Jaipur Engineering College & Research Centre, Jaipur

Session 2020-21

Notes – Unit V

Electromagnetic Fields (3EE4-08)

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Vision and Mission of Institute

Vision of institute

To become a renowned centre of outcome based learning, and work towards, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of institute

M1.Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.

M2.Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.

M3.Offer opportunities for interaction between academia and industry.

M4.Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge in a range of professions

Vision and Mission of Electrical Engineering Department

Vision of department

The Electrical Engineering department strives to recognized globally for outcome based technical knowledge and produce quality human being who can manage the advance technologies and contribute to society.

Mission Of department

M1. To impart quality technical knowledge to the learners to make them globally competitive Electrical Engineers.

M2. To provide the learners ethical guidelines along with excellent academic environment for a long productive career.

M3. To promote industry- institute relationship.

Syllabus of Electromagnetic fields

unit 1- Review of Vector Calculus

Vector algebra- addition, subtraction, components of vectors, scalar and vector multiplications, triple products, three orthogonal coordinate systems (rectangular, cylindrical and spherical). Vector calculus differentiation, partial differentiation, integration, vector operator del,gradient, divergence and curl; integral theorems of vectors. Conversion of a vector from one coordinate system to another.

Unit 2- Static Electric Field

Coulomb's law, Electric field intensity, Electrical field due to point charges. Line, Surface and Volume charge distributions. Gauss law and its applications. Absolute Electric potential, Potential difference, Calculation of potential differences for different configurations. Electric dipole, Electrostatic Energy and Energy density.

Unit 3- Conductors, Dielectrics and Capacitance

Current and current density, Ohms Law in Point form, Continuity of current, Boundary conditions of perfect dielectric materials. Permittivity of dielectric materials, Capacitance, Capacitance of a two wire line, Poisson's equation, Laplace's equation, Solution of Laplace and Poisson's equation, Application of Laplace's and Poisson's equations.

unit 4- Static Magnetic Fields

Biot-Savart Law, Ampere Law, Magnetic flux and magnetic flux density, Scalar and Vector Magnetic potentials. Steady magnetic fields produced by current carrying conductors.

Unit5- Magnetic Forces, Materials and Inductance

Force on a moving charge, Force on a differential current element, Force between differential current elements, Nature of magnetic materials, Magnetization and permeability, Magnetic boundary conditions, Magnetic circuits, inductances and mutual inductances.

Unit 6- Time Varying Fields and Maxwell's Equations

Faraday's law for Electromagnetic induction, Displacement current, Point form of Maxwell's equation, Integral form of Maxwell's equations, Motional Electromotive forces. Boundary Conditions

Unit 7- Electromagnetic Waves

Derivation of Wave Equation, Uniform Plane Waves, Maxwell's equation in Phasor form, Wave equation in Phasor form, Plane waves in free space and in a homogenous material. Wave equation for a conducting medium, Plane waves in lossy dielectrics, Propagation in good conductors, Skin effect. Poynting theorem.

Course outcomes for Electromagnetic fields

CO1- Acquire basic understanding of vectors , their representation and conversion in different coordinate systems.

CO2- Able to compute the force, fields & energy of the electrostatic & magneto static fields. Able to analyze the materials, conductors, dielectrics, inductances and capacitances.

CO3- Understand the concept of time varying field and able to solve electromagnetic relation using Maxwell equations. Also able to analyze the electromagnetic waves.

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Magnetic forces, materials and Inductance

Force due to magnetic field ⇒

There are at least three ways in which force due to magnetic fields can be experienced. The force can be-

(a) Due to a moving charged particle in a B field.

(b) On a current element in an external B field or

(c) between two current elements.

(A) Force on a charged Particle ⇒

The electric force F_e on a stationary or moving electric charge Q in an electric field is given by Coulomb's experimental law and is related to the electric field intensity $\bar{E}$ as

$$F_e = Q\bar{E} \quad — (1)$$

This shows that if Q is positive, F_e and $\bar{E}$ have same direction.

A magnetic field can exert force only on a moving charge. From experiments, it is found that the magnetic force F_m experienced by a charge Q moving

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with a velocity u in a magnetic field $\bar{B}$ is

$$\bar{F}_m = Q\bar{u} \times \bar{B} \quad - (2)$$

This clearly shows that $\bar{F}_m$ is perpendicular to both $\bar{u}$ and $\bar{B}$.

From eq. (1) & (2), by comparision b/w $\bar{F}_e$ and $\bar{F}_m$ we can see that $\bar{F}_e$ is independent of the velocity of the charge and can perform work on charge and change it's kinetic energy. unlike $\bar{F}_e$, $\bar{F}_m$ depends on the charge velocity and is normal to it. however F_m can not perform work because it is at right angle to the direction of motion of charge ($\bar{F}_m \cdot d\bar{l} = 0$); it does not cause an increase in kinetic energy of the charge.

The magnitude of F_m is generally small in comparison to $\bar{F}_e$ except at high velocity.

For a moving charge Q in the presence of both electric and magnetic fields, the total force on the charge is given by

$$\bar{F} = \bar{F}_e + \bar{F}_m$$

or

$$\boxed{\bar{F} = Q(\bar{E} + \bar{u} \times \bar{B})} \quad - (3)$$

This is known as lorentz force equation. it relates mechanical force to Electrical force. if the mass of charged particle moving in $\bar{E} \times \bar{B}$ fields is m, by newton's second law of motion -

$$\bar{F} = m\frac{d\bar{u}}{dt} = q(\bar{E} + \bar{u} \times \bar{B}) \quad \text{--- (4)}$$

The solution to this equation is important in determining the motion of charged particles in $\bar{E}$ and $\bar{B}$ fields. Note that in such fields, energy can be transferred only by means of electric field. A summary on force exerted on a charged particle is given in table below.

State of Particle	$\bar{E}$ fields	$\bar{B}$ Fields	Combine $\bar{E}$ and $\bar{B}$ fields
Stationary	$q\bar{E}$	–	$q\bar{E}$
moving	$q\bar{E}$	$q\bar{u} \times \bar{B}$	$q(\bar{E} + \bar{u} \times \bar{B})$

(B) Force on a current element ⇒

To determine the force on a current element $I\,dl$ of a current-carrying conductor due to the magnetic field $\bar{B}$, modify eq. (2) using the fact that for convection current density

$$\boxed{\bar{J} = \rho_v\,\bar{u}} \quad \text{--- (5)}$$

we recall the relationship b/w current elements:

$$I\,dl = K\,ds = \bar{J}\,dv \quad \text{--- (6)}$$

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Combine eq. 5 and 6

$$I\,dl = \rho_v\,\bar{u}\,dV = dQ\,\bar{u} \quad \left[\because \rho_v = \frac{dQ}{dV}\right]$$

Alternatively, $I\,dl = \frac{dQ}{dt}\,dl = dQ\,\frac{dl}{dt} = dQ\,\bar{u}$

Hence,

$$\boxed{I\,dl = dQ\,\bar{u}} \quad \text{— (7)}$$

This shows that an elemental charge dQ moving with velocity $\bar{u}$ (thereby producing convection current element $dQ\bar{u}$) is equivalent to a conduction current element $I\,dl$.

Thus the force on a current element $I\,dl$ in a magnetic field $\bar{B}$ is found from eq. (2) by merely replacing $Q\bar{u}$ by $I\,d\bar{l}$. That is

$$d\bar{F} = I\,d\bar{l} \times \bar{B} \quad \text{— (8)}$$

If the current I is through a closed path L or circuit, the force on the circuit is given by

$$\boxed{\bar{F} = \oint_L I\,d\bar{l} \times \bar{B}} \quad \text{— (9)}$$

In using eq. 8 and 9 note that the magnetic field produced by the current element $I\,d\bar{l}$ does not exert force on the element itself, just as a point charge does not exert force on itself. The $\bar{B}$ field that exerts force on $I\,d\bar{l}$ must be due to another element.

