

## DC motor characteristics

### The Equivalent Circuit of a Separately-Excited DC Motor

A separately-excited d.c. motor is a motor whose field circuit is supplied from a separate constant voltage power supply. Fig.11.5 shows the electrical equivalent circuits of a separately-excited d.c. motor. In this figure, the armature circuit is represented by an ideal voltage source  $E_a$  and a resistor  $R_a$  in series with armature inductance  $L_a$ . This representation is really the Thevenin equivalent of the entire rotor structure, including rotor coils, inter poles and compensating windings, if present.

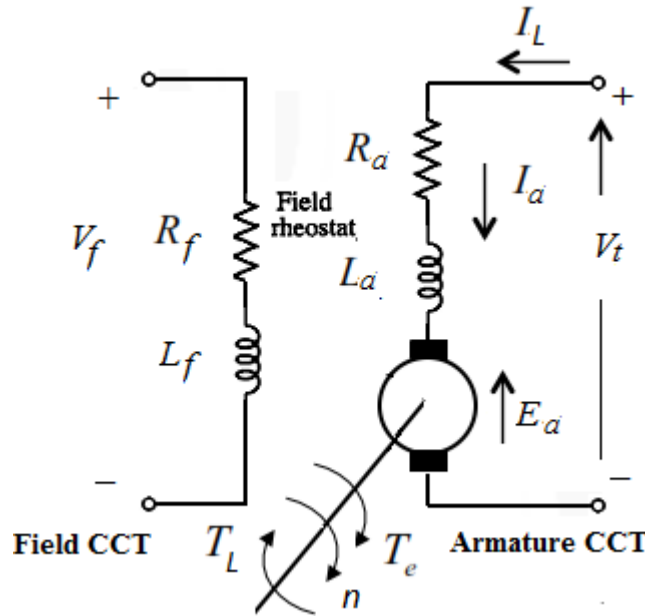


Fig.1.1 Electrical equivalent circuit of a separately-excited d.c. motor.

The field coils, which produce the magnetic flux in the motor, are represented by inductor  $L_f$  and resistor  $R_f$ . This circuit is approximate because we made some of the few simplifications:

- i- The brush drop voltage is often only a very tiny fraction of the generated voltage in the machine. Thus, in cases where it is not too critical, the brush drop voltage may be left out or included in the  $R_a$ .
- ii- The internal resistance of the field coils is sometimes lumped together with the variable resistor and the total is called  $R_f$ .

The principal equations of d.c. machine are:

The internal induced (back) *emf* is given by:

$$E_a = K_e \Phi n \dots\dots\dots 1.1$$

and the electromagnetic (developed) torque  $T_e$  is

$$T_e = K_T \Phi I_a \dots\dots\dots 1.2$$

The relation between the terminal voltage and the induced *emf* is

$$V_t = E_a + I_a R_a \dots\dots\dots 1.3$$

Input electrical power to the armature circuit

$$P_{in} = V_t I_a \dots\dots\dots 1.4$$

Developed power

$$P_d = T_e \omega \dots\dots\dots 1.5$$

Output mechanical power

$$P_{out} = T_L \omega \dots\dots\dots 1.6$$

where

- $n$  = speed of the motor in revolution per minute (rpm),
- $\Phi$  = flux per pole in Weber (Wb),
- $K_e$  = machine constant =  $p.Z / 60 a$ ,
- $P$  = number of poles,  $Z$  = total number of conductors in the armature circuit ,  $a$  = number of parallel paths (  $a = p$  for lap winding,  $a = 2$  for wave winding).
- $K_T$  = torque constant =  $9.55 K_e$  ,
- $I_a$  = armature current (A).
- $\omega$  = angular speed =  $2\pi n / 60$  (rad /s)

**Speed and Torque Equations**

The output characteristic (speed-torque relationship) of a separately- excited d.c. motor can be derived from the induced voltage equation (1.1) and torque equation (1.2) of the motor plus the motor general equation (1.3) as follows :

From Eq.(1.2) current  $I_a$  can be expressed as:

$$I_a = \frac{T_e}{K_T \Phi} \dots\dots\dots 1.7$$

Combining the  $V_t$  ,  $E_a$  and  $I_a$  equations:

$$V_t = K_e \Phi n + \frac{T_e}{K_T \Phi} R_a \dots\dots\dots 1.8$$

Finally, solving for the motor speed:

$$n = \frac{V_t}{K_e \Phi} - \frac{R_a}{K_e K_T \Phi^2} T_e \dots\dots\dots 1.9$$

The later is called the fundamental equation of the speed of d.c. motor. The no load

speed  $n_o$  is found when  $T_e = 0$ , hence the no load speed is,

$$n_o = \frac{V_t}{K_e \Phi} \dots\dots\dots 1.10$$

**Representation of the speed equation of the d.c. motor in terms of the angular velocity  $\omega$**

Referring to the two basic equations of d.c. motor Eqs.(1.1) and (1.2), in which the constants  $K_e$  and  $K_T$  are defined previously as the machine constant and the torque constant respectively. The relation between these two constants is,  $K_T = 9.55 K_e$  for all types of d.c. machine. In the SI system of units the constants  $K_e$  and  $K_T$  are identical ( $K_e = K_T = K$ ) and have the dimensions Newton metres per Weber ampere or Volt seconds per Weber radian. Since the angular velocity  $\omega = 2\pi n / 60$  rad /s, equations (1.1) and (1.2) can be re-written as,

$$E_a = K_e \Phi n = K_e \Phi \frac{60}{2\pi} \omega = 9.55 K_e \Phi \omega \dots\dots\dots 1.11$$

Or  $E_a = K_T \Phi \omega = K \Phi \omega \dots\dots\dots 1.12$

And  $T_e = K \Phi I_a \dots\dots\dots 1.13$

From which,

$$I_a = \frac{T_e}{K \Phi} \dots\dots\dots 1.14$$

Substituting Eq.(1.1) and Eq.(1.2) in Eq.(1.3) yields,

$$V_t = K \Phi \omega + \frac{T_e}{K \Phi} R_a \dots\dots\dots 1.15$$

Finally, solving for the motor speed:

$$\omega = \frac{V_t}{K \Phi} - \frac{R_a}{K^2 \Phi^2} T_e \dots\dots\dots 1.16$$

Equation (1.16) represents the general equation of the speed of d.c. motors in terms of the angular velocity  $\omega$  in rad per second which is used instead of  $n$  in rpm in many text books.

