$$\boxed{T = \frac{2\pi}{\omega_0}}$$

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# Fourier Series ⇒

NORMAL ~~FOER~~ form

$$\boxed{x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos(n\omega_0 t) + \sum_{n=1}^{\infty} b_n \sin(n\omega_0 t)}$$

Average Value ⇒ DC Value ⇒

$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot dt$$

$$x(t) = a_0 + \sum_{n=1} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \cos n\omega_0 t \cdot dt$$

$$b_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t)\cdot \sin(n\omega_0 t)\cdot dt$$

$$a_0 = \int_{-T/2}^{T/2} x(t)\,dt \qquad a_n = \int_{-T/2}^{T/2} x(t)\cos n\omega t\,dt$$

(i) When $x(t) \longrightarrow$ even signal

$a_0$ = find = constant

$a_n$ = Constant

$b_n = 0$

→ fourier series $a_0$ + Cosine term

(ii) When $x(t) \longrightarrow$ Odd signal

$a_0 = 0$

$a_n = 0$

$b_n$ = Constant

→ fourier series only Sinusodial term

$$\boxed{x(t) = x\left(t \pm \frac{T}{2}\right)}$$

Half wave Symmetric ⇒ (Odd Harmonics present in fourier series) (always)

$a_0 = 0$

Signal even → Cosine series → 1, 3, 5 .....

odd → Sine wave Series → 1, 3, 5 .....

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \cos$$

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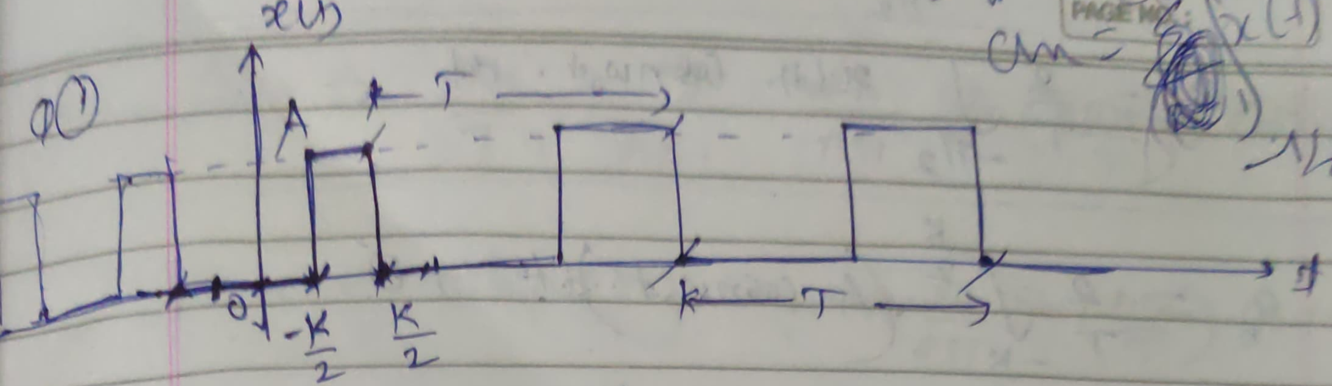
$a_n = \frac{2}{T}\int x(t) \cos n$

Q(1)



$$-\frac{T}{2} \le t \le -\frac{K}{2} \longrightarrow 0$$

$$-\frac{K}{2} \le t \le \frac{K}{2} \longrightarrow A$$

$$\frac{K}{2} \le t \le \frac{T}{2} \longrightarrow 0$$

fourier series

$$= a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + \sum_{n=1}^{\infty} b_n \sin n\omega_0 t$$

$$\Rightarrow a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\, dt$$

$$\Rightarrow a_0 = \frac{1}{T}\left[\int_{-T/2}^{-K/2} 0 + \int_{-K/2}^{K/2} A\, dt + \int_{K/2}^{T/2} 0\, dt\right]$$

$$a_0 = \frac{1}{T}\left[0 + A\int_{-K/2}^{K/2} dt + 0\right]$$

$$a_0 = \frac{1}{T} A\left[t\right]_{-K/2}^{K/2}$$

$$a_0 = \frac{A}{T}\left[\frac{K}{2} - (-K/2)\right]$$

$$\boxed{a_0 = \frac{A}{T} \cdot K}$$

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$$a_n = \frac{2}{T}\int_{-T/2}^{T/2} x(t) \cdot \cos n\omega_0 t \cdot dt$$

$$a_n = \frac{2}{T}\int_{-\frac{K}{2}}^{\frac{K}{2}} (A \cdot \cos n\omega_0 t \cdot) dt + 0 \cdots$$

$$= \frac{2A}{T} \cdot \left[\frac{\sin n\omega_0 t}{n\omega_0}\right]_{-K/2}^{K/2}$$

$$= \frac{2A}{T}\frac{1}{n\omega_0}\left[\sin\left(\frac{n \cdot \omega_0 K}{2}\right) - \sin\left(\frac{-n\omega_0 K}{2}\right)\right]$$

$$= \frac{2A}{T}\frac{1}{n\omega_0}\left[\sin\left(\frac{n\omega_0 K}{2}\right) + \sin\left(\frac{n\omega_0 K}{2}\right)\right]$$

$$= \frac{2A}{T} \cdot \frac{1}{n\omega_0} \cdot 2 \cdot \sin\left(\frac{n\omega_0 K}{2}\right)$$

$$\because T = \frac{2\pi}{\omega_0}$$

$$= \frac{\cancel{2}A}{\cancel{2}\pi} \cdot \cancel{\omega_0} \cdot \frac{1}{n\cancel{\omega_0}} 2 \; \sin\left[n \cdot \frac{\cancel{2}\pi}{\cancel{2}T} \cdot K\right]$$

$$\boxed{a_n = \frac{2A}{n\pi}\sin\left(\frac{n\pi K}{T}\right)}$$

Given signal is even (symmetric to y axis)

$$b_n = 0$$

$$\text{Fourier series} = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega_0 t + 0$$

