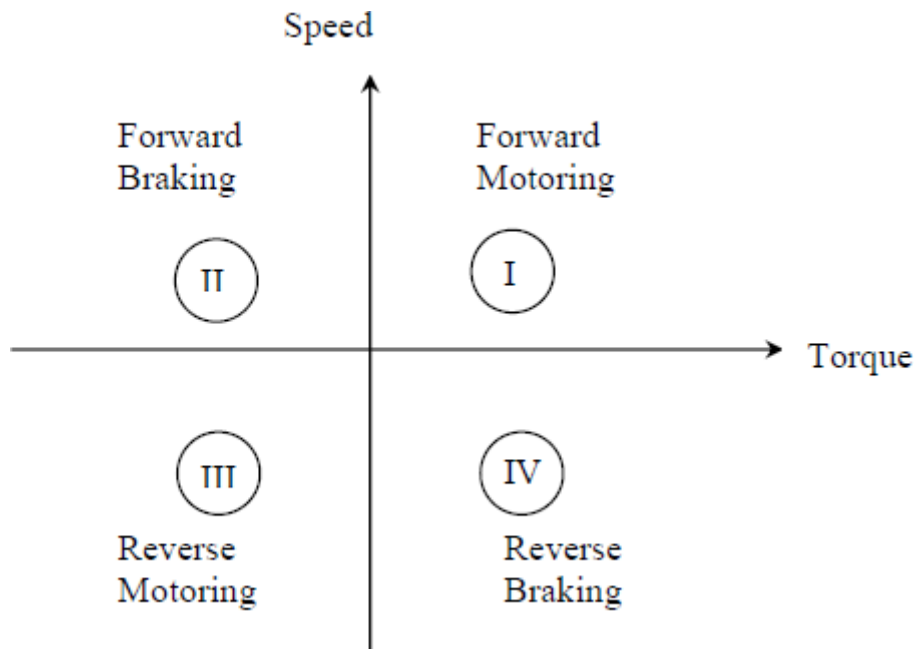


## Multi quadrant Operation

For consideration of multi quadrant operation of drives, it is useful to establish suitable conventions about the signs of torque and speed. A motor operates in two modes – Motoring and braking.

In motoring, it converts electrical energy into mechanical energy, which supports its motion. In braking it works as a generator converting mechanical energy into electrical energy and thus opposes the motion.

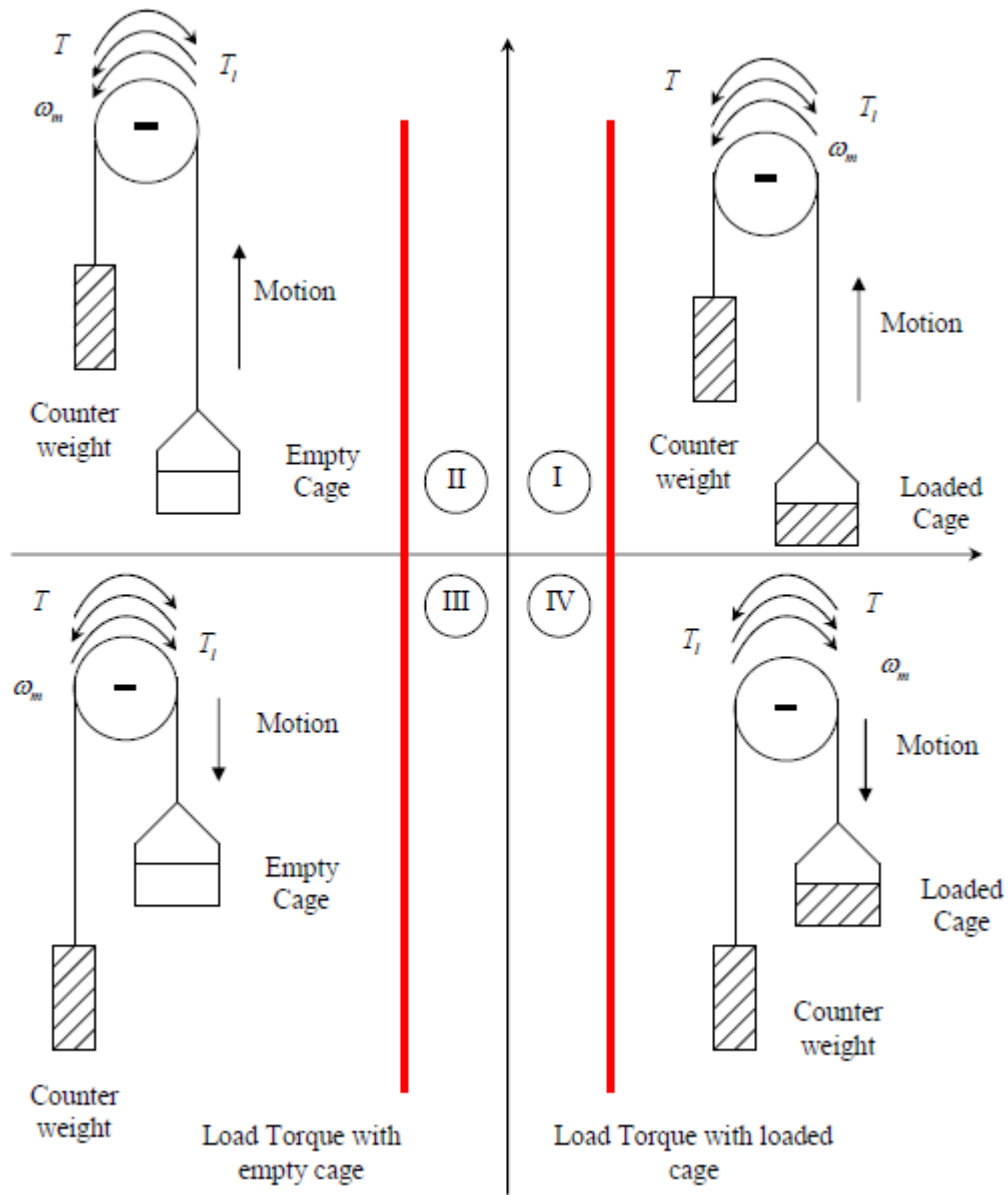
Motor can provide motoring and braking operations for both forward and reverse directions. Figure shows the torque and speed co-ordinates for both forward and reverse motions. Power developed by a motor is given by the product of speed and torque. For motoring operations power developed is positive and for braking operations power developed is negative.