The no load speed is when  $T_e = 0$ , hence the no load speed  $\omega_o$  :

$$\omega_o = \frac{V_t}{K \Phi} \dots\dots\dots 1.17$$

During transient periods where  $n \neq$  constant

$$V_t = E_a + R_a I_a + L_a \frac{di_a}{dt} \dots\dots\dots 1.18$$

For steady-state ,  $n =$  constant and  $\frac{di_a}{dt} = 0$ .

## **MECHANICAL CHARACTERISTICS OF DC MOTORS IN DRIVING CONDITIONS**

When a d.c. motor is used in driving system, its basic operational characteristics are determined by both the values of the resisting torque  $T_L$  created by the load, and the electromechanical properties of the motor itself. In the steady-state operation of the drive, it has been shown in Chapter Ten, Section, that the value of the torque  $T_m$  developed by the motor should equal to the load torque, i.e .....As it is seen from Eq.(1.2) that the electromagnetic torque is proportional to the armature current  $I_a$  and the effective machine flux per pole  $\Phi$ . Thus the variation of the load torque should result in variation of the motor torque as well, i.e. variation of armature current  $I_a$  and the magnetic flux  $\Phi$ . The relation between the motor torque and the armature current,  $T_m = f(I_a)$  , is called the electrical characteristic or internal characteristic of the motor .

In drive systems, the internal characteristic of the motor is not very important, since we are interested in the motor shaft speed and not the current. Therefore, to find out the motor's actual speed corresponding to specific value of the motor torque, one should know the actual relation between the speed and torque  $n = f(T_m)$ , which is called the external or mechanical characteristic of the motor. In general, the mechanical characteristic of a d.c. motor depends on its type whether it is separately, shunt, series or compound. Each type has its own mechanical characteristic which is different from the others as it will be explained hereinafter.

### **Mechanical Characteristics of a Separately-Excited d.c. Motor**

The output characteristic of a separately excited d.c. motors which is the relation between the torque and speed is given in Eq.(1.9) or

Eq.(1.16). This equation is called the d.c. motor speed equation. Now since  $V_t, \Phi, K_e, K_T$  and  $R_a$  are all assumed constants, Eq.(1.9) can be expressed analytically as,

$$n = \alpha - \frac{1}{\beta} T_e = \alpha - m T_e \quad \dots\dots\dots 1.19$$

which is just a straight line with a negative slope  $m = \frac{1}{\beta}$ . However, In this equation

$$\alpha = \frac{V_t}{K_e \Phi} = n_o \quad \dots\dots\dots 1.20$$

Where  $n_o$ = no load speed , i.e. when  $T_e = 0$ .

$$\beta = \frac{K_T K_e \Phi^2}{R_a} \quad \dots\dots\dots 1.21$$

$\beta$  is a constant called the coefficient of hardness of the motor.

The resulting mechanical characteristics (speed-torque characteristics) of the separately-excited motor is shown in Fig.1.6 (Curve-1).

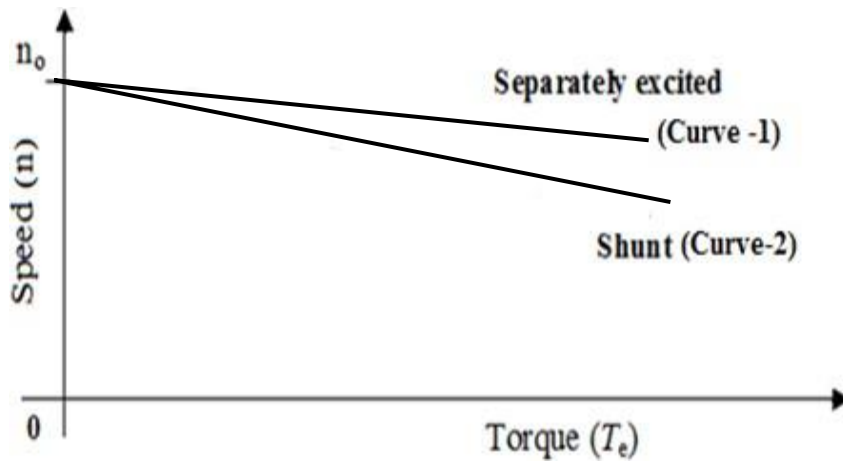
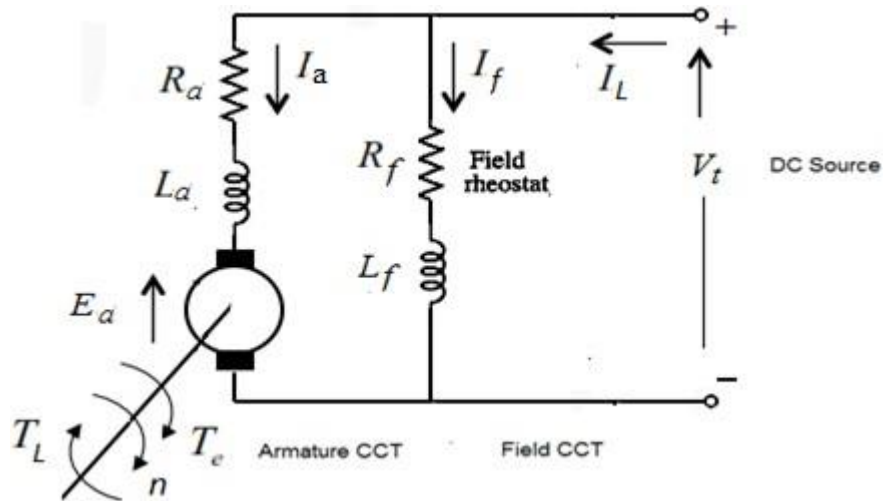


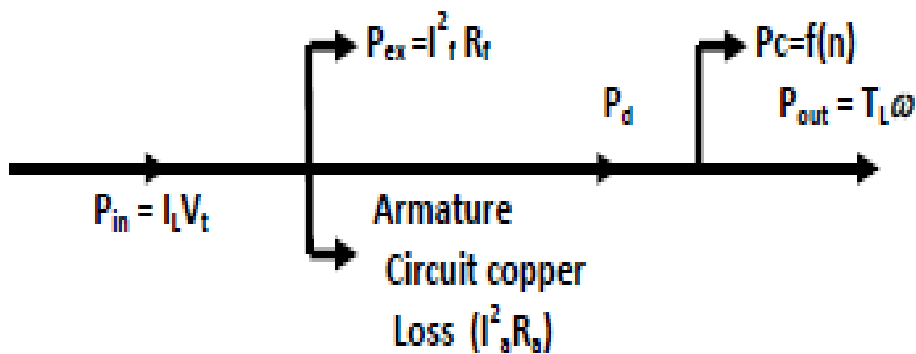
Fig.1.6 Speed-torque characteristic of separately-excited d.c. motor (Curve-1) and shunt (self-excited) d.c. motor (Curve-2).

