Lecture -Notes

Electrical Circuit Analysis

(3EE5-04)

B.Tech III SEM Electrical Engineering

Prepared By: Ram Singh Assistant Professor, Electrical Engineering, JECRC Jaipur

Vision of JECRC

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of JECRC

M1. Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
M2. Identify, based on informed perception of Indian, regional and global needs, areas of focus and provide platform to gain knowledge and solutions.
M3. Offer opportunities for interaction between academia and industry.
M4. Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

Vision of EE Department

Electrical Engineering Department strives to be recognized globally for outcome based knowledge and to develop human potential to practice advance technology which contribute to society.

Mission of EE Department

M1. To impart quality technical knowledge to the learners to make them globally competitive Electrical Engineers.
M2. To provide the learners ethical guidelines along with excellent academic environment for a long productive career.
M3. To promote industry-institute relationship.

PSO of EE Department

PSO1 Graduates will be able to contribute for the development of automation.
PSO2 Graduates will be able to contribute towards integration of the green energy.

PROGRAM OUTCOMES

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis:** Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions:** Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems:** Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage:** Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society:** Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability:** Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics:** Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one’s own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes	
CO1	Analyze the basic rule of electric network theorems.
CO2	Analyze the transient and steady state conditions of AC and DC circuits
CO3	Analyze the two port network functions and Laplace transform of electrical circuits

CO-PO Mapping

PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12	PSO1	PSO1
Co1	3	3	1	2	2	1	2	-	-	2	1	-	-	-
Co2	2	2	2	1	1	1	1	-	-	3	1	-	-	--
Co3	3	2		2	2	2	1	-	-	2	1	-	-	---

Teaching and Examination Scheme

S.NO	Course Type	Course		Hours Per Week			Marks				Cr
		Code	Name	L	T	P	Exam Hrs	IA	ETE	Total	
1	PCC/PEC	Code-3EE4-05	Electrical Circuit Analysis	3	0	0	3	30	120	150	3

RAJASTHAN TECHNICAL UNIVERSITY, KOTA

SYLLABUS

2nd Year - III Semester: B.Tech. (Electrical Engineering)

3EE4-05 Electrical Circuit Analysis

Credit: 3 — **Max. Marks: 150 (IA:30, ETE:120)**

3L+0T+0P — **End Term Exam: 3 Hours**

SN	CONTENTS	Hours
1.	**Network Theorems** Superposition theorem, Thevenin theorem, Norton theorem, Maximum power transfer theorem, Reciprocity theorem, Compensation theorem. Analysis with dependent current and voltage sources. Node and Mesh Analysis. Concept of duality and dual networks.	10
2.	**Solution of First and Second order networks** Solution of first and second order differential equations for Series and parallel R-L, R-C, RL- C circuits, initial and final conditions in network elements, forced and free response, time constants, steady state and transient state response.	8
3.	**Sinusoidal steady state analysis** Representation of sine function as rotating phasor, phasor diagrams, impedances and admittances, AC circuit analysis, effective or RMS values, average power and complex power. Three-phase circuits. Mutual coupled circuits, Dot Convention in coupled circuits, Ideal Transformer.	8
4.	**Electrical Circuit Analysis Using Laplace Transforms** Review of Laplace Transform, Analysis of electrical circuits using Laplace Transform for standard inputs, convolution integral, inverse Laplace transform, transformed network with initial conditions. Transfer function representation. Poles and Zeros. Frequency response (magnitude and phase plots), series and parallel resonances	8
5.	**Two Port Network and Network Functions** Two Port Networks, terminal pairs, relationship of two port variables, impedance parameters, admittance parameters, transmission parameters and hybrid parameters, interconnections of two port networks.	6
	TOTAL	**40**

Network Theory

May 30, 2007.

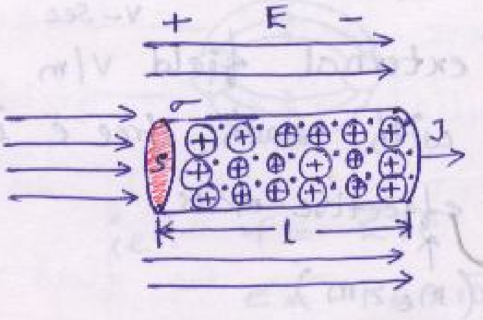
1. Basics
2. Theorems
3. L.T.
4. Transients < dc / ac
5. Ac Analysis

Ref:

1. Network Analysis – Van valkenburg
2. ✓ Engg. circuit analysis – Hayt & kemmerly
3. previous papers : GK pub.
   (i). GATE < EE / EC / INSTRUMENTAL (1990-2007) (UPSC)
   (ii). IES < EE / EC
   (iii). IAS – Prelims – EE (1994-2006) (Raj kamal Text book / Cengg Koti)

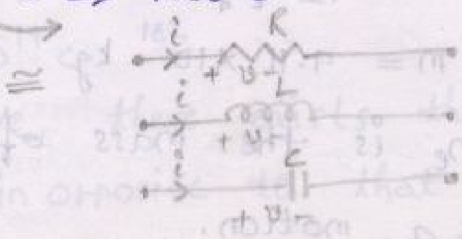
Basics.

→ The mechanism of energy ~~flow~~ through the conductor and ohm's law:-



⊕ → Ag^+ ion, immobile, Larger in size ie 10^3 times than $\bar{e}$.

• → free e^-



→ $Ag^{+1} \rightarrow +1$

$Cu^{+2} \rightarrow +2$

$Au^{+2} \rightarrow +2$

$Al^{+3} \rightarrow +3$

→ The mobility of free $\bar{e}$'s in a Ag, is several times to that of other conductors so its conductivity is very high.

→ Generally in any conductor, there are 10^{18} to 10^{23} atoms per unit volume (ie per unit cube) and hence there are 10^{18} to 10^{23} free $\bar{e}$s per unit volume in a Ag conductor. ie every conductor is a very rich of free $\bar{e}$s.

→ In the presence of external field different free e^- will undergo diff. forces [due to a large no. of free $\bar{e}$s] and hence they will move with diff. velocity. But only one velocity is defined, so called drift velocity. It is an avg. velocity of all the free $\bar{e}$s within a conductor. and is given by $v_d = \mu E$ m/s.

μ = mobility of free $\bar{e}$s $\frac{m^2}{V\text{-}sec}$

E – Applied external field V/m

→ The K.E. associated with each free $\bar{e}$ is

$KE = \frac{1}{2} m_e v_d^2$ J — effective mass

$m = 9.11 \times 10^{-31}$ kg $(m_e \approx m)$

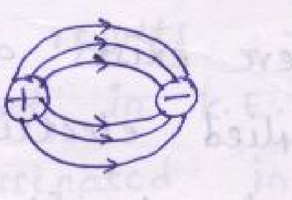
m_e is the mass of free $\bar{e}$ while it is in a motion.

The first Half of the Ohm's experiment when the conductor not carrying electrical energy $E=0$:-

→ when $E=0 \Rightarrow v_d=0 \Rightarrow K.E.=0$
ie all the free $\bar{e}$ are in the rest.

→ Since the conductor is operating at room temp. (27°C or 300°K), diff. free $\bar{e}$'s will acquire diff. thermal energies [due to a Large no. of free $\bar{e}$] and hence they will move in diff. directions in a random manner the net flow of $\bar{e}$ motion, in any direction zero, ie the charge motion is zero and the i is zero and also the current density [J] is zero.
ie when $E=0$, then $J=0$.

Second Half of Ohm's experiment, when the conductor is carrying electrical energy $[E \neq 0]$:-



when the conductor is subjected to an axial electric field, the force will be exerted on every free $\bar{e}$.

ie. $\vec{f} = \vec{E} . e$ N
$e = -1.6 \times 10^{-19}$ C

Since 'e' is −ve, there exists the direction of force is in opposite to that of E. and hence there exists a net $\bar{e}$ motion ie the charge motion in the direction opposite so that of 'E'.

The magnitude of charge is given by

$q = ne$ C , n = no. of free $\bar{e}$'s crossing a reference cs area, a variable quantity due a large no. of free $\bar{e}$.

$e = -1.6 \times 10^{-19}$ C

→ The time rate of flow of electric charges is nothing but the electric i ie

$$i = \frac{dq}{dt} \text{ A}$$

Since q is -ve, the conventional current direction is opposite that of the charge motion ie $\bar{e}$ motion [ie in the dire. of 'E']

The current per unit cs area is nothing but the current density resulted within a conductor

ie $J = \frac{i}{s}$ A/m²

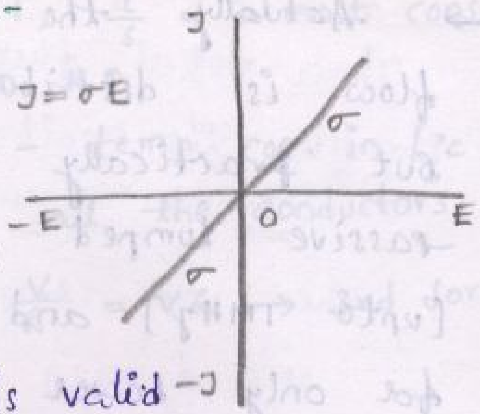
Since 's' is a scalar, the dire. of 'J' is in the dire. of 'i', ie in the dire. of E.

Acc. to Ohm, there exists a linear relation b/w the applied electric field and resulting current density by $J \propto E$

$J = \sigma E$ → Ohm's law in the field theory form.

σ → conductivity of the conductor.

J – E characteristics :-



At the origin E = 0 ⇒ J = 0 and σ is not equal to zero.

Limitation:-

The ohm's law is valid only when proportionality const. σ is const. ie the temp. is kept constant condition.

At the const. E, as temp. increases from room temp. there exists an increase in collisions among the free $\bar{e}$'s and hence the mobility falls, so the conductivity decreases. [Here the collisions b/w the free $\bar{e}$'s and +ve ions are assumed to be const., since E is kept constant.]

At a const. TEMP. as 'E' increases there exists an increase in collisions b/w the free $\bar{e}$s and the +ve ions [larger in size], which results the fall lost in v_d and hence the lost in K.E. This losted energy will be dissipated in the form of heat, which results the volt. drop across the conductor. [Here the collisions amount, the free $\bar{e}$s are assumed to be const, since the temp. is kept const.

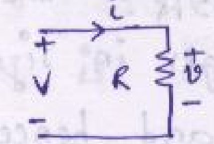
→ Actually the opposition for the energy flow is distributive through the conductor. But practically this is approximated into passive lumped R, L, C's for lower freq.s [upto 1MHz] and hence n/w theory valid for only lower freq.s.

At higher freq.s we can't derive the lumped elements so no lumped electric n/w, so no n/w theory ie field theory is applicable.

Field theory approach of solving the distributive electric n/w's are valid for all freq.s starting from zero [DC].

So the currents through all the 3 passive lumped elements will always flows from +ve to −ve terminals.

Resistance R :-



$\longrightarrow$ Since $J = \sigma E$

$\Rightarrow \frac{i}{s} = \sigma\left(\frac{v}{l}\right)$

$\Rightarrow v = \left(\frac{l}{\sigma s}\right) i$

$\Rightarrow v = Ri \longrightarrow$ Ohm's law in ckt theory form

$\therefore R = \frac{l}{\sigma s}$

Limitation :

The Ohm's law is valid when R is kept const ie temp. is kept const.

→ As $T\uparrow \Rightarrow l\uparrow, S\uparrow, \frac{l}{s}$ = almost const.
$\sigma\downarrow$ so $R\uparrow$

→ $R_t = R_0(1+\alpha t)$, α – temp. coe. in /°c, which is +ve for all the conductors.

→ Since $V = Ri \Rightarrow i = \frac{V}{R} = VG$ → 3rd form of ohm's law.

G = conductance $\mho$

Since $i = \frac{dq}{dt}$, $v = R.\frac{dq}{dt}$ → 4th form ohm's

→ $R = \frac{l}{\sigma s} \Rightarrow \sigma = \frac{l}{RS} = \frac{m}{\Omega - m^2} = \mho/m$ (or) S/m

→ Resistivity $\rho = \frac{1}{\sigma} = \frac{RS}{l} = \frac{\Omega - m^2}{m} = \Omega - m$

→ power $P = \frac{dw}{dt} = \frac{dw}{dq} \cdot \frac{dq}{dt}$

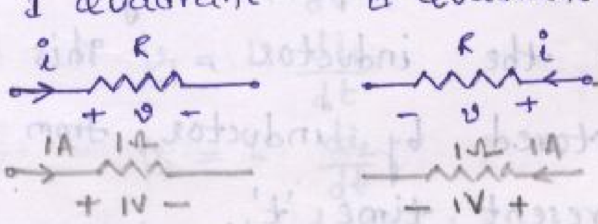
→ $P = i^2R = v^2/R$ (w) $= v.i$ (w)

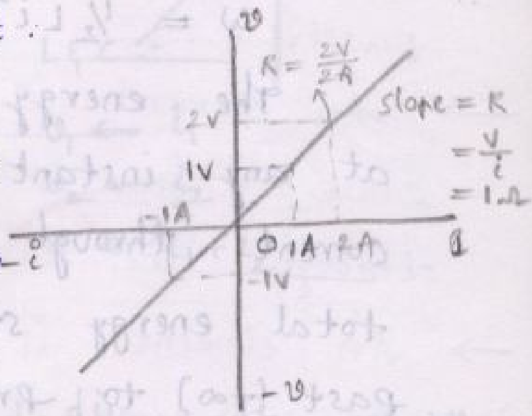
→ Energy $dw = p\,dt \Rightarrow w = \int p\,dt$ (J)

$w = \int i^2R\,dt = \int \frac{v^2}{R}\,dt$.

V- I characteristics:-

I Quadrant III Quadrant

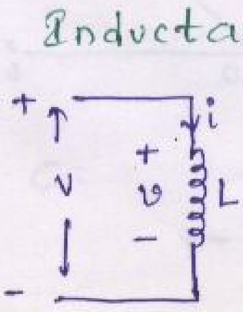




Observations:-

1. Resistor is a linear, passive, bilateral and time invariant in V-I plane.

Inductance L:-



when a time varying i is flowing through the coil, a time varying magnetic flux will be produced. The total flux produced $\phi N = \psi$ (wb)

ϕ - flux per turn, N - no. of turns.

The total flux is proportional to the i through the coil ie $\psi \propto i$

$$\Rightarrow \psi = Li$$

The volt. drop across the coil is $v = \frac{d\psi}{dt}$

$$v = \frac{d}{dt}(Li) = L.\frac{di}{dt}$$

$$i = \frac{1}{L}\int_{-\infty}^{t} v.dt \quad \underset{-\infty \quad 0 \quad t \quad \infty}{\longleftrightarrow}$$

power $p = vi = L.\frac{di}{dt}.i = Li.\frac{di}{dt}$ (ω)

Energy $\omega = \int p\,dt$

$$= \int Li.\left(\frac{di}{dt}\right).dt \quad (J)$$

$$p = Li\frac{di}{dt} = \frac{d}{dt}\left(\tfrac{1}{2}Li^2\right)$$

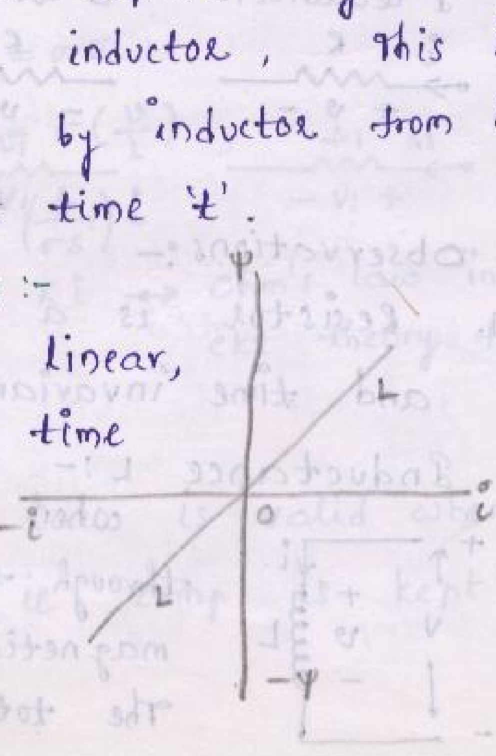
$$\omega = \int \frac{d}{dt}\left(\tfrac{1}{2}Li^2\right)dt$$

$$\omega = \tfrac{1}{2}Li^2 \ (J)$$

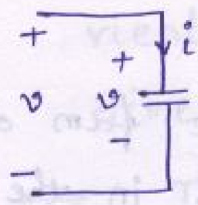
The energy stored in the inductor at any instant will depends only on the current through the inductor, this is total energy stored by inductor from infinite past (-∞) to present time 't'.

$\psi - I$ characteristics :-

→ The inductor is a linear, passive, bilateral, time invariant element in $\psi - i$ plane.



Capacitor C :-



$$i = \frac{dq}{dt}, \quad q \propto v$$

$$q = Cv$$

C - capacitor parameter.

$\rightarrow \quad i = C \cdot \frac{dv}{dt} \qquad i = \frac{d}{dt}(Cv)$

$\rightarrow \quad v = \frac{1}{C} \int_{-\infty}^{t} i\, dt$

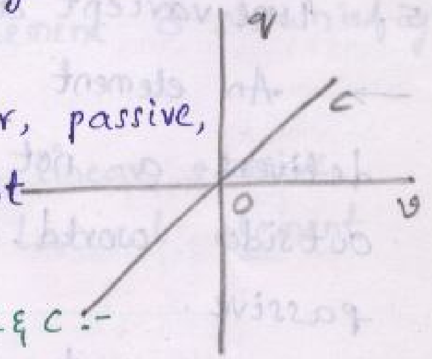
$\rightarrow \quad p = C \cdot v \frac{dv}{dt} = \frac{C}{2} \cdot \frac{dv^2}{dt}$

$\rightarrow \quad \omega = \int \frac{d}{dt}\left(\frac{1}{2}Cv^2\right) dt = \frac{1}{2}Cv^2 \cdot \text{(J)}$

So energy stored in capacitor at any instant depends on voltage at that instant.

q - v characteristics :-

The capacitor is a linear, passive, bilateral, time invariant in q-v plane.



Relation b/w v & i in L & C :-

L : $v = L \cdot \frac{di}{dt}$

$v_1 \leftarrow i_1$

$v_2 \leftarrow i_2$

$v_1 = L \cdot \frac{di_1}{dt}$ $\quad ? \leftarrow i_1 + i_2$

$v_2 = L \cdot \frac{di_2}{dt}$

$\therefore v = L \cdot \frac{d}{dt}(i_1 + i_2) = L \cdot \frac{di_1}{dt} + L \cdot \frac{di_2}{dt} = v_1 + v_2$

So the relation b/w v & i in L is linear

and hence $v = L \cdot \frac{di}{dt} \rightarrow$ 5th form Ohm's law

$i = \frac{1}{L} \int_{-\infty}^{t} v\, dt \rightarrow$ 6th form Ohm's

C : $\quad i = C \cdot \frac{dv}{dt} \rightarrow$ 7th

$v = \frac{1}{C} \int_{-\infty}^{t} i\, dt \rightarrow$ 8th

NOTE :-

① . $W_L = \frac{1}{2} L i^2$ and $i = \int H \cdot dl$

② $W_c = \frac{1}{2} C v^2$ and $v = \int E \cdot dl$

so inductor stores energy in the form of magnetic field and capacitor → in the form of electric field.

Types of Elements :-

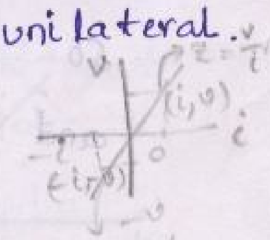
1. Active and passive
2. Linear and Non- linear
3. Bilateral and unilateral
4. Distributed and lumped
5. Time varient and invariant.

→ An element is said to be active if it delivers a net amont of energy to the outside world. otherwise it is said to be passive.

→ An element is said to be linear if its char.s for all time 't', is a st. line, through the origin, otherwise → Non linear

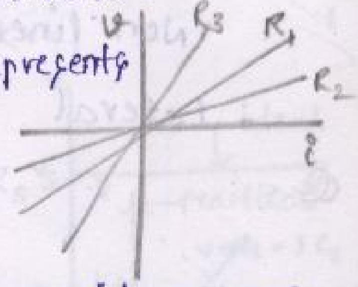
→ An element is said to be bilateral if it offers same impedance for either dire. of i flow, → otherwise → unilateral.

In other words for a bilateral element, if (i, v) is on the char.s then (-i, -v) must also be on the char.s.



→ An element is said to be time invarient if its char.s doesn't change with time otherwise → time varient.

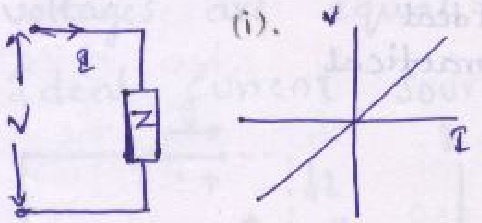
→ The besides char.s also represents passive, linear and bilateral.



NOTE:- The resistors, inductors, capacitors are passive if and only if $R \geq 0$, $L \geq 0$ & $C \geq 0$. Otherwise they are said to be active ie $R < 0$, $L < 0$ & $C < 0$.

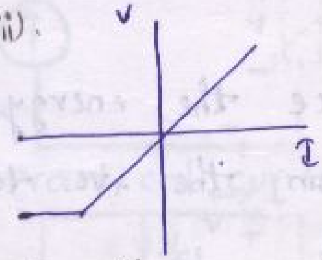
The v-I char.s of an element is shown in fig(b) then the element — ?

(i).



→ linear, passive, bilateral element.

(ii).

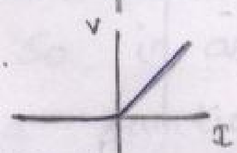


→ Non-linear, passive, unilateral element

(iii)

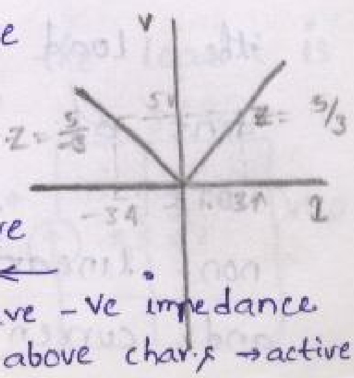
→ Non-linear, passive, Bi-lateral element

(iv).



→ Non-linear, passive unilateral.

(v).

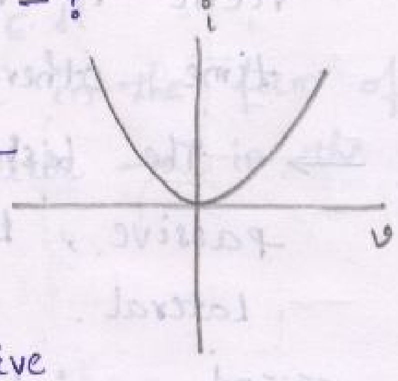


Non linear, active unilateral ←

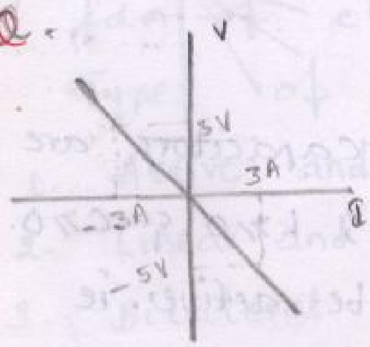
NOTE:- No passive element will have -ve impedance in any portion of its char.s. So above char.s → active

Q. The voltage-current relations in a resistor $i = z v^2$ then that element — ?

Non linear, active, uni-lateral ←



Q.



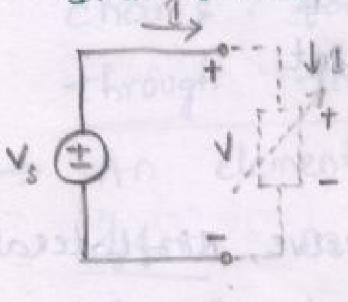
→ linear, active, bi-lateral $(i, v) = (-3, 5)$
$= (3, -5)$.

Obs:- All the linear elements are always bi-lateral and converse need not be true.

SOURCES:-

- independent
  - V
    - Ideal
    - practical
  - I
    - Ideal
    - practical
- dependent
  - VCVS
  - VCCS
  - CCVS
  - CCCS

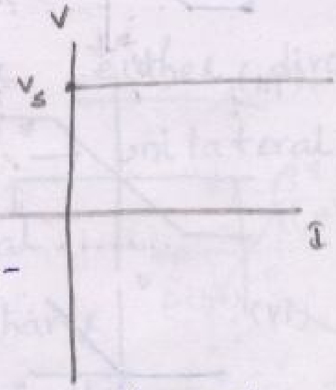
Ind. Ideal voltage sources:-



from any source the energy delivery is from the +ve terminal.

Source voltage $= V_s$

$V = V_s$ for all 'I'.

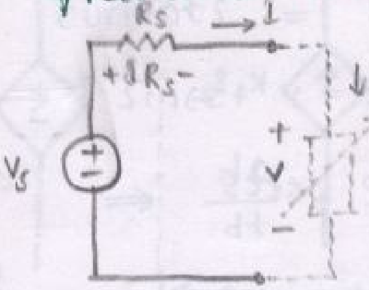


So in an ideal voltage source, the load voltage is independent of load i drawn.

NOTE :- All the sources are inherently non-linear in nature, since the voltage and current relation is non-linear.