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$$\boxed{\text{Fourier Series} = \frac{AK}{T} + \sum_{n=1}^{\infty} \frac{2A}{n\pi} \sin\left(\frac{n\pi K}{T}\right) \cdot \cos n\omega_0 t}$$

$$= \frac{AK}{T} + \frac{2A}{\pi} \sin\left(\frac{\pi K}{T}\right) \cdot \cos\omega_0 t + \frac{2A}{2\pi} \sin\left(\frac{2\pi K}{T}\right) \cos 2\omega_0 t + \cdots$$

$$\Rightarrow \quad C_0 = a_0$$

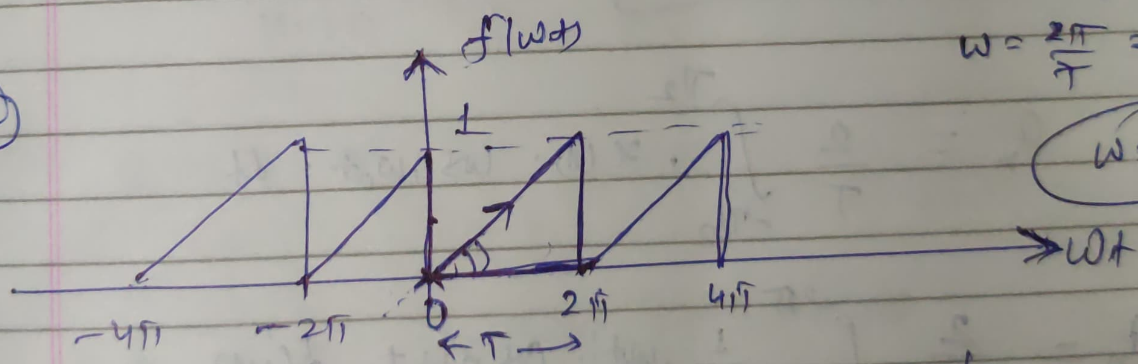
$$C_n = \sqrt{a_n^2 + b_n^2}$$

$$\phi = \tan^{-1}\left(\frac{b_n}{a_n}\right)$$

Power Form

Fourier Series. $\quad x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cos(n\omega_0 t - \phi)$

Q (2)




$$\omega = \frac{2\pi}{T} = \frac{2\pi}{2\pi}$$

$$\omega = 1$$

$$m = \text{Slope} = \frac{y\text{-axis Value}}{x\text{-axis Value}}$$

$$m = \frac{1}{2\pi}$$

$y = mx + c$

$(y - y_1) = m(x - x_1)$

$y = mx + c$ { straight line passing from origin)

$c = 0$

Equation of line

$$f(\omega t) = \frac{1}{2\pi} \cdot \omega t$$

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$$a_0 = \frac{1}{T}\int_{-T/2}^{T/2} x(t)\cdot dt = \frac{1}{T}\int_0^T f(\omega t)\cdot dt$$

$$a_0 = \frac{1}{2\pi}\int_0^{2\pi} \frac{1}{2\pi}\cdot \omega t \cdot d\omega t$$

$$a_0 = \frac{1}{2\pi\cdot 2\pi}\cdot\int_0^{2\pi}(\omega t)\cdot d(\omega t)$$

$$= \frac{1}{4\pi^2}\left[\frac{(\omega t)^2}{2}\right]_0^{2\pi}$$

$$= \frac{1}{(2\pi)(2\pi)}\left(2\pi - 0\right)$$

$$\boxed{a_0 = \frac{1}{\cancel{2\pi}\,\cancel{2\pi}}\cdot\frac{\cancel{2\pi}\cdot\cancel{2\pi}}{2}} = \frac{1}{2} = 0.5$$

$$\Rightarrow \quad a_n = \frac{2}{T}\int_{-T/2}^{T/2}\cdot x(t)\cdot \cos n\omega_0 t\cdot dt$$

$$a_n = \frac{2}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}(\omega t)\cdot \cos n\omega_0 t\,.d(\omega t)$$

$$a_n = \frac{1}{\pi}\cdot\frac{1}{2\pi}\Big[\qquad\qquad\qquad\Big]$$

$$\int I\cdot II\cdot dt = I\int II\cdot dt - \int \frac{d}{dt} I\int II\, dt\, .dt$$

$$\frac{d}{dt}(I\cdot II) = I\cdot\frac{d\,II}{dt} + II\,\frac{d\,I}{dt}$$

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$$a_n = \frac{2}{\pi}$$

$$a_n = \frac{1}{\pi \cdot 2\pi}\left[(\omega t)\cdot\int_0^{2\pi}\cos n\omega_0 t\, d\omega t - \int_0^{2\pi}\frac{d(\omega t)}{d\omega t}\int \cos n\omega_0 t\cdot d\omega t\, d\omega t\right]$$

$$a_n = \frac{1}{\pi \cdot 2\pi}\left[\left[\omega t\cdot\frac{\sin n\omega_0 t}{n}\right]_0^{2\pi} - \int_0^{2\pi} 1\cdot\frac{\sin n\omega_0 t}{n}\cdot d\omega t\right]$$

$$a_n = \frac{1}{\pi \cdot 2\pi}\left[\frac{1}{n}\cdot 2\pi\left[\sin(2\pi n) - \sin 0\right] - \left[\frac{\cos n\omega_0 t}{(-n)(n)}\right]_0^{2\pi}\right]$$

$$a_n = \frac{1}{\pi \cdot 2\pi}\left[0 + \frac{1}{n\cdot n}\left[\cos n\cdot 2\pi - \cos 0\right]\right]$$

$$a_n = \frac{1}{\pi \cdot 2\pi}\left[0 + \frac{1}{n^2}\left[1 - 1\right]\right]$$

$$a_n = 0$$

$$b_n = \frac{2}{T}\int_0^{2\pi} x(t)\cdot\sin n\omega_0 t\cdot dt$$

$$= \frac{2}{2\pi}\int_0^{2\pi}\frac{1}{2\pi}(\omega t)\sin n\omega_0 t\cdot d\omega t$$

$$= \frac{1}{\pi \cdot 2\pi}\left[I\int II\cdot dt - \int\frac{d}{dt}I\cdot\int II\cdot dt\cdot dt\right]$$

$$= \frac{1}{\pi \cdot 2\pi}\left[(\omega t)\int_0^{2\pi}\sin n\omega_0 t\cdot d\omega t - \int 1\cdot\int\sin n\omega_0 t\, d\omega t\, d\omega t\right]$$

$$= \frac{1}{\pi \cdot 2\pi}\left[\left[\omega t\cdot\frac{\cos n\omega_0 t}{-n}\right]_{\omega t=0}^{2\pi} - \int_0^{2\pi}\frac{\cos n\omega_0 t}{-n}\cdot d\omega t\right]$$