## Mechanical Characteristics of Shunt d.c. Motor

The equivalent circuit of a shunt d.c. motor is shown in Fig.1.7(a). The output characteristic of a shunt and separately-excited d.c. motors are approximately the same. As it has been mentioned that the separately-excited d.c. motor is a motor whose field circuit is supplied from a separate constant-voltage d.c. source, whereas a shunt d.c. motor is a motor whose field circuit gets its power directly across the armature terminals of the motor as shown in Fig.1.7(a). This means that, when the



(a)



(b)

Fig.1.7 Shunt d.c. motor : (a)The equivalent circuit, (b) Power flow.

supply voltage  $V_t$  to a motor is assumed constant, there is no practical difference in behaviour between these two machines. Unless otherwise specified, whenever the behaviour of a shunt motor is described, the separately-excited motor is included too. Hence for shunt motor :

The KVL equation for the armature circuit is

$$V_t = E_a + I_a R_a \tag{1.22}$$

The currents relations is

$$I_L = I_a + I_f \tag{1.23}$$

The power flow diagram in shunt motor is shown in Fig.1.7(b).

$$P_d = T_e \omega = I_L V_t - (I_f V_t + I_a^2 R_a)$$

.....1.24

Note that:  $P_c$  = mechanical loss+ iron loss and both are speed dependent.

The speed equation of the shunt motor is the same equation of the separately-excited motor (Eqs.(1.9) and (1.16)). These equations are just a straight line with a negative slope. The resulting torque-speed characteristic of a shunt d.c. motor is also shown in Fig.1.6 (Curve-2).

To explain how does a shunt d.c. motor respond to a load, suppose that the load on the shaft of a shunt motor is increased, then the load torque  $T_L$  will exceed the developed torque  $T_e$  in the machine, and the motor will start to slow down. When the motor slows down, its internal generated voltage drops ( $E_a = K_e n \phi \downarrow$ ), so the armature current in the motor  $I_a = (V_t - E_a \downarrow) / R_a$  increases.

As the armature current increases, the developed torque in the motor increases ( $T_e = K_T I_a \phi$ ) and finally the developed torque will equal the load torque at a lower mechanical speed of rotation.

### Mechanical Characteristics of Series d.c. Motor

A series d.c. motor is a d.c. motor whose field windings consist of relatively few turns connected in series with the armature circuit. The equivalent circuit of this type of motor is shown in Fig.11.8. A distinct feature of a series motor is that the field current is equal to the armature current and the current drawn from the supply , i.e.

$$I_S = I_a = I_L \tag{1.25}$$

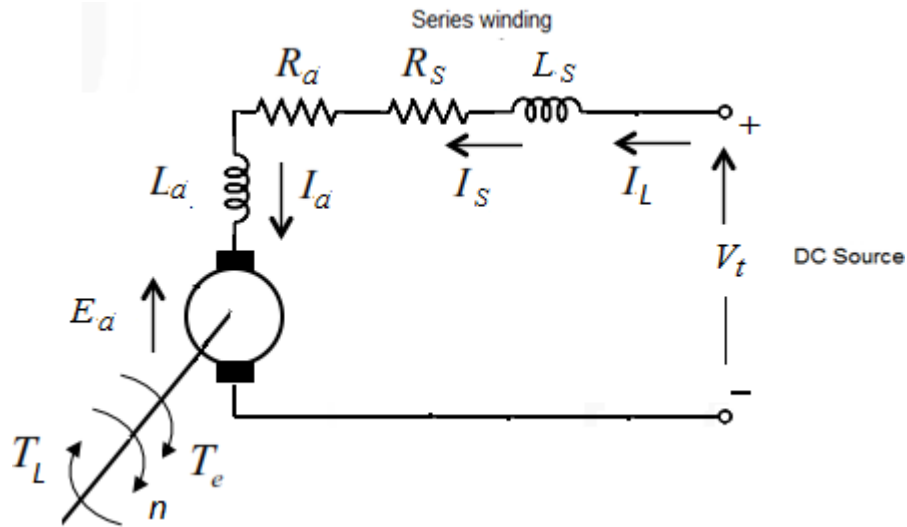


Fig.1.8 Series d.c. motor equivalent circuit.

### Induced Torque in a Series d.c. Motor

The basic behavior of a series d.c. motor is due to the fact that the flux is directly proportional to the armature current, at least until saturation is reached, (Fig.11.9). As the load on the motor increases, its flux increases too. As seen earlier, an increase in flux in the motor causes a decrease in its speed. The result is that a series d.c. motor has a sharply drooping speed-torque characteristic. The developed torque is

$$T_e = K_T \phi I_a \quad \dots\dots\dots 1.26$$

The flux in this machine is directly proportional to its armature current (at least until iron saturates). Therefore, the flux in the machine can be given by

$$\phi = K_f I_a \quad \dots\dots\dots 1.27$$



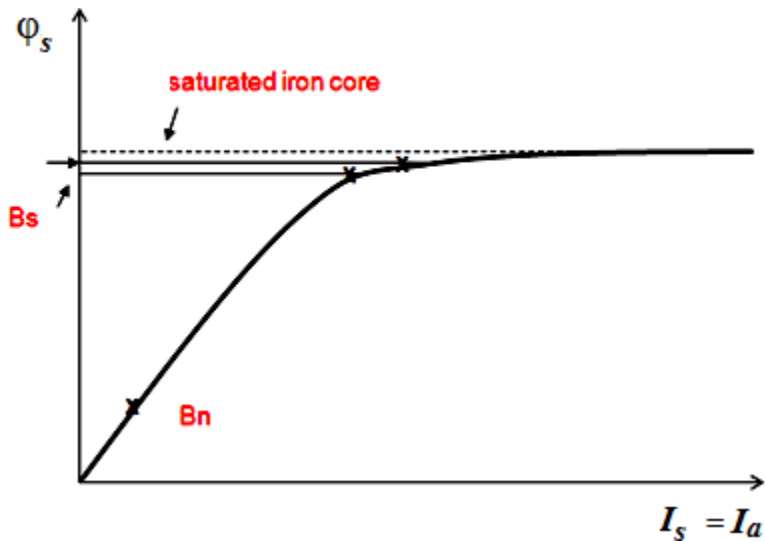


Fig.1.9 Magnetization curve of a series motor.

where is a constant. Thus,

$$T_e = K_T \phi I_A = K_T K_f I_a^2 \dots\dots\dots 1.28$$

Series d.c. motors are therefore used in applications requiring very high torques. Example: starter motors in cars, elevator motors, tractor motors etc.

