

Signal -

A function of one or more independent variables which contains some information

eg:- Radio signal, T.V. signal etc.

One Dimensional Signal -

When function depends on a single variable,

eg:- Speech signal whose amplitude varies with time.

Multi Dimensional Signal -

When functions depends on two or more variables.

eg:- image as it is a 2-dimensional signal with horizontal and vertical co-ordinates.

Classification of Signals -

Based upon their nature and characteristics in the time domain, s/g is classified as

1) Continuous-time s/g

2) Discrete-time s/g.

Subject

1) Continuous time signal - (Analog signal)

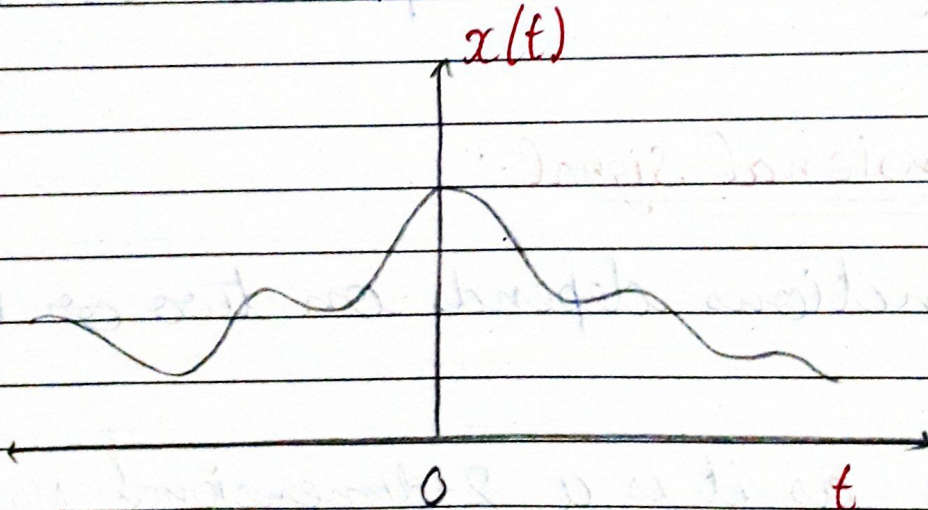
A s/g of continuous amplitude and time.

* This s/g will have certain value at every instant of time.

* It is represented by $x(t)$

where $x \rightarrow$ represent shape of s/g

$t \rightarrow$ represent time. It is the Independent Variable.



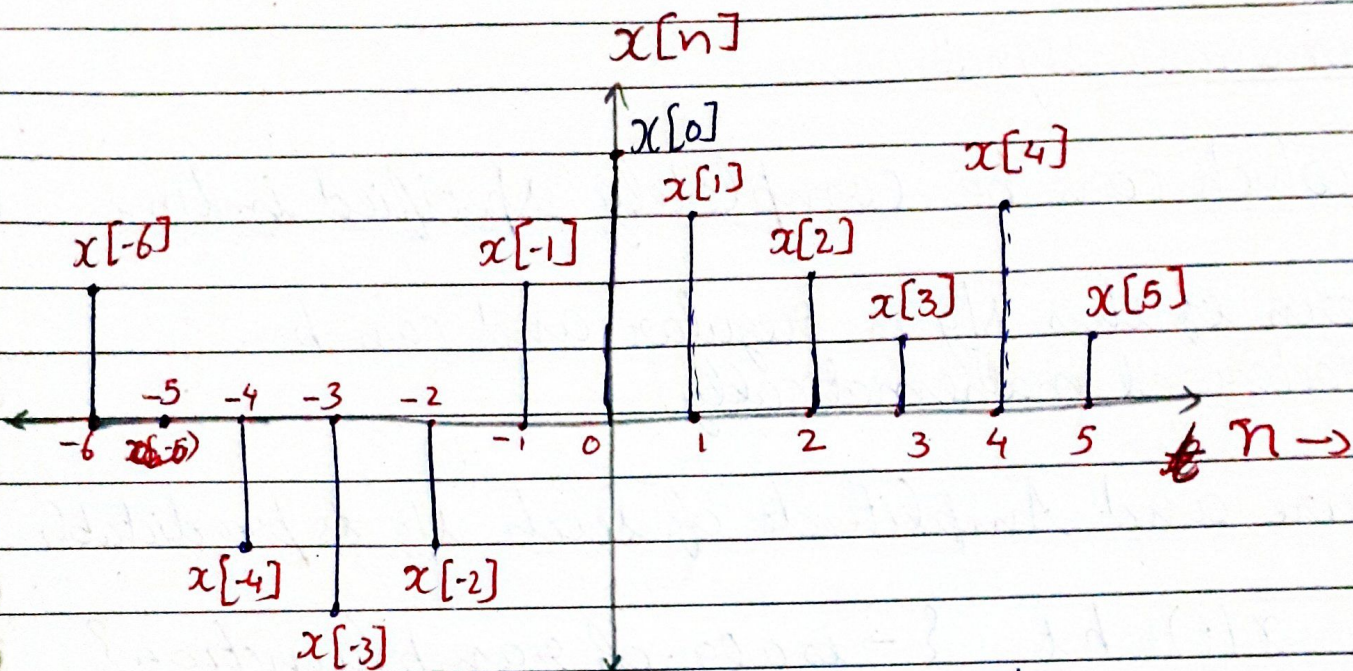
A continuous Time s/g.

2) Discrete time signals -

A s/g which represent only at discrete instant of time. (it has values at only certain intervals).

Subject

* It is represented by $x[n]$ square bracket
independent variable



A discrete time s/g.

* Further both continuous time s/g and discrete time s/g can be classified as -

- 1) Deterministic and non-deterministic (random) signal.
- 2) Periodic and non-periodic s/g.
- 3) Even and odd s/g.
- 4) Energy and Power s/g.

1) Deterministic and Non-Deterministic

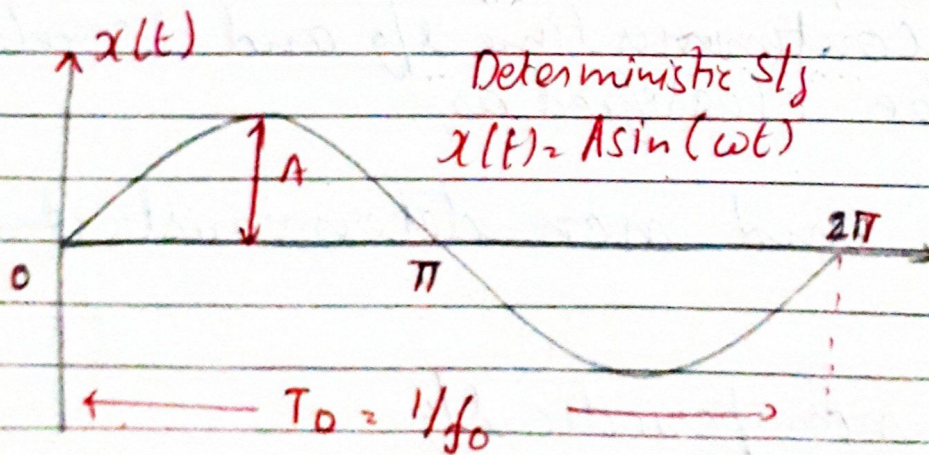
Deterministic signal-

s/g which can be completely specified in time.

- * Pattern of this s/g is regular and can be characterized mathematically.
- * Nature and Amplitude of such s/g is predictable.

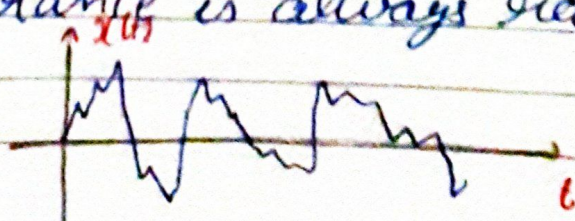
eg - $x(t) = bt$ $\left\{ \rightarrow \text{is a eq. of ramp function?} \right.$

$x(t) = a \sin \omega t$ $\left\{ \rightarrow \text{sinusoidal s/g with Amp } A? \right.$



Random s/g - (Non Deterministic s/g) -

A s/g whose occurrence is always random in nature.



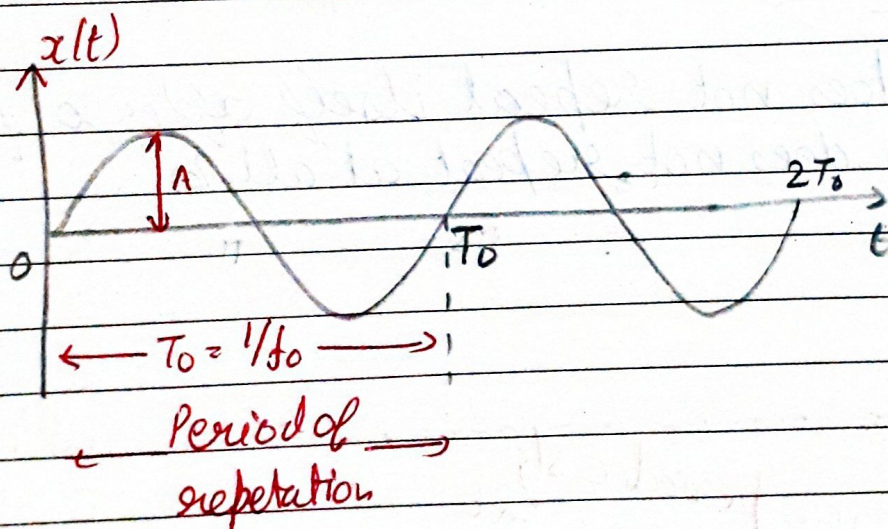
Subject

2) Periodic and Non Periodic s/g :-1) Periodic s/g - (continuous s/g)

A s/g which repeat itself after a fixed time period.