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$$= \frac{1}{\pi \cdot 2\pi}\left[\left(\frac{2\pi\,[\cos n2\pi - \overset{0}{[\cos 0}]}{-n}\right) + \frac{1}{n}\cdot\left[\frac{\sin n\omega_0 t}{n}\right]_0^{2\pi}\right]$$

$$= \frac{1}{\pi \cdot 2\pi}\left[0 + \frac{1}{n^2}\cdot\left[\sin n \cdot 2\pi - \sin 0\right]\right]$$

$$b_n = \frac{1}{2\pi^2}\cdot\frac{1}{n^2}(0) = 0$$

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Complex Fourier Exp. Series ⇒

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}}$$

$$\boxed{C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{-jn\omega_0 t} \cdot dt}$$

$$\Rightarrow \quad C_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{-j(-n)\cdot\omega_0 t} \cdot dt$$

Complex Conjugate ⇒ $j \rightarrow -j$

$a \rightarrow a^*$ $\qquad 3+4j \longrightarrow 3-4j$

$$C^*_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} \cdot x(t) \cdot e^{-(-j)(-n)\cdot\omega_0 t} \cdot dt$$

$$\boxed{C^*_{-n} = \frac{1}{T}\int_{-T/2}^{T/2} \cdot x(t) \cdot e^{-jn\omega_0 t} \cdot dt}$$

$$\boxed{C_n = C^*_{-n}}$$

$$\Rightarrow \boxed{C^*_n = C_{-n}}$$

$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$\cos\theta = \frac{e^{j\theta} + e^{-j\theta}}{2} \quad ; \quad \sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

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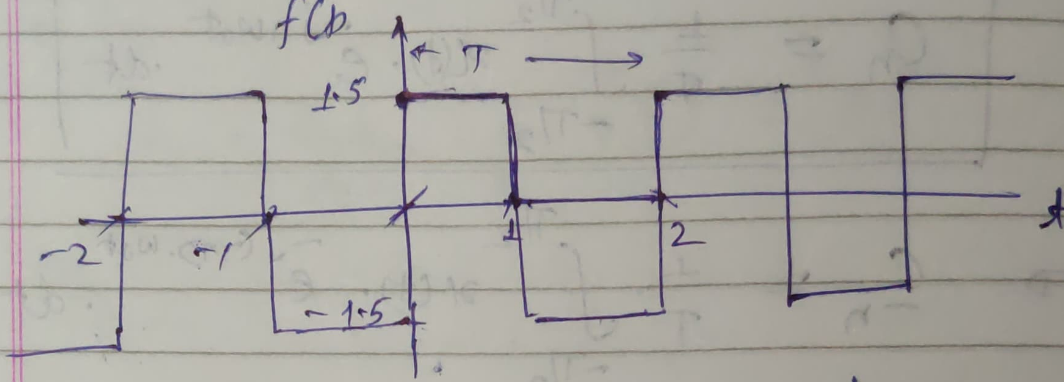
Cos and Sin → self fourier series

Q(3) $f(t) = \begin{cases} 1.5 & ; \ 0 \le t \le 1 \\ -1.5 & ; \ 1 \le t \le 2 \end{cases}$




expo. fourier series = $\sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$

⇒ Coefficient $C_n = \frac{1}{T}\int_{-T/2}^{T/2} x(t) \cdot e^{-jn\omega_0 t}\, dt$

$T = \frac{2\pi}{\omega_0} \Rightarrow \omega_0 = \frac{2\pi}{T} = \frac{2\pi}{2} = \pi$

$T = 2$ sec

$$C_n = \frac{1}{2}\left[ \int_0^1 x(t) \cdot e^{-jn\omega_0 t}\, dt + \int_1^2 x(t) \cdot e^{-jn\omega_0 t}\, dt \right]$$

$$= \frac{1}{2}\left[ \int_0^1 1.5\, e^{-jn\omega_0 t} \cdot dt + \int_1^2 (-1.5)\, e^{-jn\omega_0 t} \cdot dt \right]$$

$$= \frac{1}{2}\left\{ 1.5 \left[ \frac{e^{-jn\omega_0 t}}{(-jn\omega_0)} \right]_0^1 + 1.5 \left[ \frac{e^{-jn\omega_0 t}}{-jn\omega_0} \right]_1^2 \right\}$$

$$= \frac{1}{2}\left\{ \frac{-1.5}{jn\omega_0} \left[ e^{-jn\omega_0} - e^0 \right] + \frac{1.5}{jn\omega_0} \left[ e^{-jn\omega_0 2} - e^{-jn\omega_0} \right] \right\}$$

$$= \frac{1}{2}\left[ \frac{-1.5}{jn\omega_0} \cdot e^{-jn\omega_0} + \frac{1.5}{jn\omega_0} + \frac{1.5}{jn\omega_0} \cdot e^{-jn\omega_0 2} - \frac{1.5}{jn\omega_0} e^{-jn\omega_0} \right]$$

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$$\omega_0 = \pi$$

$$C_n = \frac{1}{2}\left[\frac{2(-1.5)\cdot e^{-jn\pi}}{jn\pi} + \frac{1.5}{jn\pi} + \frac{1.5}{jn\pi}\cdot e^{-jn2\pi}\right]$$

$$\because e^{-jn\pi} = \cos n\pi - j\sin(n\pi)$$

$$e^{-jn\pi} = (-1)^n - 0 = (-1)^n$$

$$\because e^{-jn2\pi} = \cos 2n\pi - j\sin 2n\pi =$$

$$= \cos 2n\pi - 0$$

$$n = 1;\ 2;\ 3; \quad \{\cos 2\pi;\ \cos 4\pi;\ \cos 6\pi;$$

$$e^{-jn2\pi} = 1$$

$$C_n = \frac{(-1.5)\cdot(-1)^n}{jn\pi} + \frac{1.5}{jn\pi}\frac{1}{(2)} + \frac{1.5}{jn\pi}\frac{1}{(2)}$$

$$C_n = \frac{1.5}{jn\pi} - \frac{1.5}{jn\pi}(-1)^n$$

Condition $\Rightarrow$ $n = 2, 4, 6 \ldots$ even no.