**The Terminal Characteristic of a Series d.c. Motor**

The assumption of a linear magnetization curve implies that the flux in the motor will be given by Eq.(1.27). This equation will be used to derive the speed-torque characteristic curve for the series motor.

**Derivation of the speed-torque characteristic:**

Referring to Fig.11.8, the KVL for this motor is

$$V_t = E_a + I_a (R_a + R_s) \dots\dots\dots 1.29$$

The armature current  $I_a$  is given by,

$$I_a = \sqrt{\frac{T_e}{K_T K_f}} \dots\dots\dots 1.30$$

Also,  $E_a =$  , thus substituting this and Eq.(1.30) in Eq. (1.29) yields

$$V_t = K_e \phi n + \sqrt{\frac{T_e}{K_T K_f}} (R_a + R_s) \dots\dots\dots 1.31$$

If the flux can be eliminated from this expression, it will directly relate the torque of a motor to its speed  $n$ . Notice that  $I_a = \phi^2 / K_f$  and  $T_e = (K_T/K_f) \phi^2$  . Thus ,

$$\phi = \sqrt{\frac{K_f}{K_T}} \sqrt{T_e} \dots\dots\dots 1.32$$

Substituting Eq. (1.32) into Eq. (1.31), and solve for  $n$  results in:

$$n = \frac{V_t}{K_e \sqrt{\frac{K_f}{K_T}}} \times \frac{1}{\sqrt{T_e}} - \frac{(R_a + R_s)}{K_e K_f} \dots\dots\dots 1.33$$

and the torque equation is

$$T_e = T_m = K_T K_f I_a^2 \dots\dots\dots \mathbf{1.34}$$

The speed-torque curve of series motor will vary, according to Eq.(1.33) as shown in Fig.1.10. It can be easily, from Eq. (1.33), being noted that the speed-torque characteristic of series motor is a hyperbola

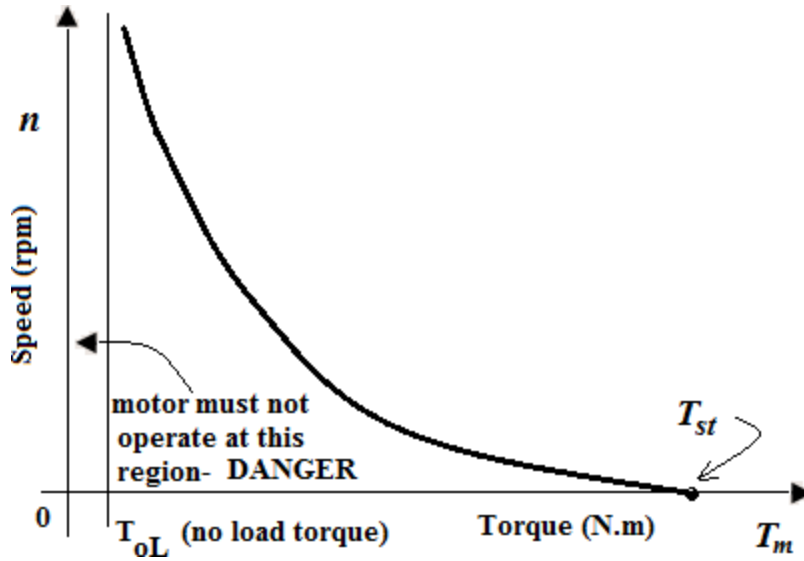


Fig.1.10 The speed-torque characteristic of a series d.c. motor.

with asymptote at the speed axis. To plot this hyperbola, substituting  $T_e = 0$  in Eq.(1.33) yields,  $n = n_o = \infty$  i.e. , the no load speed at the ideal no load running of the series motor is infinite (excessive speed). Therefore, at very small load torques are likely to cause rapid increase in the motor speed  $n$  which is dangerous to the motor mechanical construction (armature winding, bearings, and commutator structure).

Series motors can usually protected against the danger of excessive speeds by a positive connection to their load before starting. However, for small motors, below 200W, the mechanical losses in the motor may be sufficient to prevent excessive no load speed. In high power motors, even a threefold increase,  $n = 3n_n$ , where  $n_n$  is the normal (rated) speed may result in mechanical over stresses dangerous to the armature . Therefore, these machines should not be run at no load condition.

On the other hand, the starting torque can be calculated by setting  $n = 0$  in Eq. (1.33), thus

$$T_{st} = \frac{V_t^2}{(R_a + R_s)^2} K_T K_f \dots\dots\dots 1.35$$

Since the values of  $R_a$  and  $R_s$  are usually very small, hence the starting torque of the series motor is considerably large compared with that of separately-excited and

shunt motors. This feature makes it preferable to be used to start heavy loads such as electric vehicles, electric trains and elevators.

### Mechanical Characteristics of Compound d.c. Motor

The connection diagram of a compound motor is shown in Fig.1.11. The mechanical characteristics of the compound motor occupy intermediate position between the characteristics of the shunt and the series motors due to the fact that the compound motors contains both the shunt and the series windings.

Fig.1.12 depicts the speed-torque characteristics of the three types of d.c. motors namely, shunt, series and compound. As is clear from these characteristics, the compound motor at ideal idle running has an ultimate no load speed  $n_o$ . Beside, when compared with shunt motor, the compound machine develop stronger starting electromagnetic torque, and when compared with series motor, it exhibits more “rigid” mechanical characteristics.