Mathematically,

$$x(t) = x(t + T_0) \rightarrow \text{condition of Periodicity}$$

where $T_0 =$ Period of s/g $x(t)$ 

Periodic s/g.

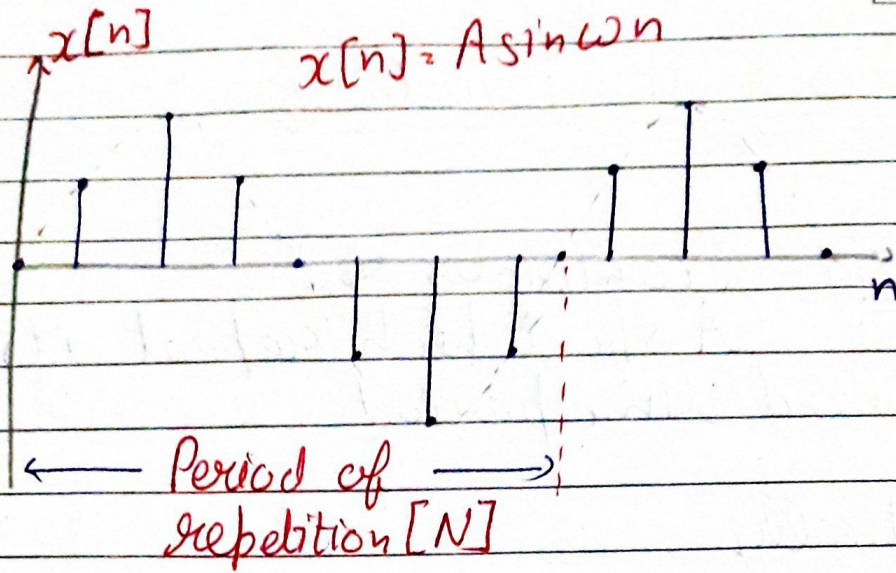
Per Periodic s/g (Discrete s/g)

Mathematically,

$$x[n] = x[n + N] \rightarrow \text{condition of Periodicity}$$

where $N \rightarrow$ period of s/g.

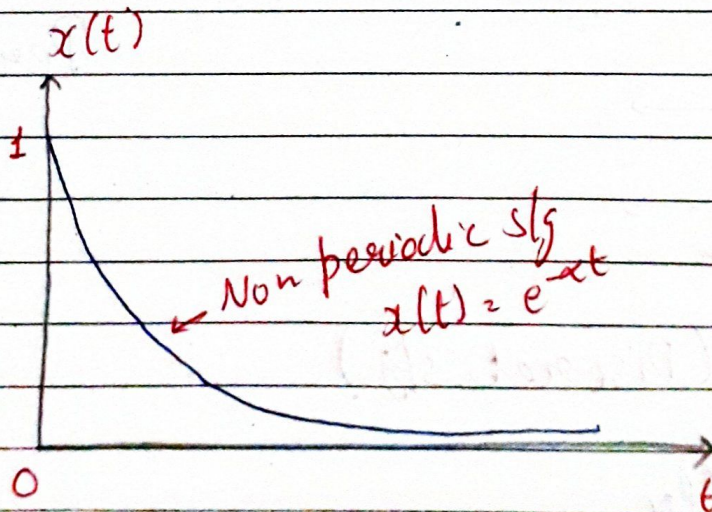
Subject



Discrete time periodic s/g.

2) Non-Periodic s/g : (Aperiodic s/g)

A s/g which does not repeat itself after a fixed time period or does not repeat at all.



Mathematically, (condition for Non periodic s/g)

$x(t) \neq x(t + T_0)$	→	not CT s/g
$x[n] \neq x[n + N]$	→	not DT s/g.

Q Condition for periodicity of a discrete-time s/g :-

Sol. A discrete time sinusoidal s/g is periodic only if its frequency f_0 is rational.

ie. $f_0 \rightarrow$ should be in form of ratio of two integers.

Proof :-

Let for D.T. s/g condition of periodicity is given by

$$x[n+N] = x[n] \quad - (1)$$

Let $x[n]$ be a cosine wave.

$$\therefore x[n] = A \cos [2\pi f_0 n + \theta] \quad - (2)$$

where $A \rightarrow$ Amplitude
 $\theta \rightarrow$ Phase shift

Now replace n by $[n+N]$ in eq (2), we get

$$x[n+N] = A \cos [2\pi f_0 (n+N) + \theta] \quad - (3)$$

comparing eq (1), (2) & (3), we get

$$A \cos [2\pi f_0 n + \theta] = A \cos [2\pi f_0 (n+N) + \theta]$$

$$A \cos [2\pi f_0 n + \theta] = A \cos [2\pi f_0 n + 2\pi f_0 N + \theta] \quad - (4)$$

Let us consider K (an integer). If we have to satisfy the above eq.

$$2\pi f_0 N = 2\pi K$$

$$\therefore f_0 = \frac{K}{N} \rightarrow \text{integers}$$

Periodicity condition for $x[n] = x_1[n] + x_2[n]$

Let $x_1[n]$ & $x_2[n]$ are both periodic discrete sig. sequence.

Let f_1 & f_2 be the corresponding frequency.

\therefore Acc. to condition of periodicity

$$f_1 = \frac{K_1}{N_1} \quad \& \quad f_2 = \frac{K_2}{N_2}$$

\therefore resultant sig $x[n]$ is periodic if $\frac{N_1}{N_2}$ is

ratio of two integers.

* Period of $x[n]$ will be least common multiple of N_1 & N_2 .

Formula for fundamental angular freq.

$$\omega = \frac{2\pi}{N}$$

$N \rightarrow$ no. of samples in one cycle.

1) Prove that the sine wave is a periodic s/g.

Sol. Let us consider a sine wave $x(t)$ which is mathematically represented as

$$x(t) = A \sin \omega_0 t$$

Condition for periodicity:

$$x(t+T_0) = A \sin \omega_0 (t+T_0)$$

$$= A \sin (\omega_0 t + \omega_0 T_0) \quad \text{--- (1)}$$

As, we know $\omega_0 = 2\pi f_0$

$$\text{and } T_0 = \frac{1}{f_0}$$

$$\therefore \omega_0 T_0 = 2\pi f_0 \times \frac{1}{f_0}$$

$$\boxed{\omega_0 T_0 = 2\pi}$$

Subst. the value of $\omega_0 T_0$ in eq (1)

$$x(t+T_0) = A \sin [\omega_0 t + 2\pi] \quad \left\{ \begin{array}{l} \sin(A+B) = \sin A \cos B \\ + \cos A \sin B \end{array} \right.$$

$$= A \{ \sin(\omega_0 t) \cos 2\pi + \cos(\omega_0 t) \sin 2\pi \}$$

$$x(t+T_0) = A \{ \sin \omega_0 t \}$$

Hence sine wave is a periodic s/g.

$$\left\{ \begin{array}{l} \cos 2\pi = 1 \\ \sin 2\pi = 0 \end{array} \right.$$

2) Prove that the exponential signal is a non-periodic s/g.

Sol. Let $x(t) = e^{-\alpha t}$ be the exponential s/g. — (1)

Let $t = (t + T_0)$

Sub. the value of t in equation of exponential s/g.

$$x(t + T_0) = e^{-\alpha(t + T_0)} = e^{-\alpha t} \cdot e^{-\alpha T_0}$$

but $T_0 = \infty$

$$e^{-\alpha T_0} = e^{-\infty} = 0$$

$$x(t + T_0) = e^{-\alpha t} \cdot 0 = 0 \quad \text{--- (2)}$$

comparing eq (1) & (2)

$$x(t) \neq x(t + T_0)$$

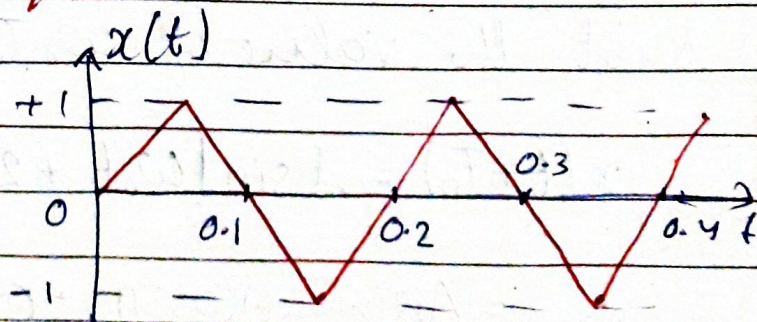
Hence exponential s/g is not a periodic s/g.

3) What is the fundamental freq. of the waveform shown below in Hz & rad/sec.

Sol.

If we see the diagram clearly.

One cycle = 0.2 sec.



$$T_0 = 0.2 \text{ sec}$$

$$\text{Frequency } (f_0) = \frac{1}{T_0} = \frac{1}{0.2} = \underline{\underline{5 \text{ Hz}}}$$

$$\begin{aligned} \text{Frequency in rad/sec } \omega_0 &= 2\pi f_0 \\ &= 2 \times 3.14 \times 5 \\ &= \underline{\underline{31.4 \text{ rad/sec}}} \end{aligned}$$

4) Determine whether the following discrete-time s/g are periodic or not. If periodic determine fundamental period.