$$C_n = \frac{1.5}{jn\pi} - \frac{1.5}{jn\pi}\cdot 1 = 0$$

$n = 1;\ 3, 5 \ldots$ odd no.

$$C_n = \frac{1.5}{jn\pi} - \frac{1.5}{jn\pi}(-1) = \frac{3}{jn\pi}$$

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$$C_n \Rightarrow \begin{cases} \frac{3}{jn\pi} & ; n = \text{odd} \\ 0 & ; n = \text{even} \end{cases}$$

$$C_0 = a_0 = \frac{1}{T}\int_0^T x(t)\cdot dt$$

$$C_0 = \frac{1}{2}\left[\int_0^1 1.5\, dt + \int_1^2 (-1.5)\cdot dt\right]$$

$$= \frac{1}{2}\left[(1.5)\left(t\right)_0^1 - 1.5\,(t)_1^2\right]$$

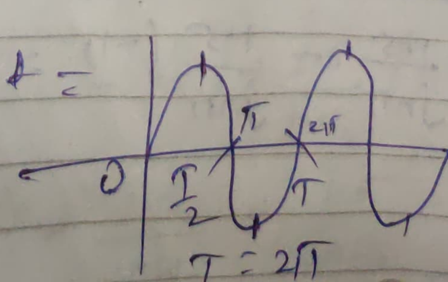
$$= \frac{1}{2}\left[1.5\,[1-0] - 1.5\,(2-1)\right]$$

$$C_0 = \frac{1}{2}\left[1.5 - 1.5\right] = 0$$

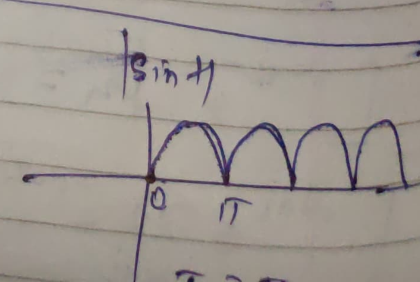
$$\text{Series} = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}$$

$$\text{Series} = \begin{cases} \sum_{n=-\infty}^{\infty} \frac{3}{jn\pi}\cdot e^{jn\omega_0 t} & ; n = \text{odd no.} \\ 0 & ; n = \text{even no} \end{cases}$$

$\sin t =$




$|\sin t|$



$\cos \to 0, 2\pi, 4\pi \cdots = 1$

$\cos \to \pi, 3\pi, 5\pi \cdots = -1$

$\sin \to \frac{\pi}{2} ; 2\pi + \frac{\pi}{2} \cdots = 1$

$\sin 3\pi/2 \cdots = -1$

$A \sin t$

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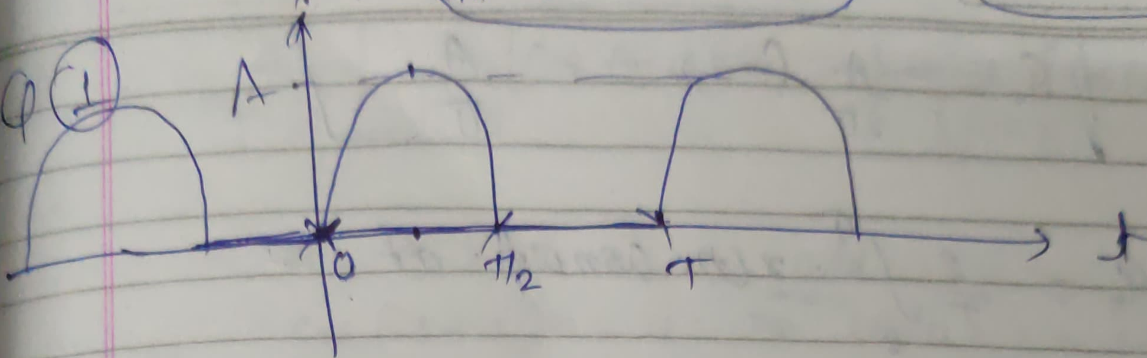
Q 1




$T.P. = T$

$\boxed{\omega = \frac{2\pi}{T}} \Rightarrow \omega T = 2\pi$

$$x(t) \begin{cases} A \sin \omega t & ; \ 0 \le t \le T/2 \\ 0 & ; \ \frac{T}{2} \le t \le T \end{cases}$$

Trigo. Fourier Series

$$x(t) = a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t \quad — (1)$$

$$a_0 = \frac{1}{T} \int_0^T x(t) \cdot dt$$

$$a_0 = \frac{1}{T} \left[ \int_0^{T/2} A \sin \omega t \cdot dt + \int_{T/2}^{T} 0 \cdot dt \right]$$

$$= \frac{1}{T} A \int_0^{T/2} \sin \omega t \cdot dt + 0$$

$$= \frac{A}{T} \left[ -\frac{\cos \omega t}{\omega} \right]_0^{T/2}$$

$$= \frac{-A}{\omega T} \left[ \cos \omega \cdot \frac{T}{2} - \cos 0 \right]$$

$$= \frac{-A}{2\pi} \left[ \cos\left(\frac{2\pi}{2}\right) - \cos 0 \right]$$

$$= \frac{-A}{2\pi} [-1 - 1]$$

$\cos n\pi = (-1)^n$

$\cos \pi = -1$

$\cos 2\pi = 1$

$\cos 3\pi = -1$

$\cos 4\pi = 1$

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$$\boxed{a_0 = -\frac{A}{2\pi}(-2) = \frac{A}{\pi}}$$

$$a_n = \frac{2}{T}\int_0^T x(t).\cos n\omega t \cdot dt$$

$$a_n = \frac{2}{T}\left[\int_0^{T/2} A\cdot \sin\omega t \cdot \cos n\omega t \cdot dt + \int_{T/2}^{T} 0\right]$$