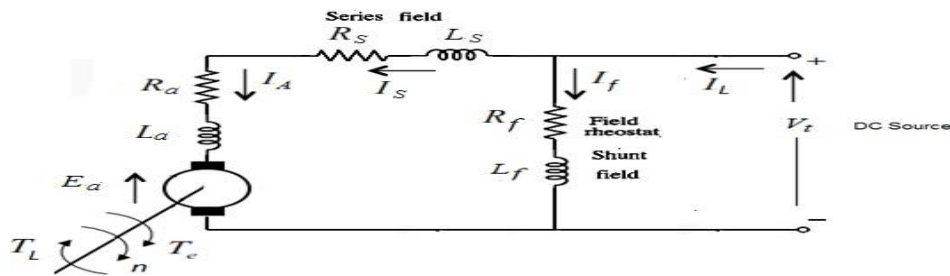
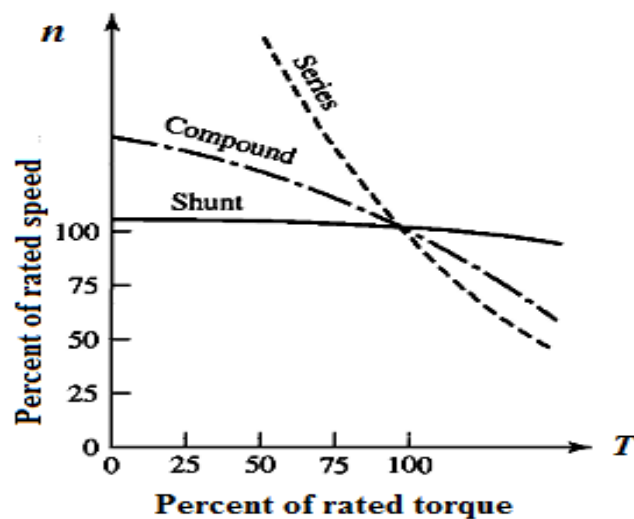


Fig.1.11: Compound motor connection diagram.

Fig.1.12  
Compound motor mechanical characteristics as compared with shunt and series motors.



## DC MOTORS SPEED CONTROL

The speed control of d.c. motors is very important subject in the consideration of the application of these motors. The various methods of speed control follow directly from the fundamental equation of the d.c. motor speed equation (11.9), from which one can predict the ways to control  $n$  if we write down it in the following simple form,

$$n = \frac{V_t - I_a R_a}{K_e \Phi} \dots\dots\dots 1.36$$

This equation shows that the speed is directly proportional to the applied voltage ( $V_t$ ), inversely proportional to the flux per pole and changing the armature resistance  $R_a$  by adding an external resistance in series with it. Therefore, it is clear that the motor speed  $n$  can be varied by the following methods:

- (i) Varying the terminal voltage  $V_t$ , hence varying the applied voltage to the armature  $V_a$ .
- (ii) Adjusting the field resistance  $R_f$  (and thus varying the field flux).
- (iii) Inserting a resistor in series with the armature circuit ( $R_a + R_{add} = \sum R_a$ ), (rheostat control).

The first method is the most common method to decrease or increase the speed, while the third one is rarely used now days since it results in excessive losses.

### Motor Speed Control of Shunt and Separately Excited d.c. Motors

#### (A) *Changing the Armature Voltage*

This method involves changing the voltage applied to the armature of the motor without changing the voltage applied to the field. If the voltage  $V_a$  is increased, then  $I_a$  rises, since  $[I_a = (V_a \uparrow - E_a) / R_a]$ . As  $I_a$  increases, the developed torque  $T_e = K_T \Phi I_a$  increases, making  $T_e > T_L$ , and the speed of the motor increases. Motor accelerated to new speed: (1→2→3) as shown in Fig.1.13. Now, as the speed increases,  $E_a (= \uparrow)$  increases, causing the armature current to decrease. This decrease in  $I_a$  decreases the developed torque, causing  $T_e = T_L$  at a higher rotational speed.

#### (B) *Changing the Field Resistance ( Field weakening method*

If the field resistance increases, then the field current decreases ( $I_f \downarrow = V_f / R_f \uparrow$ ), and as the field current decreases, the flux decreases as well. A decrease in flux causes an instantaneous decrease in the internal

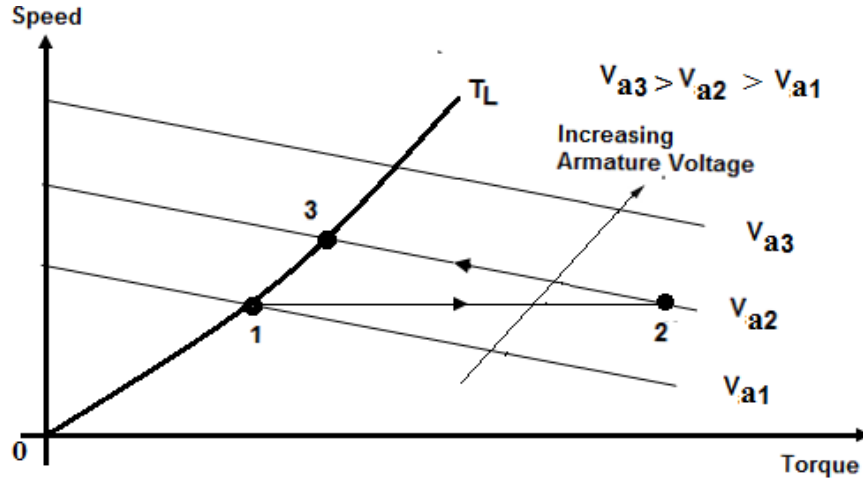


Fig.1.13 The effect of armature voltage variation on speed control of d.c. separately-excited and shunt motor.

generated voltage  $E_a \downarrow (=K_e \phi \downarrow n)$ , which causes a large increase in the machines armature current since, The developed torque in a motor is given by  $T_e = K_T \Phi I_a$  Since the flux in this machine decreases while the current  $I_a$  increases, so which way does the developed torque change?

To understand what is happening, the following example will illustrate the sequence of events for the motor shown in Fig.1.14:

$$I_a \uparrow = \frac{V_t - E_a}{R_a}$$

Fig.1.14 Shunt motor with added resistance in the field circuit.