1) $\cos(0.01\pi n)$

Sol. Let $x[n] = \cos(0.01\pi n)$ — (1)

Sub. comparing with standard eq. $x[n] = \cos 2\pi f n$ — (2)

$$2\pi f \cdot n = 0.01\pi n$$

$$f = \frac{0.01}{2} = \frac{1}{200} = \frac{K}{N}$$

$\therefore f$ is expressed in the ratio of two integers with $K=1$ & $N=200$.

Hence s/g is periodic with $N=200$.

2) $x(n) = \cos 3\pi n \sin 3n$

Comparing with standard eq. $x[n] = \cos 2\pi f n$

$$2\pi f n = 3\pi n \Rightarrow f = \frac{K}{N} = \frac{3}{2\pi} \text{ which is not an integer}$$

3) Even (Symmetrical) or Odd (Antisymmetrical) s/g

Symmetrical Signal (continuous Time)

A s/g $x(t)$ is said to be symmetrical or even if it satisfies the condition.

$$x(t) = x(-t)$$

where $x(t)$ = value of s/g for $+ve(t)$.

$x(-t)$ = value of s/g for $-ve(t)$.

Asymmetrical s/g (continuous Time)

A s/g $x(t)$ is said to be antisymmetrical or odd if it satisfies the condition.

$$x(t) = -x(-t)$$

Even Discrete Time s/g :

$$x[n] = x[-n]$$

Odd Discrete Time s/g :

$$x[n] = -x[-n]$$

Decomposing a s/g into Even and Odd parts

Let $x(t)$ be the s/g which is expressed in even & odd part.

$$x(t) = x_e(t) + x_o(t) \quad - (1)$$

where $x_e(t)$ = Even component of s/g $x(t)$

$x_o(t)$ = Odd component of s/g $x(t)$

put $t = -t$ in eq (1)

$$x(-t) = x_e(-t) + x_o(-t) \quad - (2)$$

Expression for even part $x_e(t)$

$$x_e(t) = x_e(-t) \quad \{\text{for even s/g}\}$$

$$x_o(t) = -x_o(-t) \quad \{\text{for odd s/g}\}$$

Sub. the above value in eq. (2)

$$x(-t) = x_e(t) - x_o(t) \quad - (3)$$

adding eq (1) & (3) we get -

$$x(t) + x(-t) = 2x_e(t)$$

$$x_e(t) = \frac{1}{2} \{x(t) + x(-t)\}$$

Expression for the odd part $x_o(t)$

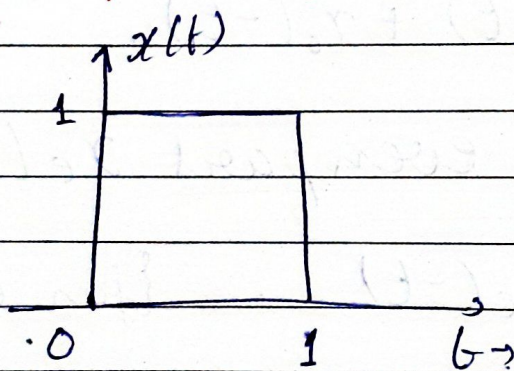
Subtracting eq (3) ~~from~~ ^{from} eq (1).

$$x(t) - x(-t) = 2x_o(t)$$

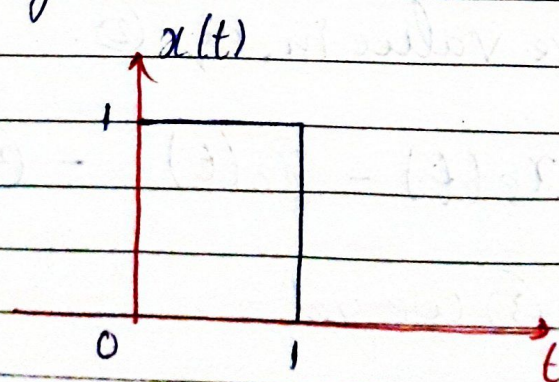
$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$



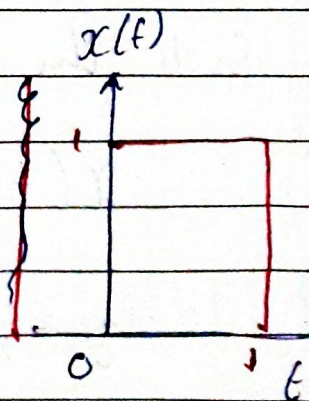
Q. Draw even & odd part of $x(t)$



Sol. Step 1 → Draw s/g $x(t)$



(a)

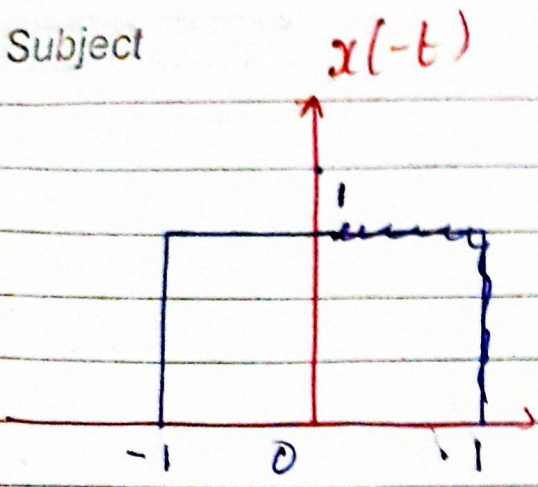


(b)

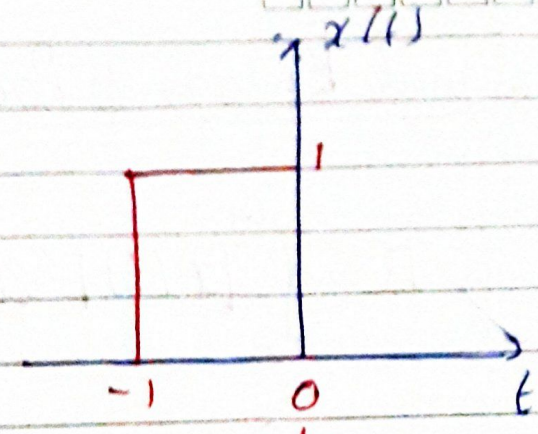
Step 2 →

Draw the folded version (mirror image) $x(-t)$.

Subject

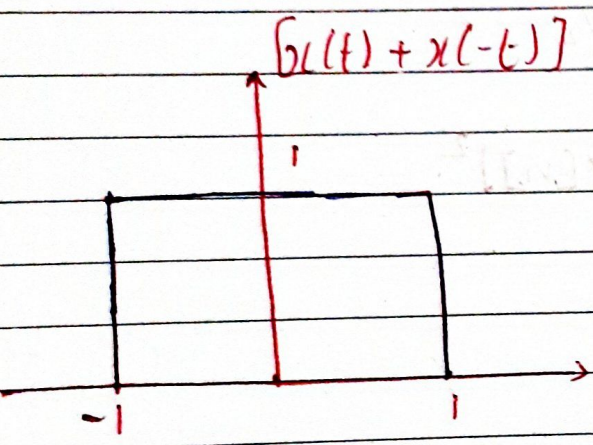


(c)

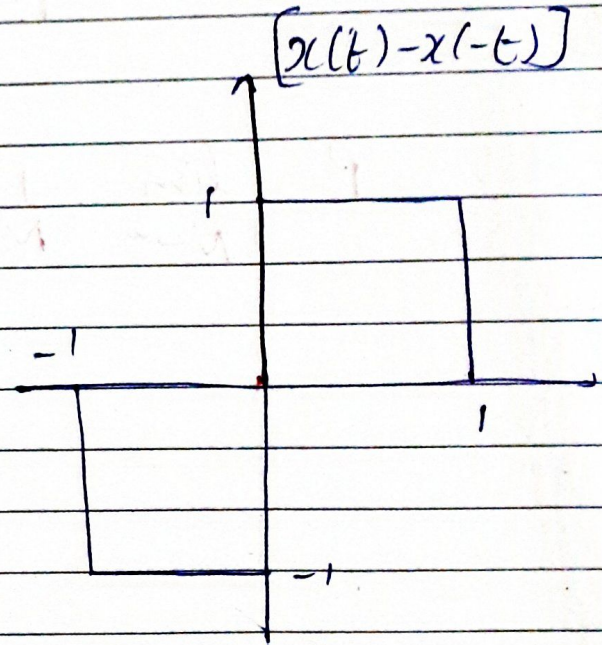


(d)

Step 3: Next, we add $x(t)$ and $x(-t)$ or subtract $x(-t)$ from $x(t)$



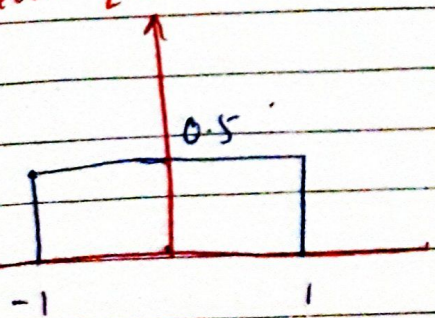
(e)



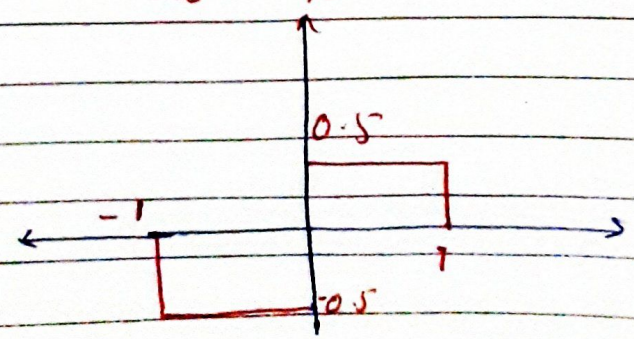
(f)

Step 4: divide the addition & sub. by 2 to get $x_e(t)$ & $x_o(t)$

$$x_e(t) = \frac{1}{2} [x(t) + x(-t)]$$



$$x_o(t) = \frac{1}{2} [x(t) - x(-t)]$$



4) Signal Energy & Power

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

$$P(\text{Power}) = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x^2(t)| dt$$

$$P = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} |x[n]|^2$$

Energy and Power signals.Energy signal:-

which has finite energy and zero average power.

