Unit 5

AC Bridges

INTRODUCTION: Alternating current bridges are most popular, convenient and accurate instruments for measurement of unknown inductance, capacitance and some other related quantities. In its simplest form, ac bridges can be thought of to be derived from the conventional dc Wheatstone bridge. An ac bridge, in its basic form, consists of four arms, an alternating power supply, and a balance detector.

SOURCES AND DETECTORS IN AC BRIDGES: For measurements at low frequencies, bridge power supply can be obtained from the power line itself. Higher frequency requirements for power supplies are normally met by electronic oscillators. Electronic oscillators have highly stable, accurate yet adjustable frequencies. Their output waveforms are very close to sinusoidal and output power level sufficient for most bridge measurements.

When working at a single frequency, a tuned detector is preferred, since it gives maximum sensitivity at the selected frequency and discrimination against harmonic frequencies. Vibration galvanometers are most commonly used as tuned detectors in the power frequency and low audio-frequency ranges. Though vibration galvanometers can be designed to work as detectors over the frequency range of 5 Hz to 1000 Hz, they have highest sensitivity when operated for frequencies below 200 Hz.

Head phones or audio amplifiers are popularly used as balance detectors in ac bridges at frequencies of 250 Hz and above, up to 3 to 4 kHz.

Transistor amplifier with frequency tuning facilities can be very effectively used as balance detectors with ac bridges. With proper tuning, these can be used to operate at a selective band of frequencies with high sensitivity. Such detectors can be designed to operate over a frequency range of 10 Hz to 100 kHz.

General form of A.C. bridge: AC bridge are similar to D.C. bridge in topology(way of connecting).It consists of four arm AB,BC,CD and DA .Generally the impedance to be measured is connected between ‘A’ and ‘B’. A detector is connected between ‘B’ and ’D’. The detector is used as null deflection instrument. Some of the arms are variable element. By varying these elements, the potential values at ‘B’ and‘D’ can be made equal. This is called balancing of the bridge.

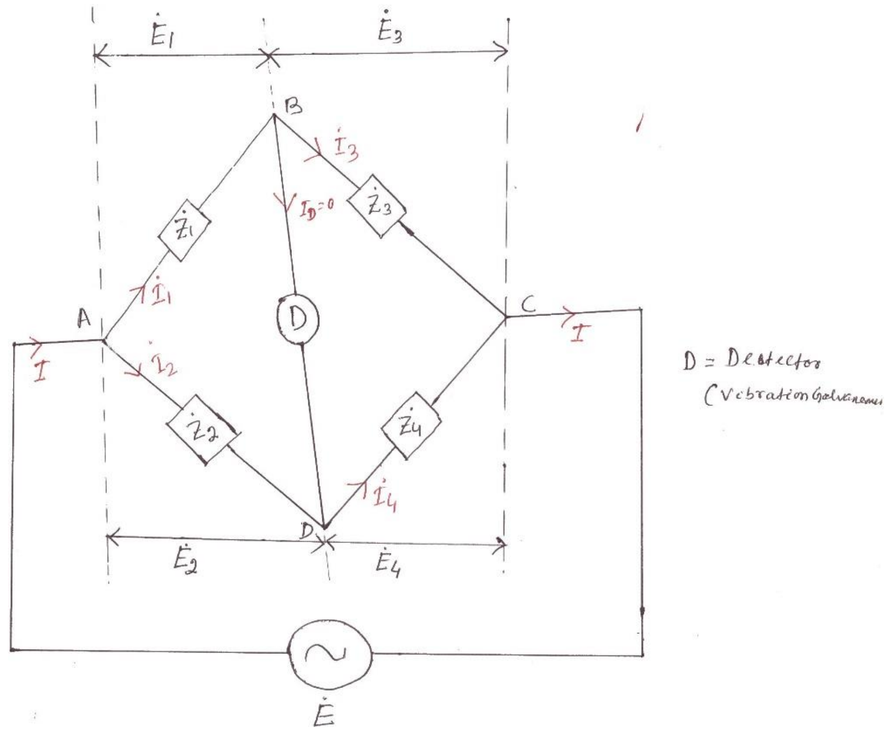


Fig. General form of A.C. bridgeAt

the balance condition, the current through detector is zero.

$$I_1=I_3$$

$$I_2=I_4$$

$$\frac{I_1}{I_2}=\frac{I_3}{I_4}$$

At balance condition,

Voltage drop across ‘AB’=voltage drop across ‘AD’.

$$E_1 = E_2$$

$$I_1 Z_1 = I_2 Z_2$$

Similarly, Voltage drop across ‘BC’=voltage drop across ‘DC’

$$E_3 = E_4$$

$$I_3 Z_3 = I_4 Z_4$$

Form above equation

$$\frac{I_1}{I_2}=\frac{z_2}{z_1}$$

$$\frac{I_3}{I_4} = \frac{z_4}{z_3}$$

From above equation we can write

$$\therefore \frac{\dot{Z}_2}{\dot{Z}_1} = \frac{\dot{Z}_4}{\dot{Z}_3}$$

$$\therefore \dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$$

Products of impedances of opposite arms are equal.

$$\therefore |Z_1|\angle\theta_1 |Z_4|\angle\theta_4 = |Z_2|\angle\theta_2 |Z_3|\angle\theta_3$$

$$\Rightarrow |Z_1||Z_4|\angle\theta_1 + \theta_4 = |Z_2||Z_3|\angle\theta_2 + \theta_3$$

$$|Z_1||Z_4| = |Z_2||Z_3|$$

$$\theta_1 + \theta_4 = \theta_2 + \theta_3$$

* For balance condition, magnitude on either side must be equal.

* Angle on either side must be equal.

Summary

For balance condition,

- $\dot{I}_1 = \dot{I}_3, \quad \dot{I}_2 = \dot{I}_4$
- $|Z_1||Z_4| = |Z_2||Z_3|$
- $\theta_1 + \theta_4 = \theta_2 + \theta_3$
- $\dot{E}_1 = \dot{E}_2 \quad \& \quad \dot{E}_3 = \dot{E}_4$

5.1 Measurements of inductance

5.1.1 Maxwell's inductance bridge

The choke for which R_1 and L_1 have to measure connected between the points 'A' and 'B'. In this method the unknown inductance is measured by comparing it with the standard inductance.

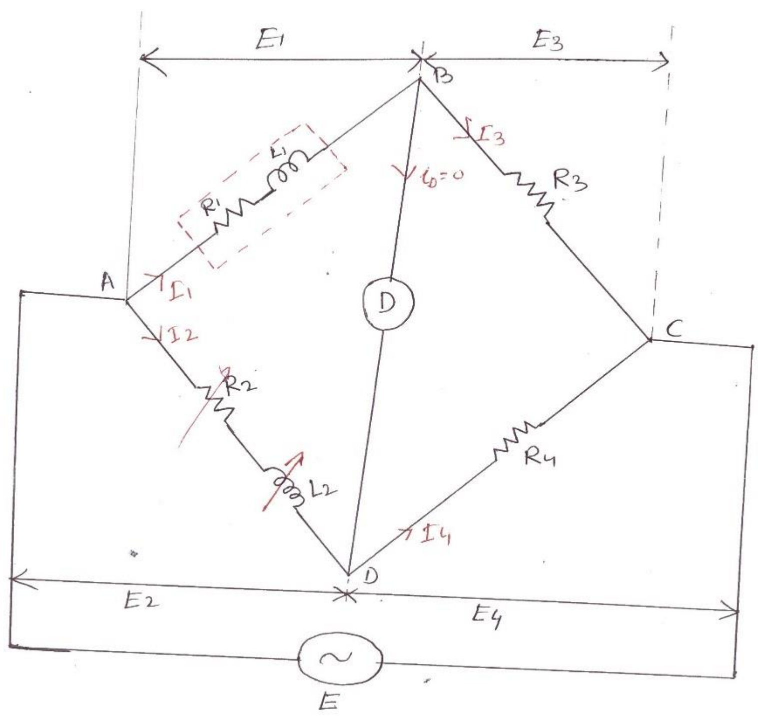


Fig. Maxwell's inductance bridge

L_2 is adjusted, until the detector indicates zero current.

