Jaipur Engineering College & Research Centre, Jaipur

Session 2020-21

Notes – Unit VI

Electromagnetic Fields (3EE4-08)

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Vision and Mission of Institute

Vision of institute

To become a renowned centre of outcome based learning, and work towards, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of institute

M1.Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.

M2.Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.

M3.Offer opportunities for interaction between academia and industry.

M4.Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge in a range of professions

Vision and Mission of Electrical Engineering Department

Vision of department

The Electrical Engineering department strives to recognized globally for outcome based technical knowledge and produce quality human being who can manage the advance technologies and contribute to society.

Mission Of department

M1. To impart quality technical knowledge to the learners to make them globally competitive Electrical Engineers.

M2. To provide the learners ethical guidelines along with excellent academic environment for a long productive career.

M3. To promote industry- institute relationship.

Syllabus of Electromagnetic fields

unit 1- Review of Vector Calculus

Vector algebra- addition, subtraction, components of vectors, scalar and vector multiplications, triple products, three orthogonal coordinate systems (rectangular, cylindrical and spherical). Vector calculus differentiation, partial differentiation, integration, vector operator del,gradient, divergence and curl; integral theorems of vectors. Conversion of a vector from one coordinate system to another.

Unit 2- Static Electric Field

Coulomb's law, Electric field intensity, Electrical field due to point charges. Line, Surface and Volume charge distributions. Gauss law and its applications. Absolute Electric potential, Potential difference, Calculation of potential differences for different configurations. Electric dipole, Electrostatic Energy and Energy density.

Unit 3- Conductors, Dielectrics and Capacitance

Current and current density, Ohms Law in Point form, Continuity of current, Boundary conditions of perfect dielectric materials. Permittivity of dielectric materials, Capacitance, Capacitance of a two wire line, Poisson's equation, Laplace's equation, Solution of Laplace and Poisson's equation, Application of Laplace's and Poisson's equations.

unit 4- Static Magnetic Fields

Biot-Savart Law, Ampere Law, Magnetic flux and magnetic flux density, Scalar and Vector Magnetic potentials. Steady magnetic fields produced by current carrying conductors.

Unit5- Magnetic Forces, Materials and Inductance

Force on a moving charge, Force on a differential current element, Force between differential current elements, Nature of magnetic materials, Magnetization and permeability, Magnetic boundary conditions, Magnetic circuits, inductances and mutual inductances.

Unit 6- Time Varying Fields and Maxwell's Equations

Faraday's law for Electromagnetic induction, Displacement current, Point form of Maxwell's equation, Integral form of Maxwell's equations, Motional Electromotive forces. Boundary Conditions

Unit 7- Electromagnetic Waves

Derivation of Wave Equation, Uniform Plane Waves, Maxwell's equation in Phasor form, Wave equation in Phasor form, Plane waves in free space and in a homogenous material. Wave equation for a conducting medium, Plane waves in lossy dielectrics, Propagation in good conductors, Skin effect. Poynting theorem.

Course outcomes for Electromagnetic fields

CO1- Acquire basic understanding of vectors , their representation and conversion in different coordinate systems.

CO2- Able to compute the force, fields & energy of the electrostatic & magneto static fields. Able to analyze the materials, conductors, dielectrics, inductances and capacitances.

CO3- Understand the concept of time varying field and able to solve electromagnetic relation using Maxwell equations. Also able to analyze the electromagnetic waves.

Unit 6

Time varying fields and maxwell's equations

→ Electrostatic fields denoted by $\bar{E}(x,y,z)$
→ magnetostatic fields represented by $\bar{H}(x,y,z)$
→ Time varying electromagnetic (EM) fields represented by - $\bar{E}(x,y,z,t)$ and $\bar{H}(x,y,z,t)$

→ In Static EM fields, electric and magnetic fields are independent of each other.
→ In Dynamic EM fields, the two field are inter-dependent.

Stationary charges → Electrostatic fields
Steady Currents → magnetostatic fields
time varying current → Electromagnetic fields (or waves).