If

$$0 < E < \infty \text{ \& } P = 0 \quad \text{--- (1)}$$

then $x(t)$ is an energy signal.Where E is energy
 P is Power.Power signal:-

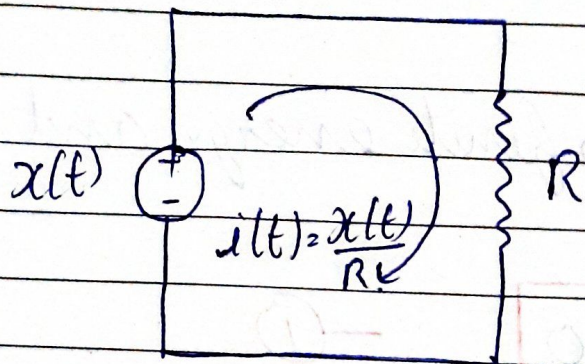
which has finite average power and infinite energy.

If

$$0 < P < \infty \text{ and } E = \infty \quad \text{--- (2)}$$

* if a s/g $x(t)$ does not satisfy any of the above two condition, then it is neither energy s/g nor a power s/g.

Derivation for Energy S/g -



Let $x(t)$ be the voltage applied across a $(R) \Omega$ resistor.

Let $i(t)$ be the current

$$i(t) = \frac{x(t)}{R}$$

Let $p(t)$ be the instantaneous power, which is given by

$$p(t) = v(t) \cdot i(t) \quad \text{--- (1)}$$

Sub. $v(t) = x(t)$ & $i(t) = \frac{x(t)}{R}$ in eq. (1), we get

$$p(t) = x(t) \cdot \frac{x(t)}{R} = \frac{x^2(t)}{R}$$

As we know that the total energy dissipated is the integral of the instantaneous power.

$$\therefore \text{Energy dissipated} = \int_{-\infty}^{\infty} \frac{x^2(t)}{R} dt$$

Subject

put $R = 1$

$$\text{Energy dissipated} = \int_{-\infty}^{\infty} x^2(t) dt = E$$

For real s/g, Energy is

$$E = \int_{-\infty}^{\infty} x^2(t) dt$$

For a complex s/g, Energy is

$$E = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

→ for continuous time s/g.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2$$

→ for discrete time s/g.

||y

For a real s/g, Average Power P

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt$$

For a complex valued s/g, Average Power P is

$$P = \lim_{T \rightarrow \infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt$$

→ for continuous time s/g.

For discrete time s/g.

$$P = \lim_{N \rightarrow \infty} \frac{1}{N} \sum_{n=-N/2}^{N/2} |x[n]|^2$$

Multichannel and Multidimensional s/g:-

Multichannel s/g -

s/g which are generated by multiple sources or multiple sensors.

The resultant s/g is the vector sum of s/g from all channels.
eg - ECG s/g.

$$x(t) = \begin{bmatrix} x_1(t) \\ x_2(t) \\ x_3(t) \end{bmatrix}$$

Multidimensional s/g -

If a s/g is a function of single independent variable, the s/g is called

One-dimensional s/g

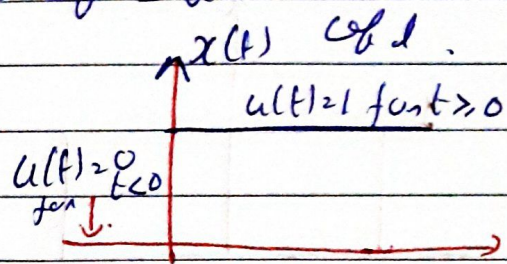
If a s/g is a function of multi (many) independent variables, it is called multidimensional s/g.

Subject

Standard S/g -① Unit Step S/g - has constant amp. of 1 for 0 on +ve value

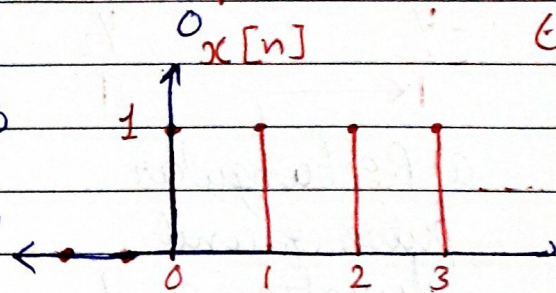
$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

(continuous Unit-
Step S/g)



$$u[n] = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases}$$

(discrete continuous
Unit step S/g)

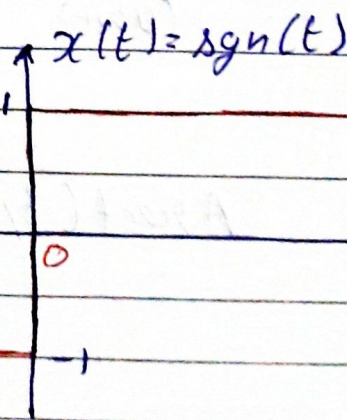


$$u[n] = \{1, 1, 1, 1, \dots\}$$

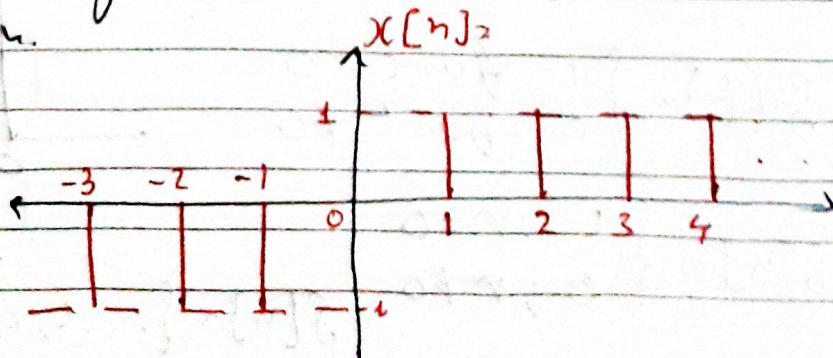
↑
arrow shows the start of $n = 0$.

② Signum Function -

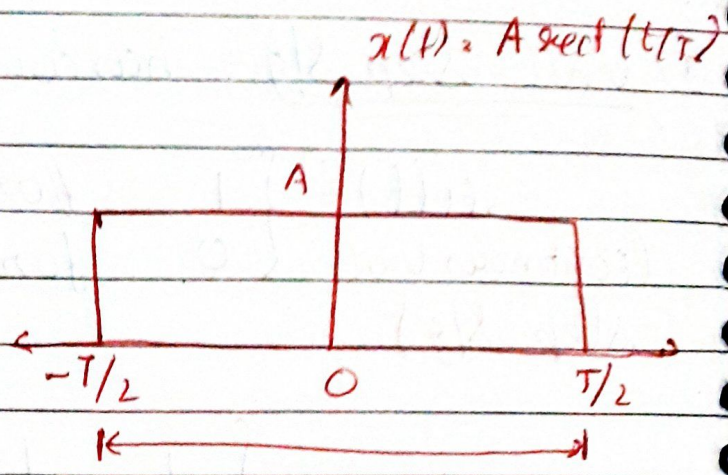
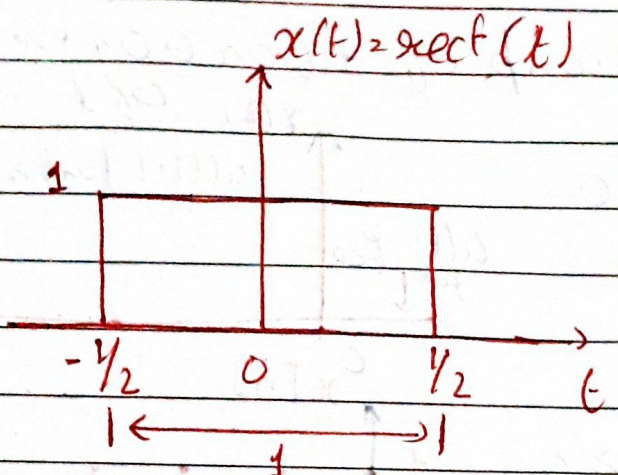
$$\text{sgn}(t) = \begin{cases} +1 & \text{for } t > 0 \\ -1 & \text{for } t < 0 \end{cases}$$



∴ This signum function is an odd or antisymmetric function.



③ Rectangular Pulse.



a) Rectangular pulse of unit duration and unit amplitude

b) Rectangular pulse of duration T and amp A.