Let R_1= unknown resistance

L_1= unknown inductance of the choke.

L_2= known standard inductance

R_1,R_2,R_4= known resistances.

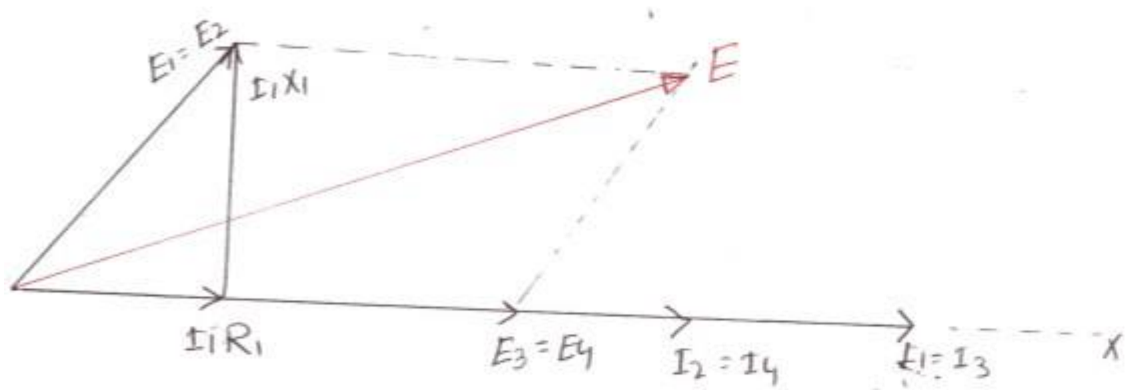


Fig Phasor diagram of Maxwell's inductance bridge

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$(R_1 + jXL_1)R_4 = (R_2 + jXL_2)R_3$$

$$(R_1 + jwL_1)R_4 = (R_2 + jwL_2)R_3$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwL_2R_3$$

Comparing real part,

$$R_1R_4 = R_2R_3$$

$$\therefore R_1 = \frac{R_2R_3}{R_4}$$

Comparing the imaginary parts,

$$wL_1R_4 = wL_2R_3$$

$$L_1 = \frac{L_2R_3}{R_4}$$

Q-factor of choke, $Q = \frac{WL_1}{R_1} = \frac{WL_2R_3R_4}{R_4R_2R_3}$

$$Q = \frac{WL_2}{R_2}$$

Advantages

- ✓ Expression for R_1 and L_1 are simple.
- ✓ Equations area simple
- ✓ They do not depend on the frequency (as w is cancelled)
- ✓ R_1 and L_1 are independent of each other.

Disadvantages

- ✓ Variable inductor is costly.
- ✓ Variable inductor is bulky.

5.1.2 Maxwell's inductance capacitance bridge

Unknown inductance is measured by comparing it with standard capacitance. In this bridge, balance condition is achieved by varying 'C_4'.

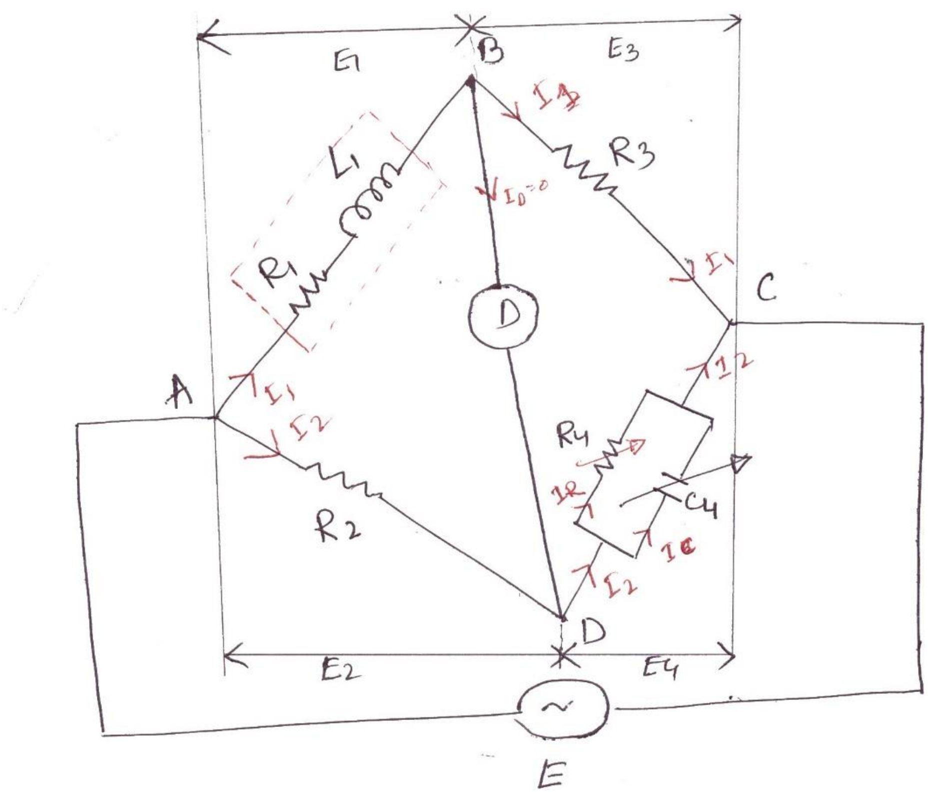


Fig Maxwell's inductance capacitance bridge

At balance condition, $Z_1Z_4=Z_3Z_2$

$$Z_4 = R_4 \parallel \frac{1}{jwC_4} = \frac{R_4 \times \frac{1}{jwC_4}}{R_4 + \frac{1}{jwC_4}}$$

$$Z_4 = \frac{R_4}{jwR_4C_4 + 1} = \frac{R_4}{1 + jwR_4C_4}$$

Substituting the value of Z_4 from Above equation and we get

$$(R_1 + jwL_1) \times \frac{R_4}{1 + jwR_4C_4} = R_2R_3$$

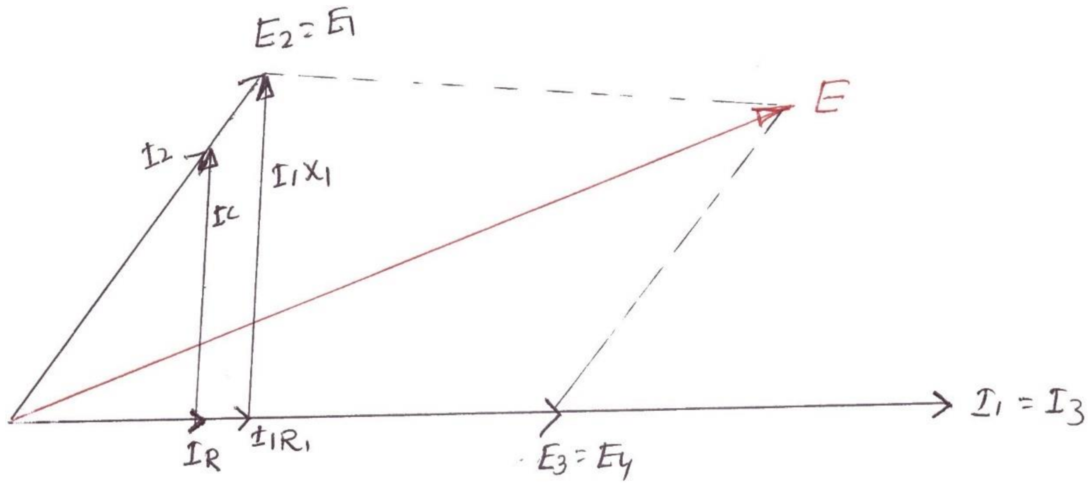


Fig Phasor diagram of Maxwell's inductance capacitance bridge

$$(R_1 + jwL_1)R_4 = R_2R_3(1 + jwR_4C_4)$$

$$R_1R_4 + jwL_1R_4 = R_2R_3 + jwC_4R_4R_2R_3$$

Comparing real parts, $R_1R_4 = R_2R_3$

$$\Rightarrow R_1 = \frac{R_2 R_3}{R_4}$$

Comparing imaginary part,

$$wL_1 R_4 = wC_4 R_4 R_2 R_3$$

$$L_1 = C_4 R_2 R_3$$

Q-factor of choke,

$$Q = \frac{WL_1}{R_1} = w \times C_4 R_2 R_3 \times \frac{R_4}{R_2 R_3}$$

$$Q = wC_4 R_4$$

Advantages

- ✓ Equation of L_1 and R_1 are simple.
- ✓ They are independent of frequency.
- ✓ They are independent of each other.
- ✓ Standard capacitor is much smaller in size than standard inductor.