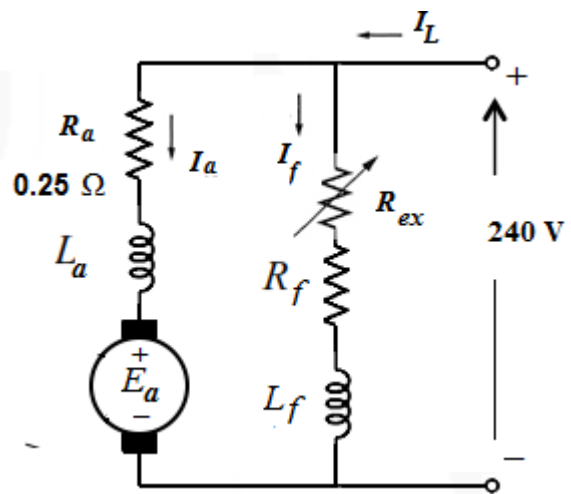


Figure 1.14 shows a shunt d.c. motor with an armature resistance of

0.25 Ω. It is currently operating with a terminal voltage of 240V and an internal generated voltage  $E_a$  of 235V. Therefore, the armature current flow is:  $I_a = (240V - 235V) / 0.25\Omega = 20A$ .

Now, what happens in this motor if there is a 1% decrease in flux?

If the flux decrease by 1%, then  $E_a$  must decrease by 1% too, because  $E_a = K_e \phi \downarrow n$ . Therefore,  $E_a$  will drop to:

$$E_{a2} = 0.99 E_{a1} = 0.99 (235) = 232.65V$$

The armature current must then rise to:

$$I_a = (240 - 232.65) / 0.25 = 29.4 A$$

Thus, a 1% decrease in flux produced a 47 % increase in armature current.

So, to get back to the original discussion, the increase in current predominates over the decrease in flux,  $T_e > T_L$  and the motor speeds up. However, as the motor speeds up,  $E_a$  rises, causing  $I_a$  to fall. Thus, developed torque  $T_e$  drops too, and finally  $T_e$  equals  $T_L$  at a higher steady- state speed than originally (1 → 2 → 3), see Fig.1.15.

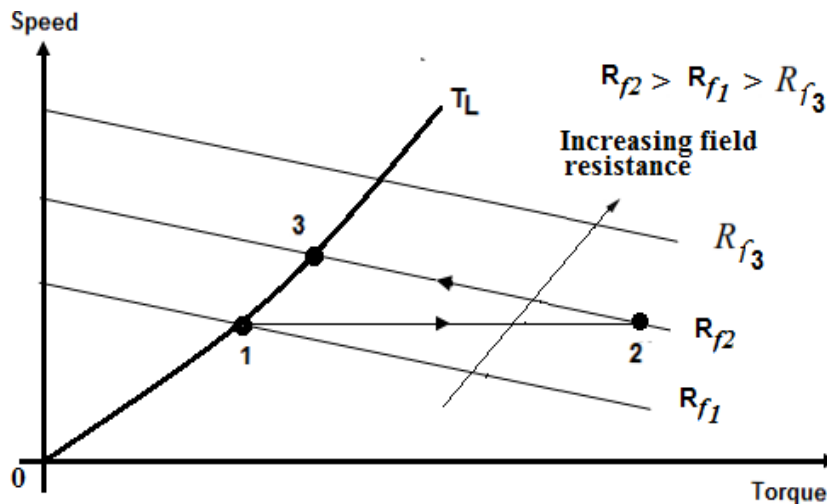


Fig.1.15 The effect of field resistance  $R_f$  variation on speed control of a shunt motor's speed-torque characteristics.

The effect of increasing the  $R_f$  is depicted in Fig.1.15 . Notice that as the flux in the machine decreases, the no-load speed of the motor increases, while the slope of the speed-torque curve becomes steeper.

**(C) Inserting a Resistor in Series with the Armature Circuit**

If a resistor is inserted in series with the armature circuit, see Fig.1.16, the effect is to drastically increase the slope of the motor's torque-speed characteristic, making it operate more slowly if loaded. This fact can be seen from the speed equation (1.9) which can be re-written as:

$$n = \frac{V_t}{K_e \Phi} - \frac{R_a + R_{add}}{K_e K_T \Phi^2} T_e \dots\dots\dots 1.37$$

If  $R_{add} \downarrow$ ,  $I_a$  and  $T_e \uparrow$ , hence motor accelerated ( $n$  increased from point 1 to point 3), see Fig.1.17.

If  $R_{add} \uparrow$ ,  $I_a$  and  $T_e \downarrow$ , hence motor decelerated ( $n$  reduced from point 4 to point 6).

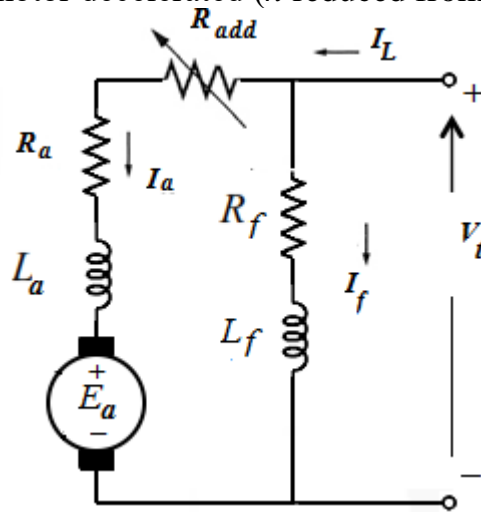


Fig.1.16 Rheostat speed control of shunt motor.

The insertion of a resistor is a very wasteful method of speed control, since the losses in the inserted resistor are very large. For this reason, it is rarely used.

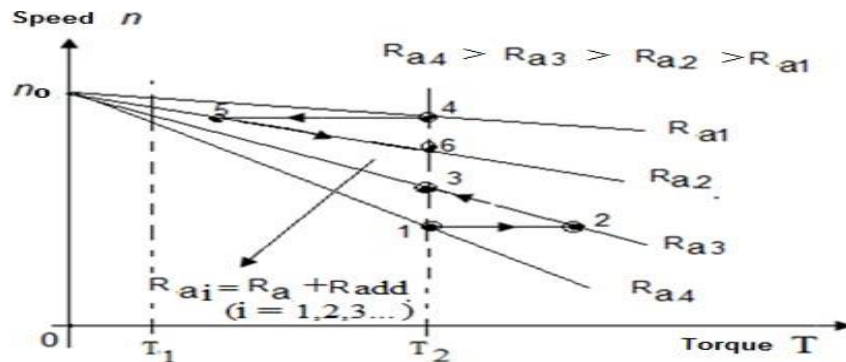




Fig.1.17 Speed variation by insertion additional resistance  $R_{add}$  in the armature circuit.