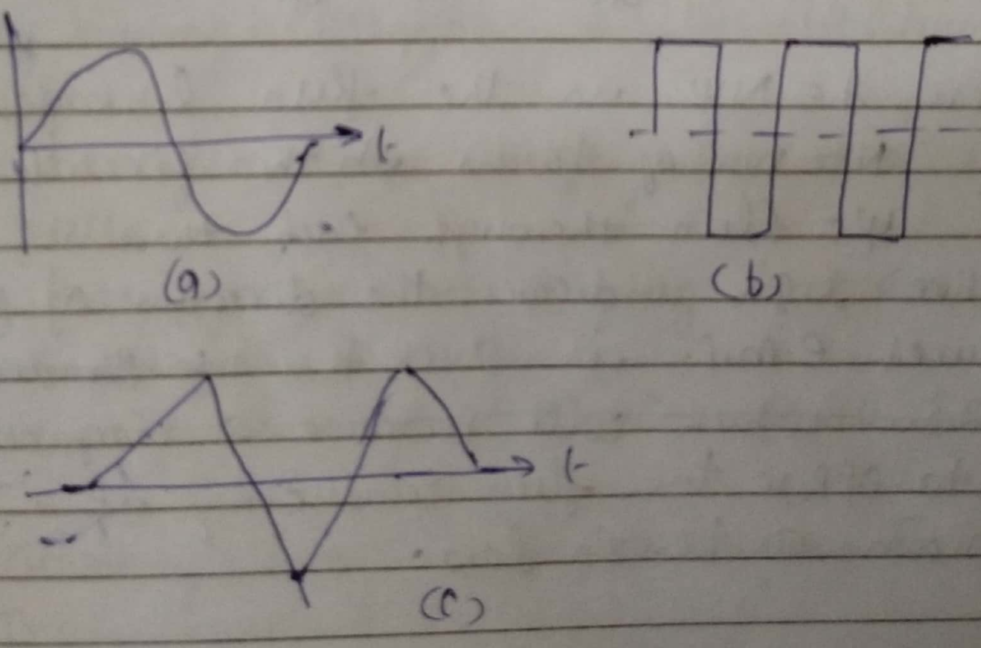

fig → Examples of time varing current:
(a) Sinusoidal (b) ractangular (c) triangular

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Faraday's law ⇒

According to faraday's experiments a static magnetic field produces no current flow, but a time-varying field produces an induced voltage (called electromotive force or Emf) in a closed circuit, which causes a flow of current.

faraday ~~discovered~~ discovered that –
"The induced emf, V_{emf} (in volts), in any closed circuit is equal to the time rate of change of the magnetic flux linkage by the circuit."

This is called faraday's law and it can be expressed as

$$V_{emf} = \frac{-d\lambda}{dt} = -N\frac{d\psi}{dt} \quad \text{—(I)}$$

where $\lambda = N\psi$ is the flux linkage,
N = no. of turns in the circuit
ψ = flux through each turn

negative sign indicates the direction of the induced e.m.f. is such that ~~to produce a current which will produce a magnetic~~ as to oppose the flux producing it. This is known as Lenz's law.

for a circuit with a single turn (N=1)

$$\boxed{V_{emf} = -\frac{d\psi}{dt}} \quad \text{—(II)}$$

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Transformer and motional Electromotive forces

as we know that the induced emf is also given by

$$V_{emf} = \oint_L \bar{E} \cdot d\bar{l} \quad \text{— (III)}$$

and the magnetic flux passing through specified area is given by -

$$\Psi = \oint_S \bar{B} \cdot d\bar{s}$$

hence $$V_{emf} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{s} \quad \text{— (IV)}$$

from eq. (III) & (IV)

$$V_{emf} = \oint_L \bar{E} \cdot d\bar{l} = -\frac{d}{dt} \int_S \bar{B} \cdot d\bar{s} \quad \text{— (V)}$$

from eq. (V) it is clear that ~~th~~ in a time-varying situation, both electric and magnetic fields are present and are interrelated.