$$= \frac{2A}{T}\int_0^{T/2} \frac{2\sin\omega t \cos n\omega t}{2} : dt$$

$$\sin(A+B) + \sin(A-B) = \sin A\cos B + \cancel{\cos A\sin B} + \sin A\cos B - \cancel{\cos A \sin B}$$

$$= 2\sin A\cos B$$

$$= \frac{2A}{T}\cdot\frac{1}{2}\left[\int_0^{T/2} \sin(\omega t + n\omega t) + \sin(\omega t - n\omega t)\, .dt\right]$$

$$= \frac{A}{T}\left[\int_0^{T/2} \sin(1+n).\omega t \;.dt + \int_0^{T/2} \sin(1-n)\omega t \;.dt\right]$$

$$= \frac{A}{T}\left[\left[\frac{-\cos(1+n)\omega t}{(1+n)\omega}\right]_0^{T/2} + \left[\frac{-\cos(1-n).\omega t}{(1-n).\omega}\right]_0^{T/2}\right]$$

$$\frac{A}{T}\left[-\frac{1}{(1+n)\omega}\left[\cos(1+n)\frac{\omega.T}{2} - \cos 0\right] + \frac{(-1)}{(1-n)\omega}\cdot\left[\cos(1-n).\frac{\omega.T}{2} - \cos 0\right]\right]$$

$$\because \omega T = 2\pi$$

$$\frac{A}{T}\left[\frac{-1}{(1+n)\omega}\left[\cos(1+n).\pi - 1\right] - \frac{1}{(1-n)\omega}\left[\cos(1-n).\pi - 1\right]\right]$$

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$$a_n = -\frac{A}{T\cdot\omega}\left[\frac{1}{(1+n)}\left[\cos(1+n)\pi - 1\right] + \frac{1}{(1-n)}\left[\cos(1-n)\pi - 1\right]\right]$$

$n = 1, 3, 5 \ldots$ odd

$n=1 \Rightarrow \because \cos(1+1)\pi - 1 \Rightarrow \cos 2\pi - 1 = 1-1 = 0$

$\because \cos(1-1)\pi - 1 = \cos 0 - 1 = 1-1 = 0$

$$a_n = -\frac{A}{\omega T}\left[0+0\right] = 0 \; ; n = \text{odd}$$

$n = \text{even} \Rightarrow n = 2, 4, 6 \ldots$

$$a_n = -\frac{A}{2\pi}\left[\frac{1}{1+n}\left[\cos(3\pi) - 1\right] + \frac{1}{(1-n)}\left[\cos(-\pi) - 1\right]\right]$$

$\because$ ~~cos~~

$$= -\frac{A}{2\pi}\left[\frac{1}{1+n}\left[-1-1\right] + \frac{1}{1-n}\left[-1-1\right]\right]$$

$$= -\frac{A}{2\pi}(-2)\left[\frac{1}{1+n} + \frac{1}{1-n}\right]$$

$$= \frac{A}{\pi}\left[\frac{1-n+1+n}{(1+n)(1-n)}\right]$$

$$a_n = \frac{A}{\pi}\left[\frac{2}{1-n^2}\right] = \frac{2A}{\pi(1-n^2)} \; ; n = \text{even}$$

$$b_n = \frac{2}{T}\int_0^T x(t)\cdot \sin n\omega t \cdot dt$$

$$b_n = \frac{2}{T}\int_0^{T/2} A\sin\omega t \cdot \sin n\omega t \cdot dt$$

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$$b_n = \frac{2A}{T}\int_0^{T/2} \frac{2\sin\omega t \cdot \sin n\omega t}{2} \cdot dt$$

$$\cos(A-B) - \cos(A+B) = \cos A\cos B + \sin A\sin B - [\cos A\cos B - \sin A\sin B]$$
$$= 2\sin A\sin B$$

$$= \frac{2A}{T}\int_0^{T/2} \frac{\cos(\omega t - n\omega t) - \cos(\omega t + n\omega t)}{2} \cdot dt$$

$$= \frac{A}{T}\left[\int_0^{T/2} \left(\cos(1-n)\omega t - \cos(1+n)\omega t\right) dt\right]$$

$$= \frac{A}{T}\left[\left[\frac{\sin(1-n)\omega t}{(1-n)\omega}\right]_0^{T/2} - \left[\frac{\sin(1+n)\omega t}{(1+n)\omega}\right]_0^{T/2}\right]$$

$$= \frac{A}{T}\left[\frac{1}{(1-n)\omega}\left[\sin(1-n)\frac{\omega T}{2} - \sin 0\right] - \left[\frac{\sin(1+n)\frac{\omega T}{2} - \sin 0}{(1+n)\omega}\right]\right]$$

$$\omega T = 2\pi$$

$$= \frac{A}{T}\left[\frac{\sin(1-n)\pi - 0}{(1-n)\omega} - \left[\frac{\sin(1+n)\pi - 0}{(1+n)\omega}\right]\right]$$

$$b_n = \frac{A}{T}[0 - 0 - 0 - 0] = 0$$

series $a_0$, $a_n$ & $b_n$ put in eq (1)

$$a_0 + \sum_{n=1}^{\infty} a_n \cos n\omega t + \sum_{n=1}^{\infty} b_n \sin n\omega t$$

$$x(t) = \frac{A}{\pi} + \sum_{n=1}^{\infty} \frac{2A}{\pi(1-n^2)} \cdot \cos n\omega t + 0 \quad ; n = \text{even}$$

$n$ = even $\Rightarrow n = 2, 4, 6 \ldots$

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$$x(t) = \frac{A}{\pi} + \frac{2A}{\pi(1-4)} \cdot \cos 2\omega t + \frac{2A}{\pi(1-16)} \cdot \cos 4\omega t + \dots$$

$$\boxed{x(t) = \frac{A}{\pi} - \frac{2A}{3\pi}\cos 2\omega t - \frac{2A}{15\pi}\cos 4\omega t + \dots \quad n = \text{even}}$$

Polar form $x(t) = C_0 + \sum_{n=1}^{\infty} C_n \cdot \cos(n\omega t - \phi)$

$C_0 = a_0$ ; $C_n = \sqrt{a_n^2 + b_n^2}$

$C_n = \sqrt{a_n^2 + 0} = a_n$

$\phi = \tan^{-1}\left(\frac{b_n}{a_n}\right) = \tan^{-1}\left(\frac{0}{a_n}\right) = \tan^{-1} 0$

$= 0$

$n = \text{even}$

Relation b/w $a_n$, $b_n$ & $C_n$ → Exp.