Disadvantages

- ✓ Standard variable capacitance is costly.
- ✓ It can be used for measurements of Q-factor in the ranges of 1 to 10.
- ✓ It cannot be used for measurements of choke with Q-factors more than 10.

We know that Q $=wC_4R_4$

For measuring chokes with higher value of Q-factor, the value of C_4 and R_4 should be higher. Higher values of standard resistance are very expensive. Therefore this bridge cannot be used for higher value of Q-factor measurements.

5.1.2 Hay's bridge

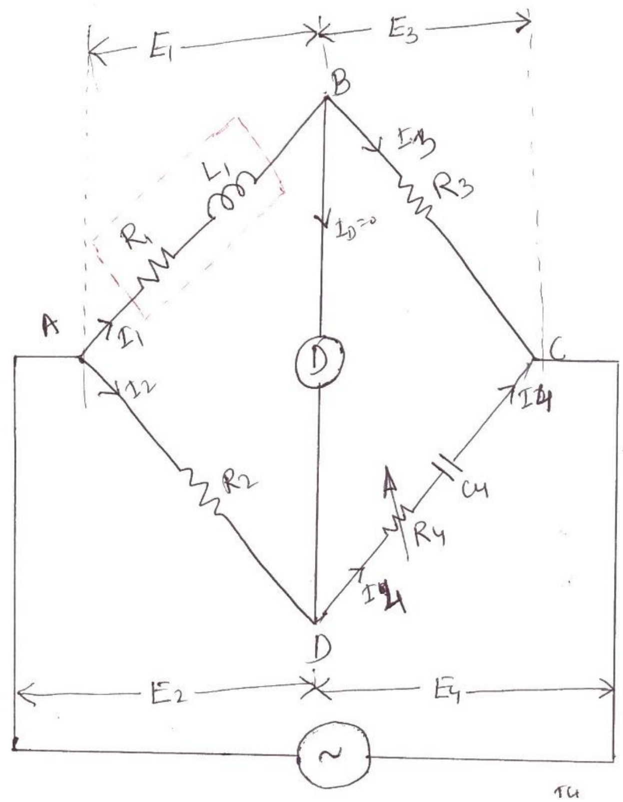


Fig Hay's bridge

- $\dot{E}_1 = I_1 R_1 + jI_1 X_1$
- $\dot{E} = \dot{E}_1 + \dot{E}_3$
- $\dot{E}_4 = \dot{I}_4 R_4 + \frac{I_4}{jwC_4}$
- $\dot{E}_3 = I_3 R_3$

$$Z_4 = R_4 + \frac{1}{jwC_4} = \frac{1 + jwR_4C_4}{jwC_4}$$

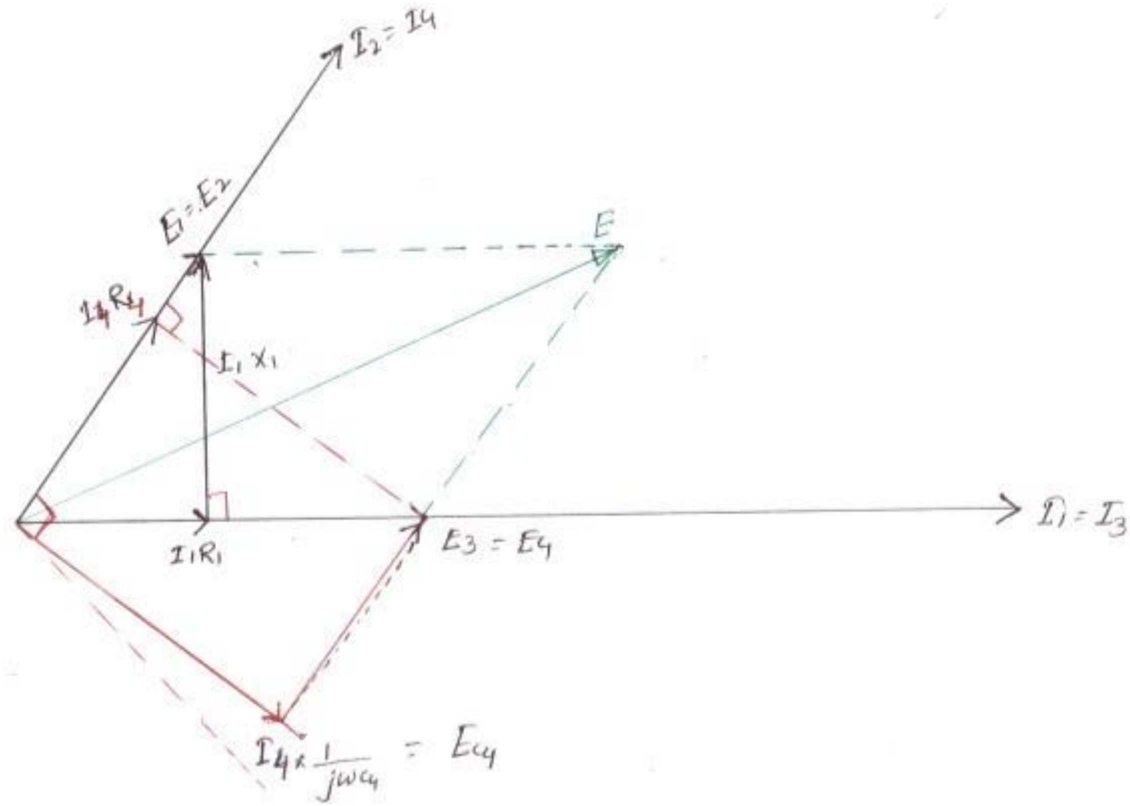


Fig Phasor diagram of Hay's bridgeAt

balance condition, $Z_1Z_4=Z_3Z_2$

At balance condition, $Z_1Z_4=Z_3Z_2$

$$(R_1 + jwL_1)\left(\frac{1 + jwR_4C_4}{jwC_4}\right) = R_2R_3$$

$$(R_1 + jwL_1)(1 + jwR_4C_4) = jwR_2C_4R_3$$

$$R_1 + jwC_4R_4R_1 + jwL_1 + j^2w^2L_1C_4R_4 = jwC_4R_2R_3$$

$$(R_1 - w^2L_1C_4R_4) + j(wC_4R_4R_1 + wL_1) = jwC_4R_2R_3$$

Comparing the real term,

$$R_1 - w^2L_1C_4R_4 = 0$$

$$R_1 = w^2L_1C_4R_4$$

Comparing the imaginary terms,

$$wC_4R_4R_1 + wL_1 = wC_4R_2R_3$$

$$C_4R_4R_1 + L_1 = C_4R_2R_3$$

$$L_1 = C_4R_2R_3 - C_4R_4R_1$$

Substituting the value of R1 from eqn. we have,

$$L_1 = C_4R_2R_3 - C_4R_4 \times w^2L_1C_4R_4$$

$$L_1 = C_4R_2R_3 - w^2L_1C_4{}^2R_4{}^2$$

$$L_1(1 + w^2L_1C_4{}^2R_4{}^2) = C_4R_2R_3$$

$$L_1 = \frac{C_4R_2R_3}{1 + w^2L_1C_4{}^2R_4{}^2}$$

Substituting the value of L_1 in above eqn., we have

$$R_1 = \frac{w^2C_4{}^2R_2R_3R_4}{1 + w^2C_4{}^2R_4{}^2}$$

$$Q = \frac{wL_1}{R_1} = \frac{w \times C_4R_2R_3}{1 + w^2C_4{}^2R_4{}^2} \times \frac{1 + w^2C_4{}^2R_4{}^2}{w^2C_4{}^2R_4R_2R_3}$$