In quadrant I, developed power is positive, hence machine works as a motor supplying mechanical energy. Operation in quadrant I is therefore called Forward Motoring.

In quadrant II, power developed is negative. Hence, machine works under braking opposing the motion. Therefore operation in quadrant II is known as forward braking.

Similarly operation in quadrant III and IV can be identified as reverse motoring and reverse braking since speed in these quadrants is negative. For better understanding of the above notations, let us consider operation of hoist in four quadrants as shown in the figure. Direction of motor and load torques and direction of speed are marked by arrows.



A hoist consists of a rope wound on a drum coupled to the motor shaft one end of the rope is tied to a cage which is used to transport man or material from one level to another level. Other end of the rope has a counter weight. Weight of the counter weight is chosen to be higher than the weight of empty cage but lower than of a fully loaded cage. Forward direction of motor speed will be one which gives upward motion of the cage. Load torque line in quadrants I and IV represents speed-torque characteristics of the loaded hoist. This torque is the difference of torques due to loaded hoist and counter weight.

The load torque in quadrants II and III is the speed torque characteristics for an empty hoist. This torque is the difference of torques due to counter weight and the empty hoist. Its sign is negative because the counter weight is always higher than that of an empty cage. The

quadrant I operation of a hoist requires movement of cage upward, which corresponds to the positive motor speed which is in counter clockwise direction here. This motion will be obtained if the motor produces positive torque in CCW direction equal to the magnitude of load torque  $TL_1$ . Since developed power is positive, this is forward motoring operation. Quadrant IV is obtained when a loaded cage is lowered. Since the weight of the loaded cage is higher than that of the counter weight, it is able to overcome due to gravity itself.

In order to limit the cage within a safe value, motor must produce a positive torque  $T$  equal to  $TL_2$  in anticlockwise direction. As both power and speed are negative, drive is operating in reverse braking operation. Operation in quadrant II is obtained when an empty cage is moved up. Since a counter weight is heavier than an empty cage, it is able to pull it up. In order to limit the speed within a safe value, motor must produce a braking torque equal to  $TL_2$  in clockwise direction. Since speed is positive and developed power is negative, it's forward braking operation.

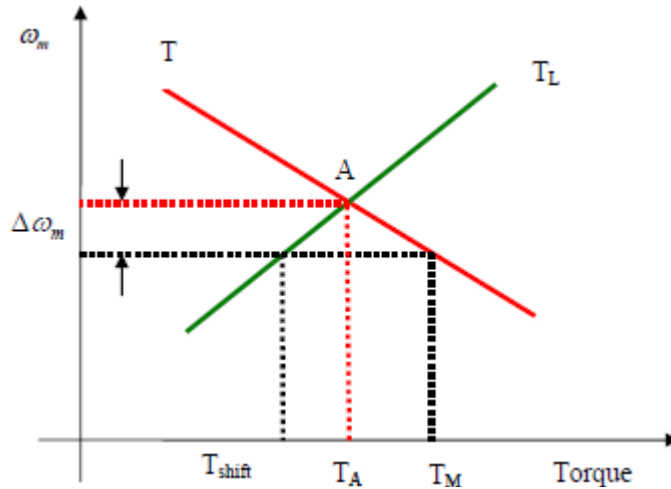
Operation in quadrant III is obtained when an empty cage is lowered. Since an empty cage has a lesser weight than a counter weight, the motor should produce a torque in CW direction. Since speed is negative and developed power is positive, this is reverse motoring operation.