They are basically active and unilateral elements.

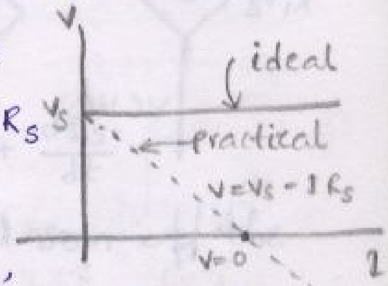
practical voltage source :-



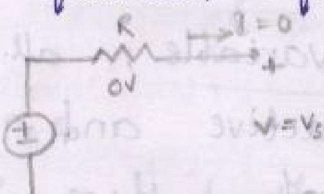
By writing KVL,

$V_s - I R_s - V = 0$

$\Rightarrow V = V_s - I R_s$



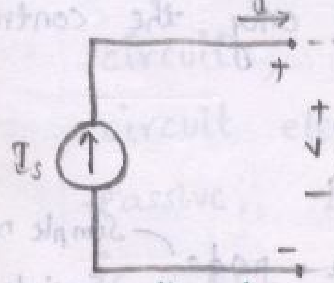
In a practical volt. source, the load voltage is a function of load i drawn.

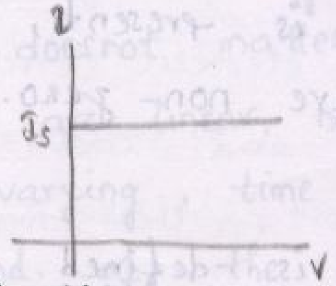


→ when $I = 0$, $V = V_s$.

ie when the i through any passive element is zero, then the two end voltages are equal. and viceversa.

Ideal current source :-

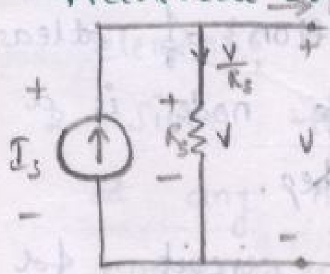


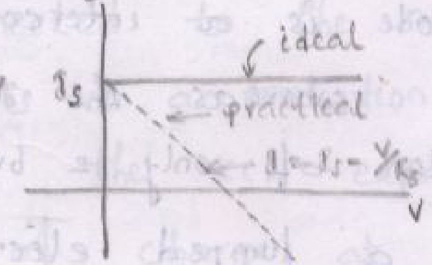


$I = I_s$ for all V.

In an ideal i source, the load V is ind. of i.

practical current source :-

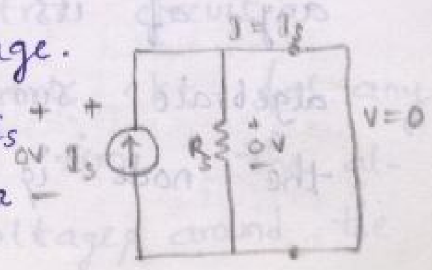




$-I_s + \frac{V}{R_s} + I = 0$

$\Rightarrow I = I_s - \frac{V}{R_s}$

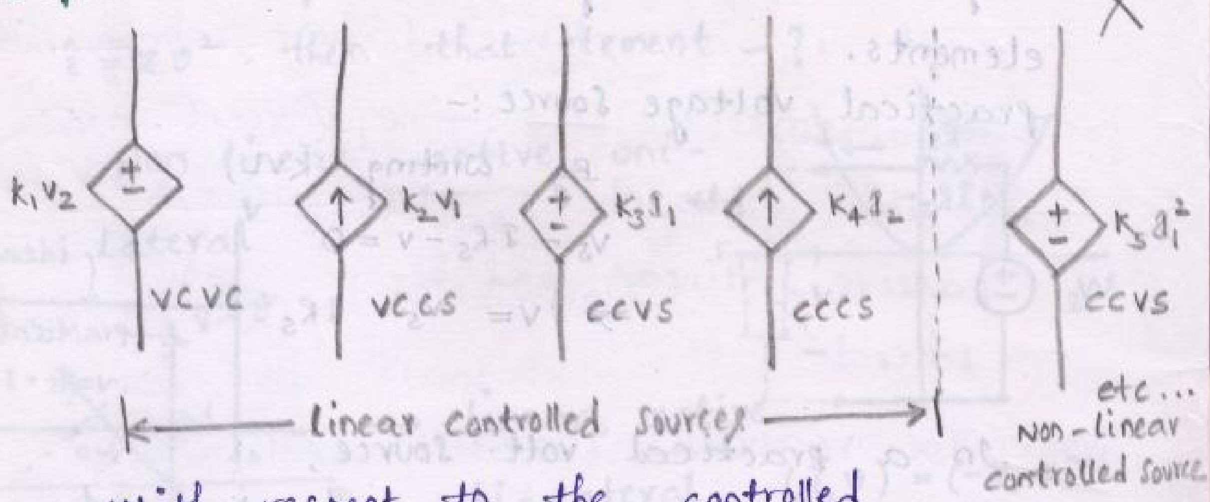
So in a practical cs, the load i is a function of load voltage.



→ when $V = 0$ then $I = I_s$

ie the current always chooses a min. resistance path.

Dependent or controlled sources:-



With respect to the controlled variable, all controlled sources are linear, active and bi-lateral elements. The presence of these elements makes the n/w a linear active and bi lateral.

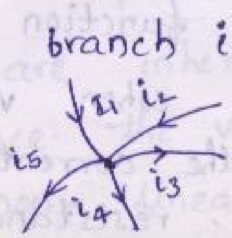
The controlled sources are said to be active elements ie the sources only when atleast one ind. source is present, then only the controlled var.s are non-zero.

K-Laws:-

1. KCL:- It is defined at a node: – Simple node / – principle node

principle node is a interconnection of atleast 3 branches, whereas the simple node is a interconnection of only 2 branches.

In a lumped electric circuit, for any of its nodes and at time 't', the algebraic sum of all the branch i's leaving the node is zero.



$$-i_1 - i_2 + i_3 + i_4 + i_5 = 0$$

$\Rightarrow i_1 + i_2 = i_3 + i_4 + i_5$ ie Sum of entering currents = sum of the leaving currents.

→ Since $i = \frac{dQ}{dt}$

$$\Rightarrow \frac{dQ_1}{dt} + \frac{dQ_2}{dt} = \frac{dQ_3}{dt} + \frac{dQ_4}{dt} + \frac{dQ_5}{dt}$$

$\Rightarrow Q_1 + Q_2 = Q_3 + Q_4 + Q_5$ ie sum of the entering charges = Sum of the leaving charges.

→ since $q = ne$,

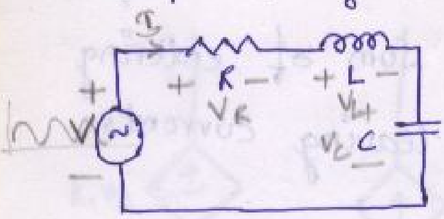
$$n_1 e + n_2 e = n_3 e + n_4 e + n_5 e$$

$\Rightarrow n_1 + n_2 = n_3 + n_4 + n_5$ ie sum of the entering e^-s = sum of the leaving e^-s.

Features:-

1. The KCL applies to any lumped electric circuit, it doesnot matter, whether the circuit elements are linear, non-linear, active, passive, time varying, time invarient etc. ie KCL is ind. of the nature of the elements connected to the node.
2. Since there is no accumulation of a charge at any node, the KCL expresses the conservasion of charge at each and every node in a lumped electric circuit.

KVL :- In a lumped electric ckt for any of its loops at any of time, the algebraic sum of branch voltages around the loop is zero. $V - V_R - V_L - V_C = 0$



$\Rightarrow V = V_R + V_L + V_C$

$\Rightarrow$ since $V = \frac{\omega}{q} = \frac{\omega_R}{q} + \frac{\omega_L}{q} + \frac{\omega_C}{q}$

$\Rightarrow \omega = \omega_R + \omega_L + \omega_C$

features :-

1. The KVL is ind. of the nature of the elements, present in a loop.
2. KVL expresses the conservation of energy in a every loop of a lumped electric ckt.

→ KCL + Ohm's law = Nodal Analysis

KVL + Ohm's law = Mesh Analysis

since KCL & KVL are ind. each other, the nodal & mesh procedures are ind. to each other.

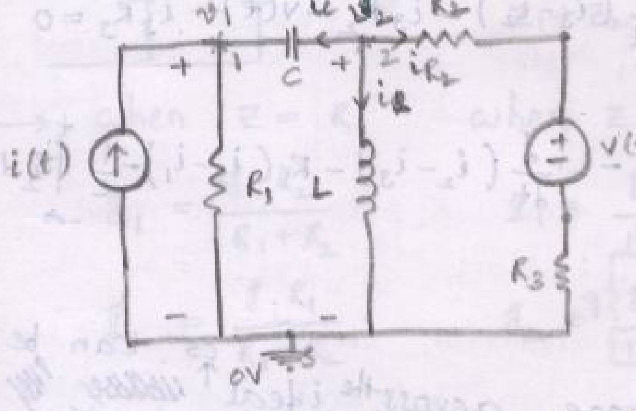
→ The above techniques are valid only for the lumped electric circuits, [where KCL, KVL are valid] and that to at a constant temp. [where the ohm's law is valid].

→ The K- laws are ind. of the nature of the elements, where as ohm's is a function of the nature of elements.

The ohm's law is defined across an element that element can be lumped or distributed $J = \sigma E$, where as the K- laws are applicable to only for the lumped electric circuits.

The ohm's law is not applicable for active elements like sources, since the V-I relation is non-linear and it is applicable to only for the linear passive elements like R, L, C.

Nodal Analysis :-



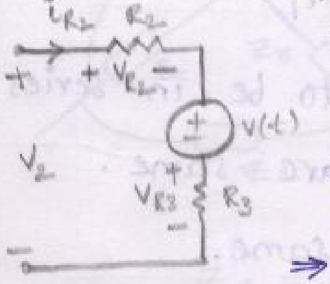
step 1 :-

1. Identify the no. of nodes.
2. Assign the node voltages with reference to ground node, whose voltage always = 0.
3. By using KCL first & ohm's next write nodal equations.

At Node 2 ; $\begin{bmatrix} V_2 > V_1 \\ V_2 > 0 \\ V_2 > v(t) \end{bmatrix}$ $i_c + i_L + i_{R_2} = 0$ (By KCL)

$$C \cdot \frac{d(V_2 - V_1)}{dt} + \frac{1}{L}\int_{-\infty}^{t} v_2 \, dt + \frac{v_2 - v(t)}{R_2 + R_3} = 0$$

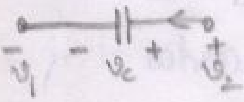
(By ohm's)



$V_2 - V_{R_2} - v(t) - V_{R_3} = 0$

$V_2 - i_{R_2}R_2 - v(t) - i_{R_2}R_3 = 0$

$$\Rightarrow i_{R_2} = \frac{v_2 - v(t)}{R_2 + R_3}$$



$v_2 - v_c - v_1 = 0$

$\Rightarrow v_2 = v_1 + v_c \Rightarrow v_c = v_2 - v_1$; $i_c = C\frac{dv_c}{dt} = C\cdot\frac{d}{dt}(v_2 - v_1)$

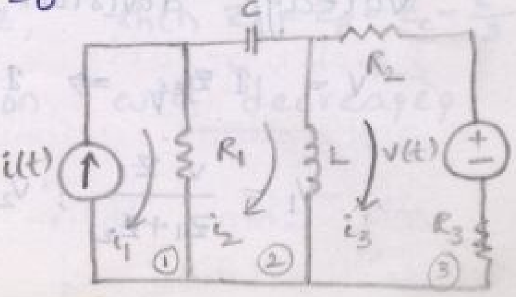
At Node 1 :- $\begin{bmatrix} v_1 > v_2 \\ v_1 > 0 \end{bmatrix}$

$$-i(t) + \frac{v_1}{R_1} + C\cdot\frac{d}{dt}(v_1 - v_2) = 0$$

Mesh Analysis :-

Steps :-

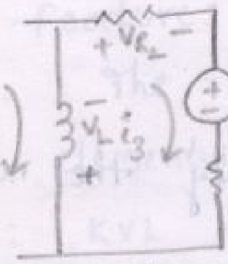
1. Identify the no. of meshes.



2. Assign mesh i's in clockwise

3. By using KVL first and ohm's law next write the mesh equations.

Mesh 3 :- $\begin{pmatrix} i_3 > i_2 \\ i_3 > i_1 \end{pmatrix}$

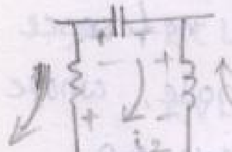


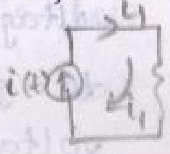
$$-V_L - V_{R_2} - v(t) - V_{R_3} = 0$$

$$-L \cdot \frac{d}{dt}(i_3 - i_2) - i_3 R_2 - v(t) - i_3 R_3 = 0$$

Mesh 2 :- $(i_2 > i_3)$ $(i_2 > i_1)$

$$-L \cdot \frac{d}{dt}(i_2 - i_3) - R_1(i_2 - i_1) - \frac{1}{C}\int_{-\infty}^{t} i_2 \, dt = 0$$





Since the voltage across the ideal cs (↑) can be any value, it is not possible to write the mesh eq. for mesh 1.

KCL :-
$$-i(t) + i_1 = 0$$
$$\Rightarrow i(t) = i_1$$

Equivivalent circuits :-

→ when 2 elements are said to be in series only the i through them are same.

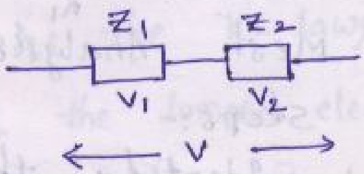
for $\parallel^{el}$ → voltages are same.

→ The impedances in series and admittances in $\parallel^{el}$ then we can add them.

$$Z_L = j\omega L \ \Omega \ ; \ Z_c = \frac{1}{j\omega C} \ \Omega$$

Voltage division principle :-

$$V = I Z_{eq} \Rightarrow I = \frac{V}{Z_{eq}}$$



$$\therefore V_1 = \frac{V \cdot Z_1}{Z_1 + Z_2} \ ; \ V_2 = \frac{V \cdot Z_2}{Z_1 + Z_2}$$

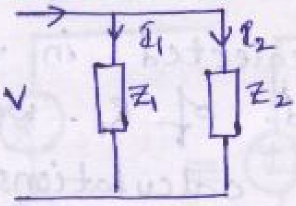
$Z_{eq} = Z_1 + Z_2$

→ when $Z = R$, when $Z = j\omega L$ when $Z = \frac{1}{j\omega C}$

$V_1 = \frac{V \cdot R_1}{R_1 + R_2}$ $V_1 = \frac{V \cdot L_1}{L_1 + L_2}$ $V_1 = \frac{V C_2}{C_1 + C_2}$

$V_2 = \frac{V R_2}{R_1 + R_2}$ $V_2 = \frac{V L_2}{L_1 + L_2}$ $V_2 = \frac{V C_1}{C_1 + C_2}$

CURRENT DIVISION :-



when taken as $Z_{eq} = \frac{Z_1 \cdot Z_2}{Z_1 + Z_2}$

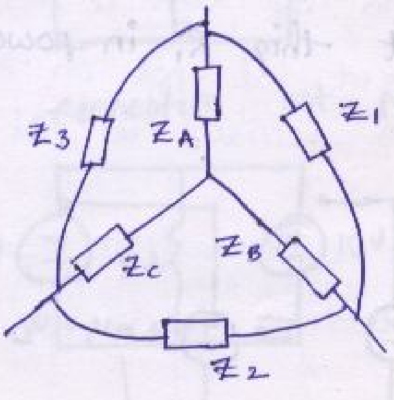
$I_1 = \frac{I \cdot Z_2}{Z_1 + Z_2}$; $I_2 = \frac{I \cdot Z_1}{Z_1 + Z_2}$

→ when $Z = R$, when $Z = j\omega L$ when $Z = \frac{1}{j\omega C}$

$I_1 = \frac{I \cdot R_2}{R_1 + R_2}$ $I_1 = \frac{I \cdot L_2}{L_1 + L_2}$ $I_1 = \frac{I \cdot C_1}{C_1 + C_2}$

$I_2 = \frac{I \cdot R_1}{R_1 + R_2}$ $I_2 = \frac{I \cdot L_1}{L_1 + L_2}$ $I_2 = \frac{I \cdot C_2}{C_1 + C_2}$.

Y- Δ conversions:-



$Z_1 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_C} = Z_B + Z_A + \frac{Z_A Z_B}{Z_C}$

$Z_2 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_A} \to Z_B + Z_C + \frac{Z_B Z_C}{Z_A}$

$Z_3 = \frac{Z_A Z_B + Z_B Z_C + Z_C Z_A}{Z_B}$. $\downarrow$ $Z_A + Z_C + \frac{Z_A Z_C}{Z_B}$

$Z_A = \frac{Z_1 Z_3}{Z_1 + Z_2 + Z_3}$

$Z_B = \frac{Z_1 Z_2}{Z_1 + Z_2 + Z_3} = \frac{Z^2}{3Z}$; $Z_C = \frac{Z_2 Z_3}{Z_1 + Z_2 + Z_3}$.

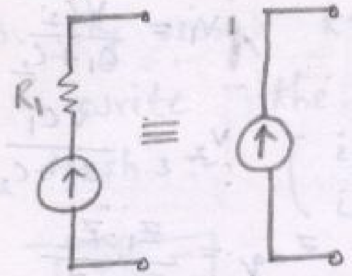
when $Z_A = Z_B = Z_C = Z$ then $Z_1 = Z_2 = Z_3 = 3Z$

ie Y-Δ transformation will increase the impedance by 3 times .

when $Z_1 = Z_2 = Z_3 = Z$, then $Z_A = Z_B = Z_C = \frac{Z}{3}$

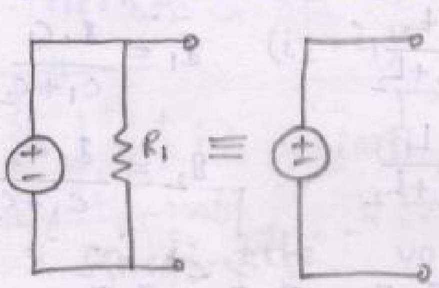
ie Δ-Y transformation will decreases the impedance by 3 times .

Equivalent circuits w.r.t. source point of view:-



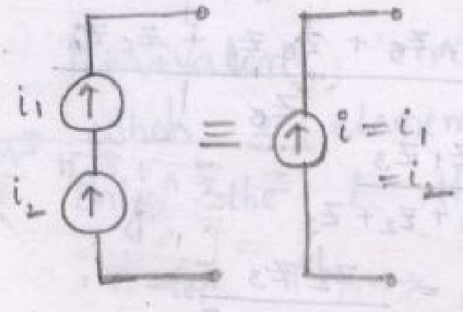
Here $R_1 \neq \infty$, since the violation of KCL.

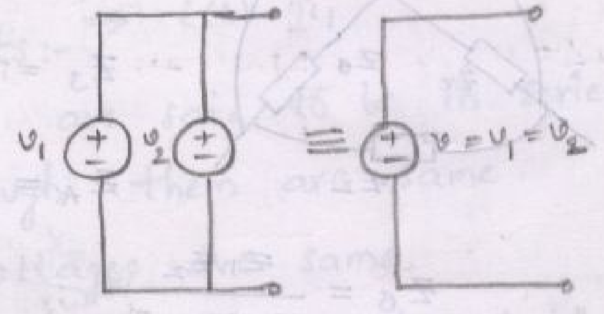
A resistor in series with an ideal CS, is neglected in the analysis. ie the load i ind. of R_1. We can't omit this R_1 in power calculations, since I^2R_1 is $\neq 0$.



Here $R_1 \neq 0$, since the violation of KVL.

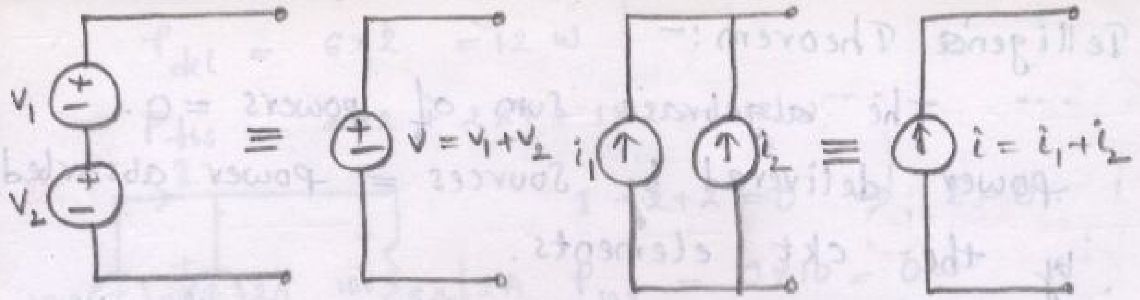
A resistor in $\parallel^{el}$ with an ideal VS can be neglected in the analysis ie the load volt. is ind. of R_1. We can't omit this R_1 in power calculations, since $V^2/R_1 \neq 0$.

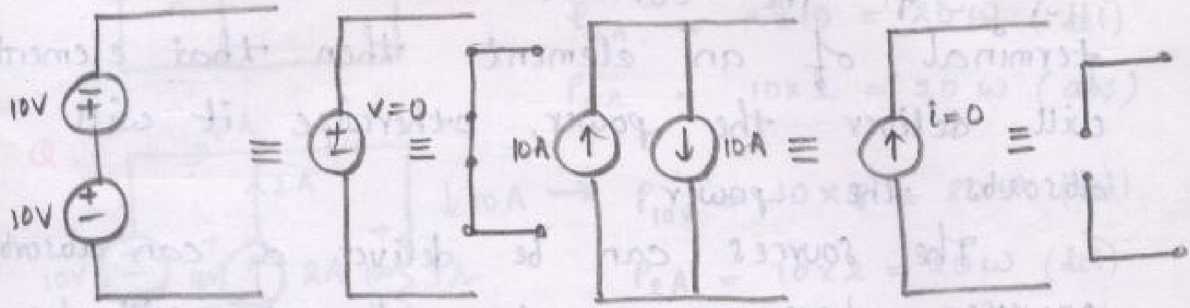




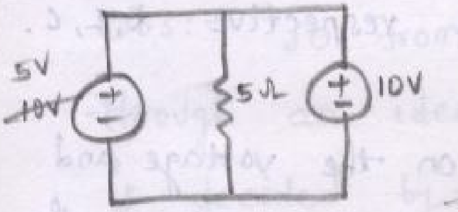
$-i_2 + i_1 = 0 \Rightarrow i_1 = i_2$ $\qquad$ $v_1 - v_2 = 0 \Rightarrow v_1 = v_2$

Two ideal CS are connected in series only when their magnitudes are equal, otherwise the violation of KCL, which results the unstability due to oscillations. Similarly 2 ideal VS are in $\parallel^{el}$ only when their magnitudes are equal, otherwise the violation of KVL.

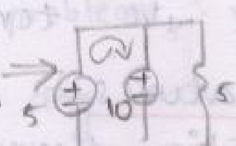




Q. Determine, the current through 5Ω.

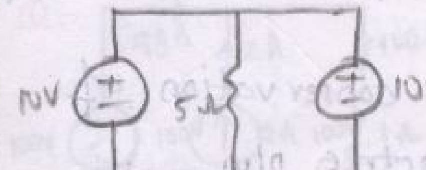


$\rightarrow$ ~~$i = \frac{10}{5} = 2A$~~ $\rightarrow$

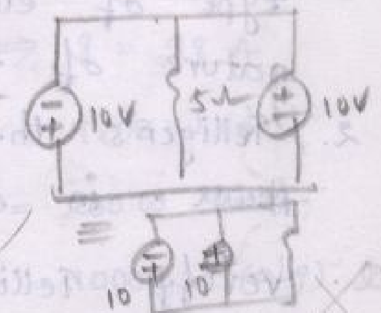


~~5 − 10 = 0~~
~~−5 = 0~~

the violation in n/w and hence physical connection not possible.



$\Rightarrow i = \frac{10}{5} = 2A$

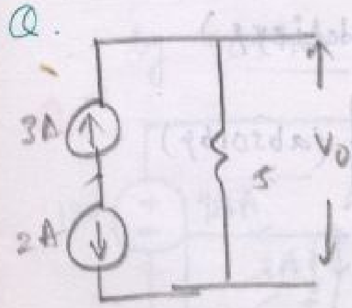


~~−10 − 10 = 0~~

By superposition theorem, n/w is not physical connected.

Not possible the physical connection.

Q.



→ not possible

~~$2 + 3 = 0$~~

violation of KCL, n/w doesn't exist.

Telligen's Theorem:-

The algebraic sum of powers = 0.

power delivered by Sources = power absorbed by the ckt elements.

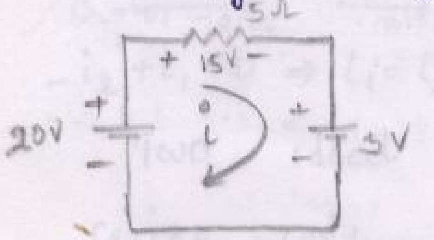
If the current enters at the -ve terminal of an element then that element will deliver the power, otherwise it will absorbs the power.