## **Safe Ranges of Operation for the Two Common Methods**

### *Field Resistance Control*

- The lower the field current in a shunt (or separately-excited) d.c. motor, the faster it turns; and the higher the field current, the slower it turns. Since an increase in field current causes decrease in speed, there is always a minimum achievable speed by field circuit control. This minimum speed occurs when the motor's field circuit has the maximum permissible current flowing through it.
- If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Field resistance control can control the speed of the motor for speeds above base speed but not for speeds below base speed. To achieve a speed slower than base speed by field circuit control would require excessive field current, possibly burning up the field windings.

### *Armature Voltage Control*

- The lower the armature voltage on a separately-excited d.c. motor, the slower it turns, and the higher the armature voltage, the faster it turns. Since an increase in armature voltage causes an increase in speed, there is always a maximum achievable speed by armature voltage control. This maximum speed occurs when the motor's armature voltage reaches its maximum permissible level.
- If a motor is operating at its rated terminal voltage, power and field current, then it will be running at rated speed, also known as base speed. Armature voltage control can control the speed of the motor for speeds below base speed but not for speeds above base speed. To achieve a speed faster than base speed by armature voltage control would require excessive armature voltage, possibly damaging the armature circuit.

These two techniques of speed control are obviously complementary. Armature voltage control works well for speeds below base speed, and field resistance control works well for speeds above base speed.

- There is a significant difference in the torque and power limits on the machine under these two types of speed control. The limiting factor in either case is the heating of the armature conductors, which places an upper limit on the magnitude of the armature current  $I_a$ .

For armature voltage control, the flux in the motor is constant, so the maximum torque in the motor is  $T_{max} = K_T \phi I_{a,max}$ . This maximum torque is constant regardless of the speed of the rotation of the motor. Since the power out of the motor is given by  $P = T\omega$ , ( $\omega = 2\pi n / 60$ ) is the angular velocity of the motor shaft), the maximum power is  $P_{max} = T_{max} \omega$ . Thus, the maximum power out is directly proportional to its operating speed under armature voltage control.

- On the other hand, when field resistance control is used, the flux does change. In this form of control, a speed increase is caused by a decrease in the machine's flux. In order for the armature current limit is not exceeded, the developed torque limit must decrease as the speed of the motor increases. Since the power out of the motor is given by  $P = T\omega$  and the torque limit decreases as the speed of the motor increases, the maximum power out of a d.c. motor under field current control is constant, while the maximum torque varies as the reciprocal of the motor's speed, see Fig.1.18.

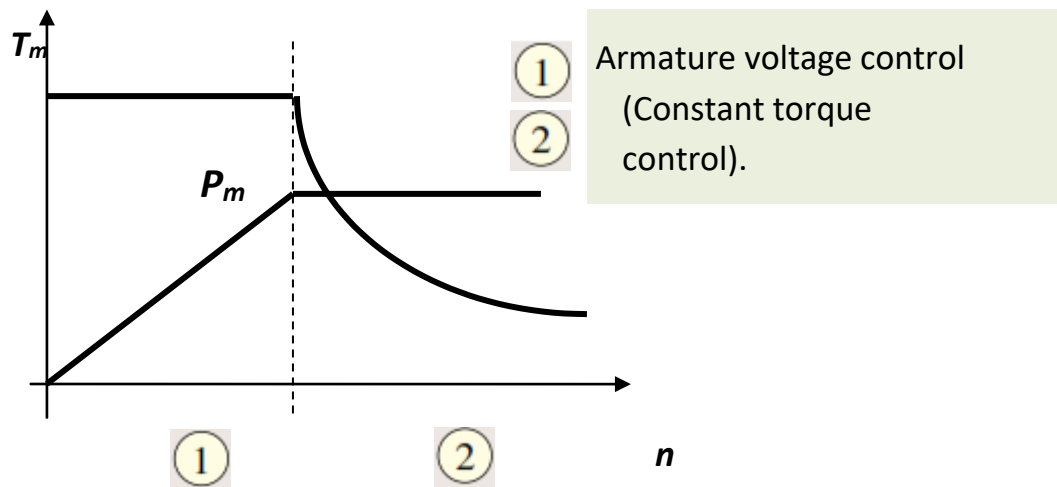


Fig.1.18 Torque and power limits for a d.c. motor.

## Speed Control of Series d.c. Motors

Speed control of series d.c. motor may be achieved through either field control, or armature control method:

**(A) Field control methods:** The speed of a series motor can be controlled by varying the flux in any one of the following ways.

**(i) Using field diverting resistor:** The field current in a series motor winding can be reduced by connecting a shunt resistance across the field winding so that a small portion of the field current is diverted to the shunt resistance thus reducing the excitation  $mmf$  and weakening the flux, Fig.1.19 (a).

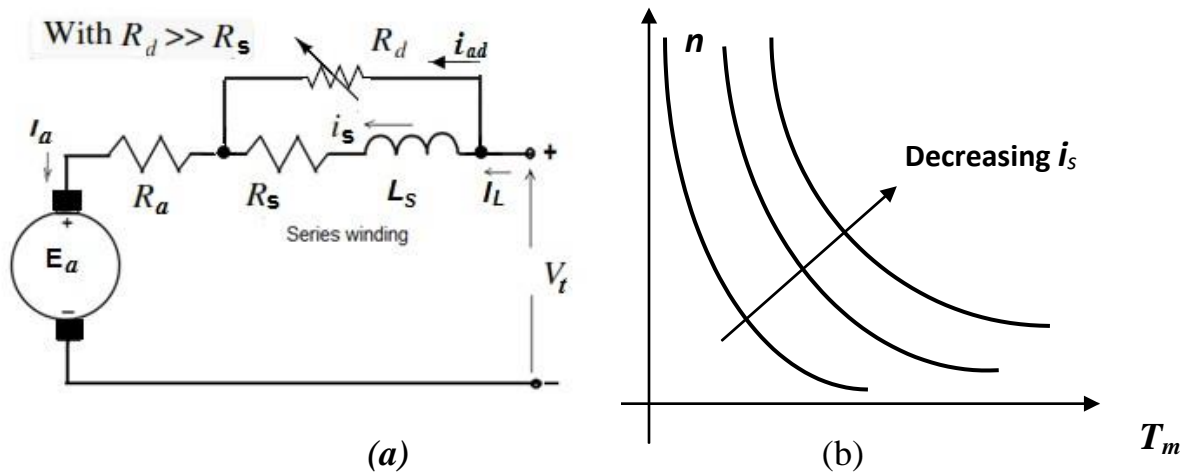


Fig.1.19 Speed control of series motor using diverter resistor

$R_d$  (field control): (a) Circuit connection, (b) Characteristics.