The variation of flux with time ~~as~~ may be caused in three ways -

1- By having a stationary loop in a time varying $\bar{B}$ field.

2. By having a time varying loop area in a static $\bar{B}$ field

3. By having a time varying loop area in a time-varying $\bar{B}$ field.

Each of these will be considered seperately.

A. Stationary loop in time varying B field— (Transformer EMF) ⇒

in fig, a stationary conducting loop is in a time varying magnetic $\bar{B}$ field.

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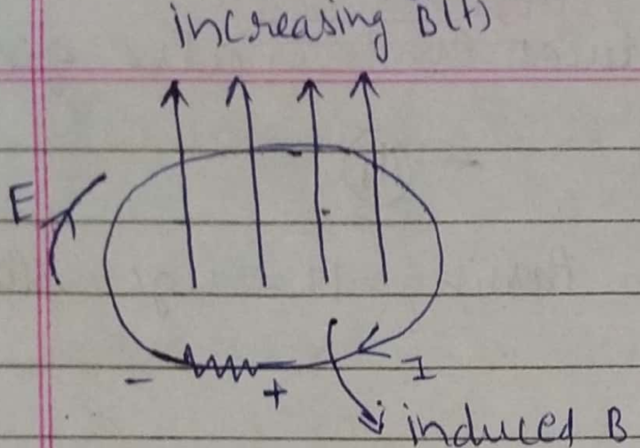

fig - Induced emf due to a stationary loop in a time varying $\bar{B}$ field

eq. (V) becomes

$$\boxed{V_{emf} = \oint_L \bar{E} \cdot d\bar{l} = \int_S \frac{\partial \bar{B}}{\partial t} \cdot ds} \quad -(6)$$

This emf induced by time varying current (Producing the time-varying $\bar{B}$ field) in a stationary loop is often reffered to as transformer emf in Power analysis, since it is due to transformer action. By applying Stoke's theorem to the middle term of eq. (6) we obtain,

$$\int_S (\nabla \times \bar{E}) \cdot ds = -\int \frac{\partial \bar{B}}{\partial t} \cdot ds \quad -(7)$$

$$\boxed{\nabla \times \bar{E} = \frac{-\partial \bar{B}}{\partial t}} \quad -(7)(a)$$

This is one of the maxwell's equation for time-varying fields. it shows that the time varying $\bar{E}$ field is not conservative ($\nabla \times \bar{E} \neq 0$)

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(B) Moving loop in Static B field (motional emf)

When a conducting loop is moving in a static $\bar{B}$ field, an emf is induced in the loop. recall the equation of force on a moving charge with velocity $\bar{u}$ in a magnetic field is

$$\bar{F}_m = Q\bar{u} \times \bar{B} \quad \text{--- (8)}$$

we define the motional electric field $\bar{E}_m$ as

$$\bar{E}_m = \frac{F_m}{Q} = \bar{u} \times \bar{B} \quad \text{--- (9)}$$

if we consider a conducting loop, moving with uniform velocity $\bar{u}$ as consisting of a large no. of free electrons, the emf induced in the loop is

$$V_{emf} = \oint_L \bar{E}_m \cdot d\bar{l} = \oint_L (\bar{u} \times \bar{B}) \cdot d\bar{l} \quad \text{--- (10)}$$

This type of emf is called motional emf or flux cutting emf because it is due to motional action. it is the kind of emf found in electrical machines.