The sources can be deliver or can absorbs powers where as passive elements will always absorbs power., since the i will enter at the +ve terminal in the respective R, L, C.

Features:-

1. This theorem depends only on the voltage and current product in an element but not on the type of element [active or passive] ie ind. of nature of the element.
2. Telligen's th. expresses the conservation of power. in every lumped electric n/w.

Q. verify Telligen's Th.



$$20 - 5i - 5 = 0$$

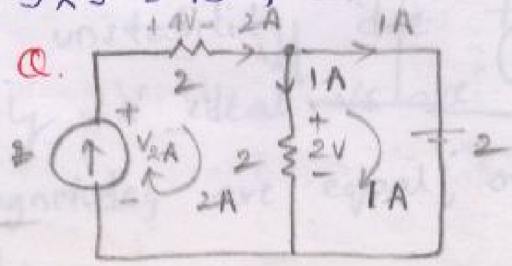
$$\Rightarrow i = \frac{20-5}{5} = 3A$$

$P_{20V} = 20 \times 3 = 60$ (delivers)

$P_R = 15 \times 3 = 45$
$P_{5V} = 5 \times 3 = 15$ } (absorbs)

$\therefore$ $P_{del} = P_{abs}$.

Q.



Sol:

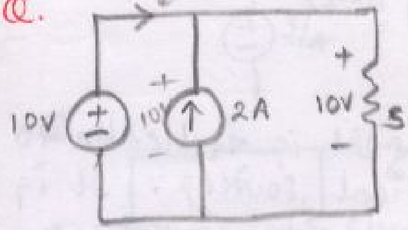
$V_{2A} - 4 - 2 = 0$

$\Rightarrow V_{2A} = 6V$

$P_{del} = 6 \times 2 = 12\ W$

$P_{Abs} = 4 \times 2 + 2 \times 1 + 1 \times 2 = 12\ W$

Q.



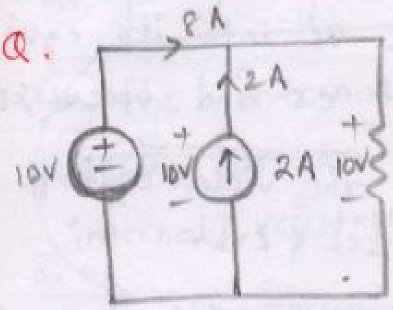
$I - 2 + 2 = 0 \Rightarrow I = 0.$

$P_{10V} = 0 \times 10 = 0\ W$

$P_{2A} = 2 \times 10 = 20\ W$ (del)

$P_{5\Omega} = 10 \times 2 = 20\ W$ (abs)

Q.



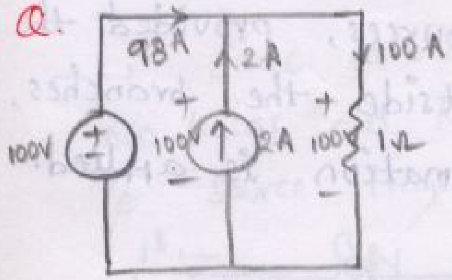
$\downarrow 10A \rightarrow$ $P_{10V} = 10 \times 8 = 80\ W$ (del)

$P_{2A} = 10 \times 2 = 20\ W$ (del)

$P_{1\Omega} = 10 \times 10 = 100\ W$ (Abs)

Obs:- so from the above 2 problems, the i through an ideal VS can be any value, it is decided by the other elements magnitudes present in the n/w.

Q.



$-I - 2 + 100 = 0 \Rightarrow I = 98\ A$

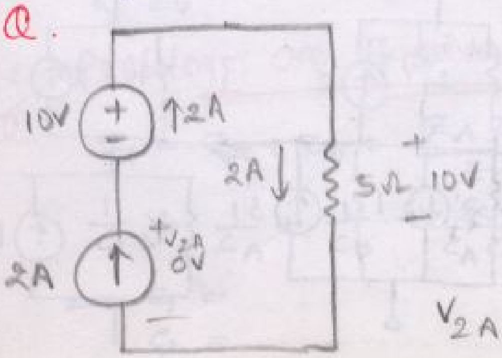
$P_{100V} = 100 \times 98 = 9800\ W$ (del)

$P_{2A} = 100 \times 2 = 200\ W$ (del)

$P_{1\Omega} = 100 \times 100 = 10000\ W$ (abs)

→ from the above problems, the voltage across CS can be any value, it is decided by other element present in the n/w.

Q.



$P_{2A} = 2 \times 0 = 0\ W$

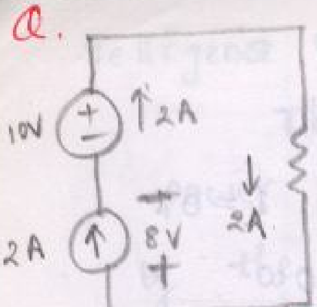
$P_{10V} = 10 \times 2 = 20\ W$ (del)

$P_{5\Omega} = 2 \times 10 = 20\ W$ (abs)

$V_{2A} + 10 - 10 = 0$

$\Rightarrow V_{2A} = 0\ V$

Q.

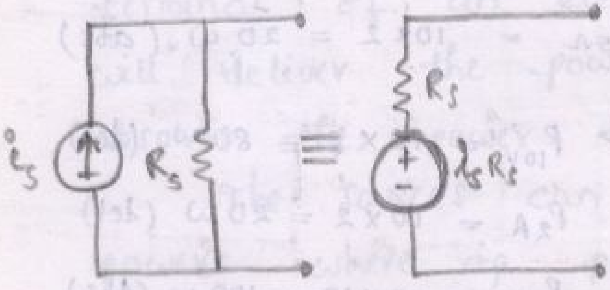


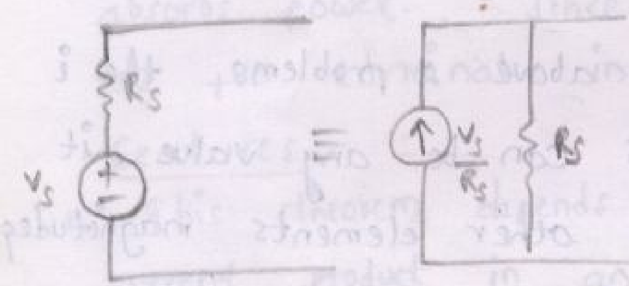
$\longrightarrow P_{10V} = 20W$ (del)

$P_{2A} = 16W$ } (ab)
$P_{1\Omega} = 4W$

Source Transformation :-

$\longrightarrow$ It is applicable to practical sources. It is impossible to convert an ideal vs into its equivalent cs and viceversa, since the violations of KCL & KVL.

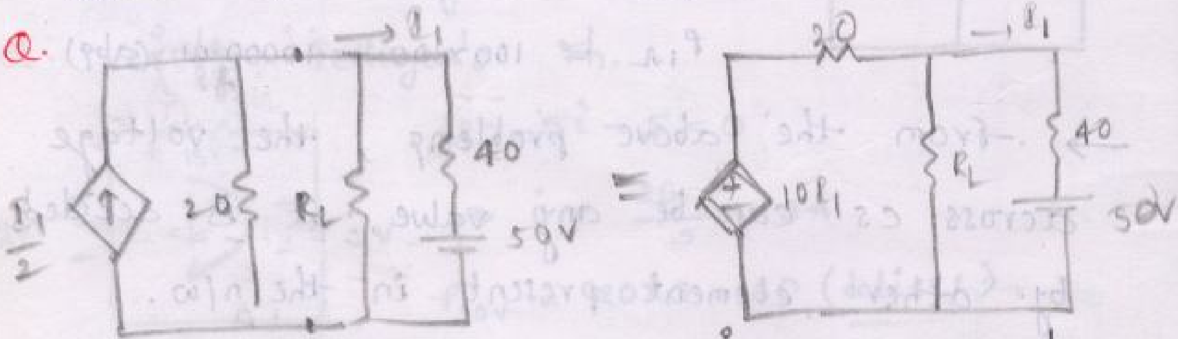




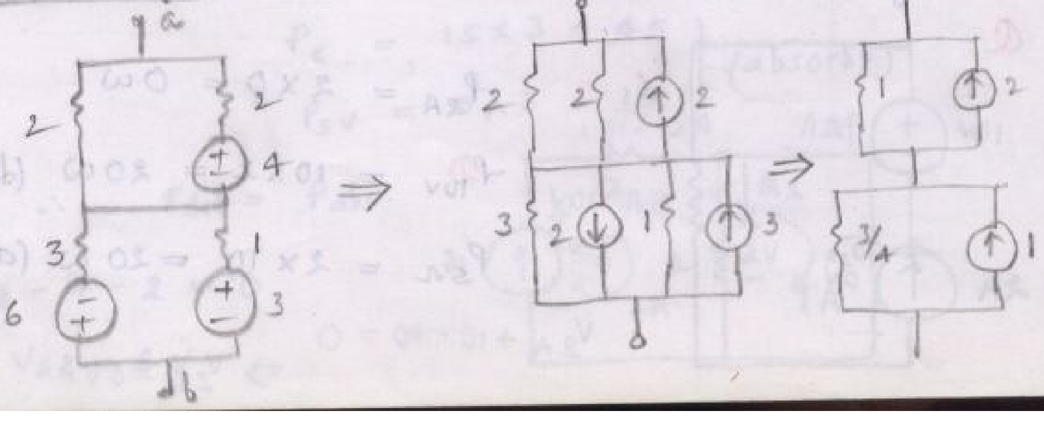
The above ckts are equal only w.r.t. the performance point of view. But the elements in the connection point of view they are not equal.

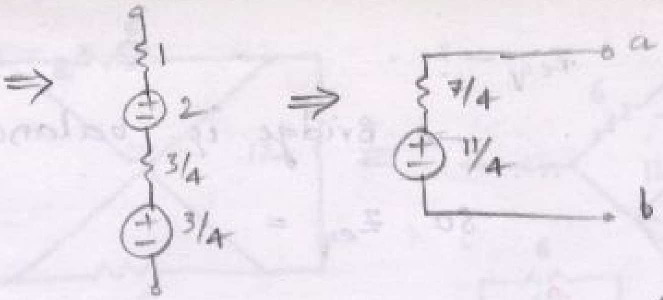
$\longrightarrow$ The source transformation is applicable even for the dependent sources, provided the controlled variable is outside the branches, where the source transformation is applied.

Q.

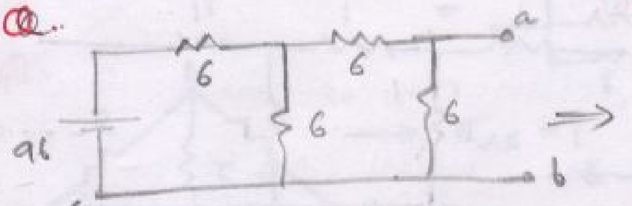


Q..



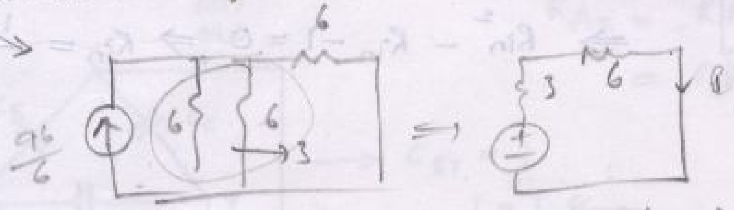


Q.

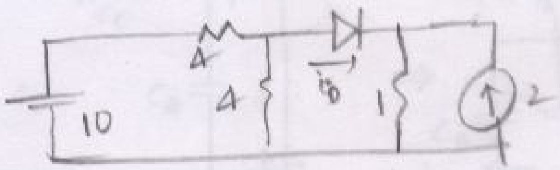


An ideal ammeter is connected across ab, then the reading of the ammeter is ?

→ The internal resistance of an ideal ammeter is zero, for an ideal voltmeter is infinite.
internal resistance for ideal CS → ∞, for ideal VS → 0

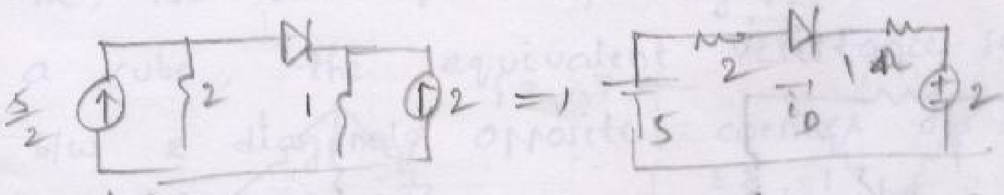


Q.



→ $i_D = ?$

→ Since diode is a non-linear element, n/w is non linear and hence superposition is not applicable ie source transfer method is applicable



ideal D
$V_f = 0$
$R_f = 0$

$5 - 2i - 1i - 2 = 0$
$3 - 3i = 0$
$\Rightarrow i = 1A$

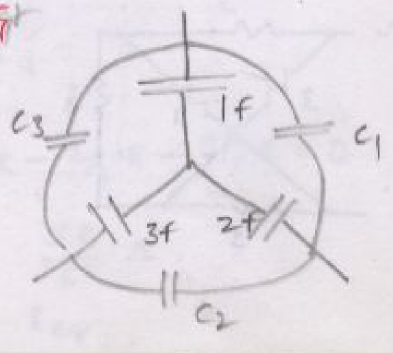
Problems on Equivalent circuits

Q.

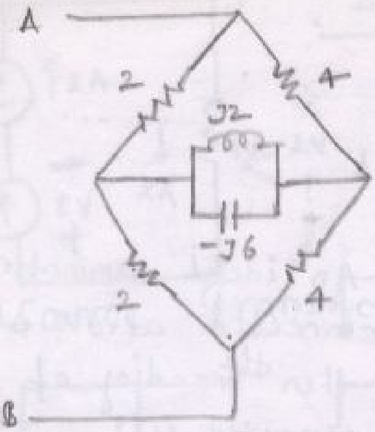
$$Z_1 = Z_A + Z_B + \frac{Z_A Z_B}{Z_C}$$

$$\frac{1}{C_1} = \frac{1}{C_A} + \frac{1}{C_B} + \frac{C_C}{C_A C_B}$$

$\Rightarrow C_1 =$
$C_2 =$
$C_3 =$



Q.

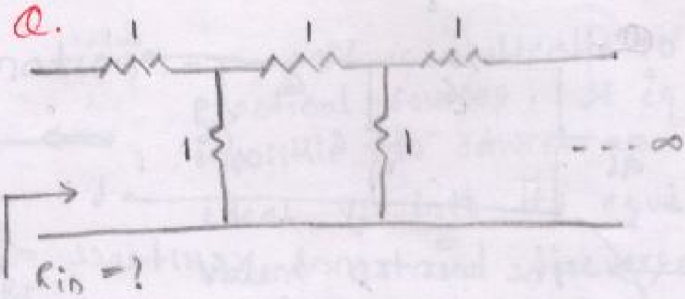


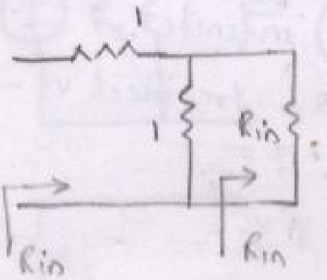
Z_{eq} = ? $Z_1 Z_3 = Z_2 Z_4$

→ Bridge is balanced,

so Z_{eq} =

Q.

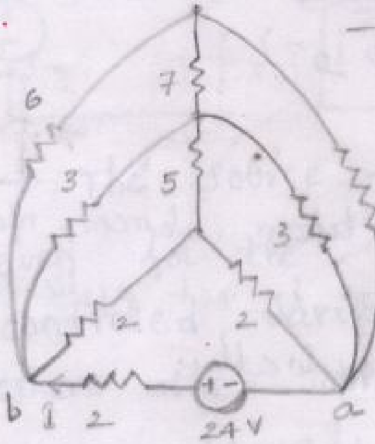




$$R_{in} = 1 + \frac{R_{in}}{1 + R_{in}}$$

$$\Rightarrow R_{in}^2 - R_{in} - 1 = 0 \Rightarrow R_{in} = \frac{1+\sqrt{5}}{2}$$

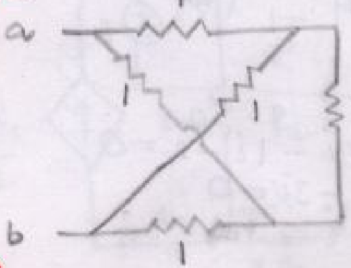
Q.



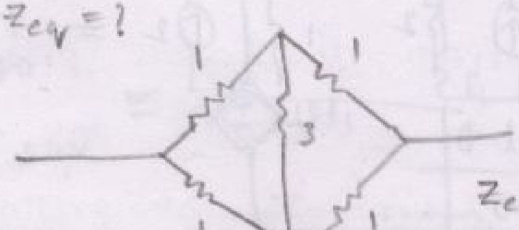
→ I = ?

$$\rightarrow I = \frac{24}{2 + 4 \| 6 \| 12} = 6A$$

Q.

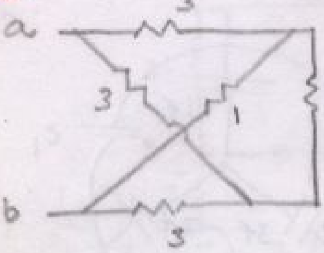


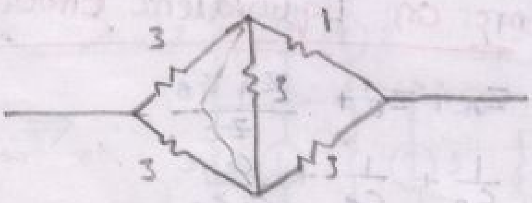
→ Z_{eq} = ?



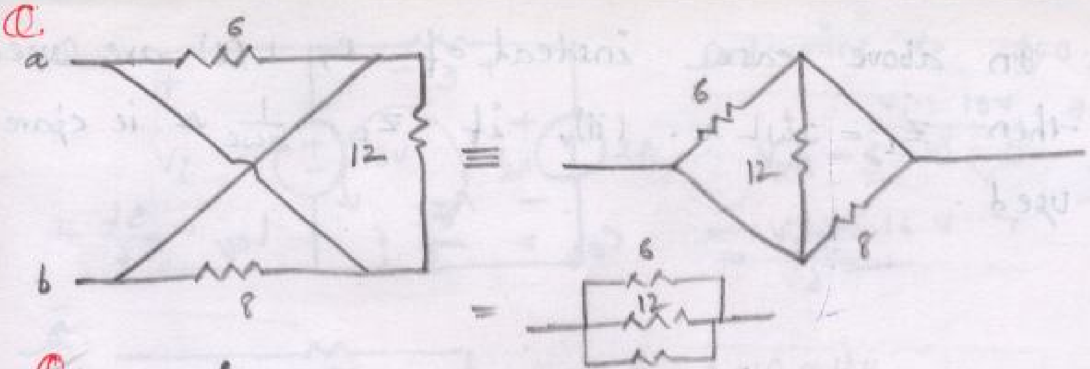
$Z_{eq} = 1\Omega$

Q.

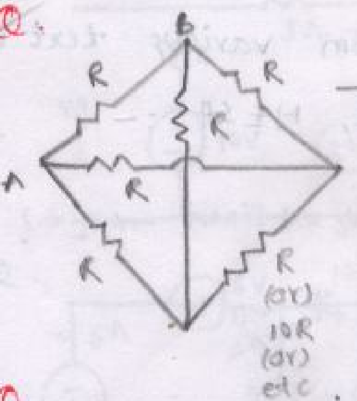




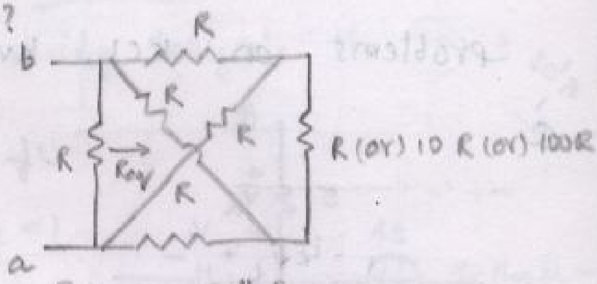
Q.



Q.



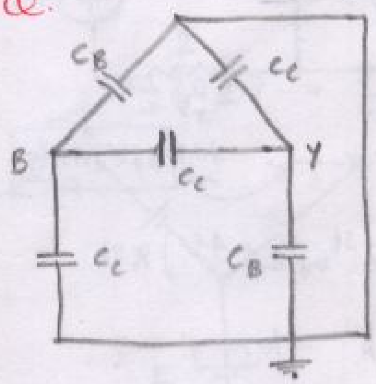
$\rightarrow R_{AB} = ?$



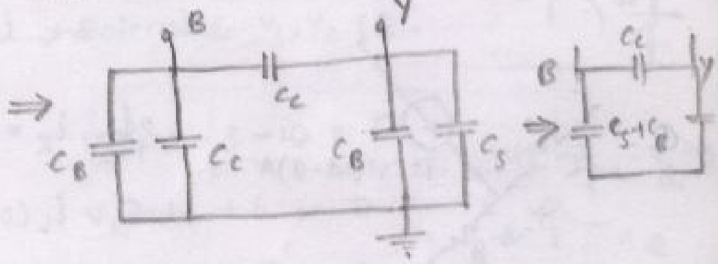
$R_{AB} = R \parallel R_{eq}$

$= R \parallel R = R/2$.

Q.

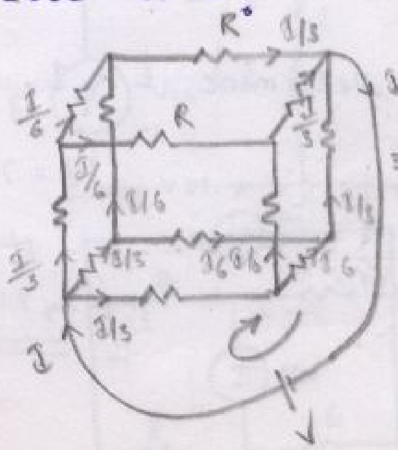


$\rightarrow C_{BY} = ?$



$$\Rightarrow C_{BY} = \frac{C_B + C_c}{2} + C_c$$

Q. 12, 1Ω are used as edges to form a cube, the equivalent resistance seen b/w 2 diagonals opposite corners of the cube is —?



$R_{eq} = \frac{V}{I}$

$V - \frac{I}{3}R - \frac{I}{6}R - \frac{I}{3}R = 0$

$\Rightarrow V/I = \frac{5R}{6}\ \Omega$.

$= R_{eq}$

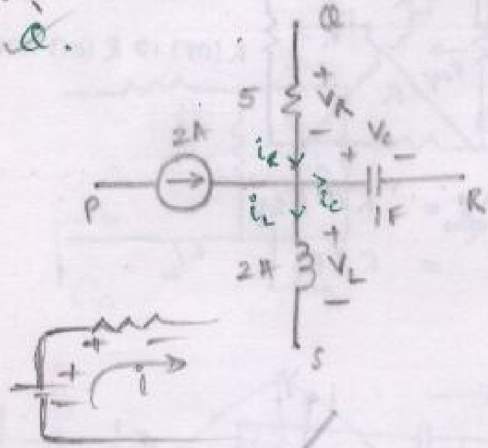
Q. In above case instead of R, L(H) are used. then $Z_L = jωL\ Ω$. (ii). if $Z_L = \frac{1}{jωC}\ Ω$ ie C's are used.

$\frac{5Z_c}{6}$ ⟹ $C_{eq} = \frac{6C}{5}\ F$ $L_{eq} = \frac{5L}{6}\ H$

→ collect all the problems from various text books.

G'03 problems on KCL, KVL, NODAL, MESH :-

Q.



If $V_R = 5V$, $V_c = 4\sin ωt$ → $V_L = ?$

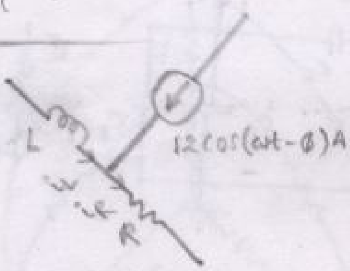
$-2 - i_R + i_c + i_L = 0$

$i_R = \frac{V_R}{R} = 1A$

$i_C = \frac{C\,dv_c}{dt} = 8\cos 2t$

⟹ i_L = ——. Ans $32\sin 2t$

Q.



If $i_R = (4e^{-3t} + 3e^{-4t})\ A$

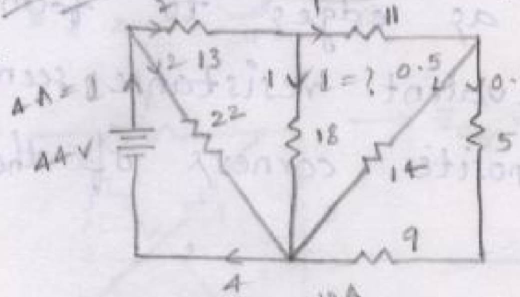
& $i_L(0) = 1A$ then $φ = ?$

$-i_L - 12\cos(ωt - φ) + i_R = 0$

At t=0 ⟹ $-i_L(0) - 12\cos(-φ) + i_R(0) = 0$

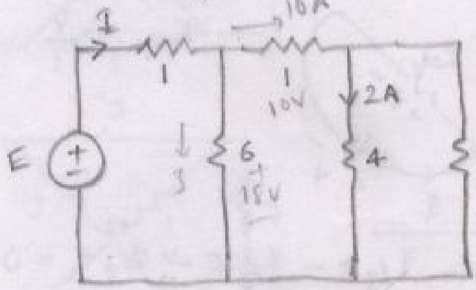
Ans $φ = 60°$

Q.