If instead of the line current element $I\,d\bar{l}$ we have surface current elements $\bar{K}\,ds$ or a volume current element $\bar{J}\,dV$, we simply make use of eq. 6 so that eq. 9 becomes

$$\bar{F} = \int_S \bar{K}\,ds \times \bar{B} \quad \text{or} \quad \bar{F} = \int_V \bar{J}\,dV \times \bar{B} \qquad (9')$$

⇒ The magnetic field $\bar{B}$ is defined as the force per unit current element.

Alternatively -

⇒ $\bar{B}$ may be defined from eq. 2 as the vector that satisfies $\bar{F}_m/q = \bar{u} \times \bar{B}$, just as we defined electric field E as the force per unit charge, $\bar{F}_e/q$. Both these definitions of $\bar{B}$ shows that $\bar{B}$ describes the force properties of a magnetic field.

(C) Force b/w two Current Elements ⇒

Let us now consider the force b/w two elements $I_1 d\bar{l}_1$ and $I_2 d\bar{l}_2$. According to Biot - Savart's law, both current elements produce magnetic fields. So we may find the force $d(dF_1)$ on element $I_1 d\bar{l}_1$ due to the field dB_2 produced by element $I_2 dl_2$ as shown in fig.

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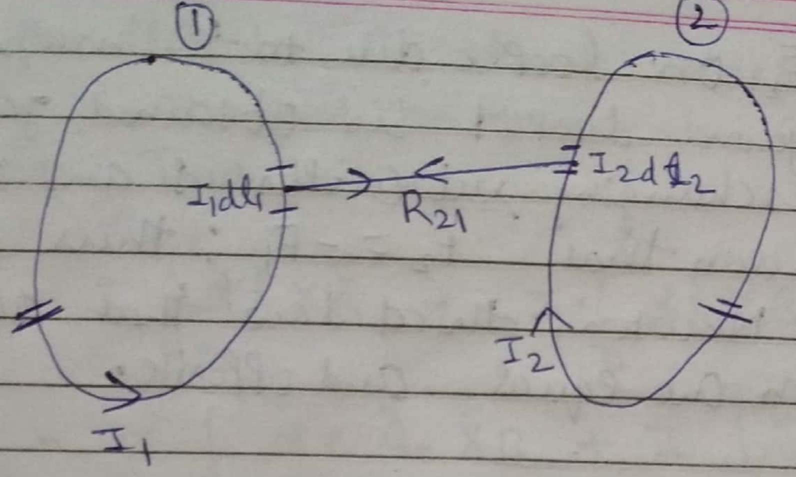

fig - Force b/w two current loops

from eq. (8)

$$d(dF_1) = I_1\, dl_1 \times d\bar{B}_2 \quad -(10)$$

But from Biot savart's law,

$$d\bar{B}_2 = \frac{\mu_0 I_2\, dl_2 \times \bar{a}_{R21}}{4\pi R_{21}^2} \quad -(11)$$

hence

$$d(d\bar{F}_1) = \frac{\mu_0 I_1 dl_1 \times (I_2 d\bar{l}_2 \times a_{R21})}{4\pi R_{21}^2} \quad -(12)$$

This equation is essentially the law of force b/w two current elements and is analogous to coulomb's law. which expresses the force b/w two stationary charges. from (12) we obtain the total force $\bar{F}_1$ on current loop 1 due to current loop 2 shown in fig as -

$$\bar{F}_1 = \frac{\mu_0 I_1 I_2}{4\pi} \oint_{L_1} \oint_{L_2} \frac{d\bar{l}_1 \times (d\bar{l}_2 \times \bar{a}_{R21})}{R_{21}^2} \quad -(13)$$

The force $\bar{F}_2$ on loop 2 due to the magnetic field B_1 from loop 1 is obtained from eq. (13) by interchanging subscripts 1 and 2. it can be shown that $F_2 = -F_1$; thus $\bar{F}_1$ and F_2 obeys Newton's third law that action and reaction are equal and opposite.

Magnetic torque and moment ⇒

Now that we have considered the force on a current loop in magnetic field, we can determine the torque on it.
The concept of a current loop experiencing a torque in a magnetic field is of para-mount importance in understanding the behavior of orbiting charged particles, dc motors, and generators. If the loop is placed parallel to a magnetic field, it experi-ences a force that tends to rotate it.
"The Torque T (or mechanical moment of forces) on the loop is the vector product of force F and the moment arm r."
That is

$$\bar{T} = \bar{r} \times \bar{F} \qquad -(1)$$

and it's unit is newton-meter (N.m)
Let us apply this to a rectangular loop of length l and width w placed in a uniform magnetic field $\bar{B}$ as shown in fig (a). From this figure, we notice that dl

is Parallel to B along sides AB and CD of the loop and no force is exerted on those sides. Thus

$$\bar{F} = I\int_B^C d\bar{l} \times \bar{B} + I\int_D^A d\bar{l} \times \bar{B}$$

$$= I\int_0^l dz\, \bar{a}_z \times B + I\int_l^0 dz\, \bar{a}_z \times \bar{B}$$

$$\bar{F} = \bar{F}_0 - \bar{F}_0 = 0 \qquad -(11)$$

where $|\bar{F}_0| = IBl$ because $\bar{B}$ is uniform. Thus no force is exerted on the loop as a whole. However $\bar{F}_0$ and $-\bar{F}_0$ act at different Points on the loop., thereby Creating a couple.

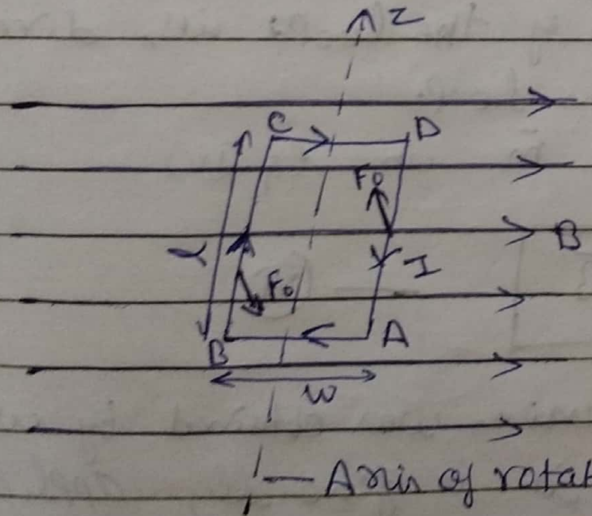

fig (a)
Rectangular Planar loop in Uniform magnetic field.

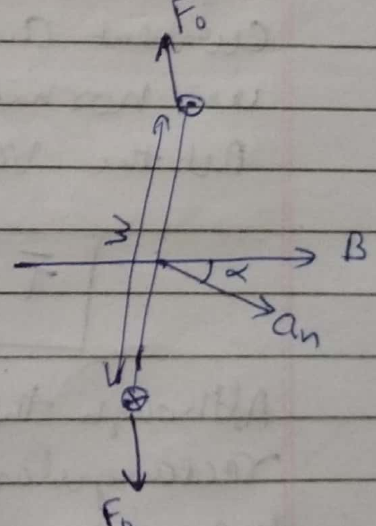

fig (b)
Cross sectional view of Part (a)

if normal to Plane of the loop makes an angle α with $\bar{B}$. as shown in fig (b) the torque on the loop is

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$$|\bar{T}| = |\bar{F}_0|\, w \sin\alpha$$

or

$$T = BIlw \sin\alpha \quad \text{—(III)}$$

But ~~lw~~ $lw = S$ the area of the loop. Hence,

$$T = BIS\sin\alpha \quad \text{(IV)}$$

we define the quantity

$$\bar{m} = IS\bar{a}_n \quad \text{(V)}$$

As the magnetic dipole moment (in Am^2) of the loop. $\bar{a}_n$ is the unit normal vector to the plane of the loop and it's direction is determined by the right hand rule.

" The magnetic dipole moment is the product of current and area of the loop; it's direction is normal to the loop.