$$Q = \frac{1}{wC_4R_4}$$

Advantages

- ✓ Fixed capacitor is cheaper than variable capacitor.
- ✓ This bridge is best suitable for measuring high value of Q-factor.

Disadvantages

- ✓ Equations of L1and R1 are complicated.
- ✓ Measurements of R_1 and L_1 require the value of frequency.
- ✓ This bridge cannot be used for measuring low Q- factor.

5.1.3 Anderson's bridge

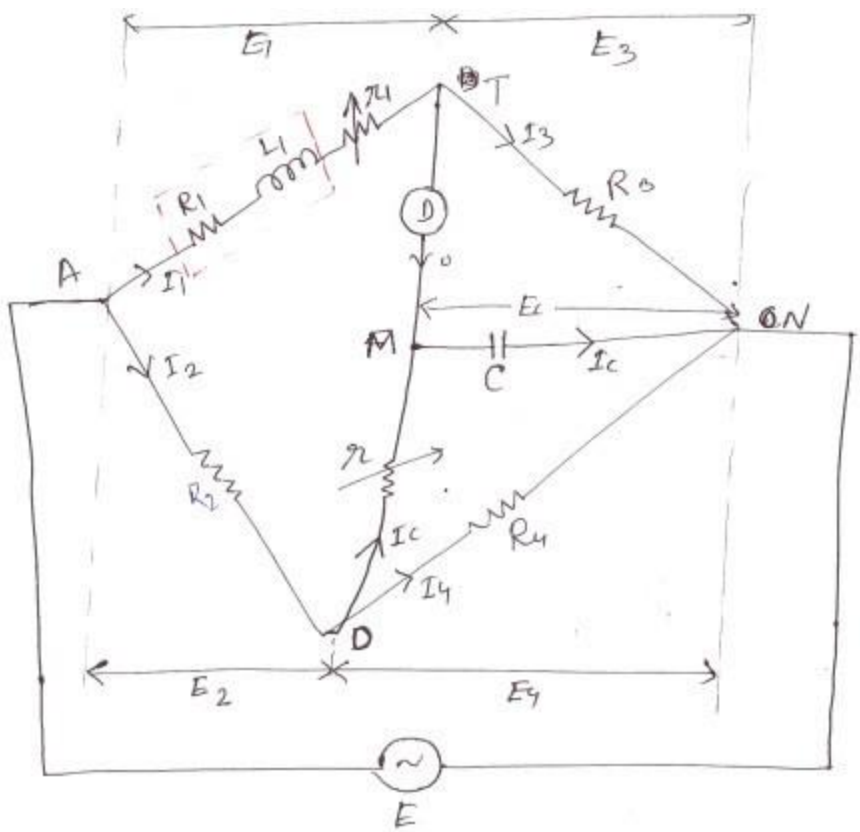


Fig Anderson's bridge

- $\dot{E}_1 = I_1(R_1 + r_1) + jI_1X_1$
- $E_3 = E_C$
- $\dot{E}_4 = \dot{I}_C r + E_C$
- $I_2 = I_4 + I_C$
- $\bar{E}_2 + \bar{E}_4 = \bar{E}$
- $\bar{E}_1 + \bar{E}_3 = \bar{E}$

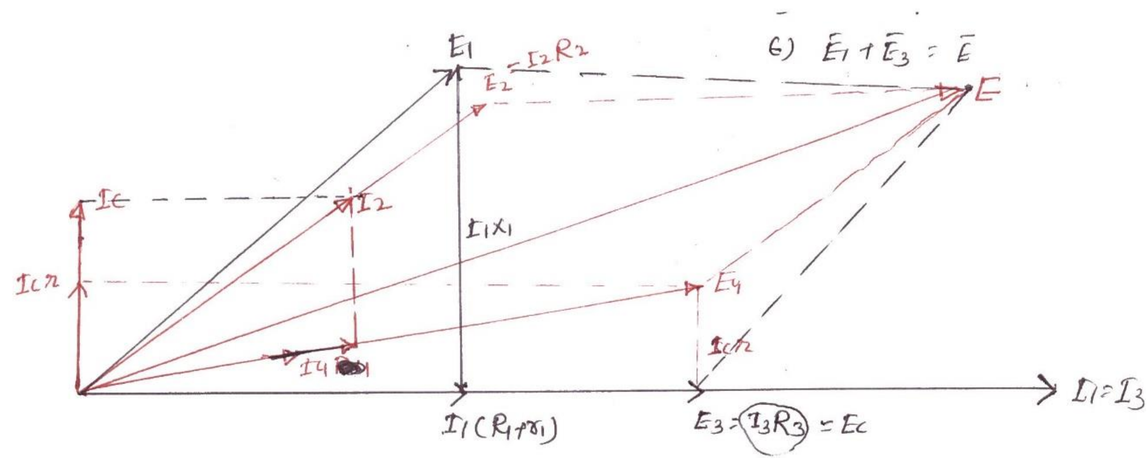


Fig Phasor diagram of Anderson's bridge

Step-1 Take I_1 as references vector .Draw $I_1R_1^1$ in phase with I_1

$R_1^1 = (R_1 + r_1)$, I_1X_1 is $\perp_r$ to $I_1R_1^1$

$E_1 = I_1R_1^1 + jI_1X_1$

Step-2 $I_1 = I_3$, E_3 is in phase with I_3, From the circuit ,

$E_3 = E_C$, I_C leads E_C by 90^0

Step-3 $E_4 = I_C r + E_C$

Step-4 Draw I_4 in phase with E_4 , By KCL , $\bar{I_2} = \bar{I_4} + \bar{I_C}$

Step-5 Draw E_2 in phase with I_2

Step-6 By KVL , $\bar{E_1} + \bar{E_3} = \bar{E}$ or $\bar{E_2} + \bar{E_4} = \bar{E}$