→ $i_{18Ω} = ?$

Q.

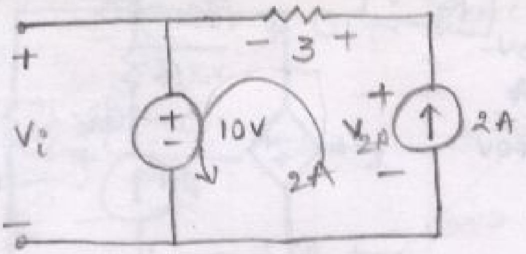


→ Determine E & I.

8V + 10V = 18V   I = 3A

E = 18 + 3V = 21 V

Q.



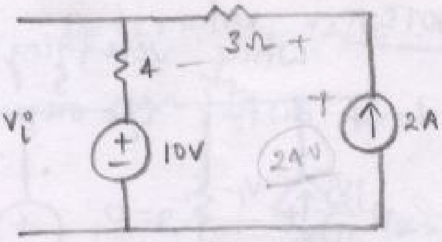
Determine V_i, also verify Telligen's theorem.

$V_i = 10V$

$V_{2A} - 6 - 10 = 0$

$\Rightarrow V_{2A} = 16\ V$

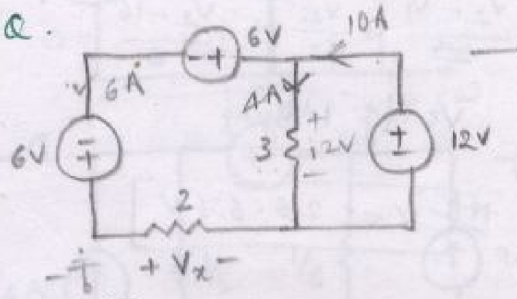
Q.



$V_i = 8 + 10 = 18V$

verify Telligen's th.

Q.

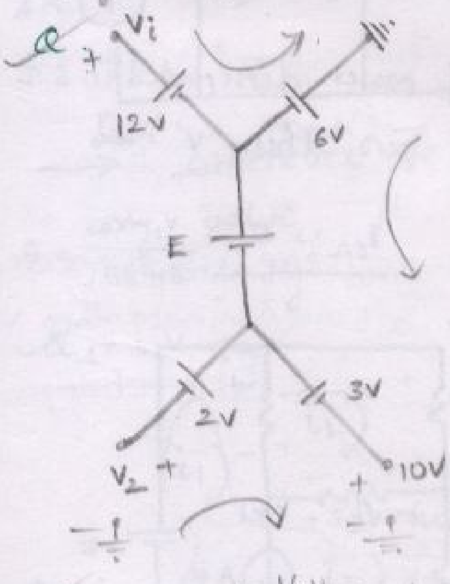


$\rightarrow V_x = ?$

$V_x - 6 + 6 - 12 = 0$ verify Telligen

$V_x = 12\ V$

Q



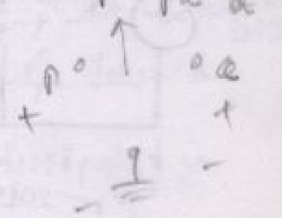
$\rightarrow$ Determine V_1, V_2 & E.

$V_2 + 2 - 3 - 10 = 0$

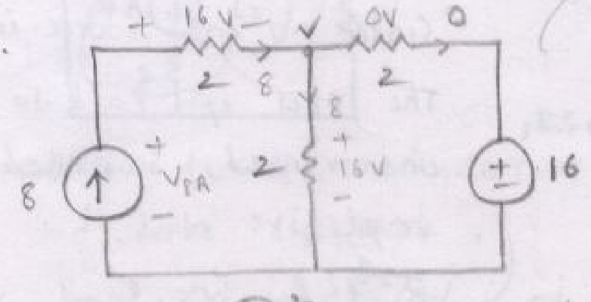
$V_1 - 12 + 6 = 0$

$-10 - 6 - E - 3 = 0$

$V_P - V_{PQ} - V_Q = 0$



Q.



$V_{8A} - 16 - 16 = 0$

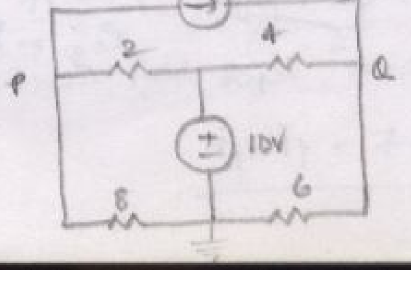
$-8 + \frac{V}{2} + \frac{V - 16}{2} = 0$

$V = 16$

$I = V/_2 = 8A$

G'03

Q.



$\rightarrow V_{PQ} = ?$

$\frac{V_Q - 0}{6} + \frac{V_Q - 10}{4} - 2 = 0$

$\Rightarrow V_Q = 10.8V$

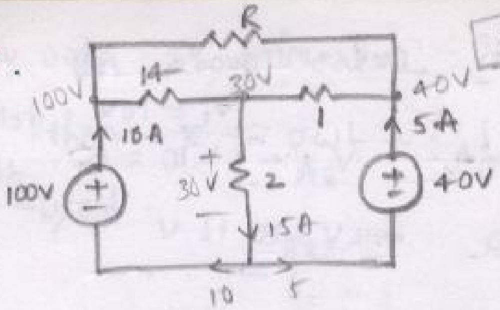
$\frac{V_P - 0}{8} + \frac{V_P - 10}{2} + 2 = 0$

$\rightarrow V_P = 4.8V$

$V_{PQ} = V_P - V_Q = -6V$

Q'03

Q.

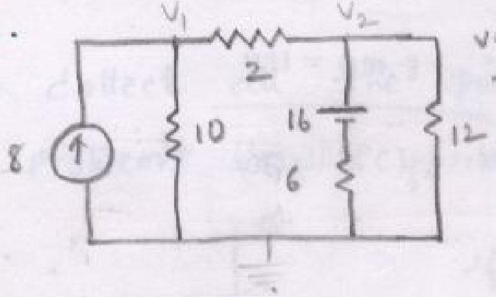


$\boxed{R = ?}$

$$-10 + \frac{100-40}{R} + \frac{100-30}{14} = 0$$

$\rightarrow R = \_\_\_.$

Q.

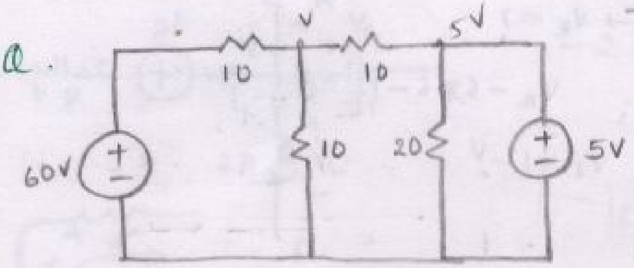


verify Tellig. **NOTE:** while verifying Telligen don't disturb the original n/w

$$-8 + \frac{V_1}{10} + \frac{V_1 - V_2}{2} = 0$$

$$\frac{V_2 - V_1}{2} + \frac{V_2}{12} + \frac{V_2 - 16}{6} = 0$$

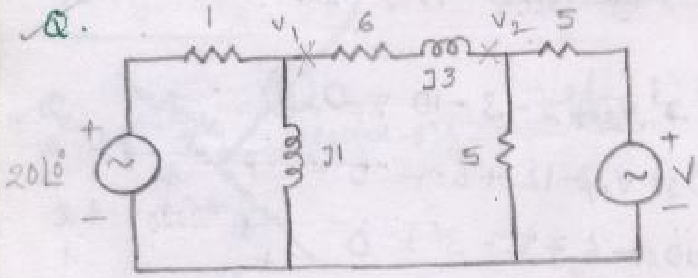
Q.



verify Telligens

$\boxed{V = 26.67\,V}$

Q.



If power dissipated in 6Ω is zero then 'V' = ?

$I_{6\Omega} = 0, \quad \frac{V_1 - V_2}{6 + J3} = 0$

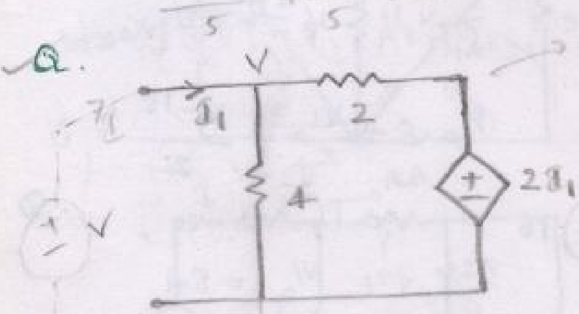
$\Rightarrow V_1 = V_2$

$$\frac{V_1 - 20\angle 0^\circ}{1} + \frac{V_1}{J1} = 0 \rightarrow V_1 =$$

$$\frac{V_2 - V}{5} + \frac{V_2}{5} = 0 \xrightarrow{\text{sub } V_2 = V_1}$$

→ what about the ckt

Q.



Since no ind. source in n/w, The n/w is said to be unenergized, so called resistive n/w.

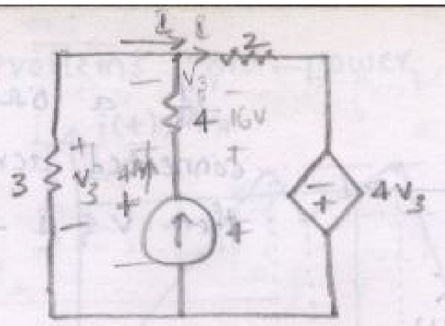
$V = R_{eq} I$

$R_{eq} = V/I = ?$

$I = I_1$

$$-I_1 + \frac{V}{4} + \frac{V - 2I_1}{2} = 0$$

$\Rightarrow V/I = \_\_\_.$

Q. 

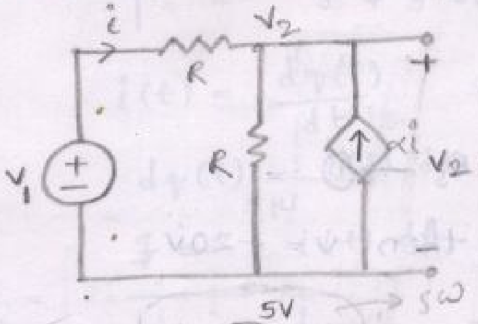
Determine I.

$$\frac{V_3}{3} - 4 + \frac{V_3 + 4V_3}{2} = 0$$

$\Rightarrow V_3 =$

$$I = \frac{V_3 + 4V_3}{2} = \frac{60}{17}\ A$$

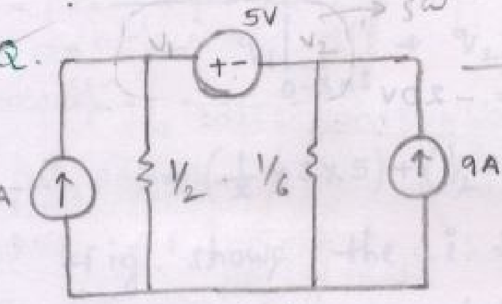
Q. Determine V_2/V_1.



$$\frac{V_2 - V_1}{R} + \frac{V_2}{R} - \alpha i = 0$$

where $i = \dfrac{V_1 - V_2}{R}$

$$V_2/V_1 = \frac{1+\alpha}{2+\alpha}$$

Q. 

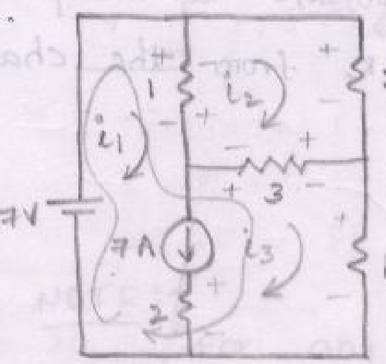
→ Determine V_1 & V_2.

$(V_1 - V_2 = 5) \Leftarrow V_1 - 5 - V_2 = 0$

$$-4 + \frac{V_1}{1/2} + \frac{V_1}{1/6} - 9 = 0$$

In side super node always KVL is written.

→ Since the i through an ideal VS can be any value, it is not possible nodal eq's at ①, ② nodes independently and hence super node procedure is followed.

Q. 

→ Determine power dissipated in 3Ω resistor.

~~Also use~~

$$-2i_2 - 3(i_2 - i_3) - 1(i_2 - i_1) = 0$$

Since the volt across any ideal CS can be any value it is not possible to write mesh eq's for the meshes ① & ③ indly and hence supermesh.

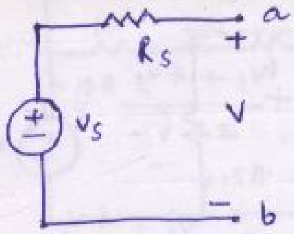
$$7 - 1(i_1 - i_2) - 3(i_3 - i_2) - 1(i_3) = 0$$

→ In side super mesh KCL

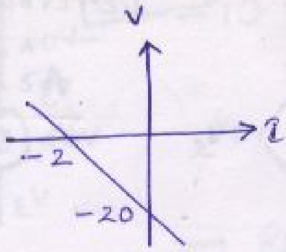
$$i_1 - i_3 = 7\ A$$

$$P_{3\Omega} = (i_2 - i_3)^2\, 3$$

$$P_{3\Omega} = 0.75\ W$$

Q. 

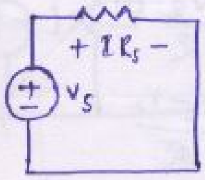
fig(a)



fig(b).

If a 10Ω is connected across a,b then V & I — ?

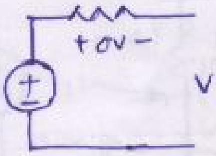
$(-2, 0) \Rightarrow$ when voltage $V = 0$, then $I = -2A$



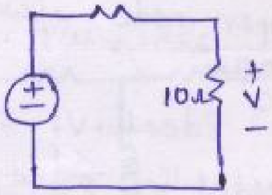
$$V_s - IR_s = 0$$

$$\Rightarrow V_s = -2R_s \longrightarrow (1)$$

$(0, -20) \Rightarrow$ when $I = 0$, then $V = -20V$

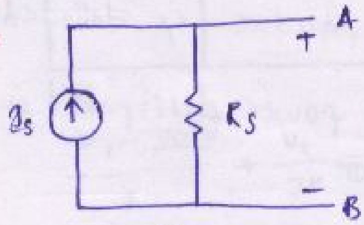


$V = V_s$ $\therefore V_s = -20V$



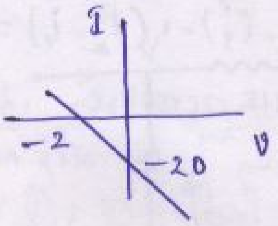
$$V = 10I$$

$$I = \frac{V_s}{10+10} = -1A$$

Q. 

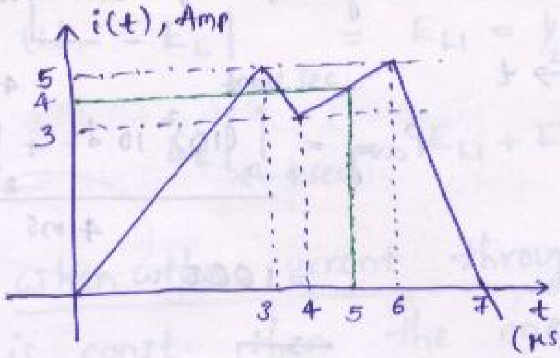
fig(a)

If a 10Ω resistor is connected across the a,b terminals then V & I — find I_s, R_s from the char.s.



Problems on power and Energy:-

Q. fig. shows the i flowing through the capacitor. Determine the charge acquired by capacitor upto the first 5 μsec — ?



$$i(t) = \frac{dq(t)}{dt}$$

$$dq(t) = i(t).dt$$

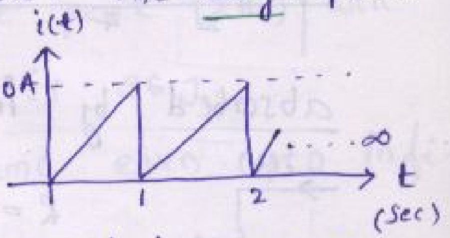
$$q = \int_0^{5\mu s} i(t).dt = \text{Area under } i(t) \text{ upto } 5\mu\text{Sec}$$

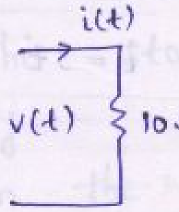
$$= q_1\Big|_{0-3\mu s} + q_2\Big|_{3-4\mu s} + q_3\Big|_{4-5\mu s}$$

$$(\tfrac{1}{2}\times1\times1+1\times3)$$

$$= \left(\frac{1}{2}\times3\times5\right) + \left(\tfrac{1}{2}\times1\times2 + 1\times3\right) + = 15\ \mu C.$$

Q. fig. shows the i through 10Ω. The avg. power dissipated by resistor is—?





P_{avg} = (Energy absorbed over one period)/(period)

$$= \frac{\int_0^1 i^2 R\,dt}{1\text{ sec}} = \frac{\int_0^1 (10t)^2 \cdot 10 \cdot dt}{1}$$

$$= \frac{1000}{3}\ J/sec.$$

NOTE:-

for any general period 'T', (1). for a voltage wave form $P_{avg} = \dfrac{\int_0^T \frac{v^2}{R}dt}{T} = \dfrac{\frac{1}{T}\int_0^T v^2 dt}{R}$

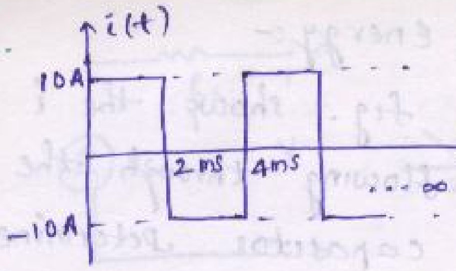
$$\Rightarrow P_{avg} = \frac{V_{rms}^2}{R}\ (\omega)$$

(2). for a current wave form $P_{avg} = \dfrac{\int_0^T i^2 R\,dt}{T}$

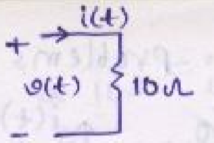
$$= \frac{1}{T}\int_0^T i^2 dt \cdot R$$

$$\therefore P_{avg} = I_{rms}^2 \cdot R\ (\omega).$$

Q.



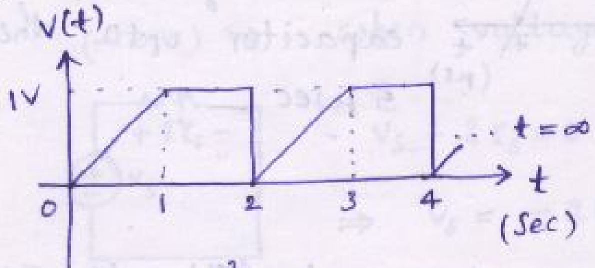
$P_{avg} = ?$ if

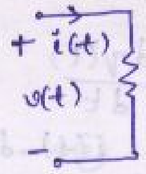


$$P_{avg} = \frac{\int_0^{2ms} (10)^2 \cdot 10\, dt + \int_{2ms}^{4ms} (-10)^2 \cdot 10\, dt}{4\, ms}$$

$= 1000$ W

Q.





$$P_{avg} = \frac{\int_0^2 \frac{v^2}{R}\, dt}{2\, sec} = \frac{1}{2}\left[\int_0^1 \frac{t^2}{10} \cdot dt + \int_1^2 \frac{1^2}{10}\, dt\right] = \frac{1}{15}\, w$$

G'03

Q.

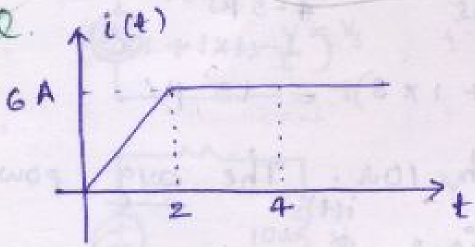
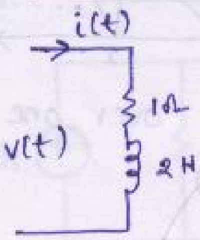


fig. shows the i flowing through an inductor of 2H and resistor of 1Ω. Determine ~~avg. power dissipated~~ energy absorbed by inductor upto the first 4 sec - ?



This is not a periodic one

$R = 1\Omega$, $0 < t \leq 2$ sec, $i = 3t$

$$E_{R_1} = \int_0^2 i^2 R\, dt \cdot J$$

$$= \int_0^2 (3t)^2 \cdot 1 \cdot dt = 24\, J$$

$$E_{R_2} = \int_2^4 6^2 \cdot 1 \cdot dt = 72\, J$$

$L = 2H$, $0 \leq t \leq 2$ sec, $i = 3t$ $\qquad v = L\frac{di}{dt}$

$$E_{L_1} = \int_0^2 \overset{v \cdot I}{L i \frac{di}{dt}} \cdot dt \cdot (J) = \int_0^2 2 \cdot 3t \cdot 3 \cdot dt = 36\, J$$

$$E_{L_2} = \int_2^4 2 \cdot 6 \cdot (0) \cdot dt = 0\, J$$

(2 to 4, $i = 6$, $\frac{di}{dt} = 0$)

$$E_{abs}\Big|_{4\, sec} = E_{R_1} + E_{R_2} + E_{L_1} + E_{L_2}$$

$= 132$ J

NOTE :-

(1) .. $E_L\Big|_{t=2\,sec} = E_{L1} = \frac{1}{2} \times 2 \times 6^2 = 36\,J$

$E_L\Big|_{t=4\,sec} = E_{L1} + E_{L2} = \frac{1}{2}$

when the current through an ideal inductor is const. then the energy absorbed zero, since the instantaneous power $P = Li\frac{di}{dt} = 0$

similarly for a const. capacitive voltage the energy absorbed is zero, since instantaneous power $\Rightarrow P = Cv \cdot \frac{dv}{dt} = 0$.

Q. In the above problem the energy stored by inductor (1Ω, 2H) upto the first 4 sec —?

Only the ideal inductive part [2H] will store the energy, so it is 36 J.

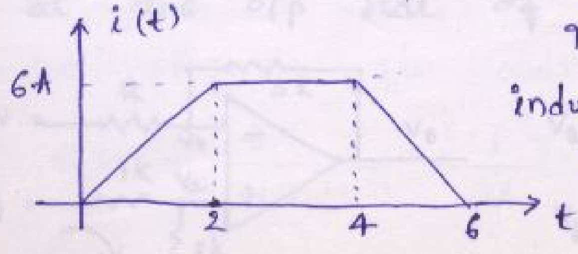
This stored energy is same even upto infinity.

Q. In the above case the energy absorbed by the inductor (1Ω, 2H) upto infinity is —?

$$E_{abs}\Big|_{t=\infty} = E_R\Big|_{t=\infty} + E_L\Big|_{t=\infty}$$

$$= 24 + \int_2^{\infty} 6^2 \cdot 1 \cdot dt + 36 + 0$$

$$= 60 + 36(\infty - 2) = \infty .$$

Q. 

The energy stored by the inductor upto the first 6 sec —?

$E_L\Big|_{t=6} = \frac{1}{2} \times 2 \times (0)^2 = 0\,J$

(or) $E_L\Big|_{\substack{stored\\t=6}} = E_{L_1} + E_{L_2} + E_{L_3} = 36 + 0 - 36$

$= 0\ J.$

Q. In the above problem, the energy absorbed by the inductor upto the first 6 sec — ?

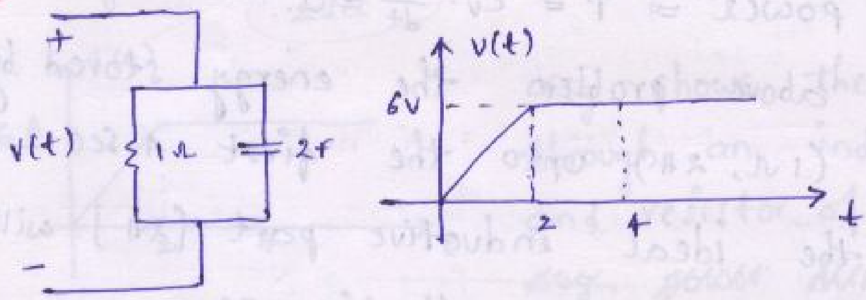
$E_{abs}\Big|_{t=6} = E_{R_1} + E_{R_2} + E_{R_3} + E_{L_1} + E_{L_2} + E_{L_3}$

$= 24 + 72 + 24 + 36 + 0 - 36$

$= 120\ J.$

$E_{R_3} = \int_4^6 i^2 R\, dt = \int_4^6 [-3(t-6)]^2 . 1 . dt = 24\ J.$

Q.



$E_R = \int \frac{v^2}{R}\, dt \cdot (J) \qquad E_c = \int cv\left(\frac{dv}{dt}\right) dt \cdot (J)$

(i). $E_{abs}\Big|_{t=4} = 132\ J$ (ii). $E_{stored}\Big|_{t=4} = 36\ J$ (or) $t = \infty$

(iii). $E_{abs}\Big|_{t=\infty} = \infty\ J$

(iv). $E_{sto}\Big|_{t=6sec} = 0\ J$

(v). $E_{abs}\Big|_{t=6sec} = 120\ J.$

LaPlace Transform in Circuit Analysis

Objectives:

- •Calculate the Laplace transform of common functions using the definition and the Laplace transform tables
- •Laplace-transform a circuit, including components with non-zero initial conditions.
- •Analyze a circuit in the s-domain
- •Check your s-domain answers using the initial value theorem (IVT) and final value theorem (FVT)
- •Inverse Laplace-transform the result to get the time-domain solutions; be able to identify the forced and natural response components of the time-domain solution.