Put the value of $\bar{m}$ in eq.(IV)

$$\boxed{\bar{T} = \bar{m} \times \bar{B}} \quad \text{—(VI)}$$

Although this expression was obtained by using a rectangular loop, it is generally applicable in determining the torque on a planar loop of any arbitrary shape. The only limitation is that the magnetic field must be uniform.

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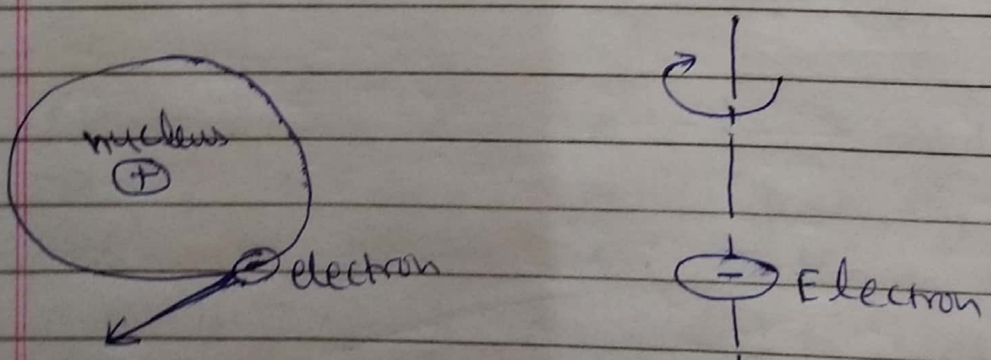
Classification of magnetic materials =)

On the basis of the magnetic behaviour, the magnetic materials are classified as - diamagnent, Paramagnetic, ferromagnetic, antiferromagnetic, ferrimagnetic and super-magnetic.

Magnetization in materials ⇒

Our discussion here will parallel that on polarization of materials in an electric field. We shall assume that our atomic model is that of an electron orbiting about a positive nucleus.

We know that the given material is composed of atoms. Each atom may be regarded as consisting of electrons orbiting about a central positive nucleus; the electron also rotate (or spin) about there ~~own~~ own axes. Thus an internal magnetic field is produced by electron orbiting around the nucleus as in fig 1(a) or electron spinning as in fig 1(b). Both these electronic motion produces internal magnetic fields B_i that are similar to the magnetic field produced by a current loop of fig 2.



(a) Electron orbiting around the nucleus

fig 1.

(b) Electron spin.

The equivalent current loop has a magnetic moment of $\bar{m} = I_b S \bar{a}_n$, where S is the area

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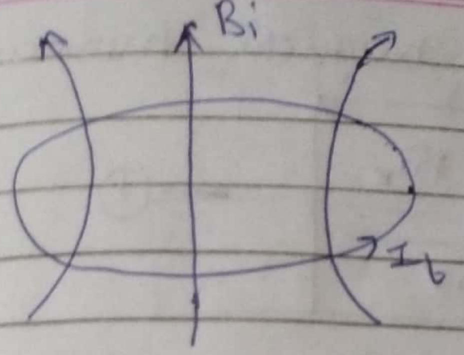
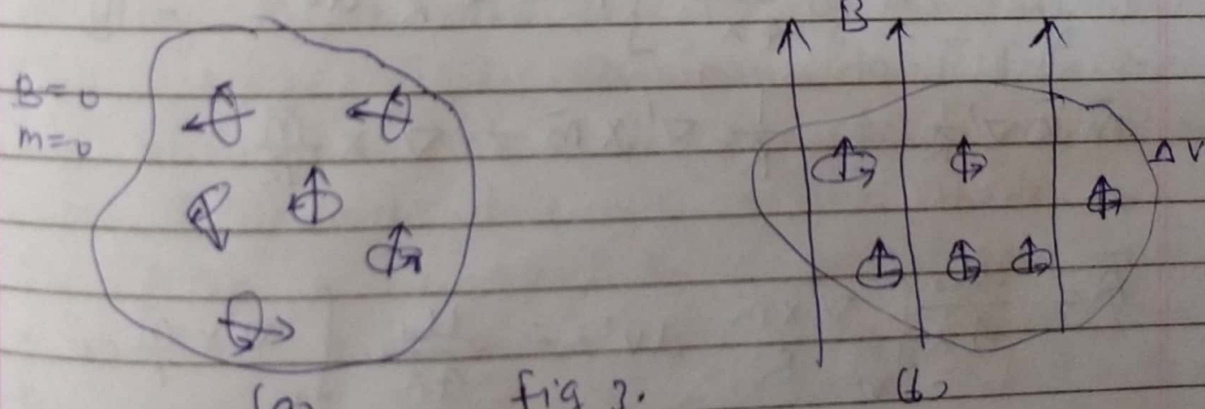


of the loop and I_b is the bound current (bo- -ound to the atoms).

fig 2. Circular current loop equivalent to electronic motion of fig 1

Without an external field $\bar{B}$ applied to the material, the sum of $\bar{m}$'s is zero due to random orientation as in fig 3(a). When an external field $\bar{B}$ is applied, the magnetic moments of electron more or less align them- -selves with $\bar{B}$ so that the net magnetic moment is zero, as illustrated in fig 3(b)



(a) fig 3. (b)

magnetic dipole moment in a volume ΔV. before B is applied

After $\bar{B}$ is applied

⇒ The magnetization M, in amperes per meter, is the magnetic dipole moment per unit volume.

If there are N atoms in a given volume ΔV and the K_{th} atom has a magnetic moment $\bar{m}_k$

$$\boxed{\bar{M} = \lim_{\Delta V \to 0} \frac{\sum_{k=1}^{N} \bar{m}_k}{\Delta V}} \quad \text{— (I)}$$

A medium for which $\bar{M}$ is not zero everywhere is said to be magnetized.

For a differential volume dv', the magnetic moment is $d\bar{m} = \bar{M} dv'$

The vector magnetic potential due to $d\bar{m}$ is

$$d\bar{A} = \frac{\mu_0 \bar{m} \times a_R}{4\pi R^2} dv' = \frac{\mu_0 \bar{m} \times \bar{R}}{4\pi R^3} dv'$$

but $\frac{\bar{R}}{R^3} = \nabla' \frac{1}{R}$

Hence

$$\bar{A} = \frac{\mu_0}{4\pi} \int \bar{m} \times \nabla' \frac{1}{R} dv' \quad \text{— (II)}$$

$$\bar{m} \times \nabla' \frac{1}{R} = \frac{1}{R} \nabla' \times \bar{m} - \nabla' \times \frac{\bar{m}}{R}$$

$$\bar{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\nabla' \times \bar{m}}{R} dv' - \frac{\mu_0}{4\pi} \int_{V'} \nabla' \times \frac{\bar{M}}{R} dv'$$

apply the vector identity

$$\int_{V'} \nabla' \times \bar{F} \, dv' = - \oint_S \bar{F} \times d\bar{s}$$

to the second integral, we obtain

$$\bar{A} = \frac{\mu_0}{4\pi} \int_{V'} \frac{\nabla' \times \bar{m}}{R} dv' + \frac{\mu_0}{4\pi} \oint_S \frac{\bar{m} \times \bar{a}_n}{R} ds'$$

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$$= \frac{\mu_0}{4\pi}\int_{v'} \frac{J_b\, dv'}{R} + \frac{\mu_0}{4\pi}\oint_{s} \frac{K_b\, ds'}{R} \quad -\text{(III)}$$

$$\boxed{\bar{J}_b = \bar{\nabla}\times\bar{m}} \quad -\text{(IV)}$$

and $$\boxed{\bar{K}_b = \bar{M}\times\bar{a}_n} \quad -\text{(V)}$$

where $\bar{J}_b$ is the bound volume current density or magnetization volume current density in ampere per meter square, $\bar{K}_b$ is the bound surface current density, in ampere per meter and $\bar{a}_n$ is unit vector normal to the surface. Equation (III) shows that the potential of a magnetic body is due to a volume current density $\bar{J}_b$ throughout the body and a surface current density $\bar{K}_b$ on the surface of the body.