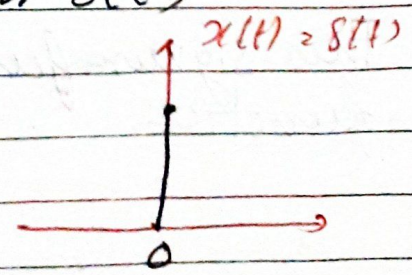
$$\text{rect}(t) = \begin{cases} 1 & \text{for } -1/2 \leq t \leq 1/2 \\ 0 & \text{otherwise} \end{cases}$$

(unit)

$$A \text{rect}(t/T) = \begin{cases} A & \text{for } -T/2 \leq t \leq T/2 \\ 0 & \text{otherwise} \end{cases}$$

④ Delta or Unit Impulse function $\delta(t)$

$$\delta(t) = \begin{cases} 1 & \text{for } t = 0 \\ 0 & \text{for } t \neq 0 \end{cases}$$



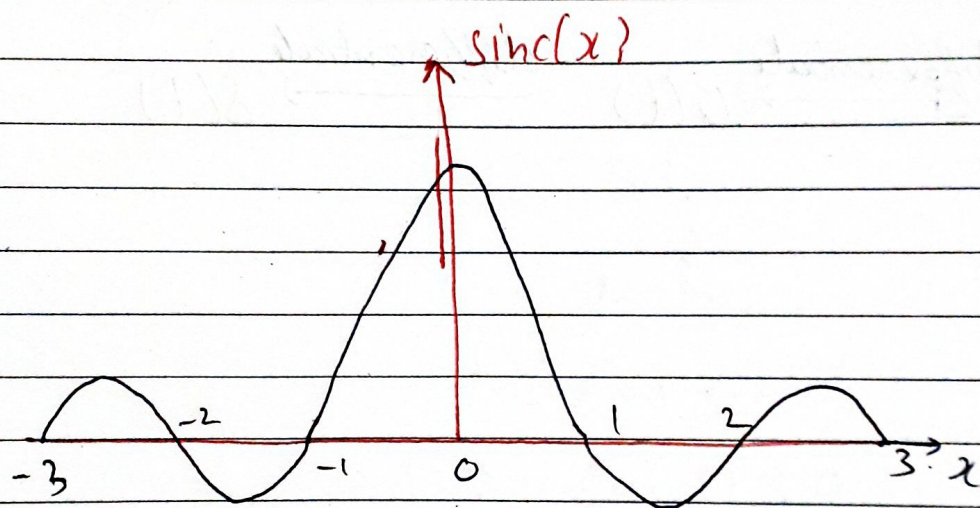
$$\delta[n] = \begin{cases} 1 & ; n = 0 \\ 0 & ; n \neq 0 \end{cases}$$

$$\delta[n] = \{ \dots, 0, 0, 0, 1, 0, 0, 0 \}$$

③ Sinc Function

$$\text{Sinc}(x) = \frac{\sin(\pi x)}{(\pi x)} \quad \text{for } x \neq 0$$

$$\text{Sinc}(x) = \begin{cases} 1 & \text{at } x = 0 \\ 0 & \text{at } x = \pm 1, \pm 2, \pm 3, \dots \end{cases}$$

Relationship between Step, Ramp & Delta Function1) Unit step and unit ramp function

$$\frac{d}{dt} r(t) = u(t) \quad \text{or} \quad \int u(t) dt = r(t)$$

2) Unit step and delta function

$$\frac{d}{dt} u(t) = \delta(t) \quad \text{or} \quad \int \delta(t) dt = u(t)$$

Rel: Unit ramp and delta function

$$g(t) = \iint f(t) dt \quad \text{or} \quad \frac{d^2}{dt^2} g(t) = f(t)$$

$$f(t) \xrightarrow{\text{integrate}} u(t) \xrightarrow{\text{integrate}} g(t)$$

$$g(t) \xrightarrow{\text{differentiate}} u(t) \xrightarrow{\text{differentiate}} f(t)$$

NumericalsSubject 1) Prove the following :-

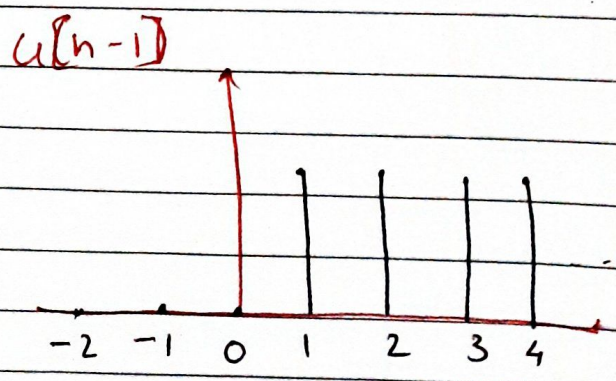
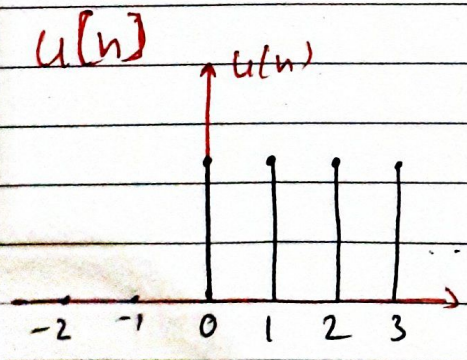
a) $\delta(n) = u(n) - u(n-1)$

Sol:- As we know

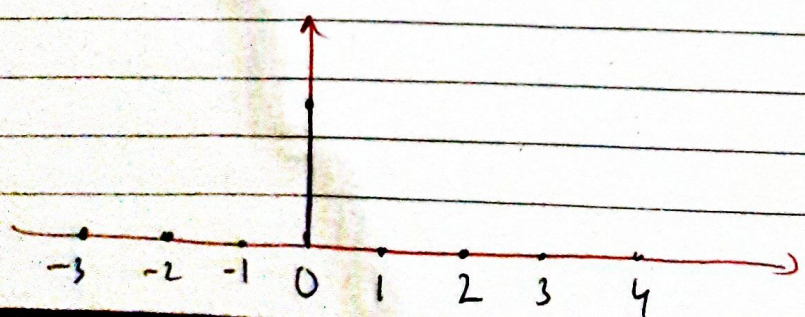
$$u(n) = \begin{cases} 1 & \text{for } n \geq 0 \\ 0 & \text{for } n < 0 \end{cases} \quad \text{--- (1)}$$

$$u(n-1) = \begin{cases} 1 & \text{for } n \geq 1 \\ 0 & \text{for } n < 1 \end{cases} \quad \text{--- (2)}$$

$$u(n) - u(n-1) = \begin{cases} 0 & \text{for } n \geq 1 \text{ i.e. } n > 0 \\ 1 & \text{for } n = 0 \\ 0 & \text{for } n < 0 \end{cases}$$



$$u[n] - u[n-1] = \delta[n]$$



$$\therefore u[n] - u[n-1] = \delta[n]$$

$$2) x(t) = \text{rect}\left(\frac{t}{T_0}\right)$$

*

Steps: [General steps for determination of energy or power s/g]

- 1) Check whether s/g is periodic or nonperiodic. If periodic and has infinite duration, then it can be power s/g. Calculate power.
- 2) If s/g is periodic and of finite duration, then it is energy s/g and calculate energy directly.
- 3) If s/g is not periodic, it is energy s/g. Calculate energy directly.

$$2) u(n) = \sum_{k=-\infty}^{\infty} \delta(k)$$

As we know $\delta(k) = \begin{cases} 0 & \text{for } n < 0 \\ 1 & \text{for } n \geq 0 \end{cases}$

If we see the eq. the R.H.S of eq. is an unit sample seq. $u(n)$.

Q. Determine whether the following s/g are energy s/g or power s/g. Calculate their energy or power.

$$1) x[n] = \left(\frac{1}{2}\right)^n u[n]$$

We can not compare it with the standard eq. to calculate ρ .

Hence it is not a periodic s/g.

$$E = \sum_{n=-\infty}^{\infty} |x(n)|^2 = \sum_{n=0}^{\infty} \left[\left(\frac{1}{2}\right)^n\right]^2 = \sum_{n=0}^{\infty} \left(\frac{1}{4}\right)^n$$

$$\sum_{n=0}^{\infty} a^n = \frac{1}{1-a}$$

$$|a| < 1$$

$$= \frac{1}{1 - \frac{1}{4}} = \frac{4}{3}$$

Operations - 11) Transformation in Independent Variable of s/s

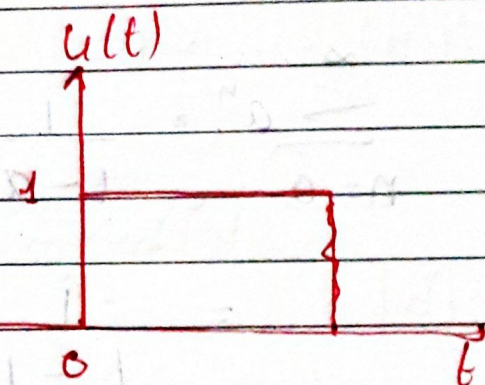
a) Delay / Advancing

b) Time Folding

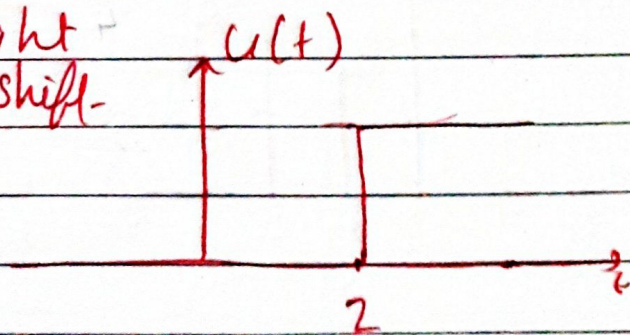
c) Time Scaling.