Applying stoke's theorem to eq. (10)

$$\int_S (\nabla \times E_m) \cdot d\bar{S} = \int_S \nabla \times (\bar{u} \times \bar{B}) \cdot d\bar{S}$$

or

$$\boxed{\nabla \times E_m = \nabla \times (\bar{u} \times \bar{B})} \quad \text{--- (11)}$$

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(c) Moving loop in time varying field =>

In general case, a moving conducting loop is in a time-varying magnetic field, both transformer emf and motional emf are present. Combining eq. (8) and (10) gives the total emf as

$$\boxed{V_{emf} = \oint_L \bar{E} \cdot d\bar{l} = -\int_S \frac{\partial B}{\partial t} \cdot d\bar{S} + \oint_L (\bar{u} \times \bar{B}) \cdot d\bar{l}} \quad -(12)$$

or from eq. 7(a) and (11)

$$\boxed{\nabla \times E = -\frac{\partial B}{\partial t} + \nabla \times (\bar{u} \times \bar{B})} \quad -(13)$$

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Displacement Current =>

For static EM fields, we recall that

$$\nabla \times \bar{H} = \bar{J} \quad \text{— (I)}$$

But the divergence of the curl of any vector field is identically zero. hence,

$$(\nabla \cdot (\nabla \times \bar{H})) = 0 = \nabla \cdot \bar{J} \quad \text{— (II)}$$

The continuity of current ~~is~~, requires that

$$\nabla \cdot \bar{J} = -\frac{\partial \rho_v}{\partial t} \neq 0 \quad \text{— (III)}$$

Thus eq. (II) & (III) are obviously incompatible for time-varying conditions. We must modify eq. (I) to agree with eq. (III). To do this, we add a term to eq. (I) so that it becomes

$$\bar{\nabla} \times \bar{H} = \bar{J} + \bar{J}_d \quad \text{— (IV)}$$

where $\bar{J}_d$ is to be determined and defined. Again, the divergence of curl of any vector is zero. hence;

$$\nabla \cdot (\nabla \times \bar{H}) = 0 = \nabla \cdot \bar{J} + \nabla \cdot \bar{J}_d \quad \text{— (V)}$$

by eq. (III) & (V)

$$\nabla \cdot \bar{J}_d = \nabla \cdot \bar{J} = \frac{\partial \rho_v}{\partial t} = \frac{\partial}{\partial t}(\nabla \cdot \bar{D}) = \nabla \cdot \frac{\partial D}{\partial t} \quad \text{— (VI)(a)}$$

or

$$\boxed{\bar{J}_d = \frac{\partial \bar{D}}{dt}} \quad \text{— (VI)(b)}$$

Substituting eq. (VI)(b) into eq. (IV)

$$\boxed{\nabla \times \bar{H} = \bar{J} + \frac{\partial \bar{D}}{\partial t}} \quad \text{— (VII)}$$

This is Maxwell's equation (based on Ampere's circuit law) for a time-varying field. The term

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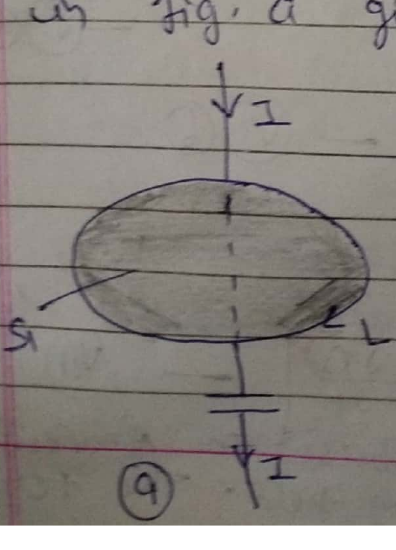
$\bar{J}_d = \partial \bar{D}/\partial t$ is known as displacement current density and $\bar{J}$ is the conduction current density ($\bar{J} = \sigma \bar{E}$). without the term J_d, the ~~prop~~ propagation of electromagnetic waves (e.g. radio or TV waves) would be impossible.

→ At low frequencies, $\bar{J}_d$ is usually neglected compared with $\bar{J}$. However, at radio frequencies the two terms are comparable.