The vector $\bar{m}$ is analogous to $\bar{P}$ polarization in dielectrics and is sometimes called mag-netic polarization density of the medium. In other sense, $\bar{m}$ is analogous to $\bar{H}$ and they both have the same units. In this respect, as $\bar{J} = \bar{\nabla}\times\bar{H}$ so $\bar{J}_b = \bar{\nabla}\times\bar{M}$ also $\bar{J}_b$ and $\bar{K}_b$ for a magnetized body are similar to ρ_{pv} and ρ_{ps} for a polarized body. As is evident in eq. (III) to (V), $\bar{J}_b$ and $\bar{K}_b$ can be derived from $\bar{M}$; therefore, $\bar{J}_b$ and $\bar{K}_b$ are not commonly used.

In free space $\bar{M}=0$ and we have

$$\bar{\nabla}\times\bar{H} = \bar{J}_f \quad \text{or} \quad \bar{\nabla}\times\left(\frac{\bar{B}}{\mu_0}\right) = \bar{J}_f \quad -\text{(VI)}$$

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where ~~f~~ $\bar{J}_f$ is the free current volume density. In a material medium $\bar{M} \neq 0$, and as a result $\bar{B}$ changes so that

$$\nabla \times \left(\frac{\bar{B}}{\mu_0}\right) = \bar{J}_f + \bar{J}_b = J$$

$$= \nabla \times \bar{H} + \nabla \times M$$

$$\bar{B} = \mu_0 (\bar{H} + \bar{M}) \quad \text{— (VII)}$$

The relationship in eq. (VII) holds for all materials whether they are linear or not. For linear materials, $\bar{M}$ (in A/m) depends linearity on $\bar{H}$ such that-

$$\boxed{\bar{M} = \chi_m \bar{H}} \quad \text{— (VIII)}$$

where χ_m is dimensionless quantity (ratio of $\bar{M}$ to $\bar{H}$) called magnetic suceptibility of the medium. It is more or less a measure of how suceptible (or sensitive) the material is to a magnetic field. Substituting eq. (VIII) in (VII) yields

~~$\bar{B} = \mu_0 [H +$~~

$$\bar{B} = \mu_0 (1+\chi_m) \bar{H} = \mu \bar{H}$$

or

$$\boxed{\bar{B} = \mu_0 \mu_r \bar{H}} \quad \text{— (IX)}$$

where $\boxed{\mu_r = 1 + \chi_m = \frac{\mu}{\mu_0}} \quad \text{— (X)}$

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The quantity $\mu = \mu_0 \mu_r$ is called the permeability of the material and is measured in henery per meter. The henery is the unit of inductance. The dimensionless quantity μ_r is the ratio of the permeability of a given material to that of free space and is known as the relative permeability of the material.

I Magnetic boundary Conditions ⇒

we define magnetic boundary conditions as the conditions that $\bar{H}$ (or $\bar{B}$) field must satisfy at the boundary b/w two different media. We make use Gauss's law for magnetic fields.

$$\oint \bar{B} \cdot d\bar{S} = 0 \quad \text{—(I)}$$

and Ampere's circuit law

$$\oint \bar{H} \cdot d\bar{l} = I \quad \text{—(II)}$$

consider the boundary b/w two magnetic media 1 and 2, characterized, respectively, by μ_1 and μ_2 as in fig.
Applying eq. (I) to the pillbox (Gaussian surface) of fig (a) and allowing $\Delta h \to 0$ we obtain

$$B_{1n} \Delta S - B_{2n} \Delta S = 0 \quad \text{—(III)}$$

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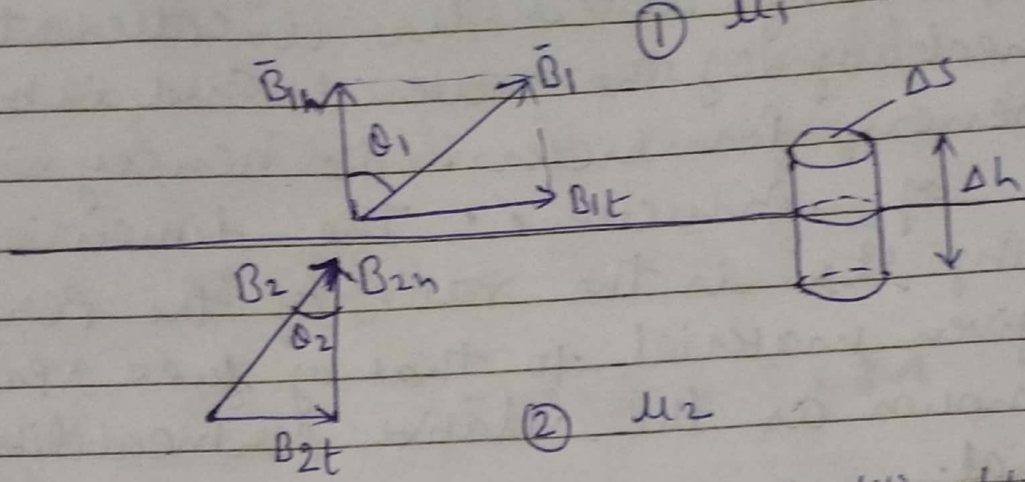

fig (a) Boundary condition b/w ~~the~~ magnetic media for $\bar{B}$

Thus $\boxed{\bar{B}_{1n} = \bar{B}_{2n}}$ or $\mu_1 \bar{H}_{1n} = \mu_2 \bar{H}_{2n}$ — (iv)

This equation shows that the normal component of $\bar{B}$ is continuous at the boundary. it also shows that the normal component of $\bar{H}$ is dis-continuous at the boundary; $\bar{H}$ undergoes some changes at the interface.

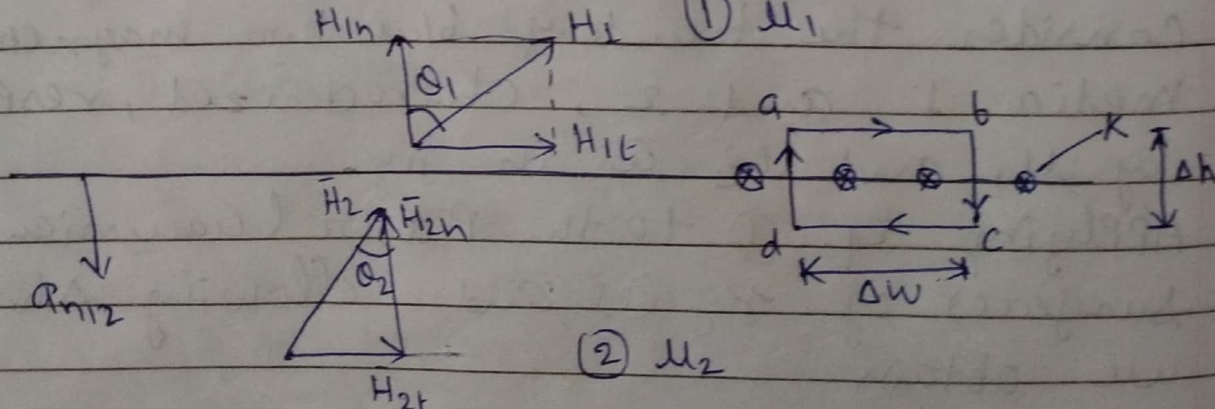

fig (b) Boundary Conditions b/w two magnetic media for $\bar{H}$

Similarly, we apply equation (II) to the closed path abcda of fig (b), where surface current K on the boundary is assumed normal to the path. we obtain:

$$K \cdot \Delta W = H_{1t} \cdot \Delta W + H_{1n} \cdot \frac{\Delta h}{2} + H_{2n} \cdot \frac{\Delta h}{2} - H_{2t} \cdot \Delta W$$
$$- H_{2n} \frac{\Delta h}{2} - H_{1n} \frac{\Delta h}{2} \quad \text{— (V)}$$

as $\Delta h \to 0$

$$H_{1t} - H_{2t} = K \quad \text{— (VI)}$$

This shows that the tangential component of H is also discontinuous. eq. (VI) may be written in terms of $\bar{B}$ as

$$\frac{B_{1t}}{\mu_1} - \frac{B_{2t}}{\mu_2} = K \quad \text{— (VII)}$$

in general case eq. (VI) becomes

$$\boxed{(\bar{H}_1 - \bar{H}_2) \times \bar{a}_{12n} = \bar{K}} \quad \text{— (VIII)}$$

where $\bar{a}_{n12}$ is a unit vector normal to the interface and is directed from medium 1 to medium 2. if the boundary is free of current or media are not conductors (for K is free current density) K=0 and eq. (VI) becomes

$$\boxed{\bar{H}_{1t} = \bar{H}_{2t}} \quad \text{or} \quad \boxed{\frac{B_{1t}}{\mu_1} = \frac{B_{2t}}{\mu_2}} \quad \text{— (IX)}$$