(Note – this material is covered in Chapter 12 and Sections 13.1 – 13.3)

LaPlace Transform in Circuit Analysis

What types of circuits can we analyze?

- Circuits with any number and type of DC sources and any number of resistors.
- First-order (RL and RC) circuits with no source and with a DC source.
- Second-order (series and parallel RLC) circuits with no source and with a DC source.
- Circuits with sinusoidal sources and any number of resistors, inductors, capacitors (and a transformer or op amp), but can generate only the steady-state response.

LaPlace Transform in Circuit Analysis

What types of circuits will Laplace methods allow us to analyze?

- Circuits with any type of source (so long as the function describing the source has a Laplace transform), resistors, inductors, capacitors, transformers, and/or op amps; the Laplace methods produce the complete response!

LaPlace Transform in Circuit Analysis

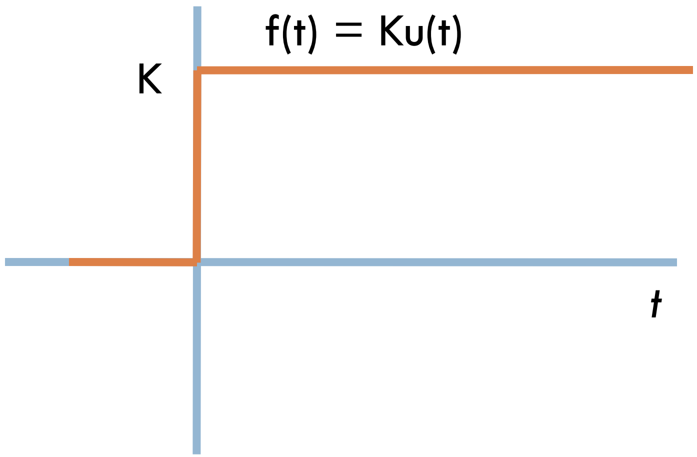
Definition of the Laplace transform:

$$\mathcal{L}\{f(t)\} = F(s) = \int_0^\infty f(t)e^{-st}dt$$

Note that there are limitations on the types of functions for which a Laplace transform exists, but those functions are "pathological", and not generally of interest to engineers!

LaPlace Transform in Circuit Analysis

Aside – formally define the “step function”, which is often modeled in a circuit by a voltage source in series with a switch.



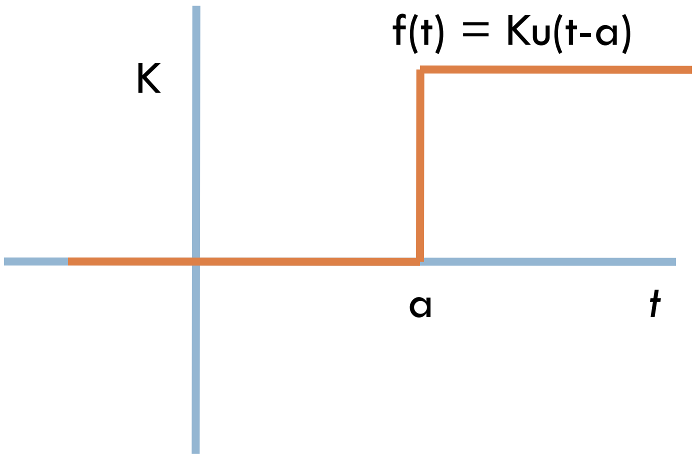
$$f(t) = 0, \quad t < 0$$
$$\quad = K, \quad t > 0$$

When K = 1, f(t) = u(t), which we call the unit step function

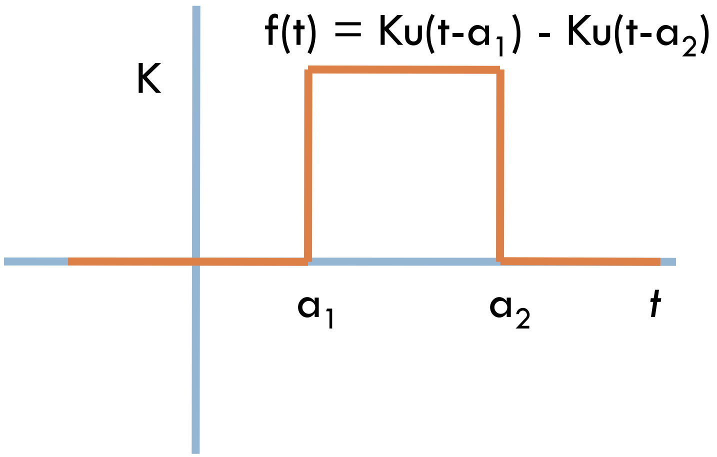
LaPlace Transform in Circuit Analysis

More step functions:

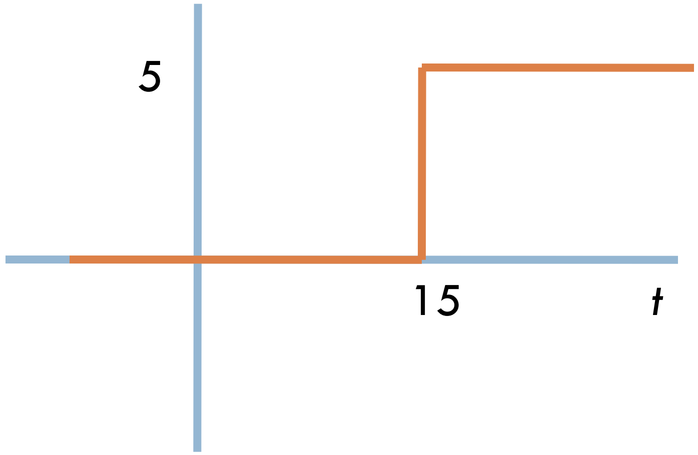
The step function shifted in time



The "window" function



Which of these expressions describes the function plotted here?



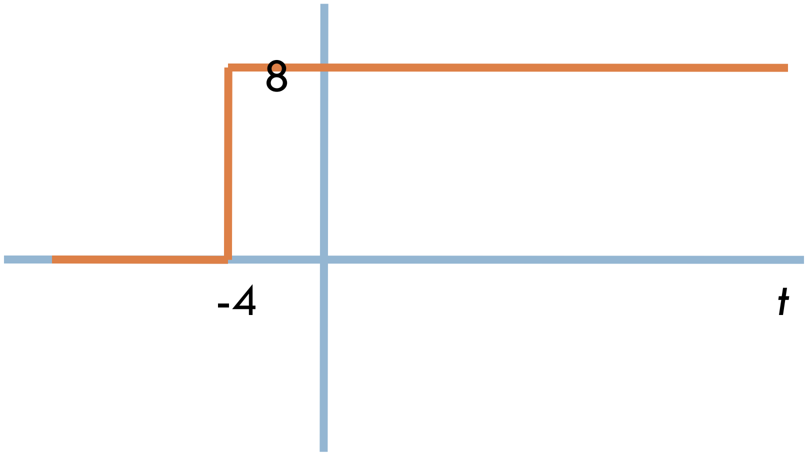
- ✗ A. u(t – 5)
- ✗ B. 5u(t + 15)
- ✓ C. 5u(t – 15)
- ✗ D. 15u(t – 5)

Which of these expressions describes the function plotted here?

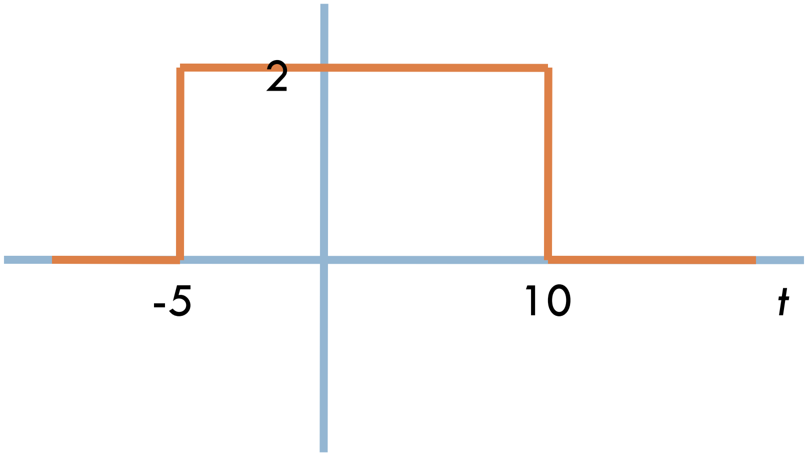
A. $8u(t+4)$

B. $4u(t-8)$

C. $8u(t-4)$



Which of these expressions describes the function plotted here?



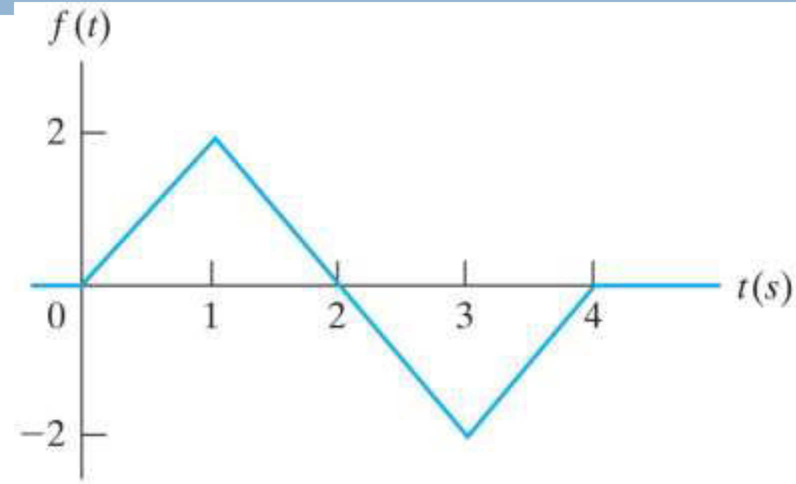
A. $2u(t + 5) + 2u(t - 10)$

B. $2u(t - 5) + 2u(t + 10)$

C. $2u(t + 5) - 2u(t - 10)$

LaPlace Transform in Circuit Analysis

Use “window” functions to express this piecewise linear function as a single function valid for all time.



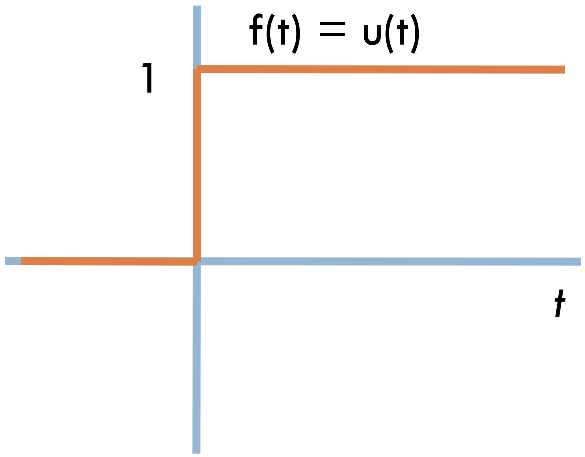
$$f(t) = \begin{array}{lll} 0, & t < 0 & \\ 2t, & 0 \le t \le 1\,\text{s} & [u(t) - u(t-1)] \\ -2t+4, & 0 \le t \le 1\,\text{s} & [u(t-1) - u(t-3)] \\ 2t-8, & 0 \le t \le 1\,\text{s} & [u(t-3) - u(t-4)] \\ 0, & t > 4\,\text{s} & \end{array}$$

$$f(t) = 2t[u(t) - u(t-1)] + (-2t+4)[u(t-1) - u(t-3)]$$
$$+ (2t-8)[u(t-3) - u(t-4)]$$

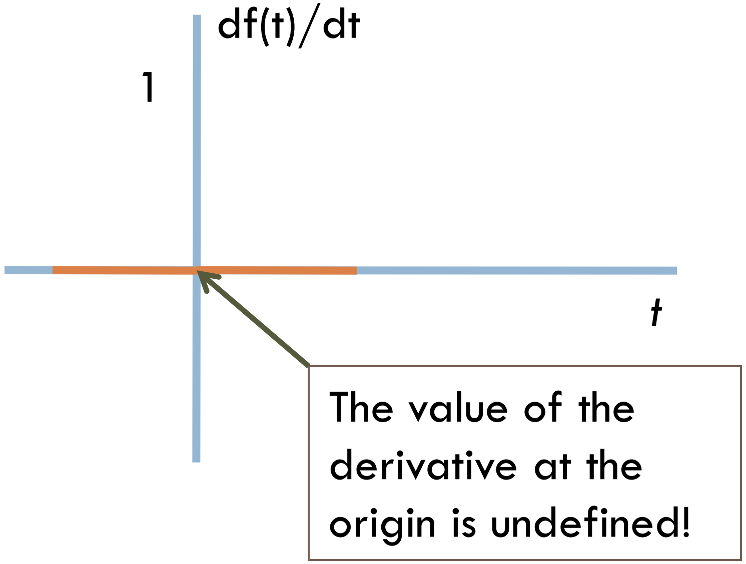
LaPlace Transform in Circuit Analysis

The impulse function, created so that the step function's derivative is defined for all time:

The step function



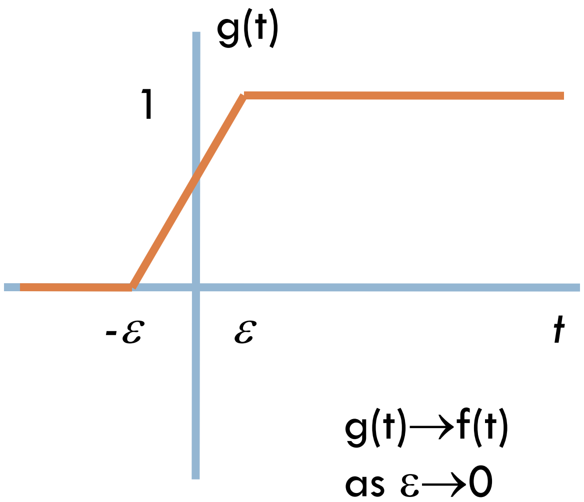
The first derivative of the step function



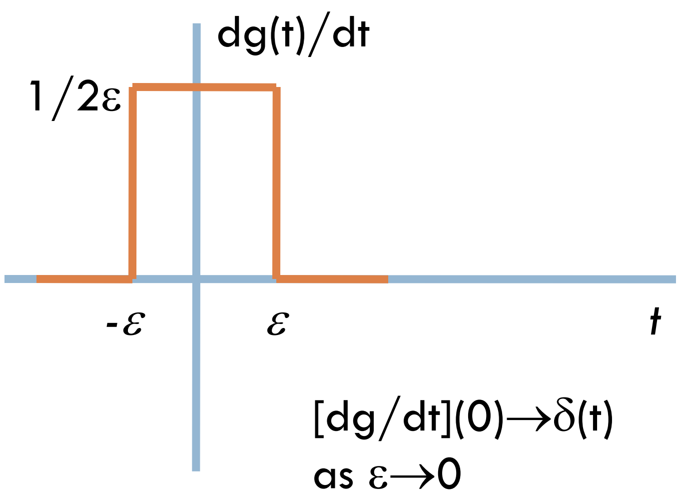
LaPlace Transform in Circuit Analysis

Use a limiting function to define the step function and its first derivative!

The step function



The first derivative of the step function



LaPlace Transform in Circuit Analysis

The unit impulse function is represented symbolically as δ(t).

Definition:

$$\delta(t) = 0 \qquad \text{for} \qquad t \neq 0$$

$$\text{and} \qquad \int_{-\infty}^{\infty} \delta(t)dt = 1$$

(Note that the area under the $g(t)$ function is

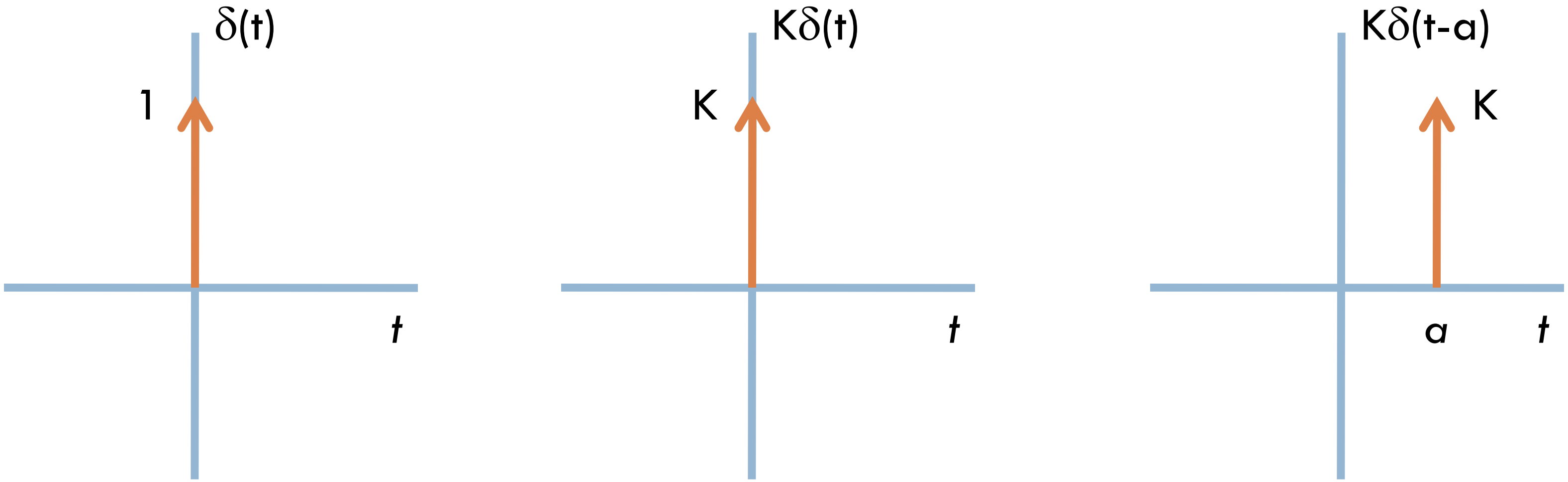
$$\frac{1}{2\varepsilon}(\varepsilon + \varepsilon), \text{ which approaches 1 as } \varepsilon \to 0)$$

Note also that any limiting function with the following characteristics can be used to generate the unit impulse function:

- Height → ∞ as ε → 0
- Width → 0 as ε → 0
- Area is constant for all values of ε

LaPlace Transform in Circuit Analysis

Another definition: $\delta(t) = \frac{du(t)}{dt}$



The sifting property is an important property of the impulse function:

$$\int_{-\infty}^{\infty} f(t)\delta(t-a)dt = f(a)$$

Evaluate the following integral, using the sifting property of the impulse function.

$$\int_{-10}^{10} (6t^2 + 3)\delta(t-2)dt$$

A. 24

B. 27

C. 3

$$6(2)^2 + 3 = 27$$

LaPlace Transform in Circuit Analysis

Use the definition of Laplace transform to calculate the Laplace transforms of some functions of interest:

$$\mathcal{L}\{\delta(t)\} = \int_0^{\infty} \delta(t) e^{-st} dt = \int_0^{\infty} \delta(t-0) e^{-st} dt = e^{-s(0)} = 1$$

$$\mathcal{L}\{u(t)\} = \int_0^{\infty} u(t) e^{-st} dt = \int_0^{\infty} 1 e^{-st} dt = \frac{1}{-s} e^{-st} \bigg|_0^{\infty} = 0 - \frac{1}{-s} = \frac{1}{s}$$

$$\mathcal{L}\{e^{-at}\} = \int_0^{\infty} e^{-at} e^{-st} dt = \int_0^{\infty} e^{-(s+a)t} dt = \frac{1}{-(s+a)} e^{-(s+a)t} \bigg|_0^{\infty} = 0 - \frac{1}{-(s+a)} = \frac{1}{(s+a)}$$

$$\mathcal{L}\{\sin \omega t\} = \int_0^{\infty} \left[\frac{e^{j\omega t} - e^{-j\omega t}}{2j} \right] e^{-st} dt = \tfrac{1}{j2} \int_0^{\infty} [e^{-(s-j\omega)t} - e^{-(s+j\omega)t}] dt$$

$$= \frac{1}{\ldots}\left[\frac{e^{-(s-j\omega)t}}{\ldots} \right]^{\infty} - \frac{1}{\ldots}\left[\frac{e^{-(s+j\omega)t}}{\ldots} \right]^{\infty} = \frac{1}{\ldots}\left[\frac{1}{\ldots} - \frac{1}{\ldots} \right] = \frac{\omega}{\ldots}$$

Look at the Functional Transforms table. Based on the pattern that exists relating the step and ramp transforms, and the exponential and damped-ramp transforms, what do you predict the Laplace transform of t^2 is?

A. $1/(s + a)$

B. s

C. $1/s^3$

LaPlace Transform in Circuit Analysis

Using the definition of the Laplace transform, determine the effect of various operations on time-domain functions when the result is Laplace-transformed. These are collected in the Operational Transform table.

$$\mathcal{L}\{K_1 f_1(t) + K_2 f_2(t) - K_3 f_3(t)\} = \int_0^\infty [K_1 f_1(t) e^{-st} + K_2 f_2(t) e^{-st} - K_3 f_3(t) e^{-st}] dt$$

$$= \int_0^\infty K_1 f_1(t) e^{-st} dt + \int_0^\infty K_2 f_2(t) e^{-st} dt - \int_0^\infty K_3 f_3(t) e^{-st} dt$$

$$= K_1 \int_0^\infty f_1(t) e^{-st} dt + K_2 \int_0^\infty f_2(t) e^{-st} dt - K_3 \int_0^\infty f_3(t) e^{-st} dt$$

$$= K_1 F_1(s) + K_2 F_2(s) - K_2 F_2(s)$$

$$\mathcal{L}\left\{\frac{df(t)}{dt}\right\} = e^{-st} f(t)\Big|_0^\infty - \int_0^\infty f(t)[-se^{-st}]dt \qquad \text{(integration by parts!)}$$

Now lets use the operational transform table to find the correct value of the Laplace transform of t^2, given that

$$\mathcal{L}\{t\} = \frac{1}{s^2}$$

- ✗ A. $1/s^3$
- ✓ B. $2/s^3$
- ✗ C. $-2/s^3$

LaPlace Transform in Circuit Analysis

Example – Find the Laplace transform of t^2e^{-at}.

Use the operational transform: $\mathcal{L}\{t^n f(t)\} = (-1)^n \dfrac{d^n F(s)}{ds^n}$

Use the functional transform: $\mathcal{L}\{e^{-at}\} = \dfrac{1}{(s+a)}$

$$\mathcal{L}\{t^2 e^{-at}\} = (-1)^2 \frac{d^2}{ds^2}\left[\frac{1}{s+a}\right] = \frac{d}{ds}\left[\frac{-1}{(s+a)^2}\right] = \frac{2}{(s+a)^3}$$

Alternatively,
Use the operational transform: $\mathcal{L}\{e^{-at} f(t)\} = F(s+a)$

Use the functional transform: $\mathcal{L}\{t^2\} = \dfrac{2}{s^3}$

$$\mathcal{L}\{t^2 e^{-at}\} = \frac{2}{\ldots^{3}}$$

LaPlace Transform in Circuit Analysis

How can we use the Laplace transform to solve circuit problems?

- Option 1:
  - Write the set of differential equations in the time domain that describe the relationship between voltage and current for the circuit.
  - Use KVL, KCL, and the laws governing voltage and current for resistors, inductors (and coupled coils) and capacitors.
  - Laplace transform the equations to eliminate the integrals and derivatives, and solve these equations for V(s) and I(s).

LaPlace Transform in Circuit Analysis

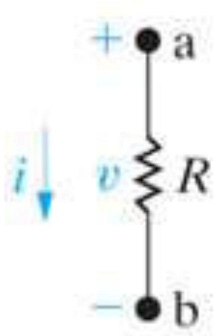
How can we use the Laplace transform to solve circuit problems?

- Option 2:
  - Laplace transform the circuit (following the process we used in the phasor transform) and use DC circuit analysis to find V(s) and I(s).
  - Inverse-Laplace transform to get v(t) and i(t).