### **Steady State Stability:**

Equilibrium speed of motor-load system can be obtained when motor torque equals the load torque. Electric drive system will operate in steady state at this speed, provided it is the speed of stable state equilibrium. Concept of steady state stability has been developed to readily evaluate the stability of an equilibrium point from the steady state speed torque curves of the motor and load system.

In most of the electrical drives, the electrical time constant of the motor is negligible compared with the mechanical time constant. During transient condition, electrical motor can be assumed to be in electrical equilibrium implying that steady state speed torque curves are also applicable to the transient state operation.

Now, consider the steady state equilibrium point A shown in figure below

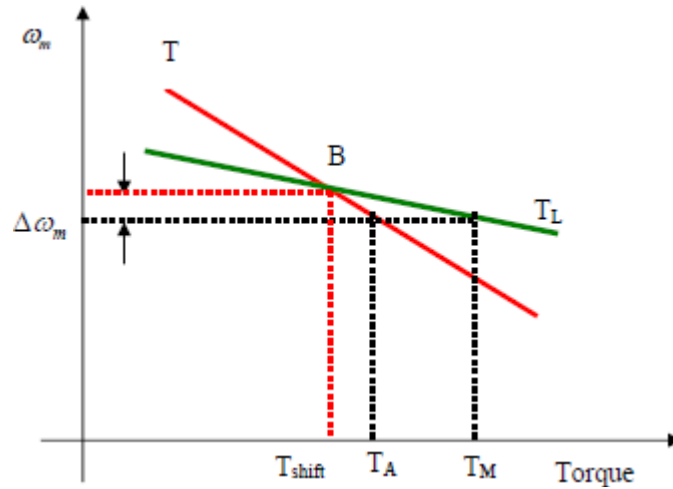


The equilibrium point will be termed as stable state when the operation will be restored to it after a small departure from it due to disturbance in the motor or load. Due to disturbance a reduction of  $\Delta m$  in speed at new speed, electrical motor torque is greater than the load torque, consequently motor will accelerate and operation will be restored to point A. Similarly an increase in speed caused by a disturbance will make load torque greater than the motor torque, resulting into deceleration and restoring of operation to point A.

Now consider equilibrium point B which is obtained when the same motor drives another load as shown in the figure. A decrease in speed causes the load torque to become greater than the motor torque, electric drive decelerates and operating point moves away from point B. Similarly when working at point B and increase in speed will make motor torque greater than the load torque, which will move the operating point away from point B.

From the above discussions, an equilibrium point will be stable when an increase in speed causes load torque to exceed the motor torque. (i.e.) When at equilibrium point following conditions is satisfied.

$$\frac{dT_L}{d\omega_m} > \frac{dT}{d\omega_m} \text{-----(1)}$$



Inequality in the above equation can be derived by an alternative approach. Let a small perturbation in speed,  $\Delta\omega_m$  results in  $\Delta T$  and  $\Delta T_L$  perturbation in  $T$  and  $T_L$  respectively. Therefore the general load torque equation becomes

$$\begin{aligned}
 (T + \Delta T) &= (T_L + \Delta T_L) + \frac{Jd(\omega_m + \Delta\omega_m)}{dt} \\
 = T + \Delta T &= T_L + \Delta T_L + \frac{Jd\omega_m}{dt} + J \frac{d\Delta\omega_m}{dt} \text{-----(2)}
 \end{aligned}$$

$$T = T_L + J \frac{d\omega_m}{dt} \text{-----(3)}$$

Subtracting (3) from (2) and rearranging

$$J \frac{d\Delta\omega_m}{dt} = \Delta T - \Delta T_L \text{-----(4)}$$

From small perturbations, the speed –torque curves of the motor and load can be assumed to be straight lines, thus

$$\Delta T = \left( \frac{dT}{d\omega_m} \right) \Delta \omega_m \text{-----} (5)$$

$$\Delta T_l = \left( \frac{dT_l}{d\omega_m} \right) \Delta \omega_m \text{-----} (6)$$

Where  $\frac{dT}{d\omega_m}$  and  $\frac{dT_l}{d\omega_m}$  are respectively slopes of the steady state speed torque curves of motor and load at operating point under considerations. Substituting (5) and (6) in (4) we get,

$$J \frac{d\Delta\omega_m}{dt} + \left( \frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m} \right) \Delta\omega_m = 0 \text{-----} (7)$$

This is a first order linear differential equation. If initial deviation in speed at  $t=0$  be  $(\Delta\omega_m)_0$  then the solution of equation (7) is

$$\Delta\omega_m = (\Delta\omega_m)_0 \exp \left\{ -\frac{1}{J} \left( \frac{dT_l}{d\omega_m} - \frac{dT}{d\omega_m} \right) t \right\} \text{-----} (8)$$

An operating point will be stable when  $\Delta\omega_m$  approaches zero as  $t$  approaches infinity. For this to happen exponential term in equation (8) should be negative.