$C_n = \frac{1}{2}(a_n - j b_n)$

$C_{-n} = \frac{1}{2}[a_n - j b_n]$

$C_{-n}^* = \frac{1}{2}[a_n + j b_n]$

$C_n^* = C_{-n}$

$$\boxed{\begin{aligned} a_n &= C_n + C_n^* \\ a_n &= 2\,\mathrm{Re}[C_n] \end{aligned}}$$

$$\boxed{b_n = j[C_n - C_n^*] = -2\,\mathrm{Im}[C_n]}$$

Fourier Exp Series

Q (1) $V = (2+3j)\cdot e^{+j\frac{\pi}{3}}$

$$\boxed{x(t) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t}}$$

for $C_n$

$C_n = 2+3j$

$C_n^* = 2-3j$

Real $= 2$

Im $= 3$

$|C_n| = \sqrt{(R)^2+(I_m)^2}$

$|C_n| = \sqrt{(2)^2+(3)^2}$

$|C_n| = \sqrt{13}$

$a_n = 2\, Re[C_n]$

$a_n = 2\times 2 = 4$

$b_n = -2\, Im[C_n]$

$b_n = -2\{3\} = -6$

Power $= \sum |C_n|^2$

Power $= (\sqrt{13})^2 = 13$

RMS $= \sqrt{\text{power}}$

RMS $= \sqrt{13}$

---

Q (2) $x(t) = \sin 3t$ define fourier series.

↑ already fourier series and we can express in exponential form

$\boxed{\omega = 3}$

$$\sin\theta = \frac{e^{j\theta} - e^{-j\theta}}{2j}$$

$$x(t) = \sin 3t = \frac{e^{j3t} - e^{-j3t}}{2j}$$

$$= \frac{1}{2j} e^{j3t} - \frac{1}{2j} e^{-j3t}$$

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$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

$$\dots + C_{-2} e^{-j2\omega t} + C_{-1} e^{-j(1)\omega t} + C_0 e^0 + C_1 e^{j(1)\omega t} + C_2 e^{j(2)\omega t} \dots$$

$\downarrow$ 0   $\downarrow$ 0   | 0   $\downarrow$ 0   $\downarrow$ 0

$$\sin 3t = -\frac{1}{2j} e^{-j(3t)} + \frac{1}{2j} e^{j1(3t)}$$

$$C_{-1} = \frac{-1}{2j} \quad ; \quad C_1 = \frac{1}{2j}$$

---

## Properties $\Rightarrow$

(1) Linearity $\Rightarrow$ $A\,x(t) \xrightarrow{FS} A\,C_{1n}$

$B \cdot y(t) \xrightarrow{FS} B\,C_{2n}$

$$A\,x(t) + B\,y(t) \xrightarrow{FS} A\,C_{1n} + B\,C_{2n}$$

(2) Time shifting $\Rightarrow$ $x(t) \longrightarrow C_n$

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega t}$$

Shift   $x(t) \xrightarrow{FSC} a_n e^{-jn\omega t}$

$x(t - t_0) \Rightarrow t - t_0 = 0$

$t = t_0$

$$= a_n \cdot e^{-jn\omega t} \cdot e^{-jn\omega t_0}$$

$$x(t - t_0) = \underline{\underline{C_n \cdot e^{-jn\omega t_0}}}$$

$s = j\omega$

$$\boxed{L\left[\frac{dx}{dt}\right] = sX(s) - x(0)}$$

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(c) Time Reversal $\Rightarrow$ $x(t) \xrightarrow{CTFS} C_n$

$x(-t) \longrightarrow C_{-n}$

(d) Time Scaling

$$x(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t}$$

$$x(at) = \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 at}$$

(e) Time Differentiation $\Rightarrow$ Continuous Time Fourier Series

$x(t) \xrightarrow{CTFS} C_n$

$\frac{d}{dt} x(t) \xrightarrow{CTFS} j \cdot \omega n \, C_n$

$\omega = \frac{2\pi}{T}$

(f) Convolution $\Rightarrow$

$x(t) \xrightarrow{FS} a_n$

$y(t) \xrightarrow{FS} b_n$

$x(t) * y(t) \xrightarrow{FS} T_0 a_n \cdot b_n$

(g) Multiplication $\Rightarrow$

$x(t) \xrightarrow{FS} a_n$

$y(t) \xrightarrow{FS} b_n$

$x(t) \cdot y(t) \xrightarrow{FS} T_0 \, a_n * b_n$

(h) Conjugate Symmetry

$\boxed{C_n^* = C_{-n}} \Rightarrow \boxed{C_n = C_{-n}^*}$

$$\underline{x(t) * y(t)} = \int_{\tau=0}^{\infty} x(\tau) \cdot y(t-\tau) \cdot d\tau$$

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## (I) Parseval's Relation / Theorem $\Rightarrow$

$$\frac{1}{T}\int_0^T |f(t)|^2 = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$\Rightarrow \quad x_1(t) \xrightarrow{\text{F.S. Coefficient}} C_n$$

$$x_2(t) \xrightarrow{\text{FS}} D_n$$

$$\frac{1}{T}\int_{t_0}^{t_0+T} x_1(t) \cdot x_2^*(t) = \frac{1}{T}\int_{t_0}^{t_0+T} \sum_{n=-\infty}^{\infty} C_n \cdot e^{jn\omega_0 t} \cdot \sum_{m=-\infty}^{\infty} D_n^* e^{-jm\omega_0 t} dt$$

fourier series

$$\begin{cases} x_1(t) = \sum_{n=-\infty}^{\infty} C_n e^{jn\omega_0 t} \\ x_2^*(t) = \sum_{m=-\infty}^{\infty} D_n^* e^{-jm\omega_0 t} \end{cases}$$

$$\frac{1}{T}\int_{t_0}^{t_0+T} x_1(t)\, x_2^*(t) = \frac{1}{T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} C_n \cdot D_n^* \int_{t_0}^{t_0+T} e^{jn\omega_0 t} \cdot e^{-jm\omega_0 t} dt$$

$$= \frac{1}{T}\sum_{n=-\infty}^{\infty}\sum_{m=-\infty}^{\infty} C_n D_n^* \int_0^T e^{j\omega_0(n-m)t} dt$$

when $n \neq m$

$$\int_0^T e^{j\omega_0(n-m)\cdot t} = 0 \;;\; m \neq n$$

$e^0 = 1 \quad \Leftarrow \quad n = m \Rightarrow \quad C_n = D_n \rightarrow C_n = D_n^*$

$$= \frac{1}{T}\sum_{n=-\infty}^{\infty}\sum_{n=-\infty}^{\infty} C_n \cdot C_n \int_0^T 1 \cdot dt$$