a) Time Delay / Advancing :-

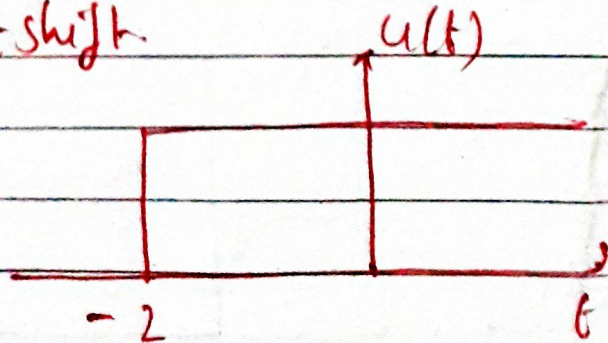
$$u(t) = \begin{cases} 1 & \text{for } t \geq 0 \\ 0 & \text{for } t < 0 \end{cases}$$

Unit step function delayed by 2

$$u(t-2) = \begin{cases} 1 & \text{for } t \geq 2 \\ 0 & \text{for } t < 2 \end{cases}$$

Unit step function advances by 2.

$$u(t+2) = \begin{cases} 1 & \text{for } t \geq -2 \\ 0 & \text{for } t < -2 \end{cases}$$



b) Time folding-

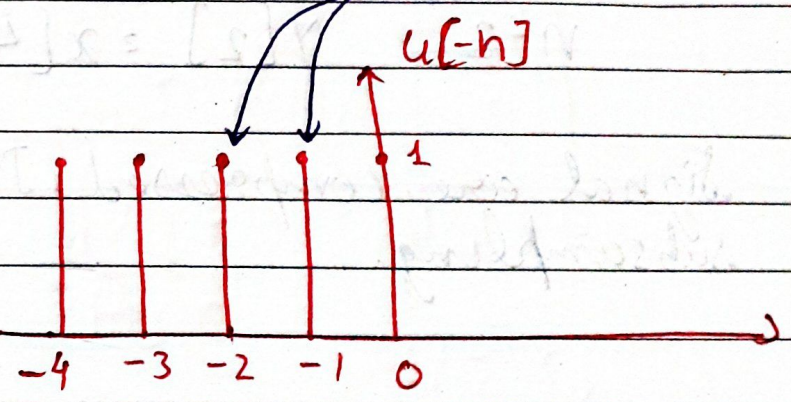
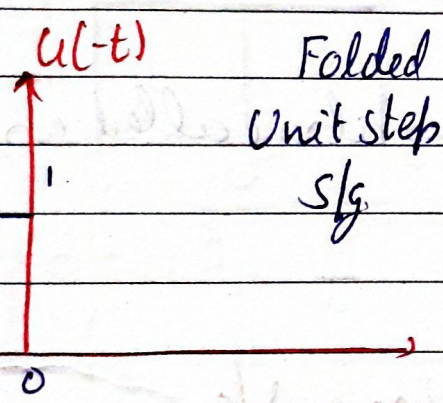
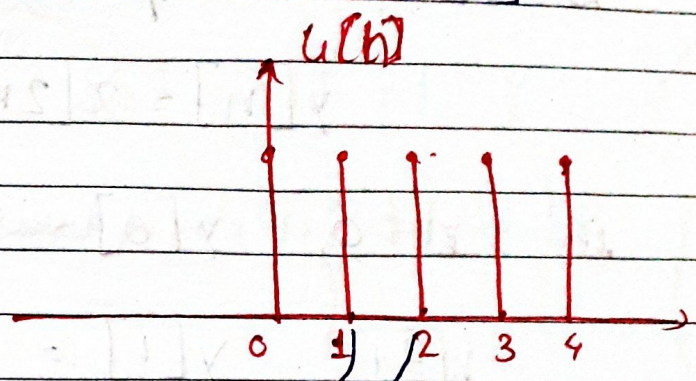
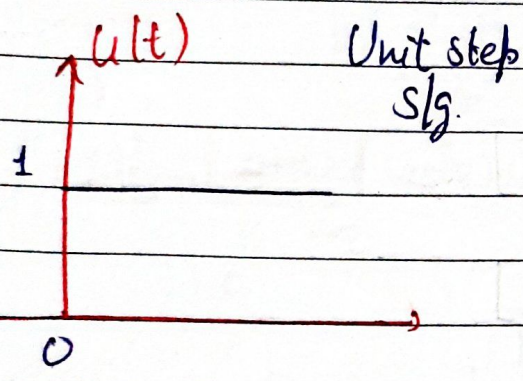
* It is used in convolution.

Let $x(t)$ is a s/g.

Time folded s/g is obtained by replacing t with $(-t)$

$y(t) = x(-t)$

$y[n] = x[-n]$



Time folding operation on unit step s/g.

c) Time Scaling-

It is divided into two parts.

a) Time compression- Time axis is compressed

$y(t) = x(2t)$

$y(t)$ will be compressed in time

b) Time expansion: Time axis is expanded.

$$y(t) = x(t/2)$$

$y(t)$ will be expanded in time.

Compression of Discrete-time s/g

Let $x[n]$ be the pulse, then

$$y[n] = x[2n]$$

ie $n=0, \quad y[0] = x[0]$

$n=1, \quad y[1] = x[2]$

$n=2, \quad y[2] = x[4]$

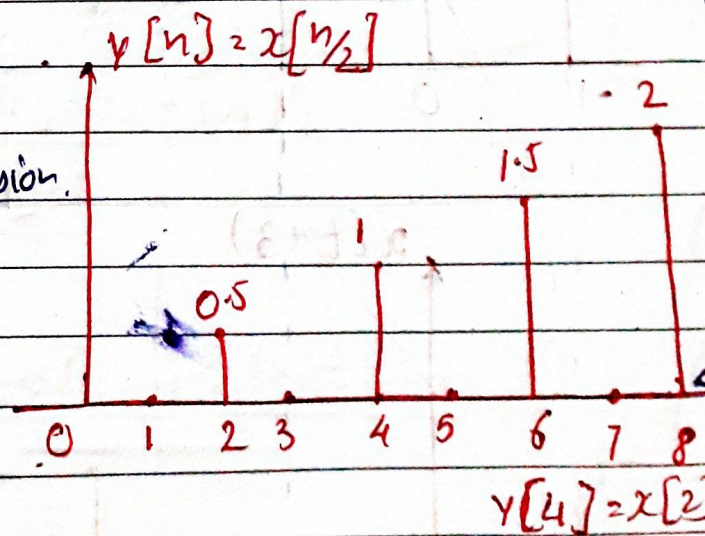
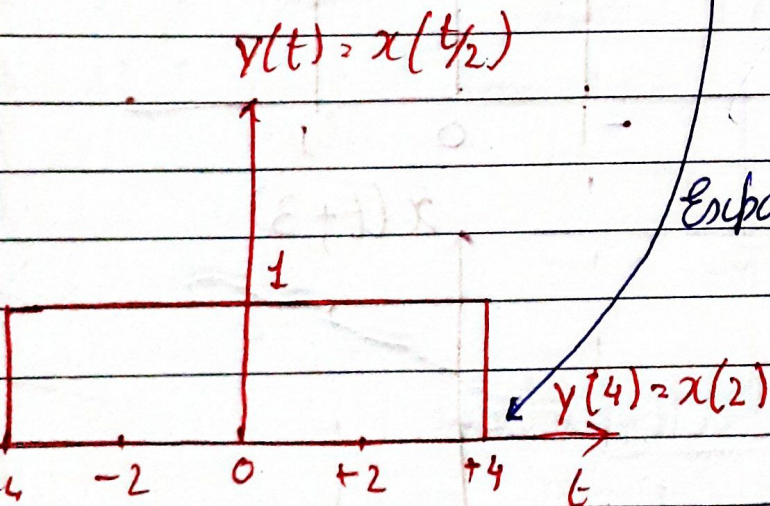
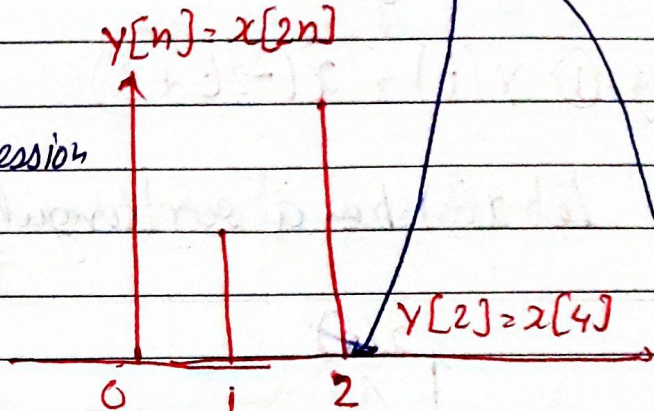
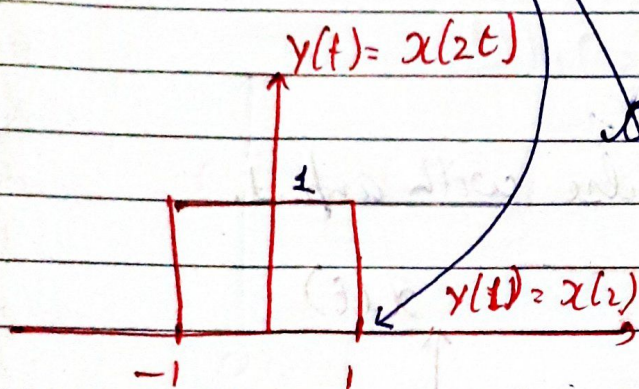
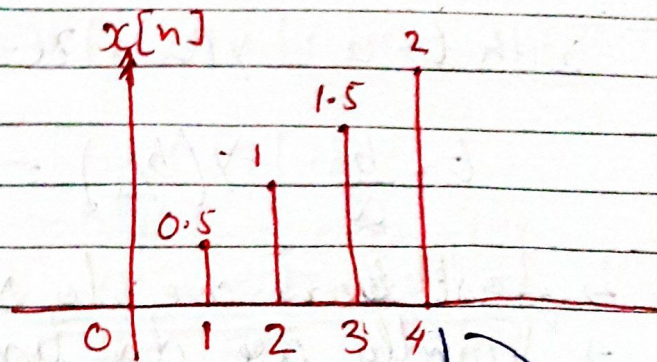
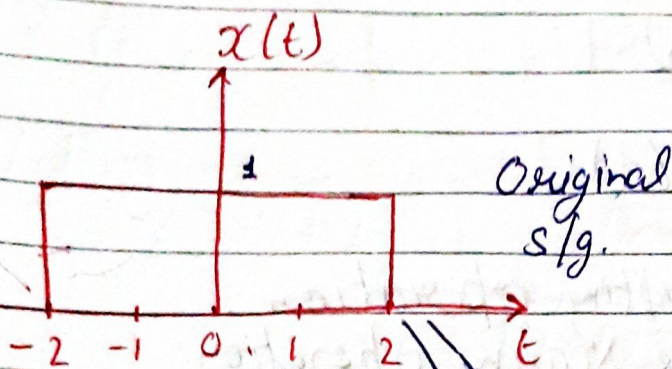
Signal are compressed. It is also called as subsampling.