Thus the tangentia component of $\bar{H}$ is continuous while that of $\bar{B}$ is discontinuous at the boundary.

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if the fields make an angle θ with the normal to the interface eq. (IV) results in

$$B_1 \cos\theta_1 = B_{1n} = B_{2n} = B_2 \cos\theta_2 \quad \text{— (X)}$$

while eq. (IX) Produces

$$\frac{B_1}{\mu_1} \sin\theta_1 = H_{1t} = H_{2t} = \frac{B_2}{\mu_2} \sin\theta_2 \quad \text{— (XI)}$$

dividing eq. (XI) by (X) gives

$$\boxed{\frac{\tan\theta_1}{\tan\theta_2} = \frac{\mu_1}{\mu_2}} \quad \text{— (XII)}$$

which is the law of refraction for magnetic flux lines at the boundary with no surface current.

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Inductors and inductances ⇒

A circuit (or closed conducting path) carrying current I produces a magnetic field $\bar{B}$ that causes a flux $\psi = \int \bar{B} \cdot d\bar{s}$ to pass through each turn of the circuit as shown in fig-(a). If the circuit has N identical turns, we define the flux linkage λ as

$$\lambda = N\psi \qquad \text{—(I)}$$

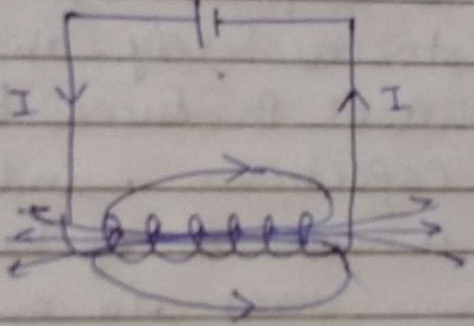

fig (a) Magnetic field $\bar{B}$ produced by a circuit.

Also, if the medium surrounding the circuit is linear, the flux linkage λ is proportional to the current I producing it; that is,

$$\lambda \propto I$$

or

$$\lambda = LI \qquad \text{—(II)}$$

Where L is constant of proportionality called the "inductance" of the circuit. The inductance L is a ~~proportionality~~ property of the physical arrangement of the circuit. A circuit or part of a circuit that has inductance is called an inductor.

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from eq. (I) & (II), we may define inductance L of an inductor as the ratio of the magnetic flux linkage λ to the current I through the inductor; that is

$$\boxed{L = \frac{\lambda}{I} = \frac{N\Psi}{I}} \quad \text{— (III)}$$

The unit of inductance is henry (H) which is the same as Wb/A. Since the henry is a fairly large unit, inductances are usually expressed in millihenrys (mH). The inductance defined by eq. (III) is commonly referred to as self-inductance, since the linkages are produced by the inductor itself. Like capacitance, inductance may be regarded as a measure of how much magnetic energy is stored in an inductor. The magnetic energy (in joules) stored in an inductor is expressed in circuit theory as

$$W_m = \frac{1}{2} L I^2 \quad \text{— (IV)}$$

or

$$\boxed{L = \frac{2W_m}{I^2}} \quad \text{— (V)}$$

Thus the self inductance of a circuit may be defined or calculated from energy consideration.

If instead of having a single circuit, we have two circuit carrying current I_1 and I_2 as shown in fig(6), a magnetic interaction exists b/w the circuits. Four component

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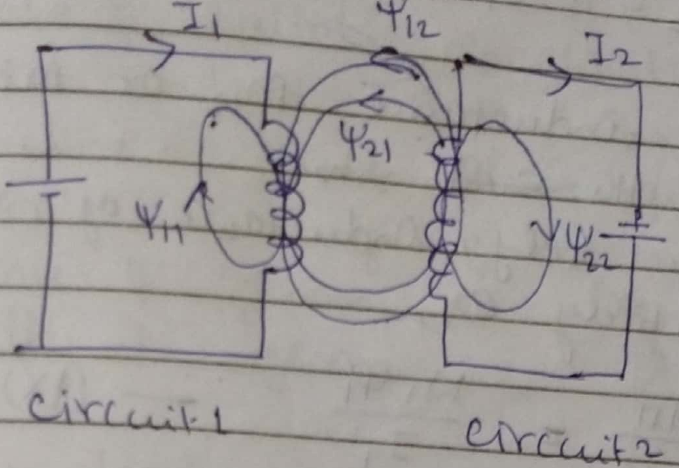
fluxes Ψ_{11}, Ψ_{12}, Ψ_{21} and Ψ_{22} are produced. The flux Ψ_{12}; for example, is the flux passing



fig(b) magnetic induction b/w two circuits.

through circuit I due to current I_2 in circuit 2. if $\bar{B}_2$ is the field due to I_2 and S_1 is the area of circuit 1, then

$$\Psi_{12} = \int_{S_1} \bar{B}_2 \cdot d\bar{S} \qquad \text{— (VI)}$$

we define the mutual inductance M_{12} as the ratio of the flux linkage $\lambda_{12} = N_1 \Psi_{12}$ on circuit 1 to current I_2, that is

$$\boxed{M_{12} = \frac{\lambda_{12}}{I_2} = \frac{N_1 \Psi_{12}}{I_2}} \qquad \text{— (VII)}$$

Similarly the mutual inductance M_{21} is defined as the flux linkages of circuit 2 per unit current I_1, that is

$$\boxed{M_{21} = \frac{\lambda_{21}}{I_1} = \frac{N_2 \Psi_{21}}{I_1}} \qquad \text{— (VIII)}$$

it can be shown by using energy concepts that if the medium surrounding the circuit linear (i.e in the absence of ferromagnetic material)

$$M_{12} = M_{21} \quad —$$

The mutual inductance M_{12} or M_{21} is expressed in H. ~~and the~~

we define the self inductance of circuit 1 and 2 respectively as,

$$L_1 = \frac{\lambda_{11}}{I_1} = \frac{N_1 \Psi_1}{I_1} \quad — (IX)$$

and $$L_2 = \frac{\lambda_{22}}{I_2} = \frac{N_2 \Psi_2}{I_2} \quad — (X)$$

where $\Psi_1 = \Psi_{11} + \Psi_{12}$ and $\Psi_2 = \Psi_{21} + \Psi_{22}$.

The total energy in the magnetic field is the sum of the energies due to L_1, L_2 and M_{12} (or M_{21}); that is

$$W_m = W_1 + W_2 + W_{12}$$

$$= \frac{1}{2} L_1 I_1^2 + \frac{1}{2} L_2 I_2^2 \pm M_{12} I_1 I_2 \quad — (XI)$$

The Positive sign is taken if I_1 and I_2 flows such that the magnetic fields of the two circuits strengthen each other. If the currents flow such that their magnetic field oppose each other, the negative sign is taken.

Typical examples of inductors are toroids, solenoids, coaxial transmission lines, and parallel wire transmission line.