$\delta(t) \rightarrow$ even $\boxed{\delta(t) = \delta(-t)}$

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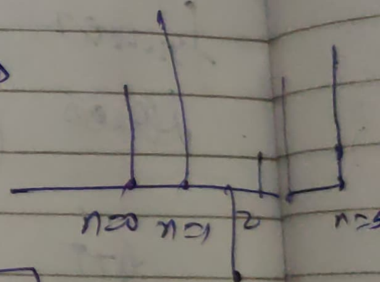
$$= \frac{1}{T} \sum_{n=-\infty}^{\infty} |C_n|^2 \, [t]_0^T$$

$$= \frac{1}{\cancel{T}} \sum_{n=-\infty}^{\infty} |C_n|^2 \, \cancel{T}$$

$$\frac{1}{T} \int_{t_0}^{t_0+T} x_1(t) \cdot x_2^*(t) \, dt = \sum_{n=-\infty}^{\infty} |C_n|^2$$

$$\boxed{\frac{1}{T} \int_{t_0}^{t_0+T} |x_1(t)|^2 \cdot dt = \sum_{n=-\infty}^{\infty} |C_n|^2}$$

Periodic Fourier Series in Discrete Time $\Rightarrow$




series $\boxed{x[n] = \sum_{k=0}^{N-1} a_k \cdot e^{j\omega_0 nk}}$

Period $= N \Rightarrow \boxed{N = \frac{2\pi}{\omega}}$ $\boxed{\omega_0 = \frac{2\pi}{N}}$

Fourier Coefficient $\Rightarrow \boxed{a_k = \frac{1}{N} \sum_{n=0}^{N-1} x[n] \cdot e^{-j\omega_0 nk}}$

Time Scaling $\hat{x}$

$\frac{n}{m}$ = Integer

$m = 2$

$n = 1; 3; 5 \Rightarrow \frac{n}{m} \neq$ Integer $= 0$

$a_k$

One difference →



$$x[n] \longrightarrow a_k$$

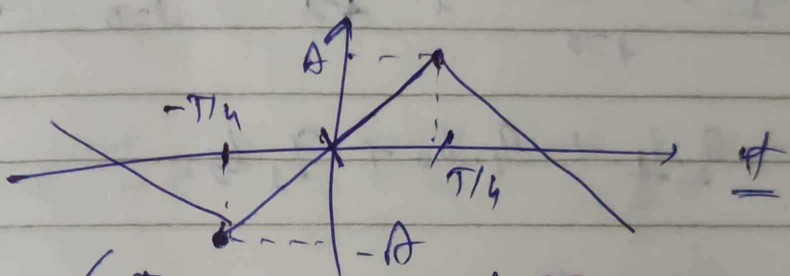
$$x[n-1] \Longrightarrow a_k \cdot e^{-j w_0 \cdot 1 \cdot k}$$

$n-1=0$

$n=1$

$$x[n] - x[n-1] \Longrightarrow a_k - a_k \cdot e^{-j w_0 k}$$

$$= a_k \left(1 - e^{-j \frac{2\pi}{N} \cdot k}\right)$$




$\left(-\frac{T}{4} ; -A\right)$ and $\left(\frac{T}{4} ; A\right)$

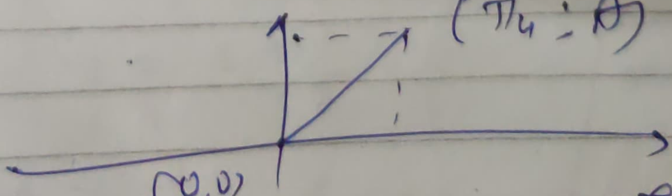
$\quad x_1 \quad y_1 \qquad\qquad x_2 \quad y_2$

$$y - y_1 = \frac{y_2 - y_1}{x_2 - x_1} \cdot (x - x_1)$$

$$x(t) - (-A) = \frac{A - (-A)}{\frac{T}{4} - (-T/4)} \cdot [t - (-T/4)]$$

$$x(t) = -A + \frac{2A}{T} \cdot 2 \cdot [t + T/4]$$

$$x(t) = -A + \frac{4A}{T}[t + T/4]$$




$$e^{j\theta} = \cos\theta + j\sin\theta$$

$$e^{-j\theta} = \cos\theta - j\sin\theta$$

$$e^{-j\pi/4} = \cos\frac{\pi}{4} - j\sin\pi/4$$

$$e^{-j\pi/4} = \frac{1}{\sqrt{2}} - j\,1/\sqrt{2}$$

$$\begin{cases} a_0, a_1, a_{-1} \\ b_0; b_1; b_{-1} \end{cases} \qquad \left\{ \frac{1}{3j} = \frac{-j}{3} \right\}$$