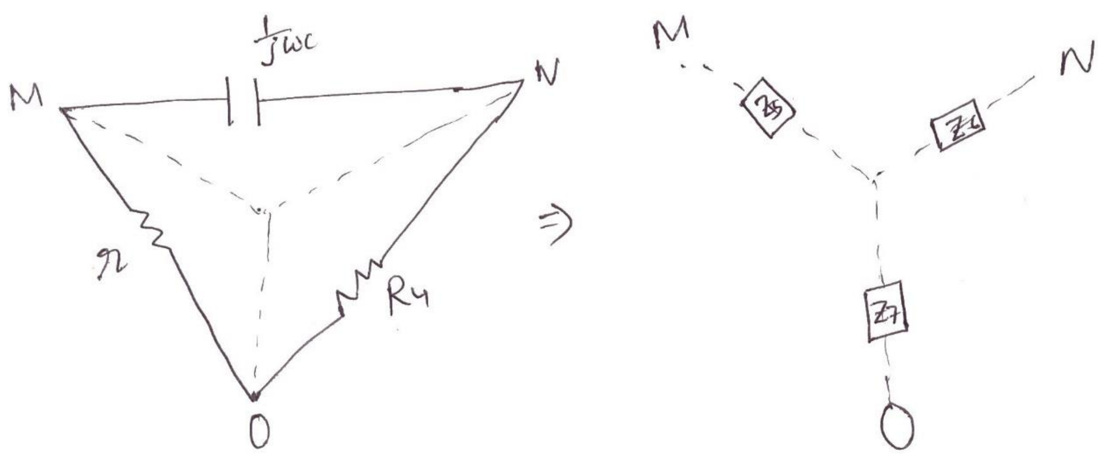


Fig Equivalent delta to star conversion for the loop MO

$$Z_7 = \frac{R_4 \times r}{R_4 + r + \frac{1}{jwc}} = \frac{jwCR_4 r}{1 + jwC(R_4 + r)}$$

$$Z_6 = \frac{R_4 \times \frac{1}{jwC}}{R_4 + r + \frac{1}{jwc}} = \frac{R_4}{1 + jwC(R_4 + r)}$$

$$(R_1^1 + jwL_1) \times \frac{R_4}{1 + jwC(R_4 + r)} = R_3(R_2 + \frac{jwCR_4 r}{1 + jwC(R_4 + r)})$$

$$\Rightarrow \frac{(R_1^1 + jwL_1)R_4}{1 + jwC(R_4 + r)} = R_3\left[\frac{R_2(1 + jwC(R_4 + r)) + jwCrR_4}{1 + jwC(R_4 + r)}\right]$$

$$\Rightarrow R_1^1 R_4 + jwL_1 R_4 = R_2 R_3 + jCwR_2 R_3(r + R_4) + jwCrR_4 R_3$$

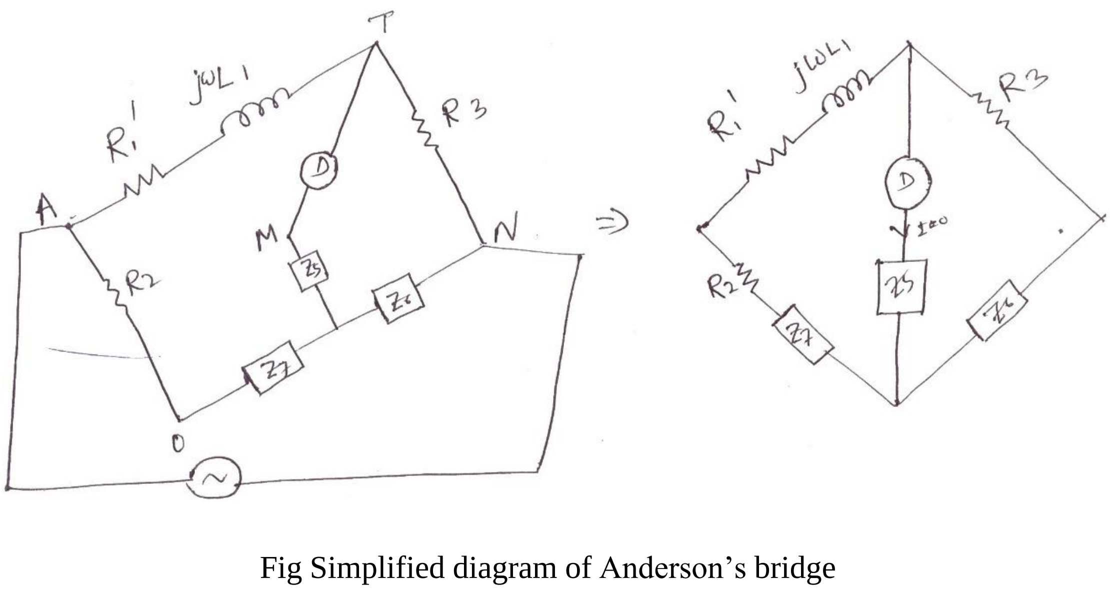


Fig Simplified diagram of Anderson’s bridge

Comparing real term,

$$R_1^1 R_4 = R_2 R_3$$

$$(R_1 + r_1)R_4 = R_2 R_3$$

$$R_1 = \frac{R_2 R_3}{R_4} - r_1$$

Comparing the imaginary term,

$$wL_1 R_4 = wCR_2 R_3 (r + R_4) + wcrR_3 R_4$$

$$L_1 = \frac{R_2 R_3 C}{R_4}(r + R_4) + R_3 rC$$

$$L_1 = R_3 C\left[\frac{R_2}{R_4}(r + R_4) + r\right]$$

Advantages

- ✓ Variable capacitor is not required.
- ✓ Inductance can be measured accurately.
- ✓ R_1 and L_1 are independent of frequency.
- ✓ Accuracy is better than other bridges.

Disadvantages

- ✓ Expression for R_1 and L_1 are complicated.
- ✓ This is not in the standard form A.C. bridge.

Heaviside's bridge for mutual inductance measurement

Before we introduce this bridge let us know more about the uses of mutual inductor in bridge circuits. Now one question must arise in our mind that why we are so much interested in mutual inductance, answer to this question is very simple we will use this mutual inductor in **Heaviside bridge circuit**. We use standard mutual inductor in finding out the the value of unknown mutual inductor in various circuits. Mutual inductor is used in various circuits as main component in determining the value of self inductance, capacitance and frequency etc.
But in many industries the use of mutual inductor in finding out the value of known self inductor is not practices because we have various other accurate methods for finding out self inductor and capacitance and these other methods may include the use of standard capacitor which are available at cheaper rate. However there may be some merits of use mutual inductor in some cases but this field is very vast.

Many researches are going on the application of mutual inductor in bridge circuits. In order to understand the mathematical part of **Heaviside bridge**, we need to derive the mathematical relation between self inductor and mutual inductor in two coils connected in series combination. Here we interested in finding out the expression for mutual inductor in terms of self inductance.
Let us consider two coils connected in series as shown in figure given below.

Such that the magnetic fields are additive, the resultant inductor of these two can be calculated as

$$L_x = L_1 + L_2 + 2M \cdots\cdots\cdots\cdots(1)$$

Where, L_1 is the self inductor of first coil,

L_2 is the self inductor of second coil,

M is the mutual inductor of these two coils.

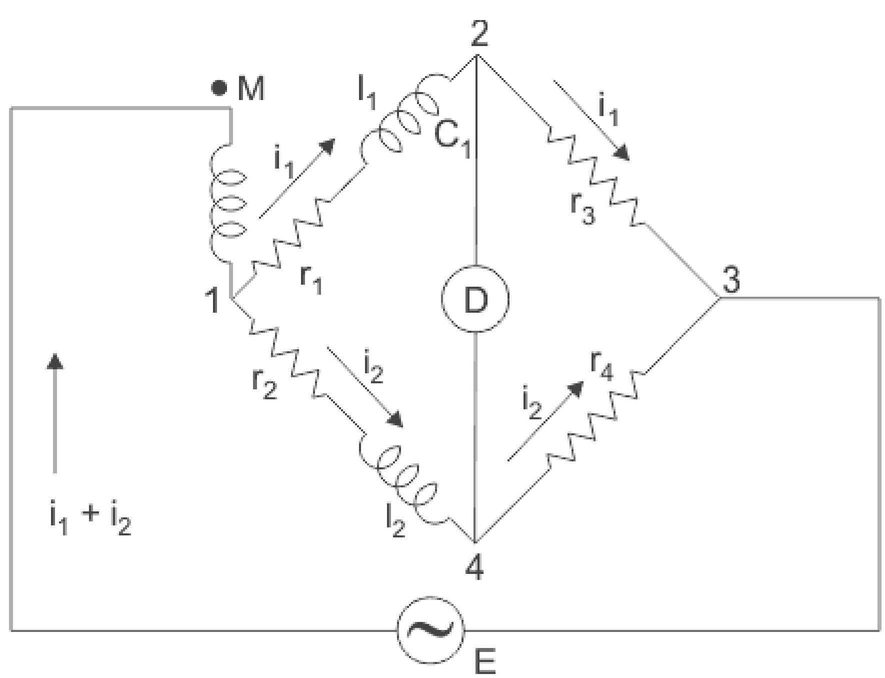
Now if the connections of any one of the coils is reversed then we have

$$L_y = L_1 + L_2 - 2M \cdots\cdots\cdots\cdots(2)$$

On solving these two equations we have

$$M = \frac{L_x - L_y}{4}$$

Thus the mutual inductor of the two coils connected in series is given by one-fourth of the difference between the measured value of self inductor when taking the direction of field in the same direction and value of self inductor when the direction of field is reversed.
However, one needs to have the two series coils on the same axis in order to get most accurate result.
Let us consider the circuit of **Heaviside mutual inductor bridge**, given below,



Main application of this bridge in industries is to measure the mutual inductor in terms of self inductance.