LaPlace Transform in Circuit Analysis

Laplace transform – resistors:

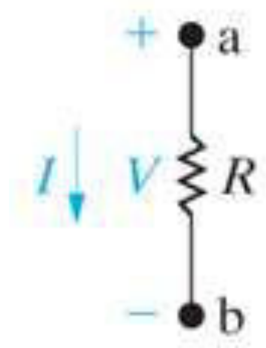
Time-domain



$$v(t) = Ri(t)$$

$$\xrightarrow{\mathcal{L}}$$

s-domain (Laplace)

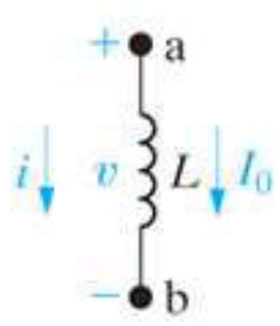


$$V(s) = RI(s)$$

LaPlace Transform in Circuit Analysis

Laplace transform – inductors:

Time-domain

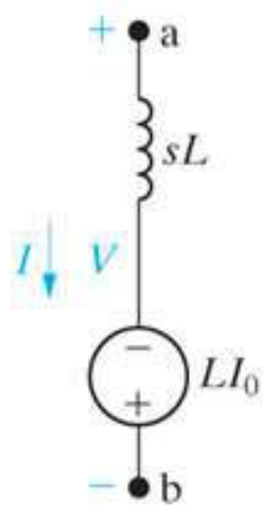


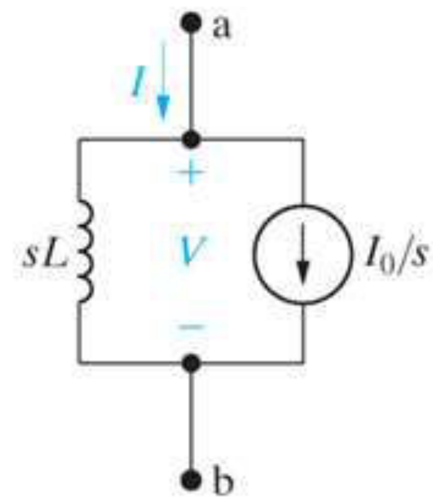
$$v(t) = L\frac{di(t)}{dt}$$

$$i(0) = I_0$$

$$\xrightarrow{\mathcal{L}}$$

s-domain (Laplace)





$$V(s) = sLI(s) - LI_0$$

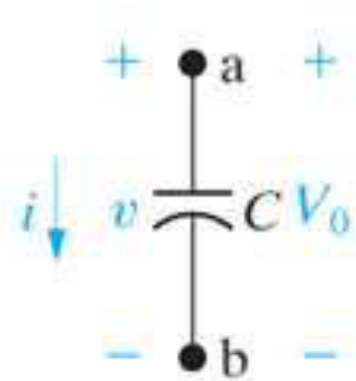
$$I(s) = \frac{V(s)}{} + \frac{I_0}{}$$

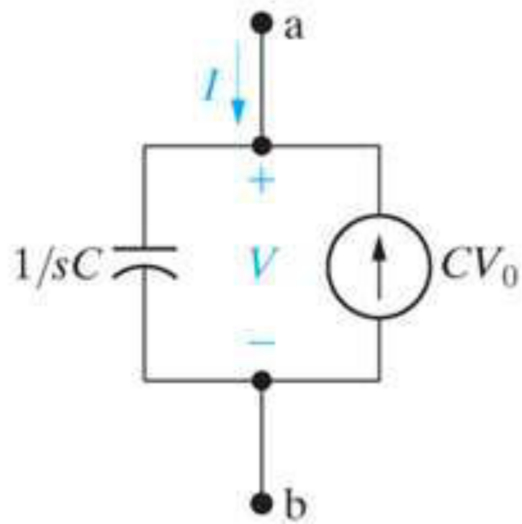
LaPlace Transform in Circuit Analysis

Laplace transform – resistors:

Time-domain

s-domain (Laplace)





$$i(t) = C\frac{dv(t)}{dt}$$

$$v(0) = V_0$$

$$\xrightarrow{\mathcal{L}}$$

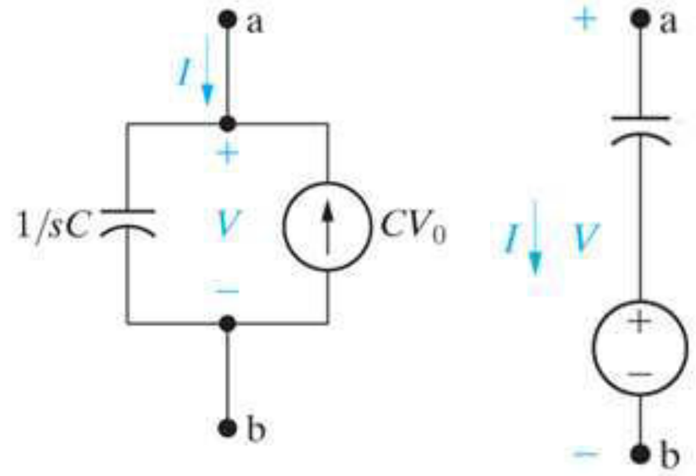
$$I(s) = sCV(s) - CV_0$$

Find the value of the complex impedance and the series-connected voltage source, representing the Laplace transform of a capacitor.

A. $sC, V_0/s$

B. $1/sC, V_0/s$

C. $1/sC, -V_0/s$



$$I(s) = sCV(s) - CV_0$$

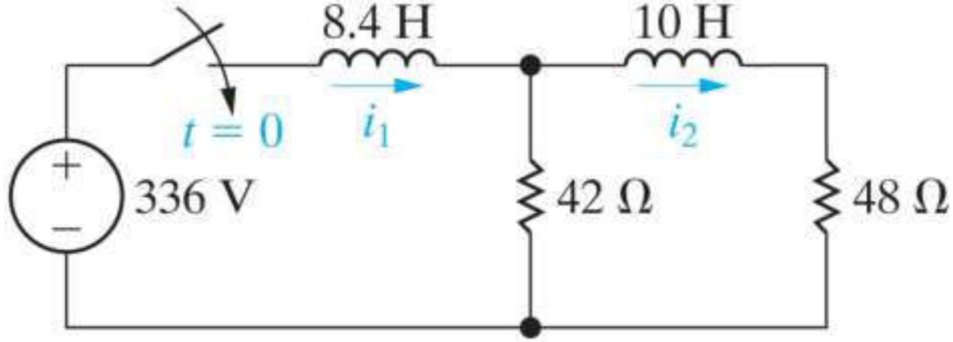
LaPlace Transform in Circuit Analysis

Recipe for Laplace transform circuit analysis:

1. Redraw the circuit (nothing about the Laplace transform changes the types of elements or their interconnections).
2. Any voltages or currents with values given are Laplace-transformed using the functional and operational tables.
3. Any voltages or currents represented symbolically, using i(t) and v(t), are replaced with the symbols I(s) and V(s).
4. All component values are replaced with the corresponding complex impedance, Z(s).
5. Use DC circuit analysis techniques to write the s-domain equations and solve them.
6. Inverse-Laplace transform s-domain solutions to get time-domain solutions.

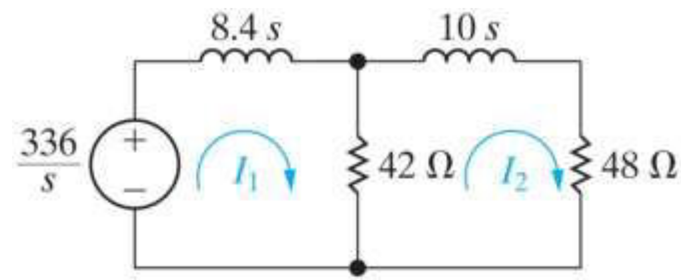
LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



$$-\frac{336}{s}+(42+8.4s)I_1-42I_2=0$$

$$(10s+90)I_2-42I_1=0 \qquad \Rightarrow \qquad I_1=\frac{10s+90}{42}I_2$$



Substituting, $$-\frac{336}{s}+\left[\frac{(42+8.4s)(10s+90)}{42}-42\right]I_2=0$$

$$\Rightarrow \qquad I_2(s)=\frac{336(42)}{s[(42+8.4s)(10s+90)-42^2]}=\frac{168}{s^3+14s^2+24s}$$

$$10s+90\left[\quad 168 \quad\right] \quad 40s+360$$

LaPlace Transform in Circuit Analysis

Recipe for Laplace transform circuit analysis:

1. Redraw the circuit (nothing about the Laplace transform changes the types of elements or their interconnections).
2. Any voltages or currents with values given are Laplace-transformed using the functional and operational tables.
3. Any voltages or currents represented symbolically, using i(t) and v(t), are replaced with the symbols I(s) and V(s).
4. All component values are replaced with the corresponding complex impedance, Z(s).
5. Use DC circuit analysis techniques to write the s-domain equations and solve them.
6. Inverse-Laplace transform s-domain solutions to get time-domain solutions.

LaPlace Transform in Circuit Analysis

Finding the inverse Laplace transform:

$$f(t) = \frac{1}{j2\pi}\int_{c-j\infty}^{c+j\infty} F(s)e^{st}ds \qquad t > 0$$

This is a contour integral in the complex plane, where the complex number c must be chosen such that the path of integration is in the convergence area along a line parallel to the imaginary axis at distance c from it, where c must be larger than the real parts of all singular values of F(s)!

There must be a better way …

LaPlace Transform in Circuit Analysis

Inverse Laplace transform using partial fraction expansion:

- Every s-domain quantity, V(s) and I(s), will be in the form

$$\frac{N(s)}{D(s)}$$

where N(s) is the numerator polynomial in s, and has real coefficients, and D(s) is the denominator polynomial in s, and also has real coefficients, and

$$O\{N(s)\} < O\{D(s)\}$$

- Since D(s) has real coefficients, it can always be factored, where the factors can be in the following forms:
  - ✓Real and distinct
  - ✓Real and repeated
  - ✓Complex conjugates and distinct

LaPlace Transform in Circuit Analysis

Inverse Laplace transform using partial fraction expansion:

- The roots of D(s) (the values of s that make D(s) = 0) are called **poles**.
- The roots of N(s) (the values of s that make N(s) = 0) are called **zeros**.

Back to the example:

$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s} = \frac{40(s+9)}{s(s+2)(s+12)}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s} = \frac{168}{s(s+2)(s+12)}$$

Find the zeros of $I_1(s)$.

$$I_1(s) = \frac{40(s+9)}{s(s+2)(s+12)}$$

A. s = –9 rad/s

B. s = –9 rad/s

C. There aren't any zeros

Find the poles of $I_1(s)$.

$$I_1(s) = \frac{40(s+9)}{s(s+2)(s+12)}$$

A. $s = 2$ rad/s, $s = 12$ rad/s

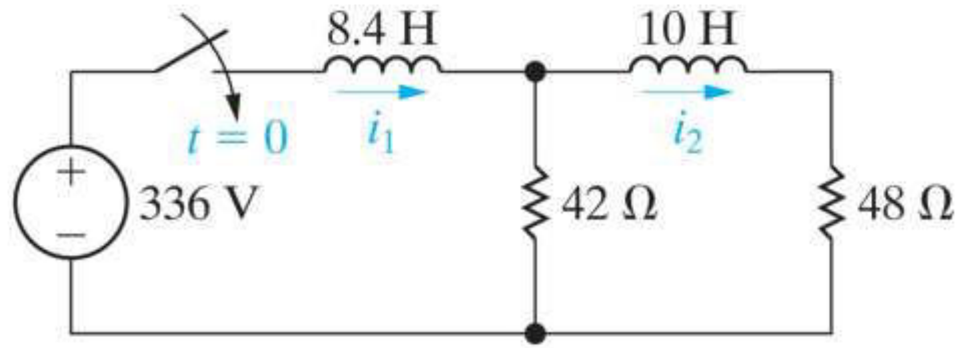
B. $s = -2$ rad/s, $s = -12$ rad/s

C. $s = 0$ rad/s, $s = -2$ rad/s, $s = -12$ rad/s

LaPlace Transform in Circuit Analysis

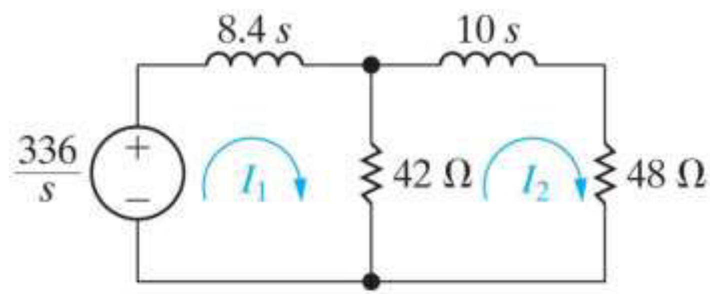
Example:

There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



$$I_1(s) = \frac{40s + 360}{s(s+2)(s+12)}$$

$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$

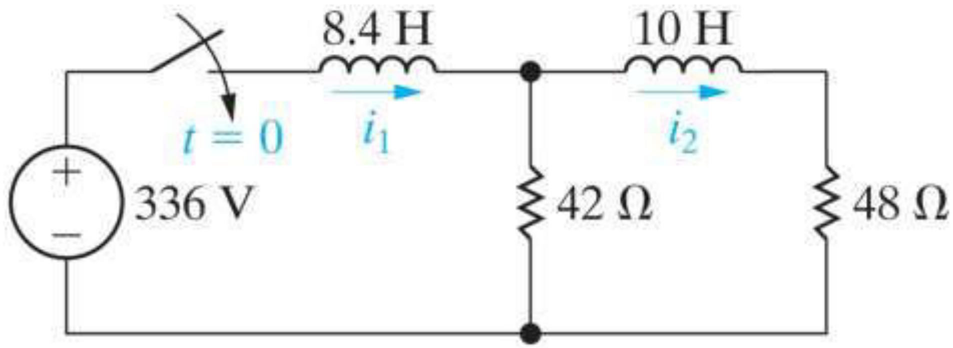


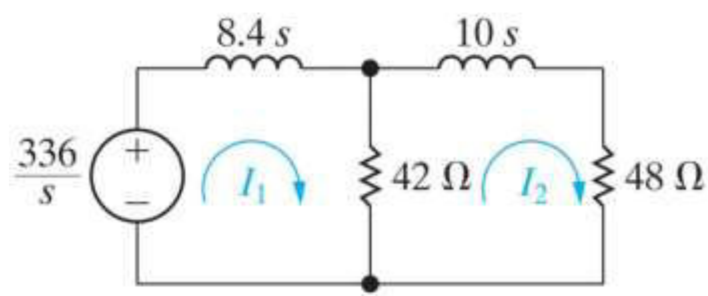
$$K_1 = \left.\frac{40s+360}{(s+2)(s+12)}\right|_{s=0} = 15; \qquad K_2 = \left.\frac{40s+360}{s(s+12)}\right|_{s=-2} = -14; \qquad K_3 = \left.\frac{40s+360}{s(s+2)}\right|_{s=-12} = -1$$

$$\therefore \qquad I_1(s) = \frac{15}{} + \frac{-14}{} + \frac{-1}{}$$

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.





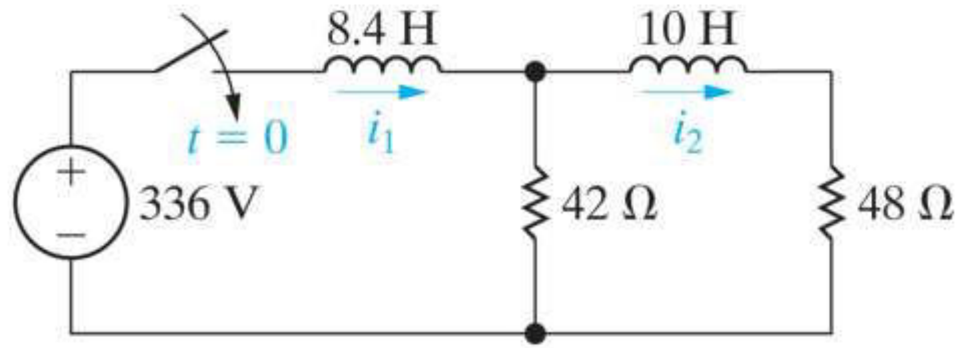
$$i_1(t) = \mathcal{L}^{-1}\left\{\frac{15}{s} + \frac{-14}{s+2} + \frac{-1}{s+12}\right\}$$

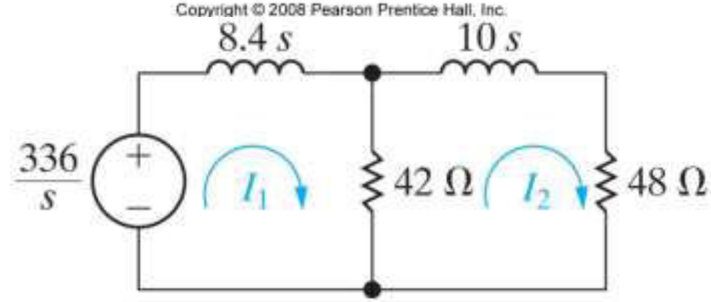
$$= [15 - 14e^{-2t} - e^{-12t}]u(t) \text{ A}$$

The forced response is $15u(t)$ A;

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.





$$I_2(s) = \frac{168}{s(s+2)(s+12)}$$

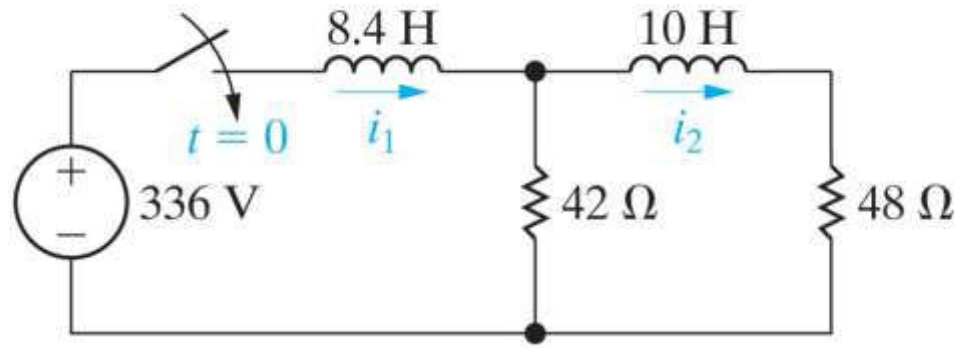
$$= \frac{K_1}{s} + \frac{K_2}{s+2} + \frac{K_3}{s+12}$$

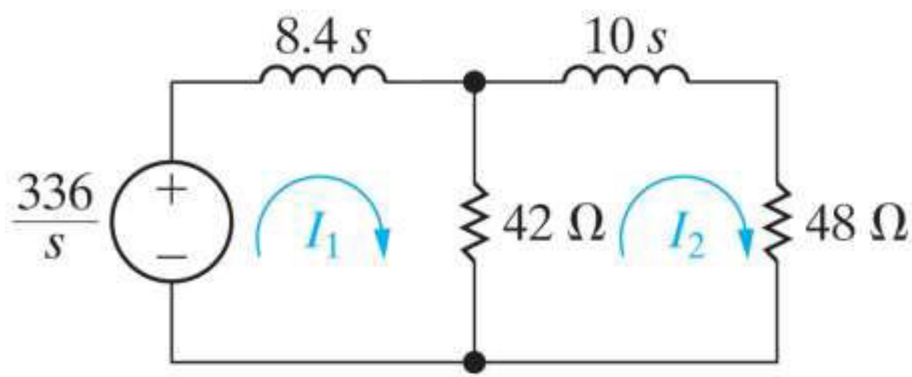
$$K_1 = \left.\frac{168}{(s+2)(s+12)}\right|_{s=0} = 7; \qquad K_2 = \left.\frac{168}{s(s+12)}\right|_{s=-2} = -8.4; \qquad K_3 = \left.\frac{168}{s(s+2)}\right|_{s=-12} = 1.4$$

$$\therefore \quad I_2(s) = \frac{7}{\ } + \frac{-8.4}{\ } + \frac{1.4}{\ }$$

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.





$$i_2(t) = \mathcal{L}^{-1}\left\{\frac{7}{s} + \frac{-8.4}{s+2} + \frac{1.4}{s+12}\right\}$$

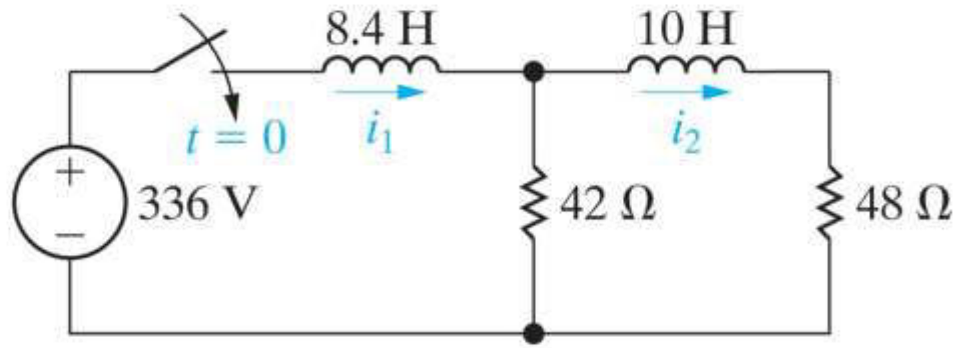
$$= [7 - 8.4e^{-2t} + 1.4e^{-12t}]u(t)\ \text{A}$$

The forced response is $7u(t)$ A;

LaPlace Transform in Circuit Analysis

Example:

There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



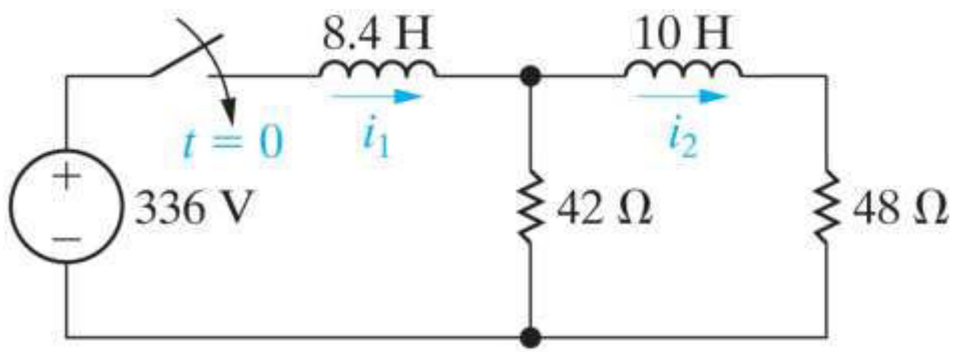
$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})u(t)A$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})u(t)A$$

Check the answers at t = 0 and t = ∞ to make sure the circuit and the equations match!

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})u(t)A$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})u(t)A$$

At $t = 0$, the circuit has no initial stored energy, so $i_1(0) = 0$ and $i_2(0) = 0$. Now check the equations:

$$i_1(0) = (15 - 14 - 1)(1) = 0$$

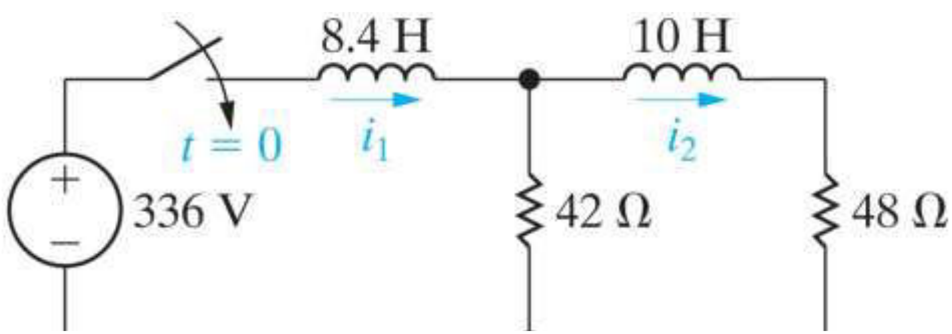
$$i_2(0) = (7 - 8.4 + 1.4)(1) = 0$$

As $t \to \infty$, the inductors behave like

A. Inductors

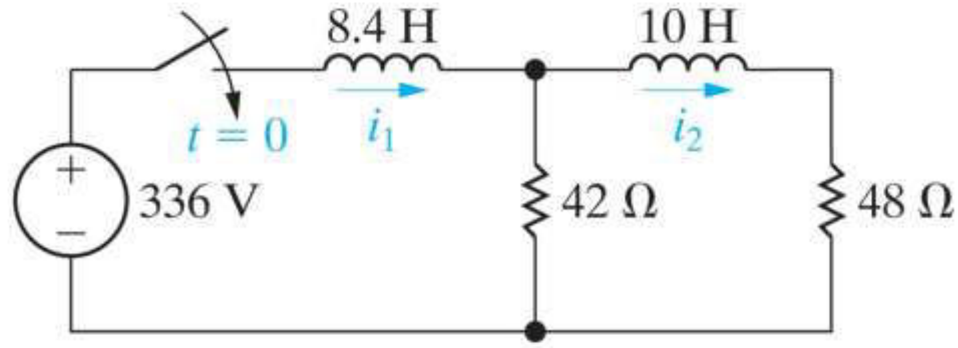
B. Open circuits

C. Short circuits



LaPlace Transform in Circuit Analysis

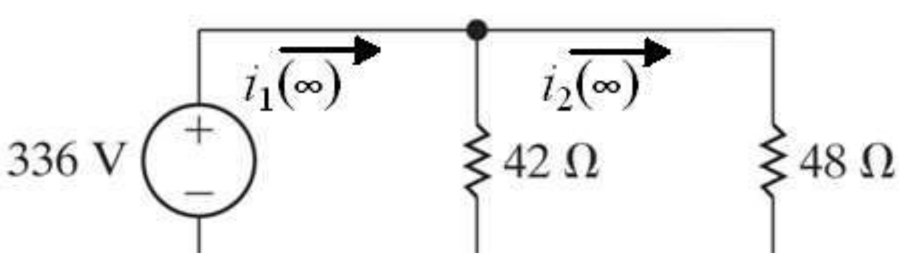
Example:
There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for t > 0.



$$i_1(t) = (15 - 14e^{-2t} - e^{-12t})u(t)A \quad \Rightarrow \quad i_1(\infty) = 15 - 0 - 0 = 15\text{ A}$$

$$i_2(t) = (7 - 8.4e^{-2t} + 1.4e^{-12t})u(t)A \quad \Rightarrow \quad i_2(\infty) = 7 - 0 - 0 = 7\text{ A}$$

Draw the circuit for t = ∞ and check these solutions.



$$42 \| 48 = 22.4\Omega$$

$$i_1(\infty) = \frac{336}{22.4} = 15\text{ A(check!)}$$

22.4

LaPlace Transform in Circuit Analysis

We can also check the initial and final values in the s-domain, before we begin the process of inverse-Laplace transforming our s-domain solutions. To do this, use the **Initial Value Theorem (IVT)** and the **Final Value Theorem (FVT)**.