Expansion of discrete-time s/g

$$y[n] = x\left[\frac{n}{2}\right]$$

$$y[0] = x[0]$$

$$y[2] = x\left[\frac{2}{2}\right] = x[1]$$



$$y(t) = x(at - b)$$

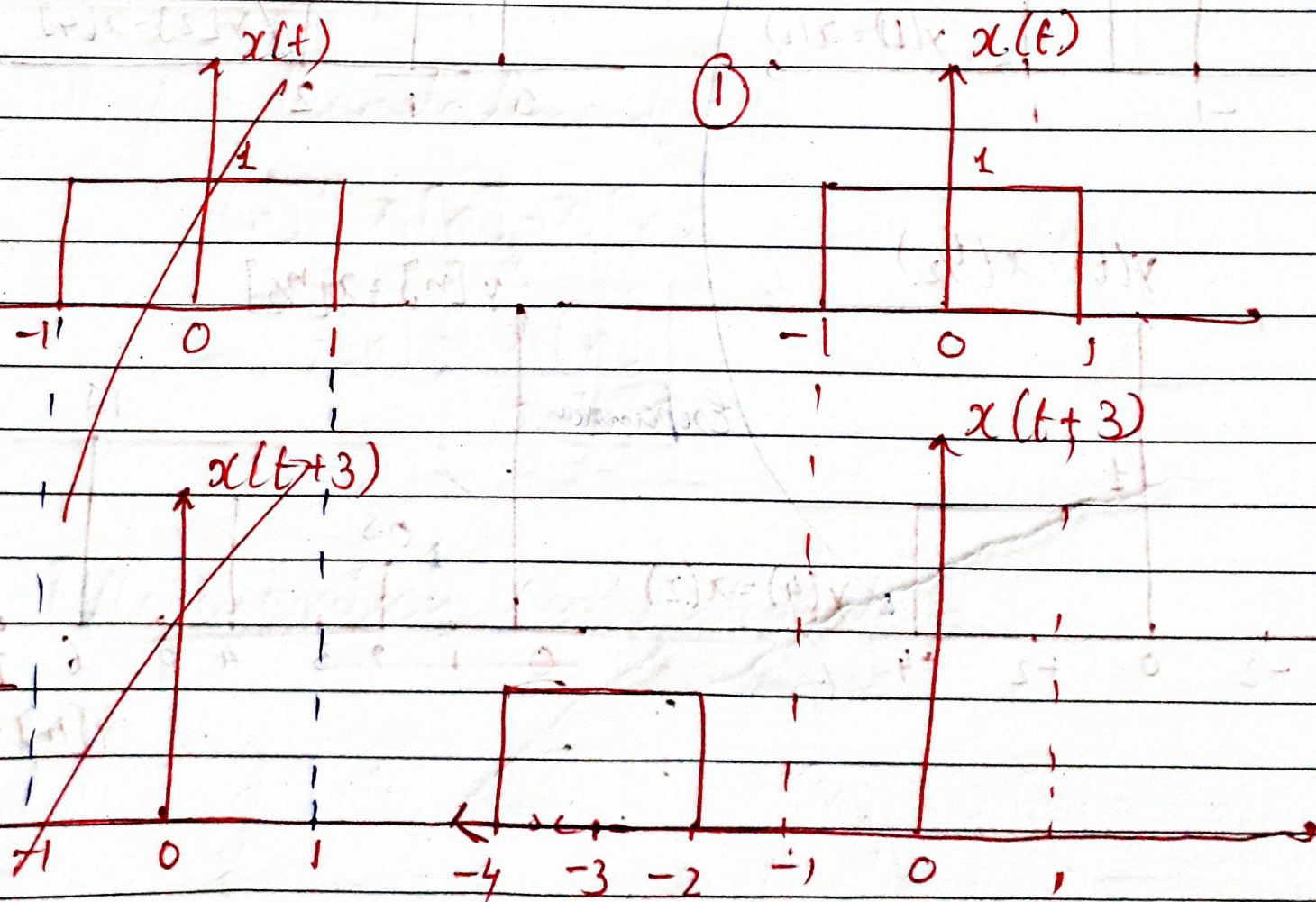
with $t=0$, $y(0) = x(-b)$

$$t = \frac{b}{a} \quad y\left(\frac{b}{a}\right) = x(0)$$

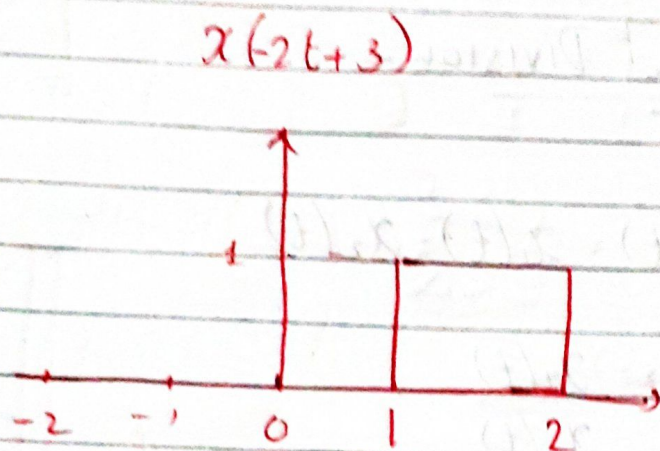
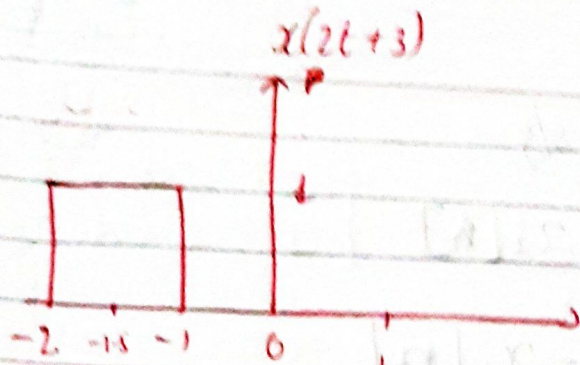
- first we do shifting operation
- secondly we do time scaling operation

eg ① $y(t) = x(-2t + 3)$

Let $x(t)$ be a rectangular pulse with amp 1.



$$x(2t)$$



2) Transformation on Amplitude of the s/s -

a) Amplitude scaling - Amplitude can be changed.

b) Addition & subtraction -

Let $x_1(t)$ & $x_2(t)$ be two continuous time s/s.
Let $y(t)$ be the resultant s/s.