Circuit of this bridge consists of four non inductive resistors r_1, r_2, r_3 and r_4 connected on arms 1-2, 2-3, 3-4 and 4-1 respectively. In series of this bridge circuit an unknown mutual inductor is connected. A voltage is applied to across terminals 1 and 3. At balance point electric current flows through 2-4 is zero hence the voltage drop across 2-3 is equal to voltage drop across 4-3. So by equating the voltage drops of 2-4 and 4-3 we have,

$$i_1 r_3 \;=\; i_2 r_4$$

Also we have,

$$(i_1 + i_2)(j\omega M) \;+\; i_1(r_1 + r_3 + j\omega l_1) \;=\; i_2(r_2 + r_4 + j\omega l_2)$$

$$Therefore,\; i_2 \frac{r_4}{r_3 + 1} j\omega M \;+\; i_2 \frac{r_2}{r_3}(r_1 + r_3 + j\omega l_1) \;=\; i_2(r_2 + r_4 + j\omega l_2)$$

$$or\; j\omega M \left(\frac{r_4}{r_3 + 1} \;+1 \right) \;+\; \frac{r_4}{r_3} r_1 \;+\; r_4 \;+\; j\omega l_1 \frac{r_4}{r_3} \;=\; r_2 + r_4 + j\omega l_2$$

$$Thus,\; r_1 \;=\; r_2 \frac{r_3}{r_4}$$

and mutual inductor is given by,

$$\frac{r_3 l_2 - r_4 l_1}{r_3 + r_4}$$

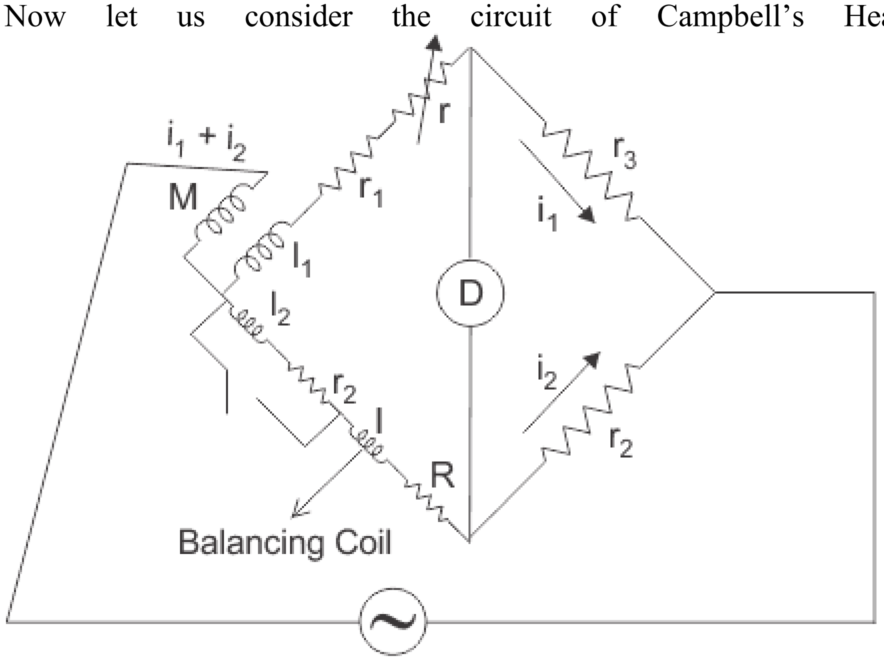
Let us consider some special case,

$$When,\; r_3 \;=\; r_4$$

In this case the mutual inductor is reduced to

$$\frac{l_1 - l_2}{2}$$

Now let us consider the circuit of Campbell's Heaviside bridge given below:



This is the modified **Heaviside bridge**. This bridge is used to measure the unknown value of self inductor in terms of mutual inductance.The modification is due to addition of balancing coil l, and R in arm 1 –

4 and also electrical resistance r is included in arm 1-2. Short circuit switching is connected across r_2 and l_2 in order to have two sets of readings one while short circuiting r_2 and l_2 and other while open circuiting r_2 and l_2.
Now let us derive the expression for self inductor for this modified Heaviside bridge. Also let us assume that the value of M and r with switch open be M_1 and r_1, M_2 and r_2 with switch closed. For open switch, we have at balance point,

$$l_2 + l = \frac{M_1(r_3 + r_4) + r_4 l_1}{r_3}$$

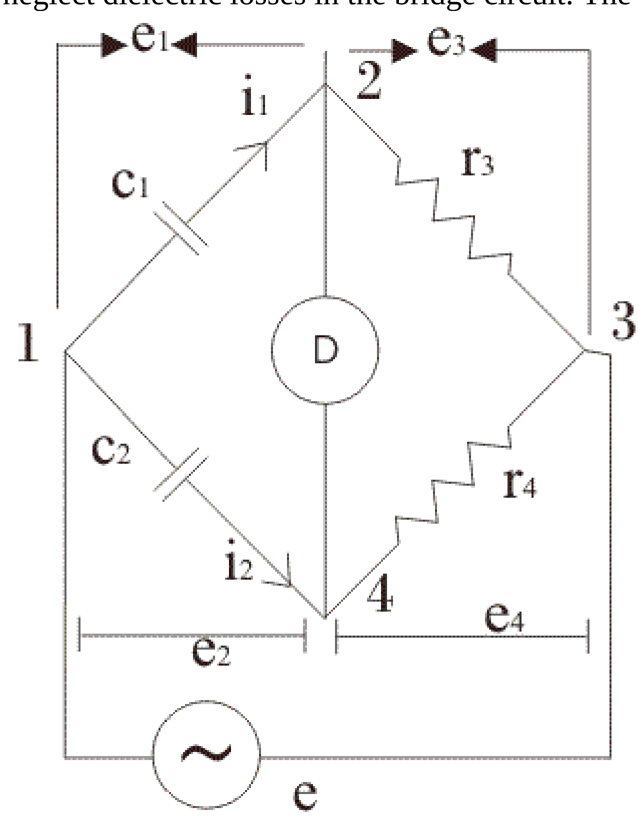
and with closed switch we can write

$$l = \frac{M_2(r_3 + r_4) + r_4 l_1}{r_3}$$

Thus we final expression for self inductor

$$l_2 \ as \ \frac{M_1 - M_2}{1 + \frac{r_4}{r_3}}$$

De Sauty Bridge for capacitance measurement
This bridge provide us the most suitable method for comparing the two values of capacitor if we neglect dielectric losses in the bridge circuit. The circuit of **De Sauty's bridge** is shown below.



Battery is applied between terminals marked as 1 and 4. The arm 1-2 consists of capacitor c_1 (whose value is unknown) which carries current i_1 as shown, arm 2-4 consists of pure resistor (here pure resistor means we assuming it non inductive in nature), arm 3-4 also consists of pure resistor and arm 4-1 consists of standard capacitor whose value is already known to us.
Let us derive the expression for capacitor c_1 in terms of standard capacitor and resistors.