The inductance of each of these inductors can be determined by following procedure.

1. Choose a suitable coordinate system
2. Let the inductor carry current I
3. Determine $\bar{B}$ from Biot-Savart law (or from Ampere's law if symmetry exists) and calculate ψ from $\psi = \int \bar{B} \cdot d\bar{s}$.
4. Finally L from $L = \frac{\lambda}{I} = \frac{N\psi}{I}$

The mutual inductance b/w two circuits may be calculated by taking a similar procedure.

In an inductor such as a coaxial or a parallel-wire transmission line, the inductance produced by the flux internal to the conductor is called the internal inductance L_{in} while that produced by the flux external to it is called external inductance L_{ext}. The total inductance L is

$$L = L_{in} + L_{ext} \quad \text{——} (XII)$$

Just as it was shown that for capacitors

$$RC = \frac{\epsilon}{\sigma} \quad \text{——} (XIII)$$

it can be shown that

$$\boxed{L_{ext}\, C = \mu \epsilon} \quad \text{——} (XIV)$$

Thus L_{ext} may be calculated using eq. (XIV) if C is known.

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Magnetic energy ⇒

Just as the Potential energy in an electro-static field was derived as

$$W_E = \frac{1}{2}\int \bar{D}\cdot\bar{E}\, dv = \frac{1}{2}\int \epsilon E^2 dv \quad \text{— (I)}$$

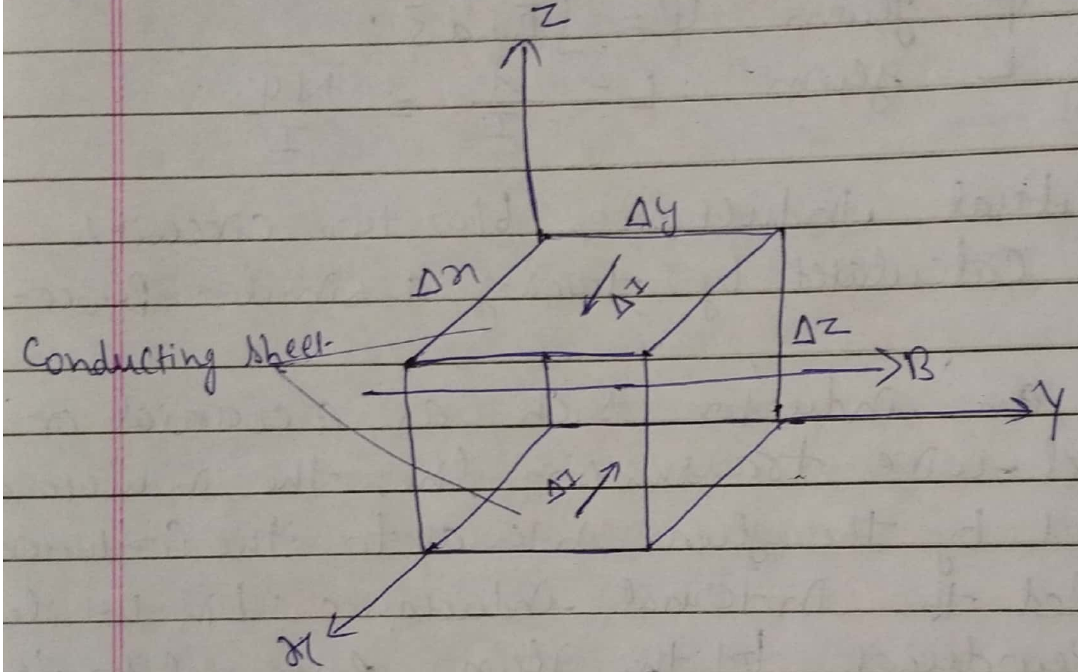

fig - A differential Volume in a magnetic field.

we would like to derive a similar expression for the energy in a magnetostatic field. A simple approach is using the magnetic energy in the field of an inductor

$$W_m = \frac{1}{2} L I^2 \quad \text{— (II)}$$

from eq. (II), the energy is stored in the magnetic field $\bar{B}$ of the inductor. we would like to express eq. (II) interms of $\bar{B}$ or $\bar{H}$.

Consider a differential volume in a magnetic field as shown in fig. Let the volume be covered with conducting sheets at the top and bottom surfaces with current ΔI. We assume that the whole region is filled with such differential volumes. from eq. $L = \frac{N\Psi}{I}$ each volume has an inductance

$$\Delta I = \frac{\Delta \Psi}{\Delta I} = \frac{\mu H \Delta x \Delta z}{\Delta I} \quad \text{—(III)}$$

where $\Delta I = H \Delta y$. substituting eq. (III) into eq. (II) we have

$$\Delta W_m = \frac{1}{2} \Delta L \Delta I^2 = \frac{1}{2} \mu H^2 \Delta x \Delta y \Delta z \quad \text{—(IV)}$$

or $$\Delta W_m = \frac{1}{2} \mu H^2 \Delta V$$

The magnetostatic energy density w_m (in J/m3) is defined as

$$w_m = \lim_{\Delta V \to 0} \frac{\Delta W_m}{\Delta V} = \frac{1}{2} \mu H^2$$

hence

$$w_m = \frac{1}{2} \mu H^2 = \frac{1}{2} \bar{B} \cdot \bar{H} = \frac{B^2}{2\mu} \quad \text{—(V)}$$

Thus the energy in a magnetic field in a linear medium is

$$W_m = \int w_m \, dV$$

or

$$\boxed{W_m = \frac{1}{2} \int \bar{B} \cdot \bar{H} \, dV = \frac{1}{2} \int \mu \bar{H}^2 \, dV} \quad \text{—(VI)}$$

which is similar to electrostatic field.

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Magnetic Circuits ⇒

The Concept of magnetic circuits is based on solving some magnetic field Problems by using a circuit approach. magnetic devices such as toroids, transformers, motors, generators, and relays may be considered as magnetic circuits.
The analysis of such circuits is made simple if an analogy b/w magnetic circuits and electric circuits is exploited. once this has bee done we can directly apply concept in electric circuits to solve their analogous magnetic circuits.
The analogy b/w magnetic and electric circuit is summerized is given table and fig.

table → Analogy b/w Electric & magnetic circuit

	Electric	magnetic
1.	Conductivity σ	Permeability μ
2.	Field intensity $\bar{E}$	Field intensity $\bar{H}$
3.	Current $I = \int \bar{J} \cdot d\bar{s}$	magnetic flux $\psi = \int \bar{B} \cdot d\bar{s}$
4.	current density $J = \frac{I}{S} = \sigma E$	Flux density $B = \frac{\psi}{S} = \mu H$
5.	EMF $= V$	magnetomotive force (mmf) $\mathcal{F}$
6.	Resistance R	Reluctance $\mathcal{R}$
7.	Conductance $G = \frac{1}{R}$	Permeance $\mathcal{P} = \frac{1}{\mathcal{R}}$
8.	ohm's law $R = \frac{V}{I} = \frac{l}{\sigma S}$	ohm's law $\mathcal{R} = \frac{\mathcal{F}}{\psi} = \frac{l}{\mu S}$
	or $V = El = IR$	or $\mathcal{F} = Hl = \psi \mathcal{R} = NI$
9.	Kirchhoff's law $\Sigma I = 0$	Kirchhoff's law $\Sigma \psi = 0$
	$\Sigma V - \Sigma RI = 0$	$\Sigma \mathcal{F} - \Sigma \mathcal{R} \psi = 0$

