(1 + 0.0208s)(1 + 0.1077s)}$$

$$\frac{\omega_m(s)}{I_a(s)} = \frac{K_b/B_t}{(1 + sT_m)} = \frac{14.5}{(1 + 0.7s)}$$

(iv) Design of current controller:

$$T_c = T_2 = 0.0208 \text{ sec}$$

Go to

$$K = \frac{T_1}{2T_r} = \frac{0.1077}{2 \times 0.001388} = 38.8$$

$$K_c = \frac{KT_c}{K_1H_cK_rT_m} = \frac{38.8 \times 0.0208}{0.0449 \times 0.355 \times 31.05 \times 0.7} = 2.33$$

(v) Current-loop approximation:

$$\frac{I_a(s)}{I_a^*(s)} = \frac{K_i}{(1 + sT_i)}$$

where

$$K_i = \frac{K_{fi}}{H_c} \cdot \frac{1}{(1 + K_{fi})}$$

$$K_{fi} = \frac{K_c K_r K_1 T_m H_c}{T_c} = 38.8$$

$$\therefore K_i = \frac{27.15}{28.09} \cdot \frac{1}{0.355} = 2.75$$

$$T_i = \frac{T_3}{1 + K_{fi}} = \frac{0.109}{1 + 38.8} = 0.0027 \text{ sec}$$

(vi) Speed-controller design:

$$T_4 = T_i + T_\omega = 0.0027 + 0.002 = 0.0047 \text{ sec}$$

$$K_2 = \frac{K_i K_b H_\omega}{B_t T_m} = \frac{2.75 \times 1.26 \times 0.065}{0.0869 \times 0.7} = 3.70$$

$$K_s = \frac{1}{2K_2T_4} = \frac{1}{2 \times 3.70 \times 0.0047} = 28.73$$

$$T_s = 4T_4 = 4 \times 0.0047 = 0.0188 = \text{sec}$$