$$y(t) = x_1(t) + x_2(t) \quad \longrightarrow \text{for addition}$$

$$\text{Ily } y(t) = x_1(t) - x_2(t) \quad \longrightarrow \text{for subtraction}$$

||y for discrete time s/g -

$$y[n] = x_1[n] + x_2[n]$$

$$y[n] = x_1[n] - x_2[n]$$

c) Multiplication and Division -

For Multiplication

$$y(t) = x_1(t) \cdot x_2(t)$$

For division

$$y(t) = \frac{x_1(t)}{x_2(t)}$$

||y for discrete time s/g -

$$y[n] = x_1[n] \cdot x_2[n]$$

$$y[n] = \frac{x_1[n]}{x_2[n]}$$

d) Differentiation and Integration

$$y(t) = \frac{d}{dt} x(t)$$

$$y[n] = x[n] - x[n-1]$$

(as direct diff doesn't exist for discrete s/g)

$$y(t) = \int_{-\infty}^t x(\tau) d\tau$$

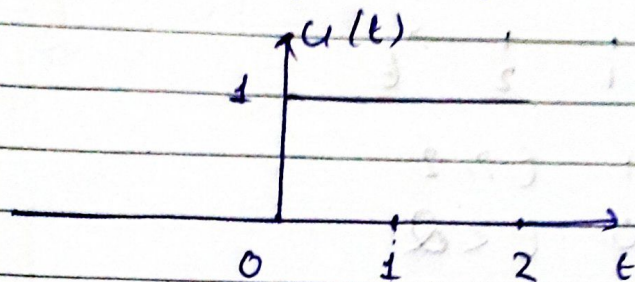
$$y[n] = \sum_{k=-\infty}^n x[k]$$

Numerical

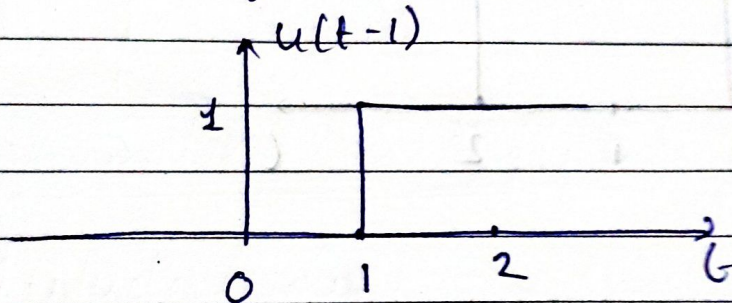
① Draw the waveform for following step function

a) $f_1(t) = 2u(t-1)$

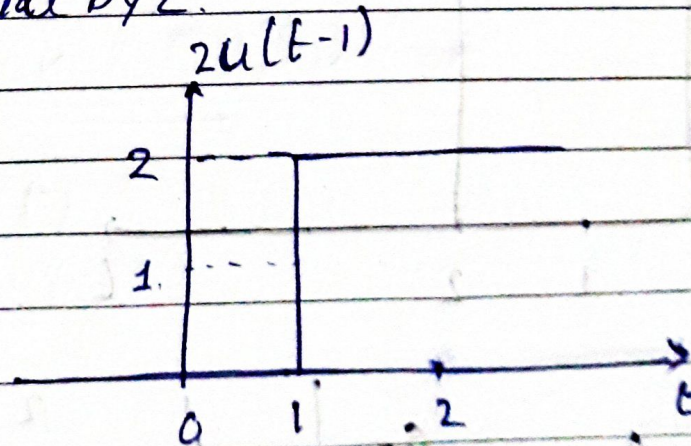
① Draw waveform of $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



② Draw waveform of $u(t-1) = \begin{cases} 1 & t \geq 1 \\ 0 & t < 1 \end{cases}$

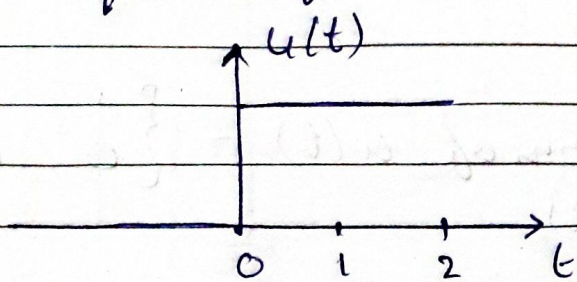


③ Draw waveform of $2u(t-1)$ in this the amp is multiplied by 2.

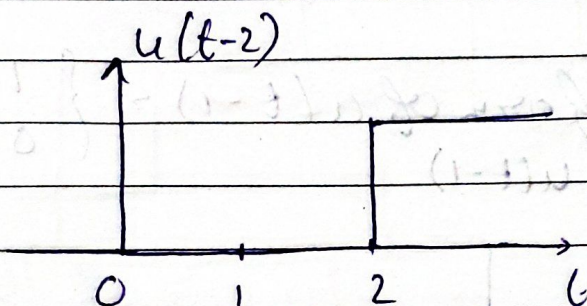


$$b) f_2(t) = -2u(t-2)$$

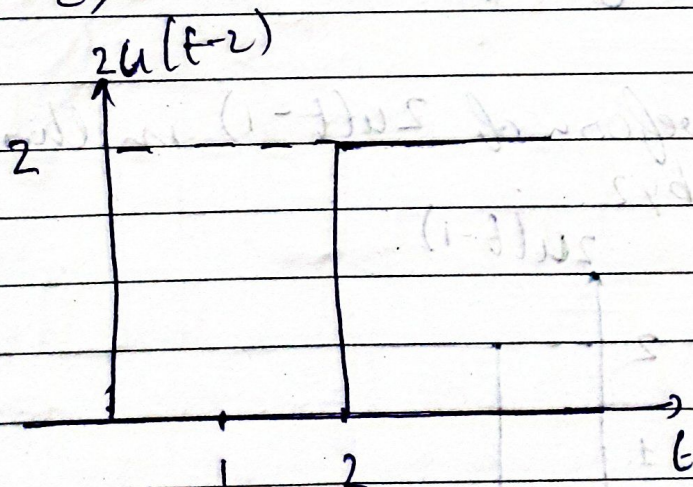
① Draw waveform of $u(t) = \begin{cases} 1 & t \geq 0 \\ 0 & t < 0 \end{cases}$



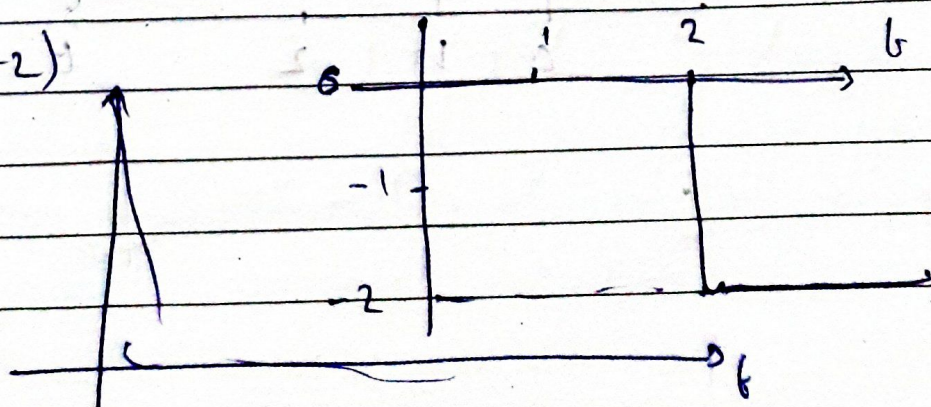
② Draw $u(t-2) = \begin{cases} 1 & t \geq 2 \\ 0 & t < 2 \end{cases}$



③ Draw $2u(t-2)$



Draw $-2u(t-2)$



System and Classification of System

System: a mathematical model of a physical process that relates the i/p (or excitation) s/g to the o/p (or response) s/g.

a) System with Memory and without Memory

A system is Memoryless if o/p at any time depends on only the i/p at the same time. eg: Resistor.

$$y(t) = x(t)$$

A system is with Memory if o/p at any time depends on future values of i/p or o/p.

eg: accumulator or summer and delay.

$$y[n] = \sum_{k=-\infty}^n x[k]$$

→ for accumulator or summer

$$y[n] = x[n-1]$$

→ for delay

b) Causal and Noncausal System

Causal = if o/p $y(t)$ at $t = t_0$ depends on only the i/p $x(t)$ for $t \leq t_0$.

if o/p of a system depends (w.r.t to time) on only the present / past values of i/p.

Noncausal = If the o/p depends on future values

eg -

$$y(t) = x(t+1)$$

whether

Q. Prove, $y[n] = x[-n]$ is causal or non causal system.

Sol:- Let us consider

$$y[n] = x[-n]$$

at $n = n_0$, $y[n_0] = x[-n_0]$ → this is past value

at $n = -4$, $y[-4] = x[4]$

Present Value

future value

Hence for this case the o/p $y[-4]$ depends on future value $x[4]$.

Hence we can conclude that the above system is non causal.

c) Linear and Non linear system -

A system is said to be linear if it satisfy two properties.

1) Additivity -

i.e. the response to $x_1(t) + x_2(t)$ is $y_1(t) + y_2(t)$

2) Homogeneity (Scaling)

i.e. response to $a x_1(t)$ is $a y_1(t)$

$$\text{or } \begin{cases} a x_1(t) + b x_2(t) \rightarrow a y_1(t) + b y_2(t) \\ a x_1[n] + b x_2[n] \rightarrow a y_1[n] + b y_2[n] \end{cases}$$

* a zero i/p yields a zero o/p.

d) Time Invariant and Time Varying system

Time Invariant - if a time shift (delay or advance) in the i/p s/s causes the same time shift in the o/p s/s.

$$a x(t - \tau) = a y(t - \tau) \quad \text{--- (1)}$$

$$a x[n - k] = a y[n - k]$$

Time Varying system - If a system does not satisfy the eq (1) given in then it is called time varying system.

* If a system is linear and time invariant, then it called linear time invariant (LTI) system.

e) Stable system:-

A system is bounded-input/bounded-output (BIBO) stable if for any bounded input x defined by

$$|x| \leq k_1,$$

the corresponding output y is also bounded.

$$|y| \leq k_2,$$

where k_1 & k_2 are finite real constants.