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$\sqrt{-1} = j$, $j^2 = -1$, $j^3 = -j$, $j^4 = 1$

$$d_k = x[n]\, y[n] \longleftrightarrow \sum_{l=0}^{N-1} a_l\, b_{k-l}$$

(i) $k=0$

$$d_0 = \sum_{l=0}^{N-1} a_l\, b_{0-l} = \sum_{l=0}^{N-1} a_l\, b_{-l}$$

$$d_0 = a_0 b_0 + a_1 b_{-1} + a_{-1} b_1$$

(ii) $k=1$

$$d_1 = \sum_{l=0}^{N-1} a_l \cdot b_{1-l} = \sum_{l=0}^{N-1} a_l\, b_{1-l}$$

$$d_1 = a_0 \cdot b_1 + a_1 \cdot b_0 + a_{-1}\, b_2$$

(iii) $k=-1 \Rightarrow$

$$d_{-1} = \sum_{l=0}^{-1:1} a_l \cdot b_{-1-l}$$

$$d_{-1} = \sum_{l=0}^{-1:1} a_l \cdot b_{-1-l}$$

$$d_{-1} = a_0 b_{-1} + a_1 \cdot b_{-2} + a_{-1} b_0$$

$$|c_2| = \sqrt{\left(\frac{1}{2\sqrt{2}}\right)^2 + \left(\frac{1}{2\sqrt{2}}\right)^2}$$

$$= \sqrt{\cancel{2} \cdot \frac{1}{4 \cdot \cancel{2}}} = \frac{1}{2}$$

$j^{-1} = j$ ... $\frac{1}{j} = -j$

$1 = -j \times j$
$1 = -j^2$
$1 = -1(-1)$
$1 = +1$

$j^2 = -1$
$j^3 = -j$
$j^4 = 1$

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4.14

$$x(t) = 1 + \sin \omega_0 t + 2\cos \omega_0 t + \cos\left(2\omega_0 t + \frac{\pi}{4}\right)$$

$$x(t) = 1 + \frac{e^{j\omega_0 t} - e^{-j\omega_0 t}}{2j} + 2\left(\frac{e^{j\omega_0 t} + e^{-j\omega_0 t}}{2}\right) + \left[\frac{e^{j(2\omega_0 t + \pi/4)} + e^{-j(2\omega_0 t + \pi/4)}}{2}\right]$$

$$x(t) = 1 + \frac{1}{2j} e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} + e^{j\omega_0 t} + e^{-j\omega_0 t} + \frac{1}{2}\left[e^{j2\omega_0 t} \cdot e^{j\pi/4} + e^{-j2\omega_0 t} \cdot e^{-j\pi/4}\right]$$

$$x(t) = 1 + \frac{1}{2j} e^{j\omega_0 t} + e^{j\omega_0 t} - \frac{1}{2j} e^{-j\omega_0 t} + e^{-j\omega_0 t} + \frac{1}{2} e^{2j\omega_0 t} \cdot \left\{\cos \pi/4 + j \sin \pi/4\right\} + \frac{1}{2} e^{-2j\omega_0 t} \cdot \left\{\cos \frac{\pi}{4} - j \sin \pi/4\right\}$$

$$= 1 + e^{j\omega_0 t}\left(1 + \frac{1}{2j}\right) + e^{-j\omega_0 t}\left[1 - \frac{1}{2j}\right] + \frac{1}{2} e^{2j\omega_0 t}\left(\frac{1}{\sqrt{2}} + j\frac{1}{\sqrt{2}}\right) + \frac{1}{2} e^{-2j\omega_0 t}\left\{\frac{1}{\sqrt{2}} - j\frac{1}{\sqrt{2}}\right\}$$

$$= 1 + e^{j\omega_0 t}\left(1 - \frac{j}{2}\right) + e^{-j\omega_0 t}\left[1 + \frac{j}{2}\right] + e^{2j\omega_0 t}\left[\frac{1}{2\sqrt{2}} + j\frac{1}{2\sqrt{2}}\right] + e^{-2j\omega_0 t}\left[\frac{1}{2\sqrt{2}} - \frac{j \cdot 1}{2\sqrt{2}}\right]$$

$C_0 = 1$ ; $C_1 = 1 - \frac{j}{2}$ $\Rightarrow |C_1| = \sqrt{1 + \left(\frac{1}{2}\right)^2}$

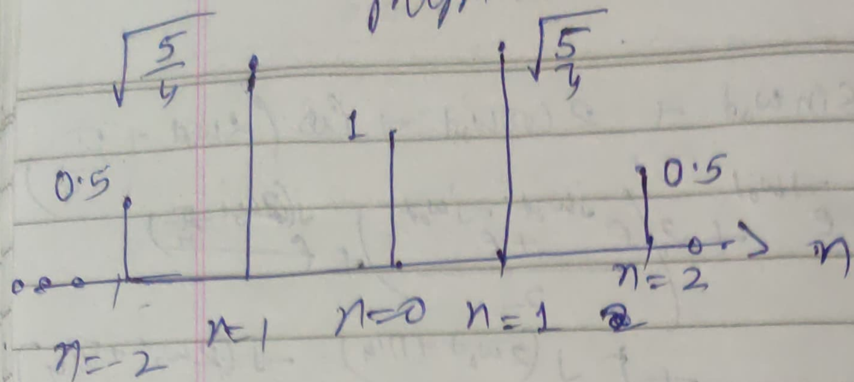
$|C_{-1}|$ $\quad C_{-1} = 1 + \frac{j}{2}$

$|C_2|$ $\quad C_2 = \frac{1}{2\sqrt{2}} + \frac{j}{2\sqrt{2}}$

$|C_{-2}|$ $\quad C_{-2} = \frac{1}{2\sqrt{2}} - \frac{j}{2\sqrt{2}}$

other $C_n = 0$
$C_n = 0$ ; $n > 2$
$C_n = 0$ ; $n < -2$
$C_n = 0$ ; $|n| > 2$

Magnitude




Phase $\angle C_n = \tan^{-1}\left(\frac{Im}{R}\right)$

$C_0 = 1 + j \cdot 0 \Rightarrow R = 1 \; ; \; Im = 0$

$\angle C_0 = \tan^{-1}\left(\frac{0}{1}\right) = 0$

$C_{-1} = 1 + \frac{j}{2} \Rightarrow \angle C_{-1} = \tan^{-1}\left[\frac{1}{2}\right]$ upside

$C_{+1} = 1 - \frac{j}{2} = \tan^{-1}\left[-\frac{1}{2}\right]$ down side

$C_2 = \frac{1}{2\sqrt{2}} + \frac{j}{2\sqrt{2}} \Rightarrow \angle C_2 = \tan^{-1}\left(\frac{2\sqrt{2}}{2\sqrt{2}}\right)$

$\angle C_2 = \pi/4$

$C_{-2} = \frac{1}{2\sqrt{2}} - \frac{j}{2\sqrt{2}} \Rightarrow C_{-2} = -\pi/4$

Phase angle