As the field current  $I_s$  is reduced,  $\phi$  will be reduced accordingly since  $\phi = K_s I_s$  and the speed  $n$  will increase according to Eq.(1.8) and Eq.(1.9). To find an equation for the speed as a function of the diverting resistance :

By KVL :

$$E_a = V_t - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L$$

$$E_a = K_e \phi n = K_e K_s I_s n = K_{es} I_s n$$

$$K_{es} I_s n = V_t - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L$$

But

$$I_s = \frac{R_d}{(R_d + R_s)} I_L$$

$$K_{es} n \frac{R_d}{(R_d + R_s)} I_L = V_T - I_L R_a - \frac{R_d R_s}{(R_d + R_s)} I_L$$

that result in :

$$n = \frac{V_t (R_d + R_s)}{K_{es} I_L R_d} - \frac{R_s}{K_{es}} - \frac{(R_d + R_s) R_a}{K_{es} R_d}$$

.1.38

This method gives speeds above normal because the reduction of the flux. As the field current is decreased, the speed-torque characteristics are shifted upward parallel to each other as shown in Fig.1.19(b). The diverting resistor should be highly inductive so that any change in the load current will not immediately affect the field winding current.

**(ii) Using tapped-field winding:** To increase the speed of the series motor, the flux is reduced by reducing the number of turns of the field winding. This is achieved by using field winding with a number of tapping brought outside, as shown in Fig.1.20.

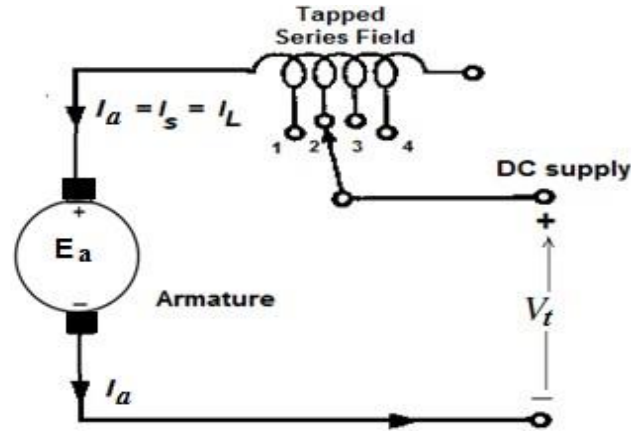


Fig.1.20 Speed control of series motor using tapped-field method.

**(B) Armature control methods:** speed control of series motor by armature control may be accomplished by one of the following methods,

**(i) Armature terminal voltage control:** Unlike with the shunt d.c. motor, there is only one efficient way to change the speed of a series d.c. motor. That method is to change the terminal voltage of the motor. If terminal voltage is increased, the speed will increase for any given torque.

From Eq. (1.38), the speed of a series motor can be controlled by variation of the terminal voltage  $V_t$  using variable d.c. supplies. Although this method was very expensive to be achieved in the past, it becomes the most common in use now a days due to the advent of the powerful power semiconductor devices which provide a cheap, small size and reliable variable d.c. supplies such as the d.c. choppers and controlled rectifiers.

**(ii) Armature resistance control:** This method is obtained by the same way as for d.c. shunt motor with the exception that the control resistance is connected in series with the supply voltage such that the total armature resistance seen by the supply (Thevenin equivalent resistance) is  $(\sum R_a = R_a + R_s + R_{add})$ , see Fig.1.21(a).

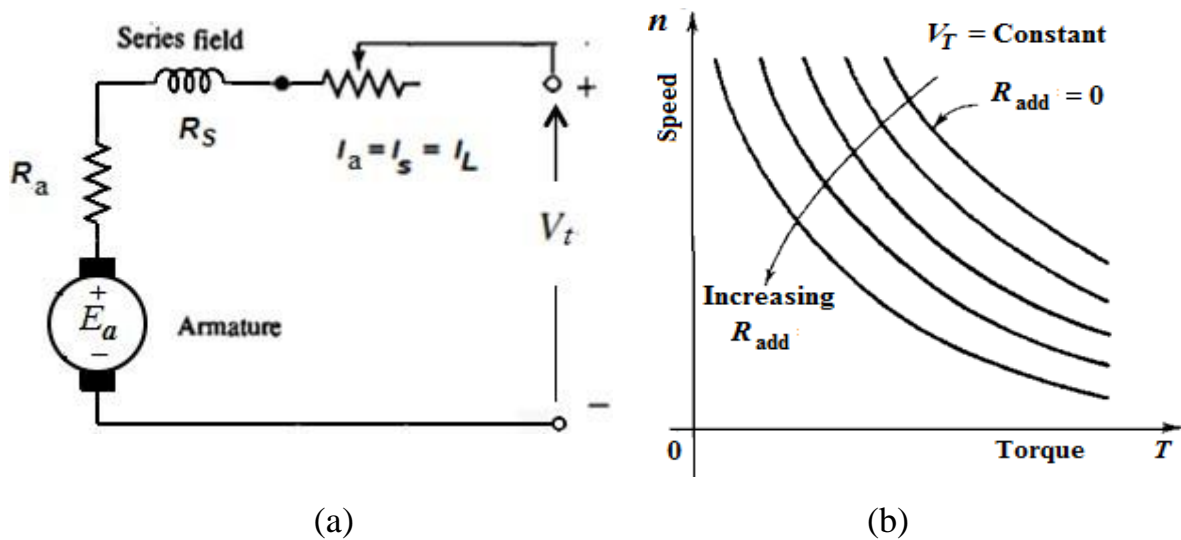


Fig.1.21 Speed control of series motor using armature resistance control.

The speed-torque characteristics for various values of  $R_{add}$  are shown in Fig.1.21(b). For particular value of  $R_{add}$ , the speed is almost inversely proportional to the square root of the torque as it can be seen from Eq.(1.39) ,

$$n = \frac{V_t}{\phi} \times \frac{1}{\sqrt{T_e}} - \frac{(R_a + R_s + R_{add})}{K_e K_f} \dots\dots\dots 1.39$$

*A high torque is obtained at low speed and a low torque is obtained at high speed. Series motors are therefore used where large starting torques are required as in hoist, cranes, etc.*