Based on the displacement current density, we define the displacement current as

$$I_d = \int \bar{J}_d \cdot d\bar{S} = \int \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S} \quad \text{— (VIII)}$$

we must bear in mind that displacement current is a result of time-varying electric field. A typical example of such current is the current through a capacitor when an alternating voltage source is applied to its plates. This ex., shown in fig., serves to illustrate the need for the displacement current. Applying an unmodified form of Ampere's circuit law to a closed path L shown in fig. a gives,



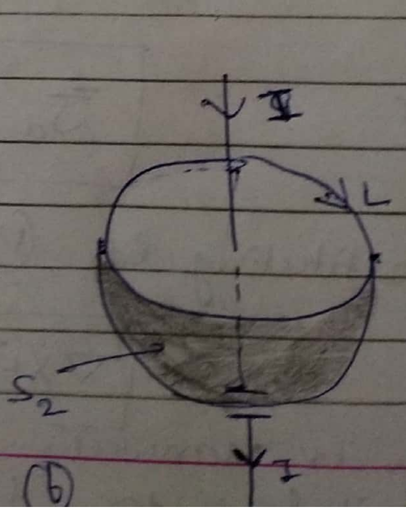


fig.

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$$\oint_L \bar{H}\cdot d\bar{l} = \int_{S_1} \bar{J}\cdot d\bar{s} = I_{enc} = I \quad \text{— (IX)}$$

where I is current through the conductor and S_1 is the flat surface bounded by L. if we use the balloon-shaped surface S_2 that passes b/w the capacitor plates, as in fig b

$$\oint_L \bar{H}\cdot d\bar{l} = \int_{S_2} \bar{J}\cdot d\bar{s} = I_{enc} = 0 \quad \text{— (X)}$$

because no conduction current ($J=0$) is flows through S_2. This is contradictory in view of the fact that the same closed path L is used.
To resolve the conflict, we need to include the displacement current in ampere's circuit law. The total current density is $\bar{J} + \bar{J}_d$. in eq. (IX), $\bar{J}_d = 0$, so that the eq. remains valid. in eq. (X), $\bar{J} = 0$, so that

$$\oint_L \bar{H}\cdot d\bar{l} = \int_{S_2} \bar{J}_d\cdot d\bar{s} = \frac{d}{dt}\int_S \bar{D}\cdot d\bar{s} = \frac{dQ}{dt} = I \quad \text{— (XI)}$$

So we obtain the same current for either surface, although it is conduction current in S_1 and displacement current in S_2.

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Maxwell's Equation in Point form =)

we have already obtained two of maxwell's equations for time-varying fields.

$$\boxed{\nabla \times \bar{E} = -\frac{\partial \bar{B}}{\partial t}} \quad \text{— (I)}$$

and

$$\boxed{\nabla \times \bar{H} = J + \frac{\partial \bar{D}}{\partial t}} \quad \text{— (II)}$$

The remaining two equations are unchanged from their non-time varying form:

$$\boxed{\nabla \cdot \bar{D} = \rho_v} \quad \text{— (III)}$$

$$\boxed{\bar{\nabla} \cdot \bar{B} = 0} \quad \text{— (IV)}$$

These four equations form the basis of all electromagnetic theory. They are partial differential equations and relate the electric and magnetic fields to each other and to their sources, charge and current density.
The auxiliary equations relating $\bar{D}$ and $\bar{E}$.

$$\boxed{\bar{D} = \epsilon \bar{E}} \quad \text{— (V)}$$

relating $\bar{B}$ & $\bar{H}$, $\boxed{\bar{B} = \mu \bar{H}}$ — (VI)

defining conduction current density, $\boxed{\bar{J} = \sigma \bar{E}}$ — (VII) and defining Convection current density in terms of the volume charge density ρ_v,

$$\boxed{\bar{J} = \rho_v v} \quad \text{— (VIII)}$$

are also required to define and relate the quantities appearing in ~~the~~ maxwell's eq. eq. (V) & (VI) may be replaced by the relationship involving the Polarization and magnetization fields,

$$\boxed{\bar{D} = \epsilon_0 \bar{E} + \bar{P}} \quad \text{— (IX)}$$

$$\boxed{\bar{B} = \mu_0 (\bar{H} + \bar{M})} \quad \text{— (X)}$$

for linear materials we may relate $\bar{P}$ to $\bar{E}$

$$\bar{P} = \chi_e \epsilon_0 \bar{E} \quad \text{— (XI)}$$

and $\bar{M}$ to $\bar{H}$

$$\bar{M} = \chi_m \bar{H} \quad \text{— (XII)}$$

Finally, because of it's fundamental importance we should include the Lorentz force equation, written in Point form as the force Per unit volume.,

$$\boxed{f = \rho_v (\bar{E} + \bar{v} \times \bar{B})} \quad \text{— (XIII)}$$

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Maxwell's equations in integral form ⇒