At balance condition we have,

$$\frac{1}{j\omega c_1} \times r_4 \;=\; \frac{1}{j\omega c_2} \times r_3$$

It implies that the value of capacitor is given by the expression

$$c_1 \;=\; c_2 \times \frac{r_4}{r_3}$$

In order to obtain the balance point we must adjust the values of either r_3 or r_4 without disturbing any other element of the bridge. This is the most efficient method of comparing the two values of capacitor if all the dielectric losses are neglected from the circuit.
Now let us draw and study the phasor diagram of this bridge. Phasor diagram of **De Sauty bridge** is shown below:

Let us mark the current drop across unknown capacitor as e_1, voltage drop across the resistor r_3 be e_3, voltage drop across arm 3-4 be e_4 and voltage drop across arm 4-1 be e_2. At balance condition the current flows through 2-4 path will be zero and also voltage drops e_1 and e_3 be equal to voltage drops e_2 and e_4 respectively.
In order to draw the phasor diagram we have taken e_3 (or e_4) reference axis, e_1 and e_2 are shown at right angle to e_1 (or e_2). Why they are at right angle to each other? Answer to this question is very simple as capacitor is connected there, therefore phase difference angle obtained is 90^o.
Now instead of some advantages like bridge is quite simple and provides easy calculations, there are some disadvantages of this bridge because this bridge give inaccurate results for imperfect capacitor (here imperfect means capacitors which not free from dielectric losses). Hence we can use this bridge only for comparing perfect capacitors.
Here we interested in modify the **De Sauty's bridge**, we want to have such a kind of bridge that will gives us accurate results for imperfect capacitors also. This modification is done by Grover. The modified circuit diagram is shown below:

Here Grover has introduced electrical resistances r_1 and r_2 as shown in above on arms 1-2 and 4-1 respectively, in order to include the dielectric losses. Also he has connected resistances R_1 and R_2 respectively in the arms 1-2 and 4-1. Let us derive the expression capacitor c_1 whose value is unknown to us. Again we connected standard capacitor on the same arm 1-4 as we have done in **De Sauty's bridge**. At balance point on equating the voltage drops we have:

$$\left(R_1 + r_1 + \frac{1}{j\omega c_1}\right) r_4 \;=\; \left(R_2 + r_2 + \frac{1}{j\omega c_2}\right) r_3 \cdots\cdots\cdots (1)$$

On solving above equation we get:

$$\frac{c_1}{c_2} \;=\; \frac{R_2 + r_2}{R_1 + r_1} \;=\; r_4 r_3$$

This the required equation.
By making the phasor diagram we can calculate dissipation factor. Phasor diagram for the above circuit is shown below

Let us mark δ_1 and δ_2 be phase angles of the capacitors c_1 and c_2 capacitors respectively. From the phasor diagram we have $\tan(\delta_1)$ = dissipation factor = $\omega c_1 r_1$ and similarly we have $\tan(\delta_2) = \omega c_2 r_2$.
From equation (1) we have

$$c_2 r_2 - c_1 r_1 = c_1 R_1 - c_2 R_2$$

on multiplying ω both sides we have

$$\omega c_2 r_2 - \omega c_1 r_1 \ = \ \omega(c_1 R_1 - c_2 R_2)$$

$$But \ \frac{c_1}{c_2} \ = \ \frac{r_4}{r_3}$$

Therefore the final expression for the dissipation factor is written as

$$tan(\delta_1) - tan(\delta_2) \ = \ \omega c_2 \left(R_1 \frac{r_4}{r_3} - R_2 \right)$$

Hence if dissipation factor for one capacitor is known. However this method is gives quite inaccurate results for dissipation factor.

2.5.1 Wein's bridge

Wein's bridge is popularly used for measurements of frequency of frequency. In this bridge, the value of all parameters are known. The source whose frequency has to measure is connected as shown in the figure.

$$Z_1 = r_1 + \frac{1}{jwC_1} = \frac{jwC_1 r_1 + 1}{jwC_1}$$

$$Z_2 = \frac{R_2}{1 + jwC_2 R_2}$$

At balance condition, $\dot{Z}_1 \dot{Z}_4 = \dot{Z}_2 \dot{Z}_3$

$$\frac{jwC_1 r_1 + 1}{jwC_1} \times R_4 = \frac{R_2}{1 + jwC_2 R_2} \times R_3$$

$$(1 + jwC_1 r_1)(1 + jwC_2 R_2) R_4 = R_2 R_3 \times jwC_1$$

$$\left[1 + jwC_2 R_2 + jwC_1 r_1 - w^2 C_1 C_2 r_1 R_2\right] = jwC_1 \frac{R_2 R_3}{R_4}$$

