# JECRC, Jaipur 

# Department of Electrical Engineering 

# Subject- Electrical Machine - II (EM/C-II) 

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### 1.51 DOUBLE REVOLVING FIELD THEORY:

This theory makes use of the idea that an alternating uni-axial quantity can be represented by two oppositely-rotating vectors of half magnitude. Accordingly, an alternating sinusoidal flux can be represented by two revolving fluxes, each equal to half the value of the alternating flux and each rotating synchronously $\left(N_{S}=\frac{120 f}{P}\right)$ in opposite direction.

As shown in figure: (a) let the alternating flux have a maximum value of $\phi_{m}$. Its component fluxes A and B will each equal to $\phi_{m} / 2$ revolving in anti-clockwise and clockwise directions respectively.

After some time, when A and B would have rotated through angle $+\Theta$ and $-\Theta$, as in figure: (b), the resultant flux would be

$$
=2 * \frac{\phi_{\mathrm{m}}}{2} \cos \frac{2 \theta}{2}=\phi_{\mathrm{m}} \cos \theta
$$

After a quarter cycle of rotation, fluxes A and B will be oppositely-directed as shown in figure: (c) so that the resultant flux would be zero.


Fig: 1.51(a)


Fig: 1.51(b)


Fig:1.51 (c)


Fig: 1.51 (d)


Fig: 1.51(e)

After half a cycle, fluxes A and B will have a resultant of $-2 * \frac{\phi_{m}}{2}=-\phi_{m}$. After three quarters of a cycle, again the resultant is zero, as shown in figure: (e) and so on. If we plot the values of resultant flux against $\Theta$ between limits $\Theta=0^{\circ}$ to $\Theta=360^{\circ}$, then a curve similar to the one shown in figure: (f) is obtained. That is why an alternating flux can be looked upon as
composed of two revolving fluxes, each of half the value and revolving synchronously in opposite directions.


Fig: 1.51(f)
It may be noted that if the slip of the rotor is $S$ with respect to the forward rotating flux (i.e. one which rotates in the same direction as rotor) then its slip with respect to the backward rotating flux is (2-S).

Each of the two component fluxes, while revolving round the stator, cuts the rotor, induces an e.m.f. and this produces its own torque. Obviously, the two torques (called forward and backward torques ) are oppositely-directed, so that the net or resultant torques is equal to their difference as shown in fig: (g)


Fig: 1.51(g) Torque-Speed characteristics
Now, power developed by a rotor is $P_{g}=\left(\frac{1-s}{S}\right) I_{2}^{2} R_{2}$

If N is the rotor r.p.s., then torque is given by, $\mathrm{T}_{\mathrm{g}}=\frac{1}{2 \Pi \mathrm{~N}}\left(\frac{1-\mathrm{S}}{\mathrm{S}}\right) \mathrm{I}_{2}^{2} \mathrm{R}_{2}$
Now, $\mathrm{N}=\mathrm{N}_{\mathrm{s}}(1-\mathrm{S})$
Therefore, $\mathrm{T}_{\mathrm{g}}=\frac{1}{2 \Pi \mathrm{~N}_{\mathrm{s}}} \frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{\mathrm{~S}}=\mathrm{k} \frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{\mathrm{~S}}$

Hence, the forward and backward torques are given by

$$
\mathrm{T}_{\mathrm{f}}=\mathrm{k} \frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{\mathrm{~S}} \quad \text { and } \quad \mathrm{T}_{\mathrm{b}}=-\mathrm{k} \frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{(2-\mathrm{S})}
$$

or $\mathrm{T}_{\mathrm{f}}=\frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{\mathrm{~S}}$ synch. Watt and $\mathrm{T}_{\mathrm{b}}=-\frac{\mathrm{I}_{2}^{2} \mathrm{R}_{2}}{(2-\mathrm{S})}$ synch. Watt

Total torque $\quad \mathbf{T}=\mathrm{T}_{\mathrm{f}}+\mathrm{T}_{\mathrm{b}}$

Fig: (g) shows both torques and the resultant torque for slips between zero and +2 . At standstill, $\mathrm{S}=1$ and $(2-\mathrm{S})=1$. Hence, $\mathrm{T}_{\mathrm{f}}$ and $\mathrm{T}_{\mathrm{b}}$ are numerically equal but, being oppositely directed, produce no resultant torque. That explains why there is no starting torque in a single-phase induction motor.

However, if the rotor is started somehow, say, in the clockwise direction, the clockwise torque starts increasing and, at the same time, the anticlockwise torque starts decreasing. Hence, there is a certain amount of net torque in the clockwise direction which accelerates the motor to full speed.

### 1.6 EQUIVALENT CIRCUIT:

The equivalent circuit of a single phase induction motor can be developed on the basis of two revolving field theory. To develop the equivalent circuit it is necessary to consider standstill or blocked rotor conditions.

## PULSATING AND REVOLVING MAGNETIC FIELDS

## The Rotating Magnetic Field

The principle of operation of the induction machine is based on the generation of a rotating magnetic field. Let us understand this idea better.