The integral forms of maxwell's equations are usually easier to recognize in terms of the experimental laws from which they have been obtained by generalization process.
The integral form of maxwell's equation are as follows.

$$\boxed{\oint \bar{E} \cdot d\bar{L} = -\int_s \frac{\partial \bar{B}}{\partial t} \cdot d\bar{S}} \quad \text{— (I)}$$

$$\boxed{\oint \bar{H} \cdot d\bar{L} = I + \int_s \frac{\partial \bar{D}}{\partial t} \cdot d\bar{S}} \quad \text{— (II)}$$

$$\boxed{\oint_s \bar{D} \cdot d\bar{S} = \int_{vol} \rho_v \, dV} \quad \text{— (III)}$$

$$\boxed{\oint_s \bar{B} \cdot d\bar{S} = 0} \quad \text{— (IV)}$$

Boundary Conditions ⇒

These four integral equations enables us to find the boundary conditions on $\bar{B}$, $\bar{D}$, $\bar{H}$ and $\bar{E}$ which are necessary to evaluate the constants obtained in solving maxwell's equation in partial differential form.
These boundary conditions are in general unchanged from their forms for static

fields and the same method may be used to obtain them. b/w any two real Physical media (where K must be zero on the boundary surface) eq. (I) of integral form of maxwell's eq. enables us to relate the tangential E-field components.

$$E_{t_1} = E_{t_2} \quad — \text{(V)}$$

And from (II)

$$H_{t_1} = H_{t_2} \quad — \text{(VI)}$$

the surface integral produce the boundary conditions on the normal components,

$$D_{N_1} - D_{N_2} = \rho_S \quad — \text{(VII)}$$

and

$$B_{N_1} = B_{N_2} \quad — \text{(VIII)}$$

However for a Perfect conductor ($\sigma \simeq \infty$) in time varying fields.

$$\bar{E} = 0 \quad , \quad \bar{H} = 0 \quad , \quad \bar{J} = 0$$

and hence

$$\bar{B}_n = 0 \quad ; \quad \bar{E}_t = 0$$

EXAMPLE 8.4

A parallel-plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage $50 \sin 10^3 t$ V applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_o$.

Solution:

$$D = \varepsilon E = \varepsilon \frac{V}{d}$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d}\frac{dV}{dt}$$

Hence,

$$I_d = J_d \cdot S = \frac{\varepsilon S}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

which is the same as the conduction current, given by

$$I_c = \frac{dQ}{dt} = S\frac{d\rho_s}{dt} = S\frac{dD}{dt} = \varepsilon S\frac{dE}{dt} = \frac{\varepsilon S}{d}\frac{dV}{dt} = C\frac{dV}{dt}$$

$$I_d = 2 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{5 \times 10^{-4}}{3 \times 10^{-3}} \cdot 10^3 \times 50 \cos 10^3 t$$
$$= 147.4 \cos 10^3 t \text{ nA}$$

PRACTICE EXERCISE 8.4

In free space, $\mathbf{E} = 20 \cos(\omega t - 50x)\, \mathbf{a}_y$ V/m. Calculate

(a) $\mathbf{J}_d$

(b) $\mathbf{H}$

(c) ω

Answer: (a) $-20\omega\varepsilon_o \sin(\omega t - 50x)\, \mathbf{a}_y$ A/m^2, (b) $0.4\, \omega\varepsilon_o \cos(\omega t - 50x)\, \mathbf{a}_z$ A/m, (c) 1.5×10^{10} rad/s.