- The initial value theorem:

$$\lim_{t \to 0^+} f(t) = \lim_{s \to \infty} sF(s)$$

This theorem is valid if and only if f(t) has no impulse functions.

- The final value theorem:

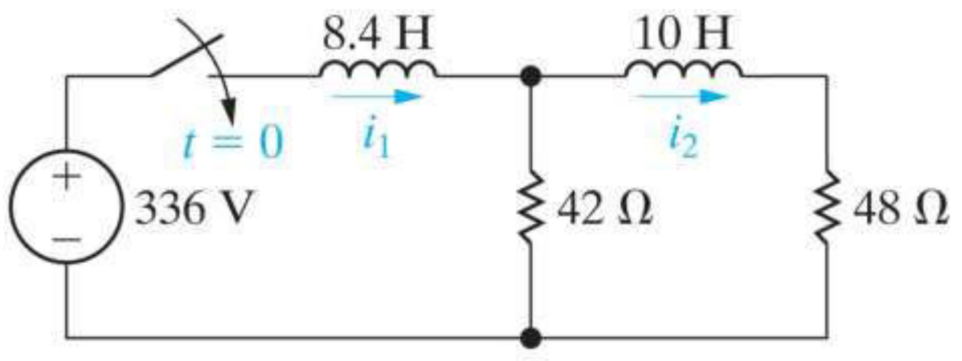
$$\lim_{t \to \infty} f(t) = \lim_{s \to 0} sF(s)$$

This theorem is valid if and only if all but one of the poles of F(s) are in the left-half of the complex plane, and the one that is not can only be at the origin.

LaPlace Transform in Circuit Analysis

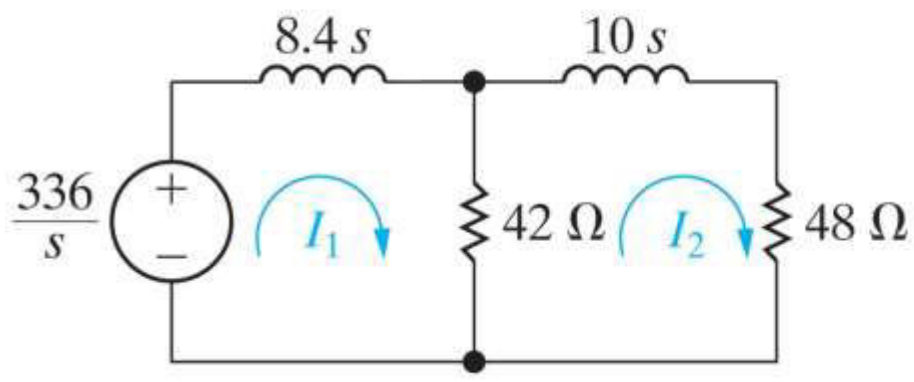
Example:

There is no initial energy stored in this circuit. Find $i_1(t)$ and $i_2(t)$ for $t > 0$.



$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s}$$

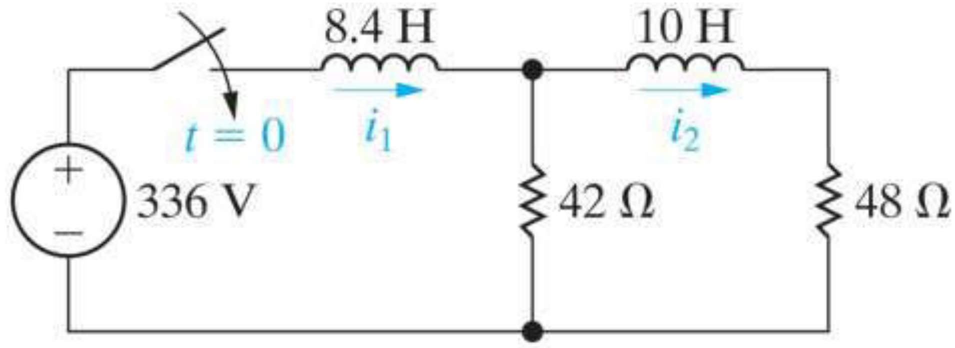


Check your answers using the IVT and the FVT.

LaPlace Transform in Circuit Analysis

IVT:

From the circuit, $i_1(0) = 0$ and $i_2(0) = 0$.



$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to 0} i_1(t) = \lim_{s\to\infty} sI_1(s)$$

$$= \lim_{s\to\infty} \frac{40s^2 + 360s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{1/s\to 0} \frac{(40/s) + (360/s^2)}{1 + (14/s) + (24/s^2)}$$

$$= 0 \text{ A (check!)}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to\infty} sI_1(s)$$

$$= \lim_{s\to\infty} \frac{168s}{s^3 + 14s^2 + 24s}$$

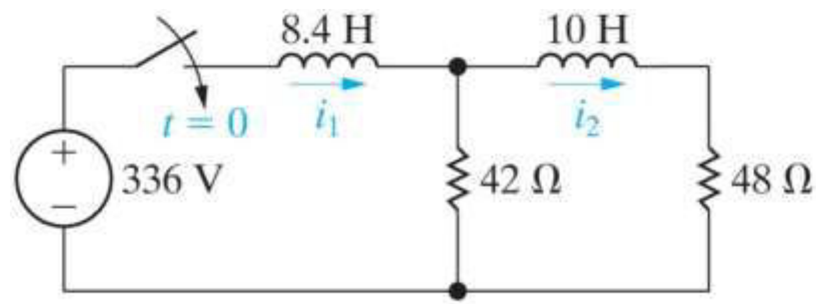
$$= \lim_{1/s\to 0} \frac{(168/s^2)}{1 + (14/s) + (24/s^2)}$$

$$= 0 \text{ A (check!)}$$

LaPlace Transform in Circuit Analysis

FVT:

From the circuit, $i_1(\infty) = 15$ A and $i_2(\infty) = 7$ A.



$$I_1(s) = \frac{40s + 360}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to 0} sI_1(s)$$

$$= \lim_{s\to 0} \frac{40s^2 + 360s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{s\to 0} \frac{40s + 360}{s^2 + 14s + 24}$$

$$= \frac{360}{\ } = 15 \text{ A(check!)}$$

$$I_2(s) = \frac{168}{s^3 + 14s^2 + 24s}$$

$$\lim_{t\to\infty} i_1(t) = \lim_{s\to 0} sI_1(s)$$

$$= \lim_{s\to 0} \frac{168s}{s^3 + 14s^2 + 24s}$$

$$= \lim_{s\to 0} \frac{168}{s^2 + 14s + 24}$$

$$= \frac{168}{\ } = 7 \text{ A(check!)}$$

LaPlace Transform in Circuit Analysis

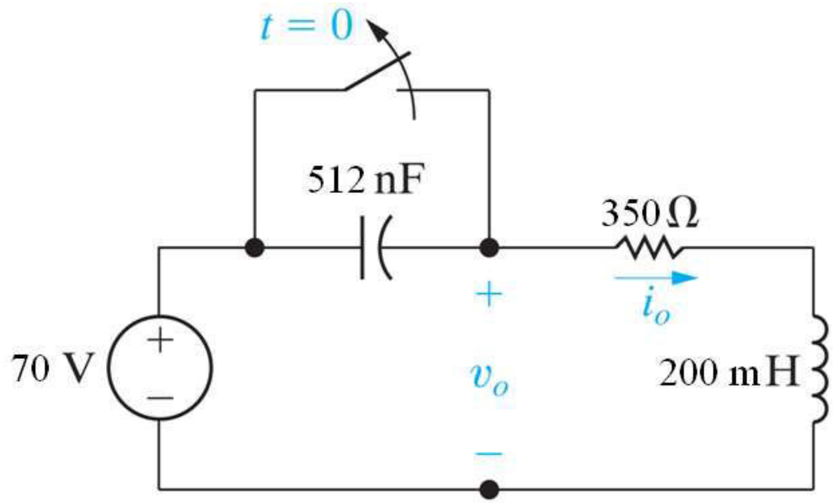
Recipe for Laplace transform circuit analysis:

1. Redraw the circuit (nothing about the Laplace transform changes the types of elements or their interconnections).
2. Any voltages or currents with values given are Laplace-transformed using the functional and operational tables.
3. Any voltages or currents represented symbolically, using i(t) and v(t), are replaced with the symbols I(s) and V(s).
4. All component values are replaced with the corresponding complex impedance, Z(s).
5. Use DC circuit analysis techniques to write the s-domain equations and solve them. Check your solutions with IVT and FVT.
6. Inverse-Laplace transform s-domain solutions to get time-

LaPlace Transform in Circuit Analysis

Example:

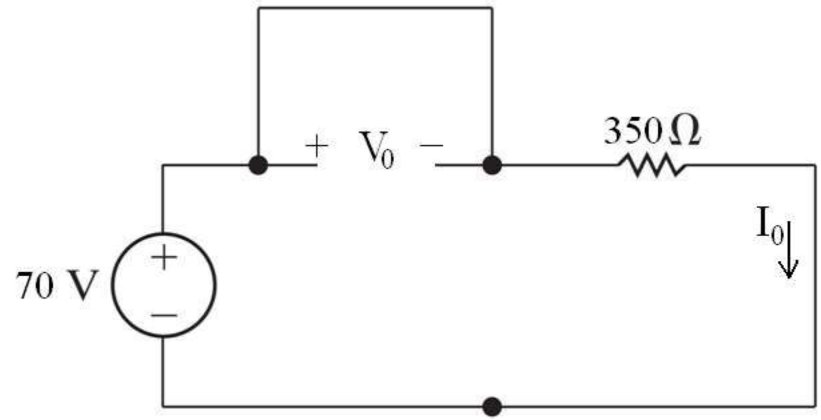
Find $v_0(t)$ for $t > 0$.



Begin by finding the initial conditions for this circuit.

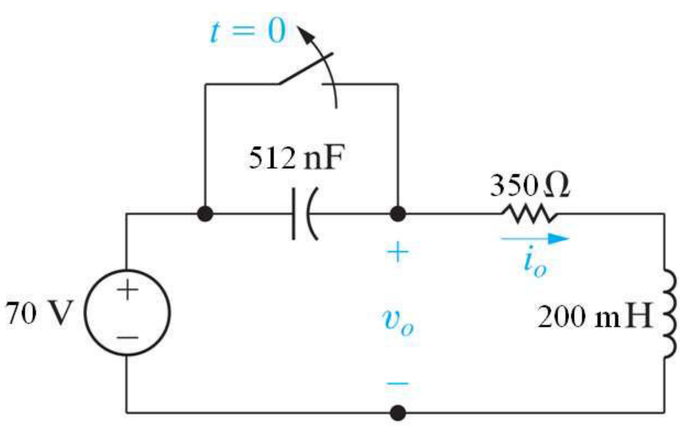
$$V_o = 0\text{ V}$$

$$I_o = \frac{70}{350} = 0.2\text{ A}$$



Give the basic interconnections of this circuit, should we use a voltage source or a current source to represent the initial condition for the inductor?

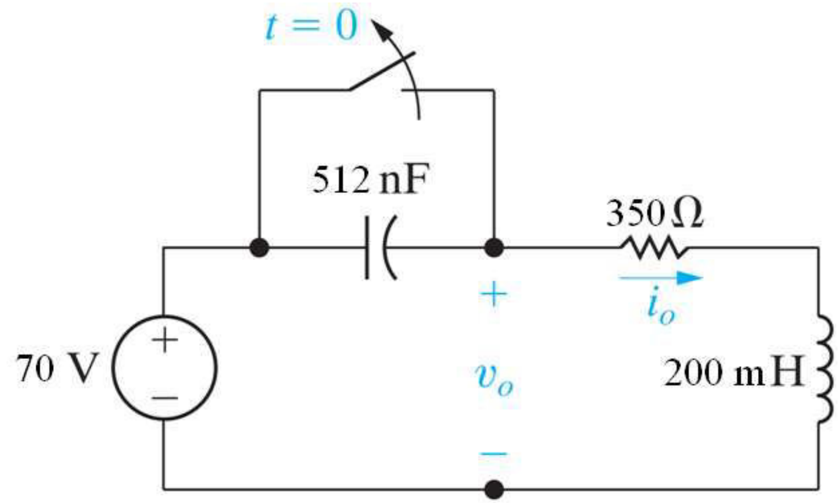
- A. Voltage source
- B. Current source
- C. Doesn't matter



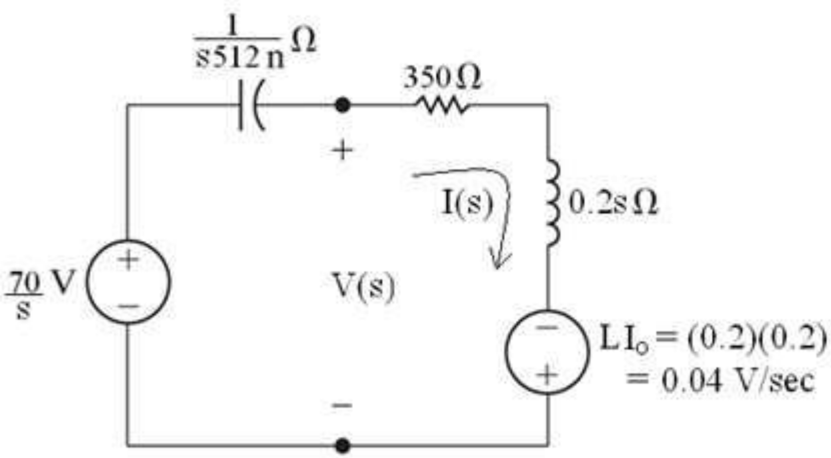
LaPlace Transform in Circuit Analysis

Example:

Find $v_0(t)$ for $t > 0$.



Laplace transform the circuit and solve for $V_0(s)$.



$$I(s) = \frac{70/s + 0.04}{1/s(512\text{n}) + 350 + 0.2s}$$

$$V(s) = (350 + 0.2s)I(s) - 0.04$$

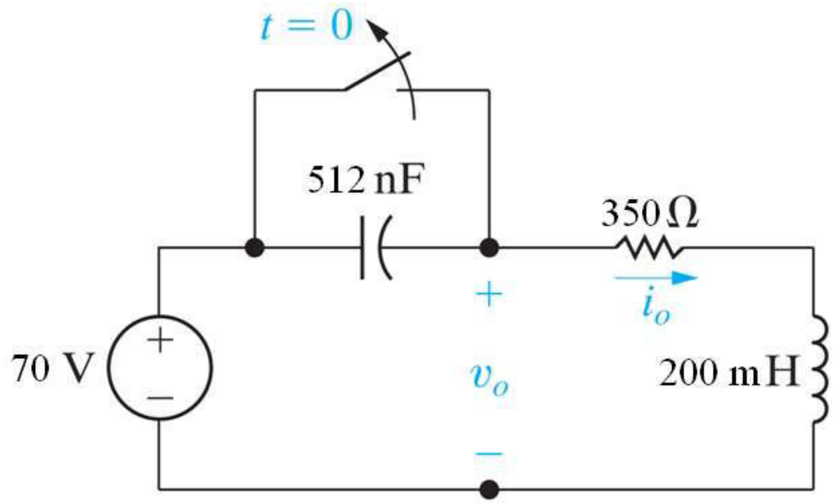
$$= \frac{(350 + 0.2s)(70/s + 0.04)}{1/s(512\text{n}) + 350 + 0.2s} - 0.04$$

$$= \frac{70s - 268{,}125}{}$$

LaPlace Transform in Circuit Analysis

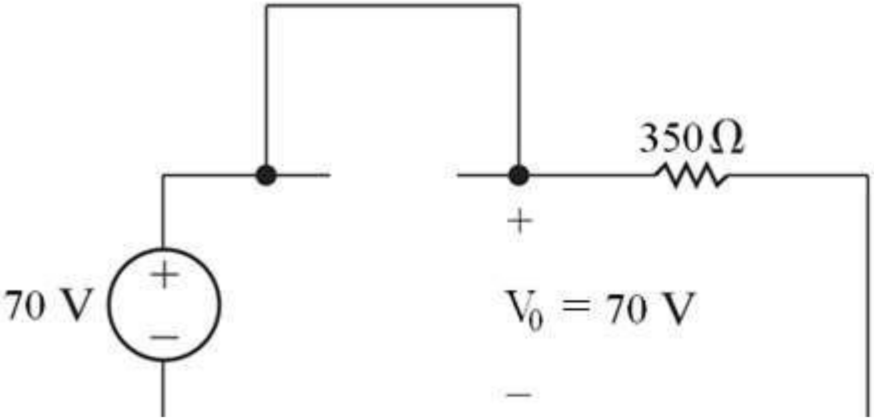
Example:

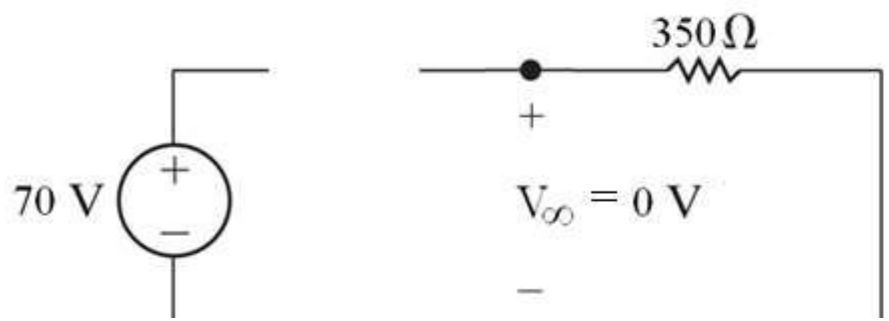
Find $v_0(t)$ for $t > 0$.



$$V_0(s) = \frac{70s - 268{,}125}{s^2 + 1750s + 9{,}765{,}625}$$

Use the IVT and FVT to check $V_0(s)$.

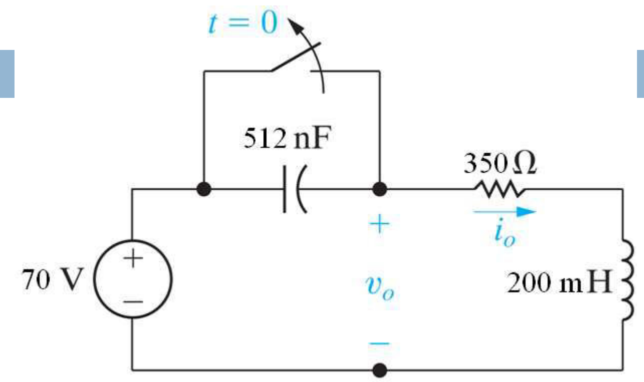




LaPlace Transform in Circuit Analysis

Example:

Find $v_0(t)$ for $t > 0$.



IVT

$$V_0(s) = \frac{70s - 268{,}125}{s^2 + 1750s + 9{,}765{,}625}$$

$$\lim_{t\to 0} v_o(t) = \lim_{s\to\infty} sV_o(s)$$

$$= \lim_{s\to\infty} \frac{70s^2 - 268{,}125s}{s^2 + 1750s + 9{,}765{,}625}$$

$$= \lim_{1/s\to 0} \frac{70 - 268{,}125/s}{1 + 1750/s + 9{,}765{,}625/s^2}$$

$$= \frac{70}{1} = 70\ \text{V(check!)}$$

FVT

$$V_0(s) = \frac{70s - 268{,}125}{s^2 + 1750s + 9{,}765{,}625}$$

$$\lim_{t\to\infty} v_o(t) = \lim_{s\to 0} sV_o(s)$$

$$= \lim_{s\to 0} \frac{70s^2 - 268{,}125s}{s^2 + 1750s + 9{,}765{,}625}$$

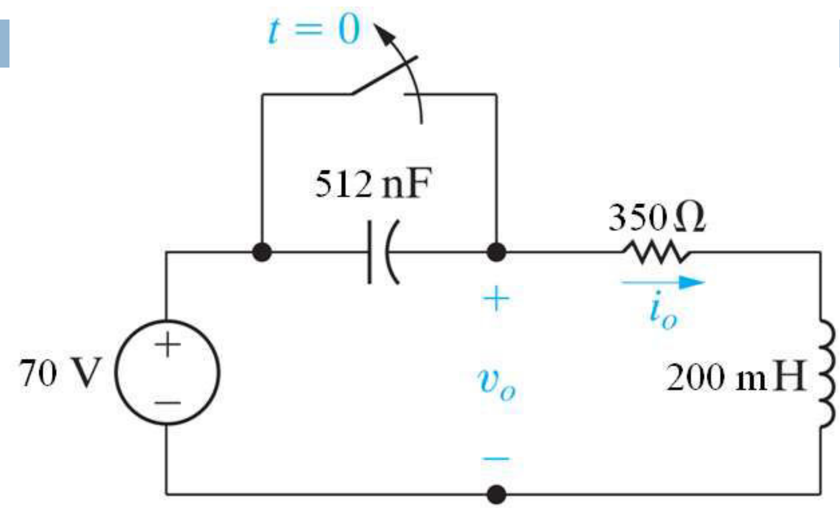
$$= \lim_{s\to 0} \frac{0}{9{,}765{,}625}$$

$$= 0\ \text{V(check!)}$$

LaPlace Transform in Circuit Analysis

Example:

Find $v_0(t)$ for t > 0.



$$V_0(s) = \frac{70s - 268{,}125}{(s+875-j3000)(s+875+j3000)}$$

Partial fraction expansion:

$$V_0(s) = \frac{K_1}{(s+875-j3000)} + \frac{K_2}{(s+875+j3000)}$$

$$K_1 = \left.\frac{70s - 268{,}125}{(s+875+j3000)}\right|_{s=-875+j3000} = \frac{70(-875+j3000)-268{,}125}{[(-875+j3000)+875+j3000]} = 65.1\angle 57.48°$$

$$K_2 = \left.\frac{70s - 268{,}125}{(s+875-j3000)}\right|_{s=-875-j3000} = \frac{70(-875-j3000)-268{,}125}{[(-875-j3000)+875+-j3000]} = 65.1\angle -57.48°$$

When two partial fraction denominators are complex conjugates, their numerators are

A. Equal
B. Unrelated
C. Complex conjugates

LaPlace Transform in Circuit Analysis

Aside – look at the inverse Laplace transform of partial fractions that are complex conjugates.

$$F(s)=\frac{10s}{s^2+2s+5}=\frac{K_1}{s+1-j2}+\frac{K_1^*}{s+1+j2}$$

$$K_1=\left.\frac{10s}{s+1+j2}\right|_{s=-1+j2}=\frac{10(-1+j2)}{-1+j2+1+j2}=5.59\angle 26.57^\circ$$

$$\therefore \qquad F(s)=\frac{5.59\angle 26.57^\circ}{s+1-j2}+\frac{5.59\angle -26.57^\circ}{s+1+j2}$$

$$\Rightarrow \qquad f(t)=5.59e^{j26.57^\circ}e^{-(1-j2)t}+5.59e^{-j26.57^\circ}e^{-(1+j2)t}$$

$$=5.59e^{-t}e^{j(2t+26.57^\circ)}+5.59e^{-t}e^{-j(2t+26.57^\circ)}$$

$$=5.59e^{-t}[\cos(2t+26.57^\circ)+j\sin(2t+26.57^\circ)]$$

$$+5.59e^{-t}[\cos(2t+26.57^\circ)-j\sin(2t+26.57^\circ)]$$

LaPlace Transform in Circuit Analysis

The parts of the time-domain expression come from a single partial fraction term:

$$F(s) = \frac{5.59\angle 26.57^\circ}{s+1-j2} + \frac{5.59\angle -26.57^\circ}{s+1+j2}$$

$$f(t) = 2(5.59)e^{-t}\cos(2t + 26.57^\circ)$$

Important – you must use the numerator of the partial fraction whose denominator has the negative imaginary part!

LaPlace Transform in Circuit Analysis

The general Laplace transform (from the table below the "Functional Transforms" table)

$$F(s) = \frac{|K|\angle\theta}{s+a-jb} + \frac{|K|\angle-\theta}{s+a-jb}$$

$$\mathcal{L}^{-1}\{F(s)\} = f(t) = 2|K|e^{-at}\cos(bt+\theta)$$

$$V_0(s) = \frac{[illegible]}{(s+875-j3000)} + \frac{[illegible]}{(s+875+j3000)}$$

The partial fraction expansion for $V_0(s)$ is shown above. When we inverse-Laplace transform, which partial fraction term should we use?

A. The first term
B. The second term
C. It doesn't matter

$$V_0(s) = \frac{[illegible]}{(s+875-j3000)} + \frac{[illegible]}{(s+875+j3000)}$$

The time-domain function for $v_o(t)$ will include a cosine at what frequency?