Click on the following steps in sequence to get a graphical picture. It is suggested that the reader read the text before clicking the link.

- Consider a cosine wave from 0 to $360^{\circ}$. This sine wave is plotted with unit amplitude.
- Now allow the amplitude of the sine wave to vary with respect to time in a simisoidal fashion with a frequency of 50 Hz .Let the maximum value of the amplitude is, say, 10 units. This waveform is a pulsating sine wave.

$$
\begin{equation*}
\mathrm{i}_{\mathrm{apk}}=\mathrm{I}_{\mathrm{m}} \cos 2 \pi .50 . \mathrm{t} \tag{1}
\end{equation*}
$$

- Now consider a second sine wave, which is displaced by $120^{\circ}$ from the first (lagging). .
- And allow its amplitude to vary in a similar manner, but with a $120^{\circ}$ time lag.

$$
\begin{equation*}
\mathrm{i}_{\mathrm{bpk}}=\mathrm{I}_{\mathrm{m}} \cos \left(2 \pi .50 . \mathrm{t}-120^{\circ}\right) \tag{2}
\end{equation*}
$$

- Similarly consider a third sine wave, which is at $240^{\circ}$ lag. . .
- And allow its amplitude to change as well with a $240^{\circ}$ time lag. Now we have three pulsating sine waves.

$$
\begin{equation*}
\mathrm{i}_{\mathrm{cpk}}=\mathrm{I}_{\mathrm{m}} \cos \left(2 \pi .50 . \mathrm{t}-240^{\circ}\right) \tag{3}
\end{equation*}
$$

Let us see what happens if we sum up the values of these three sine waves at every angle. The result really speaks about Tesla's genius. What we get is a constant amplitude travelling sine wave!

In a three phase induction machine, there are three sets of windings - phase A winding, phase B and phase C windings. These are excited by a balanced three-phase voltage supply. This would result in a balanced three phase current. Equations $1-3$ represent the currents that flow in the three phase windings. Note that they have a $120^{\circ}$ time lag between them.

Further, in an induction machine, the windings are not all located in the same place. They are distributed in the machine $120^{\circ}$ away from each other (more about this in the section on alternators). The correct terminology would be to say that the windings have
their axes separated in space by $120^{\circ}$. This is the reason for using the phase A, B and C since waves separated in space as well by $120^{\circ}$.
When currents flow through the coils, they generate MMFs. Since MMF is proportional to current, these waveforms also represent the MMF generated by the coils and the total MMF. Further, due to magnetic material in the machine (iron), these MMFs generate magnetic flux, which is proportional to the MMF (we may assume that iron is infinitely permeable and non-linear effects such as hysterisis are neglected). Thus the waveforms seen above would also represent the flux generated within the machine. The net result as we have seen is a travelling flux wave. The x -axis would represent the space angle in the machine as one travels around the air gap. The first pulsating waveform seen earlier would then represent the a-phase flux, the second represents the b-phase flux and the third represents the c-phase.

This may be better visualized in a polar plot. The angles of the polar plot represent the space angle in the machine, i.e., angle as one travels around the stator bore of the machine. Click on the links below to see the development on polar axes.

- This plot shows the pulsating wave at the zero degree axes. The amplitude is maximum at zero degree axes and is zero at $90^{\circ}$ axis. Positive parts of the waveform are shown in red while negative in blue. Note that the waveform is pulsating at the 0 $-180^{\circ}$ axis and red and blue alternate in any given side. This corresponds to the sine wave current changing polarity. Note that the maximum amplitude of the sine wave is reached only along the $0-180^{\circ}$ axis. At all other angles, the amplitude does not reach a maximum of this value. It however reaches a maximum value which is less than that of the peak occuring at the $0-180^{\circ}$ axis. More exactly, the maximum reached at any space angle $\theta$ would be equal to $\cos \theta$ times the peak at the $0-180^{\circ}$ axis. Further, at any space angle $\theta$, the time variation is sinusoidal with the frequency and phase lag being that of the excitation, and amplitude being that corresponding to the space angle.
- This plot shows the pulsating waveforms of all three cosines. Note that the first is pulsating about the $0-180^{\circ}$ axis, the second about the $120^{\circ}-300^{\circ}$ axis and the third at $240^{\circ}-360^{\circ}$ axis.
- This plot shows the travelling wave in a circular trajectory. Note that while individual pulsating waves have maximum amplitude of 10 , the resultant has

If $f_{1}$ is the amplitude of the flux waveform in each phase, the travelling wave can then be represented as

$$
\begin{align*}
f(t) & =f_{1} \cos \omega t \cos \theta+f_{1} \cos \left(\omega t-\quad \frac{2 \pi}{3}\right) \cos \left(\theta-\frac{2 \pi}{3}\right)+f_{1} \cos \left(\omega t-\frac{4 \pi}{3}\right) \cos \left(\theta-\frac{4 \pi}{3}\right) \\
& =\underline{3}_{1} \cos (\omega t-\theta)
\end{align*}
$$