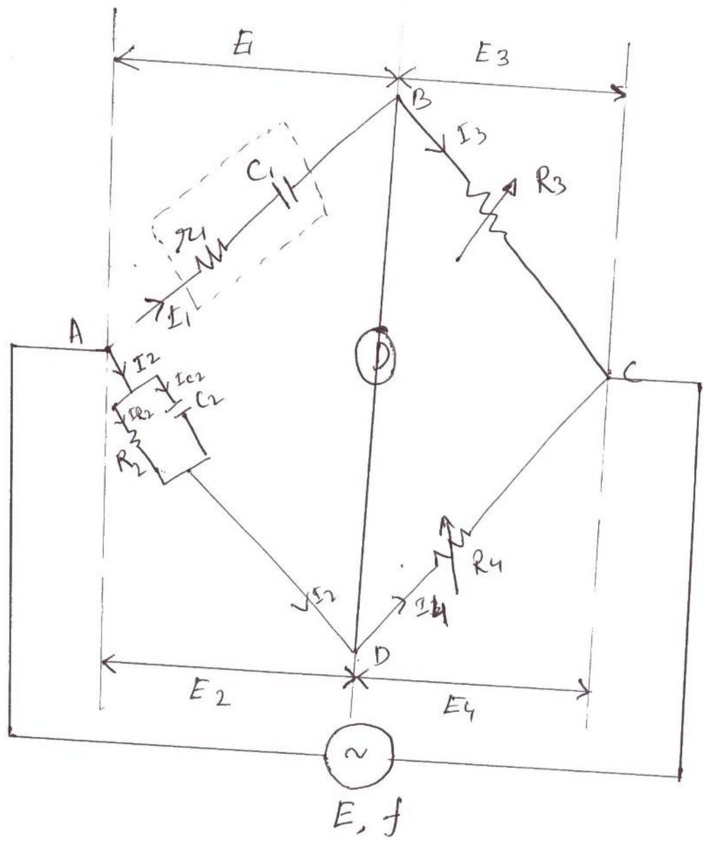


Fig Wein's bridge

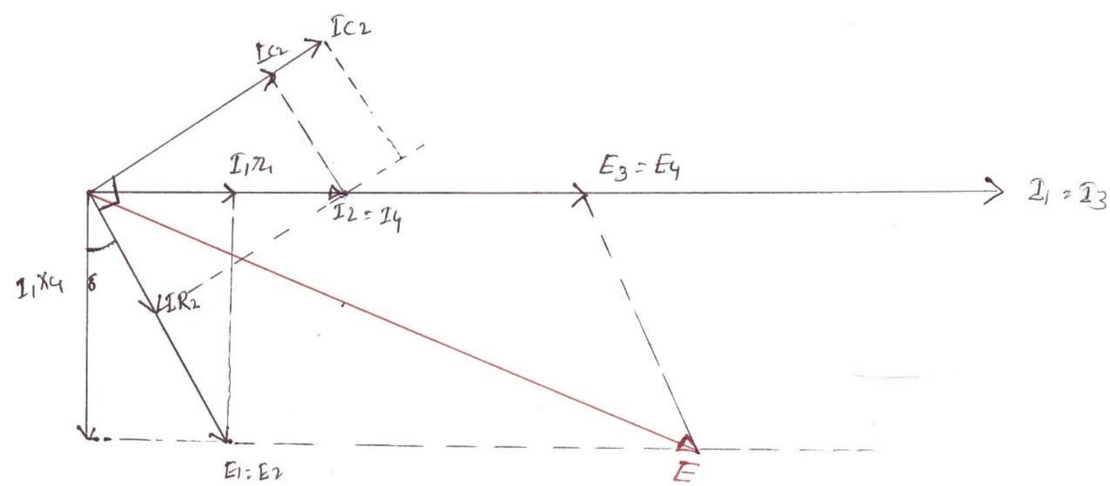


Fig Phasor diagram of Wein's bridge

Comparing real term,

$$1 - w^2 C_1 C_2 r_1 R_2 = 0$$

$$w^2 C_1 C_2 r_1 R_2 = 1$$

$$w^2 = \frac{1}{C_1 C_2 r_1 R_2}$$

$$w = \frac{1}{\sqrt{C_1 C_2 r_1 R_2}}, \quad f = \frac{1}{2\Pi\sqrt{C_1 C_2 r_1 R_2}}$$

NOTE The above bridge can be used for measurements of capacitance. In such case, r1 and C1 are unknown and frequency is known. By equating real terms, we will get R1 and C1. Similarly by equating imaginary term, we will get another equation in terms of r1 and C1. It is only used for measurements of Audio frequency.

A.F=20 HZ to 20 KHZ

R.F=>> 20 KHZ

Comparing imaginary term,

$$wC_2R_2 + wC_1r_1 = wC_1\frac{R_2R_3}{R_4}$$

$$C_2R_2 + C_1r_1 = \frac{C_1R_2R_3}{R_4} \quad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots$$

$$C_1 = \frac{1}{w^2 C_2 r_1 R_2}$$

Substituting in eqn. , we have

$$C_2R_2 + \frac{r_1}{w^2 C_2 r_1 R_2} = \frac{R_2R_3}{R_4}C_1$$

Multiplying $\frac{R_4}{R_2R_3}$ in both sides, we have

$$C_2R_2 \times \frac{R_4}{R_2R_3} + \frac{1}{w^2 C_2 R_2} \times \frac{R_4}{R_2R_3} = C_1$$

$$C_1 = \frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3}$$

$$w^2 C_1 r_1 C_2 R_2 = 1$$

$$r_1 = \frac{1}{w^2 C_2 R_2 C_1} = \frac{1}{w^2 C_2 R_2 \left[\frac{C_2 R_4}{R_3} + \frac{R_4}{w^2 C_2 R_2^2 R_3} \right]}$$

$$= \frac{1}{\left[\frac{w^2 {C_2}^2 R_2 R_4}{R_3} + \frac{R_4}{R_2 R_3} \right]}$$

$$\therefore r_1 = \frac{1}{\frac{R_3}{R_4} \left[w^2 C_2^2 R_2 + \frac{1}{R_2} \right]}$$

$$\therefore r_1 = \frac{R_3}{R_4} \left[\frac{1}{(w^2 C_2^2 R_2 + \frac{1}{R_2})} \right]$$

High Voltage Schering Bridge

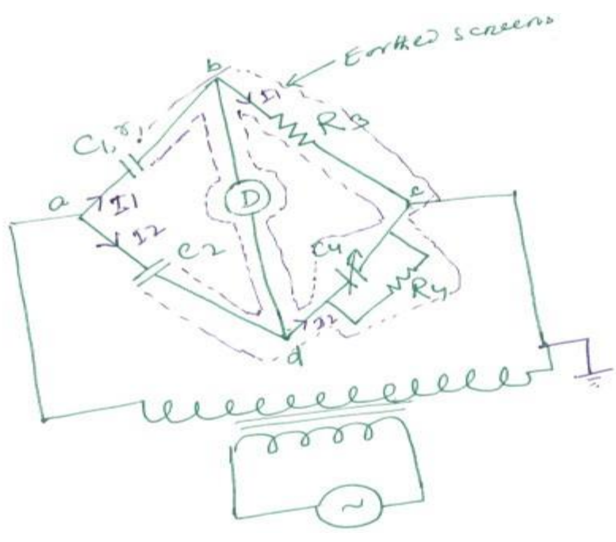


Fig High Voltage Schering bridge

(1) The high voltage supply is obtained from a transformer usually at 50 HZ.

Wagner earthing device:

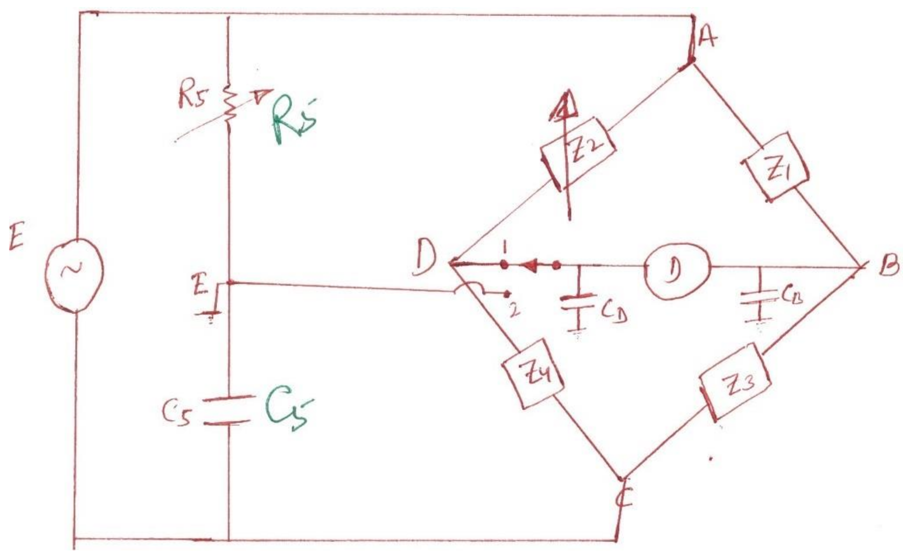


Fig Wagner Earthing device

Wagner earthing consists of 'R' and 'C' in series. The stray capacitance at node 'B' and 'D' are C_B, C_D respectively. These Stray capacitances produced error in the measurements of 'L' and 'C'. These error will predominant at high frequency. The error due to this capacitance can be eliminated using wagner earthing arm.

Close the change over switch to the position (1) and obtained balanced. Now change the switch to position (2) and obtained balance. This process has to repeat until balance is achievedin both the position. In this condition the potential difference across each capacitor is zero. Current drawn by this is zero. Therefore they do not have any effect on the measurements.

What are the sources of error in the bridge measurements?

- ✓ Error due to stray capacitance and inductance.
- ✓ Due to external field.
- ✓ Leakage error: poor insulation between various parts of bridge can produced this error.
- ✓ Eddy current error.
- ✓ Frequency error.

- ✓ Waveform error (due to harmonics)
- ✓ Residual error: small inductance and small capacitance of the resistor produce this error.

Precaution

- ✓ The load inductance is eliminated by twisting the connecting the connecting lead.
- ✓ In the case of capacitive bridge, the connecting lead are kept apart.(QC = $\frac{AÎ0Îr}{d}$)
- ✓ In the case of inductive bridge, the various arm are magnetically screen.
- ✓ In the case of capacitive bridge, the various arm are electro statically screen to reduced the stray capacitance between various arm.
- ✓ To avoid the problem of spike, an inter bridge transformer is used in between the source and bridge.
- ✓ The stray capacitance between the ends of detector to the ground, cause difficulty in balancing as well as error in measurements. To avoid this problem, we use wagner earthing device.