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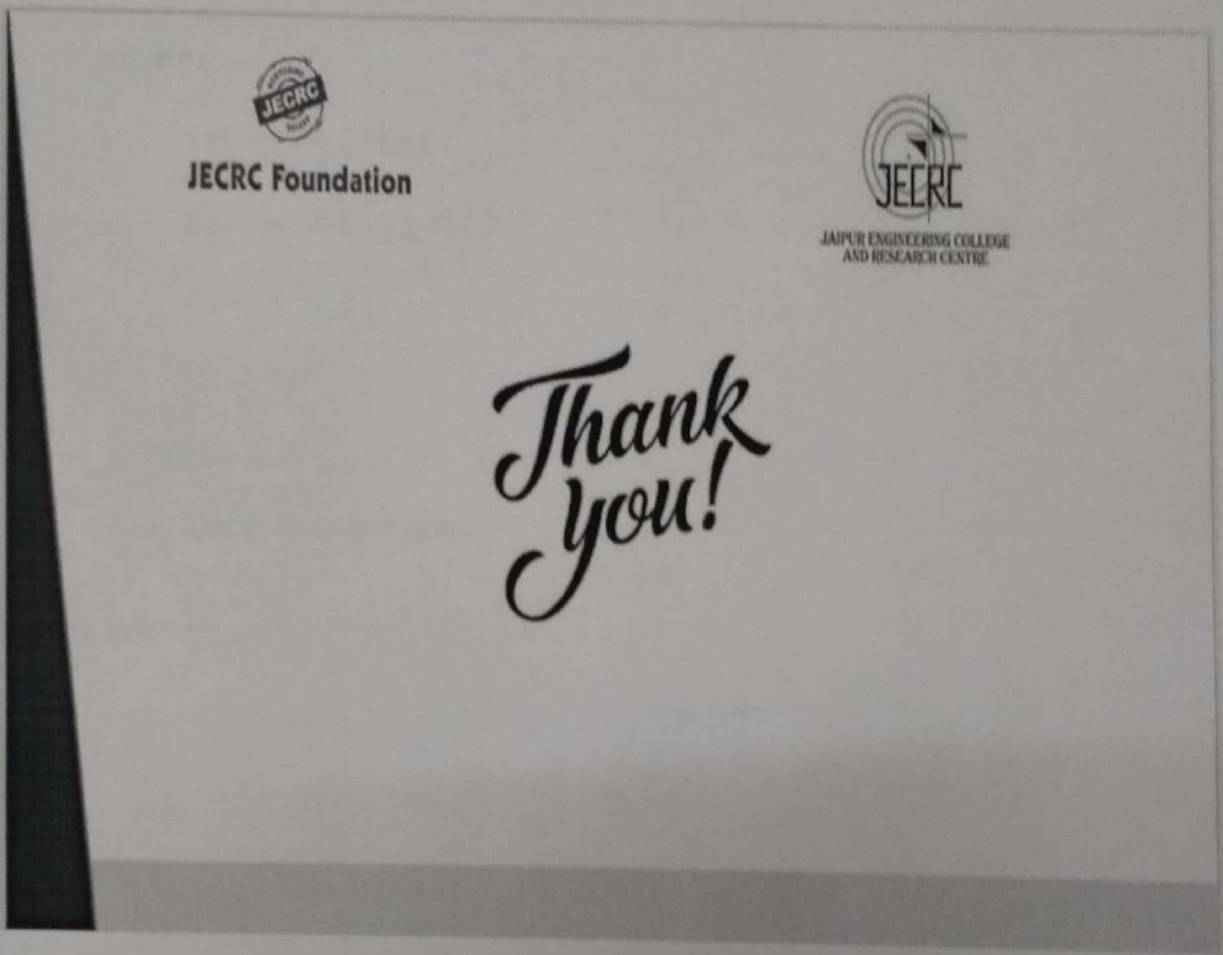
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