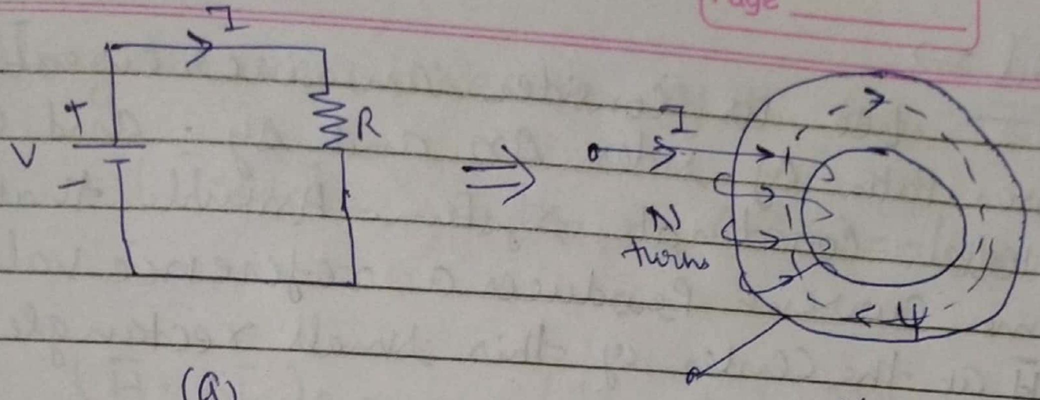

fig- Analogy b/w (a) an electric circuit and (b) a magnetic circuit.

from table - we define the magnetomotive force (mmf) $\mathcal{F}$, in amp-turns (A·t), as

$$\boxed{\mathcal{F} = NI = \oint \vec{H} \cdot d\vec{l}} \quad \text{— (I)}$$

The source of mmf in magnetic circuit is usually a coil carrying current as in fig. we also define reluctance $\mathcal{R}$ in amp turns/wb as

$$\boxed{\mathcal{R} = \frac{l}{\mu S}} \quad \text{— (II)}$$

When l and S are respectively the mean length and the cross-sectional area of the magnetic core. the reciprocal of reluctance is Permeance $\mathcal{P}$. The basic relationship for circuit element is ohm's law $(V = IR)$

$$\boxed{\mathcal{F} = \psi \mathcal{R}} \quad \text{— (III)}$$

Based on this, Kirchhoff's current and voltage law can be applied to nodes and loops of a given magnetic circuit just as in an electric circuit. The rules for adding voltages

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and for combining series and parallel resistances also hold for mmfs and reluctances. Thus for n magnetic circuit element in series

$$\psi_1 = \psi_2 = \psi_3 = ---- = \psi_n \quad -(IV)$$

and

$$\mathcal{F}_e = \mathcal{F}_{e_1} + \mathcal{F}_{e_2} + --- + \mathcal{F}_{e_n} \quad -(V)$$

For n magnetic circuit elements in parallel

$$\psi = \psi_1 + \psi_2 + \psi_3 + --- \psi_n \quad -(VI)$$

and

$$\mathcal{F}_{e_1} = \mathcal{F}_{e_2} = \mathcal{F}_{e_3} = --- = \mathcal{F}_{e_n} \quad -(VII).$$

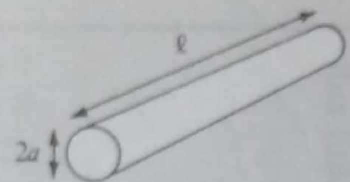
Some differences b/w electric and magnetic circuits should be pointed out.

- Unlike an electric circuit, where the current I flows, magnetic flux does not flow. Also conductivity σ is independent of J in an electric circuit, whereas permeability μ varies with flux density B in a magnetic circuit. This is because ferromagnetic (non-linear) materials are normally used in most practical magnetic devices. These differences notwithstanding, the magnetic circuit concept serves in the approximate analysis of practical magnetic devices.

Table 7.3 A Collection of Formulas for Inductance of Common Elements

1. Wire

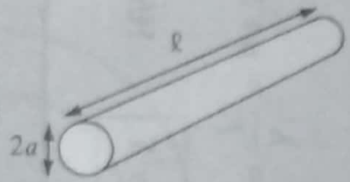
$$L = \frac{\mu_o \ell}{8\pi}$$



2. Hollow cylinder

$$L = \frac{\mu_o \ell}{2\pi}\left(\ln\frac{2\ell}{\mathbf{a}} - 1\right)$$

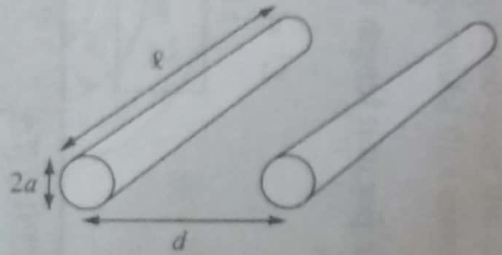
$\ell \gg \mathbf{a}$



3. Parallel wires

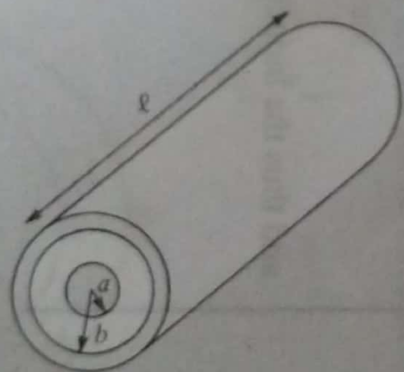
$$L = \frac{\mu_o \ell}{\pi}\ln\frac{d}{a}$$

$\ell \gg d,\ d \gg a$



4. Coaxial conductor

$$L = \frac{\mu_o \ell}{\pi}\ln\frac{b}{a}$$



5. Circular loop

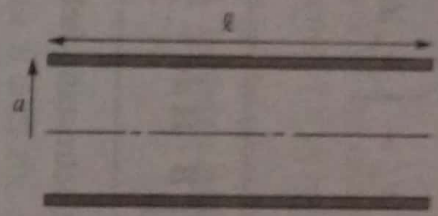
$$L = \frac{\mu_o \ell}{2\pi}\left(\ln\frac{4\ell}{d} - 2.45\right)$$

$\ell = 2\pi\rho_o,\ \rho_o \gg d$

6. Solenoid

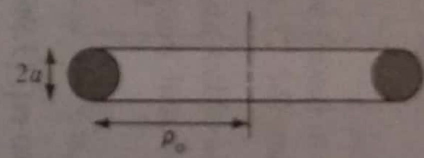
$$L = \frac{\mu_o N^2 S}{\ell}$$

$\ell \gg a$



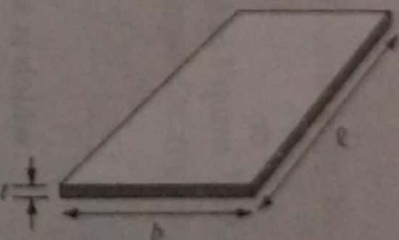
7. Torus (of circular cross section)

$$L = \mu_o N^2[\rho_o - \sqrt{\rho_o^2 - a^2}]$$



8. Sheet

$$L = \mu_o\, 2\ell\left(\ln\frac{2\ell}{b+t} + 0.5\right)$$



EXAMPLE 8.3

The magnetic circuit of Figure 8.8 has a uniform cross section of 10^{-3} m^2. If the circuit is energized by a current $i_1(t) = 3 \sin 100\pi t$ A in the coil of $N_1 = 200$ turns, find the emf induced in the coil of $N_2 = 100$ turns. Assume that $\mu = 500\,\mu_o$.

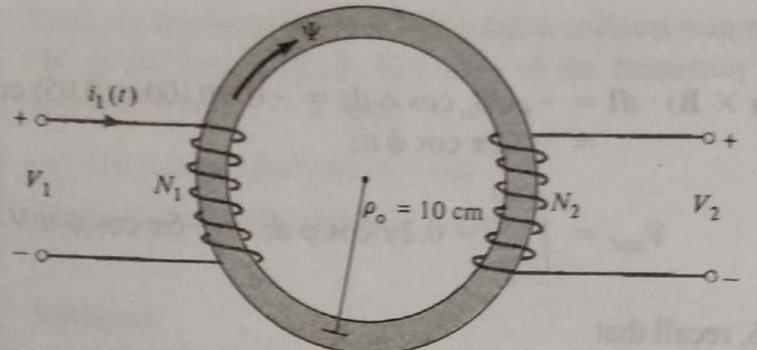


Figure 8.8 Magnetic circuit of Example 8.3.

Solution:

The flux in the circuit is

$$\Psi = \frac{\mathcal{F}}{\mathcal{R}} = \frac{N_1 i_1}{\ell/\mu S} = \frac{N_1 i_1 \mu S}{2\pi\rho_o}$$

According to Faraday's law, the emf induced in the second coil is

$$V_2 = -N_2 \frac{d\Psi}{dt} = -\frac{N_1 N_2 \mu S}{2\pi\rho_o}\frac{di_1}{dt}$$

$$= -\frac{100 \cdot (200) \cdot (500) \cdot (4\pi \times 10^{-7}) \cdot (10^{-3}) \cdot 300\pi \cos 100\pi t}{2\pi\,(10 \times 10^{-2})}$$

$$= -6\pi \cos 100\pi t \text{ V}$$

PRACTICE EXERCISE 8.3

A magnetic core of uniform cross section 4 cm^2 is connected to a 120 V, 60 Hz generator as shown in Figure 8.9. Calculate the induced emf V_2 in the secondary coil.

Answer: 72 V.

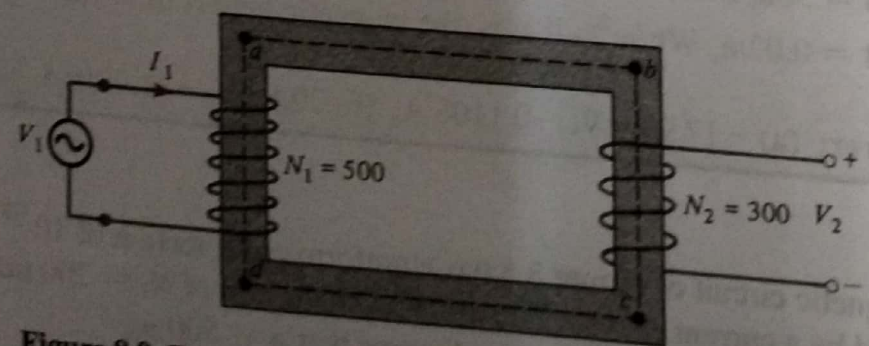


Figure 8.9 For Practice Exercise 8.3.

EXAMPLE 8.4

A parallel-plate capacitor with plate area of 5 cm^2 and plate separation of 3 mm has a voltage $50 \sin 10^3 t$ V applied to its plates. Calculate the displacement current assuming $\varepsilon = 2\varepsilon_o$.

Solution:

$$D = \varepsilon E = \varepsilon \frac{V}{d}$$

$$J_d = \frac{\partial D}{\partial t} = \frac{\varepsilon}{d} \frac{dV}{dt}$$

Hence,

$$I_d = J_d \cdot S = \frac{\varepsilon S}{d} \frac{dV}{dt} = C \frac{dV}{dt}$$

which is the same as the conduction current, given by

$$I_c = \frac{dQ}{dt} = S \frac{d\rho_s}{dt} = S \frac{dD}{dt} = \varepsilon S \frac{dE}{dt} = \frac{\varepsilon S}{d} \frac{dV}{dt} = C \frac{dV}{dt}$$

$$\begin{aligned} I_d &= 2 \cdot \frac{10^{-9}}{36\pi} \cdot \frac{5 \times 10^{-4}}{3 \times 10^{-3}} \cdot 10^3 \times 50 \cos 10^3 t \\ &= 147.4 \cos 10^3 t \text{ nA} \end{aligned}$$

PRACTICE EXERCISE 8.4

In free space, $\mathbf{E} = 20 \cos(\omega t - 50x)\, \mathbf{a}_y$ V/m. Calculate

(a) $\mathbf{J}_d$

(b) $\mathbf{H}$

(c) ω

Answer: (a) $-20\omega\varepsilon_o \sin(\omega t - 50x)\, \mathbf{a}_y$ A/m^2, (b) $0.4\, \omega\varepsilon_o \cos(\omega t - 50x)\, \mathbf{a}_z$ A/m, (c) 1.5×10^{10} rad/s.

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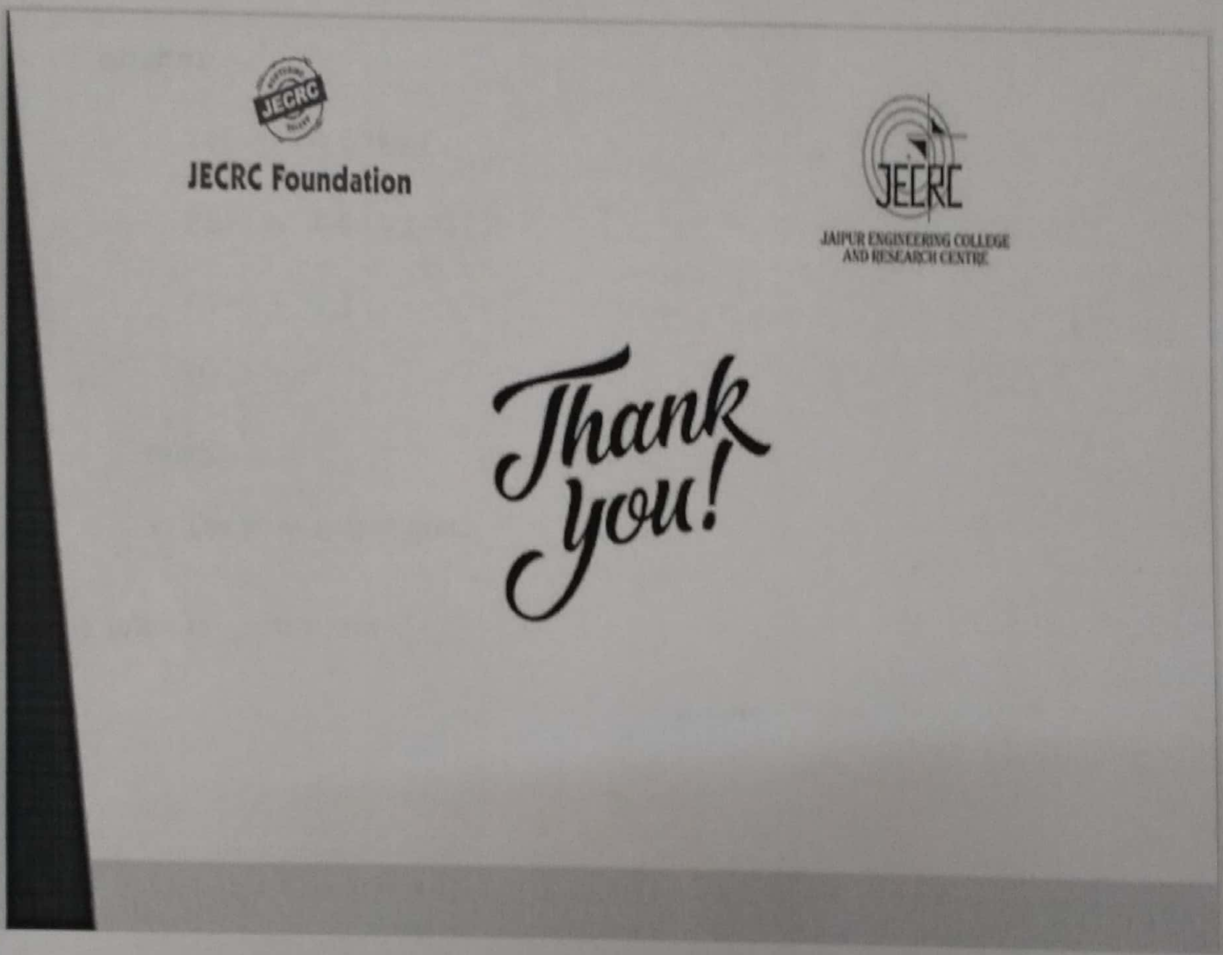
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