A. 875 rad/s

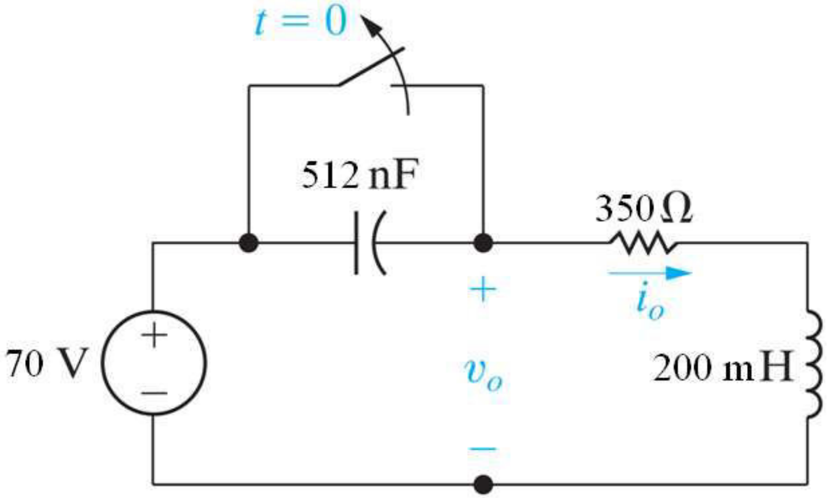
B. 130.2 rad/s

C. 3000 rad/s

LaPlace Transform in Circuit Analysis

Example:

Find $v_0(t)$ for $t > 0$.



$$V_0(s) = \frac{65.1\angle 57.48^\circ}{(s+875-j3000)} + \frac{65.1\angle -57.48^\circ}{(s+875+j3000)}$$

Inverse Laplace transform:

$$v_0(t) = 2(65.1)e^{-875t}\cos(3000t + 57.48^\circ) = 130.2e^{-875t}\cos(3000t + 57.48^\circ)\ \text{V}$$

Check at t = 0 and t → ∞:

$$v_0(0) = 130.2(1)\cos(57.48^\circ) = 70\ \text{V}$$

$$v_0(\infty) = 130.2(0)\cos(\ldots) = 0\ \text{V}$$

This example is a series RLC circuit. Its response form, repeated below, is characterized as:

$$v_0(t) = 130.2e^{-875t}\cos(3000t + 57.48°)\ \text{V}$$

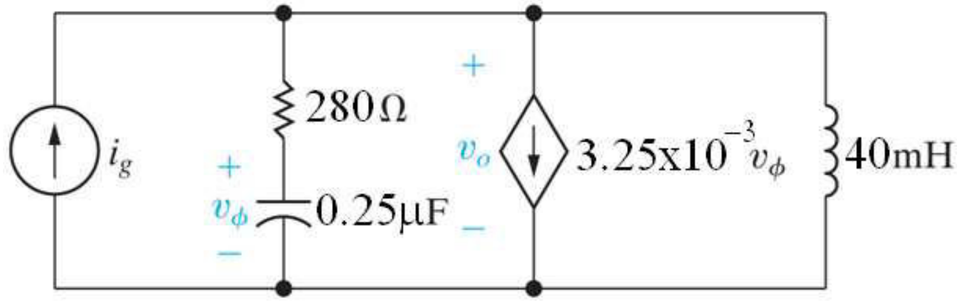
✓ A. Underdamped

✗ B. Overdamped

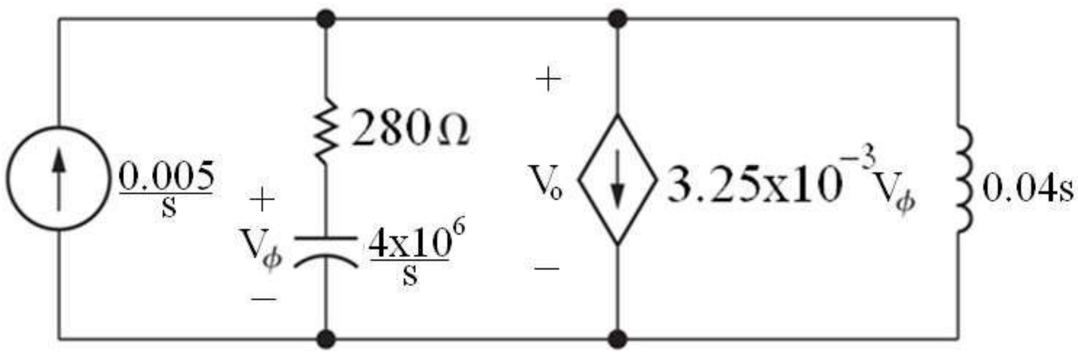
✗ C. Critically damped

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit.
Find v_o if $i_g = 5u(t)$ mA.

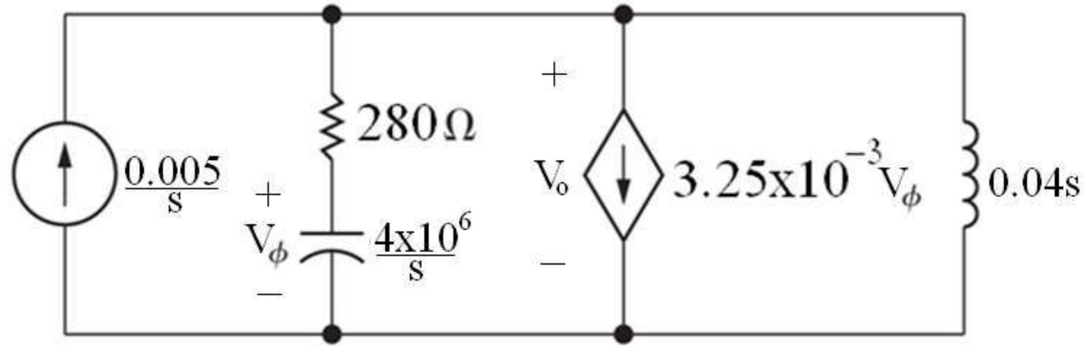


Laplace transform the circuit:



LaPlace Transform in Circuit Analysis

Example:
Find $V_o(s)$:



$$-\frac{0.005}{s}+\frac{V_o}{280+4\times10^6/s}+3.25\times10^{-3}V_\phi+\frac{V_o}{0.04s}=0$$ KCL at top node

$$V_\phi=\frac{4\times10^6/s}{280+4\times10^6/s}V_o=\frac{4\times10^6V_o}{280s+4\times10^6}$$ voltage division

$$\therefore \quad V_o\left[\frac{s}{280s+4\times10^6}+\frac{13{,}000}{280s+4\times10^6}+\frac{25}{s}\right]=\frac{0.005}{s}$$

$$\Rightarrow \quad V_o\left[\frac{s^2+13{,}000s+25\left(280s+4\times10^6\right)}{s\left(280s+4\times10^6\right)}\right]=\frac{0.005}{s}$$

$$\Rightarrow \quad V_o=\frac{1.4s+20{,}000}{\text{[illegible]}}$$

$$V_o = \frac{[illegible]}{s^2 + 20{,}000s + 10^8}$$

This s-domain expression has ____ zeros and ____ poles.

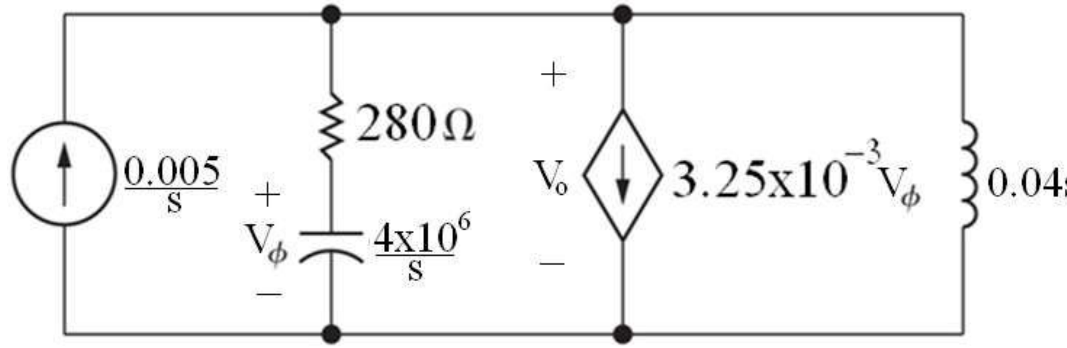
A. 0, 2

B. 1, 2

C. 2, 1

LaPlace Transform in Circuit Analysis

Example:
Check your s-domain answer:



IVT

$$V_0(s)=\frac{1.4s+20{,}000}{s^2+20{,}000s+10^8}$$

$$\lim_{t\to 0} v_0(t)=\lim_{s\to\infty} sF(s)$$

$$=\lim_{s\to\infty}\frac{1.4s^2+20{,}000s}{s^2+20{,}000s+10^8}$$

$$=\lim_{1/s\to 0}\frac{1.4+20{,}000/s}{1+20{,}000/s+10^8/s^2}$$

FVT

$$V_0(s)=\frac{1.4s+20{,}000}{s^2+20{,}000s+10^8}$$

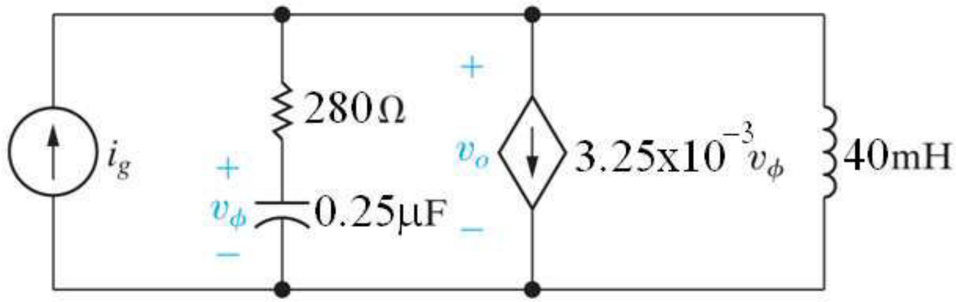
$$\lim_{t\to\infty} v_0(t)=\lim_{s\to 0} sF(s)$$

$$=\lim_{s\to 0}\frac{1.4s^2+20{,}000s}{s^2+20{,}000s+10^8}$$

$$=\frac{0}{\ldots}=0\text{ V}$$

Warning – this one's tricky!

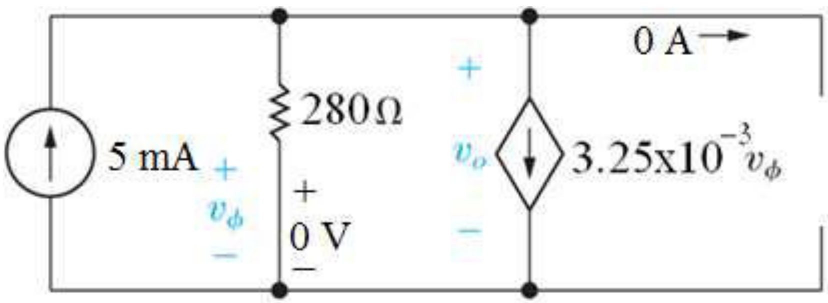
Just after t = 0, there is no initial stored energy in the circuit. Therefore, the capacitor behaves like a _____ and the inductor behaves like a ____.



- A. Open circuit/short circuit
- B. Open circuit/open circuit
- C. Short circuit/short circuit
- D. Short circuit/open circuit

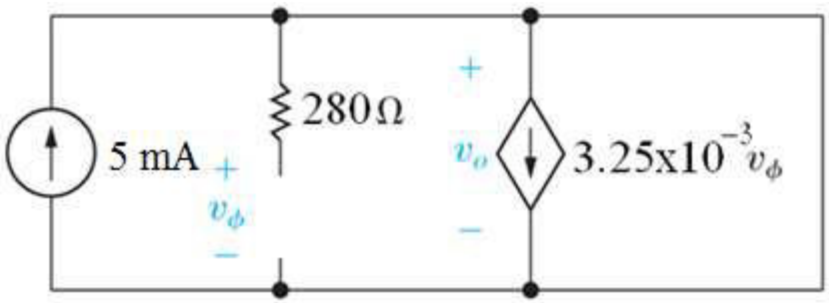
LaPlace Transform in Circuit Analysis

For t = 0



$$v_0(0) = (0.005)(280)$$
$$= 1.4\text{ V (check!)}$$

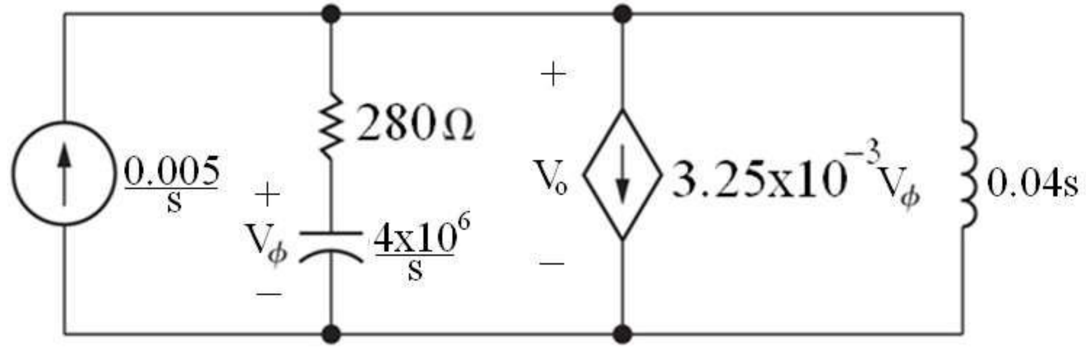
For $t \to \infty$



$$v_0(0) = 0\text{ V}$$
(it is the voltage across a wire!)

LaPlace Transform in Circuit Analysis

Example:
Partial fraction expansion:



$$V_0(s) = \frac{1.4s + 20{,}000}{s^2 + 20{,}000s + 10^8} = \frac{1.4s + 20{,}000}{(s + 10{,}000)^2}$$

$$= \frac{K_1}{(s + 10{,}000)^2} + \frac{K_2}{(s + 10{,}000)}$$

$$V_0(s) = \frac{K_1}{(s+10{,}000)^2} + \frac{K_2}{(s+10{,}000)}$$

In the partial fraction expansion given here, K_1 and K_2 are

A. Both real numbers
B. Complex conjugates
C. Need more information

LaPlace Transform in Circuit Analysis

Aside – find the partial fraction expansion when there are repeated real roots.

$$F(s) = \frac{4s^2 + 7s + 1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1}$$

$$K_1 = \left.\frac{4s^2 + 7s + 1}{(s+1)^2}\right|_{s=0} = \frac{1}{1} = 1$$

$$K_2 = \left.\frac{4s^2 + 7s + 1}{s}\right|_{s=-1} = \frac{4 - 7 + 1}{-1} = 2$$

$$K_3 = \left.\frac{4s^2 + 7s + 1}{s(s+1)}\right|_{s=-1} = \frac{4 - 7 + 1}{(-1)(0)} = \text{undefined!}$$

LaPlace Transform in Circuit Analysis

Aside – find the partial fraction expansion when there are repeated real roots. How do we find the coefficient of the term with just one copy of the repeated root?

$$(s+1)^2F(s)=\frac{K_1(s+1)^2}{s}+\frac{K_2(s+1)^2}{(s+1)^2}+\frac{K_3(s+1)^2}{s+1}$$

Eliminate these two terms

Keep this term!

$$\frac{d}{ds}\left[(s+1)^2F(s)\right]\bigg|_{s=-1}=\frac{d}{ds}\left[\frac{K_1(s+1)^2}{s}\right]\bigg|_{s=-1}+\frac{d}{ds}\left[\frac{K_2(s+1)^2}{(s+1)^2}\right]\bigg|_{s=-1}+\frac{d}{ds}\left[\frac{K_3(s+1)^2}{s+1}\right]\bigg|_{s=-1}$$

= 0 because the derivative still has

= 0 because the derivative of a

= K_3 because the derivative of

LaPlace Transform in Circuit Analysis

Aside – find the partial fraction expansion when there are repeated real roots.

$$F(s) = \frac{4s^2+7s+1}{s(s+1)^2} = \frac{K_1}{s} + \frac{K_2}{(s+1)^2} + \frac{K_3}{s+1}$$

$$K_1 = \left.\frac{4s^2+7s+1}{(s+1)^2}\right|_{s=0} = \frac{4(0)^2+7(0)+1}{(0+1)} = 1$$

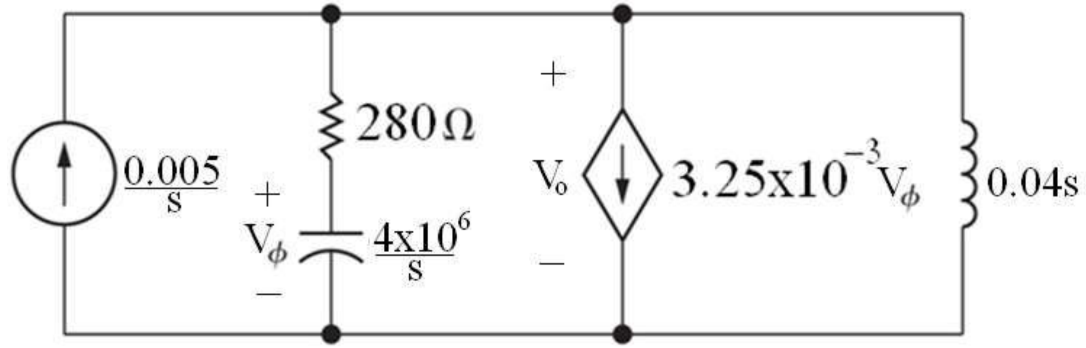
$$K_2 = \left.\frac{4s^2+7s+1}{s}\right|_{s=-1} = \frac{4(-1)^2+7(-1)+1}{(-1)} = 2$$

$$K_3 = \frac{d}{ds}\left[\frac{4s^2+7s+1}{s}\right]\bigg|_{s=-1} = \left[\frac{8s+7}{s} - \frac{4s^2+7s+1}{s^2}\right]\bigg|_{s=-1}$$

$$= \frac{8(-1)+7}{(-1)} - \frac{4(-1)^2+7(-1)+1}{(-1)^2} = 3$$

LaPlace Transform in Circuit Analysis

Back to the example; find the partial fraction expansion:



$$V_0(s) = \frac{1.4s + 20{,}000}{(s+10{,}000)^2} = \frac{K_1}{(s+10{,}000)^2} + \frac{K_2}{(s+10{,}000)}$$

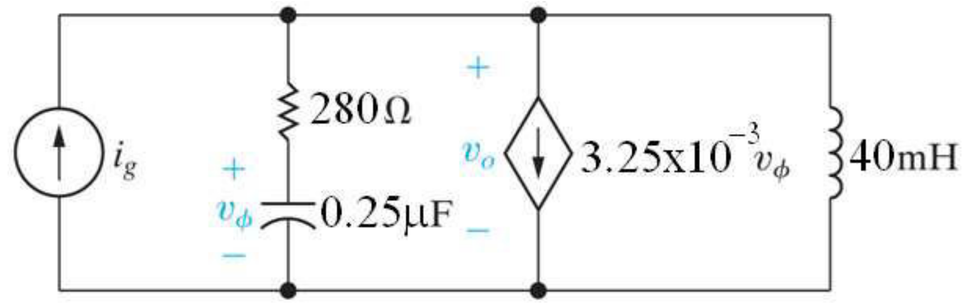
$$K_1 = 1.4s + 20{,}000\Big|_{s=-10{,}000} = 6000$$

$$K_2 = \frac{d}{ds}\left[1.4s + 20{,}000\right]\Big|_{s=-10{,}000} = 1.4$$

LaPlace Transform in Circuit Analysis

Example:

Find $v_0(t)$ for $t > 0$.



Inverse Laplace transform the result in the s-domain to get the time-domain result:

$$V_0(s) = \frac{6000}{(s+10{,}000)^2} + \frac{1.4}{(s+10{,}000)}$$

$$v_0(t) = \left[6000te^{-10{,}000t} + 1.4e^{-10{,}000t}\right]u(t) \text{ V (see the Laplace tables)}$$

$$v_0(0) = 1.4 \text{ V (check!)}$$

$$v_0(\infty) = 0 \text{ V (check!)}$$

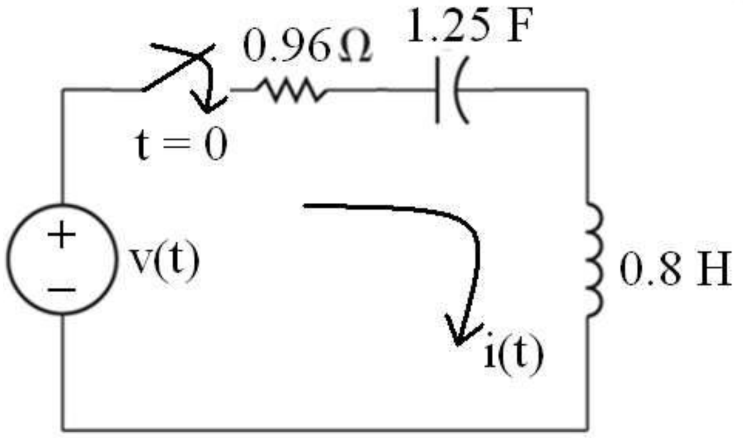
$$v_o(t) = [6000te^{-10,000t} + 1.4e^{-10,000t}]u(t) \text{ V}$$

We have seen this response form in our analysis of second-order RLC circuits; it is called:

A. Overdamped
B. Underdamped
C. Critically damped

LaPlace Transform in Circuit Analysis

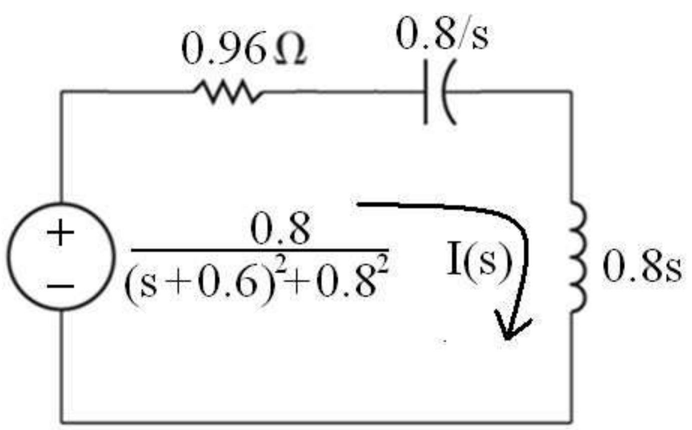
Example:
There is no initial energy stored in this circuit. Find i(t) if $v(t) = e^{-0.6t}\sin 0.8t$ V.



Laplace transform the circuit:

$$\mathcal{L}\left[e^{-0.6t}\sin 0.8t\right] = \frac{0.8}{(s+0.6)^2+0.8^2}$$

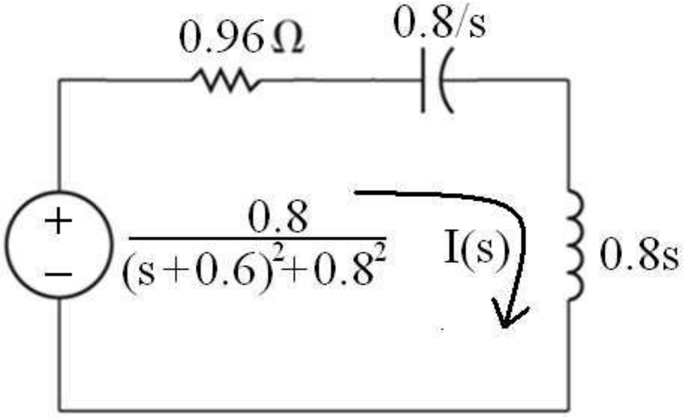
$$= \frac{0.8}{s^2+1.2s+1}$$



LaPlace Transform in Circuit Analysis

Example:

Find I(s):



$$\left(0.96+\frac{0.8}{s}+0.8s\right)I(s)=\frac{0.8}{s^2+1.2s+1}$$

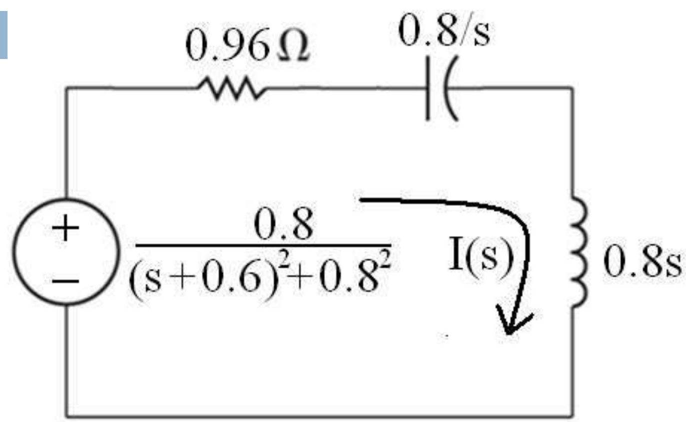
$$\therefore\left(\frac{0.8s^2+0.96s+0.8}{s}\right)I(s)=\frac{0.8}{s^2+1.2s+1}$$

$$\Rightarrow \qquad I(s)=\frac{s}{\left(s^2+1.2s+1\right)}$$

LaPlace Transform in Circuit Analysis

Example:

Check your s-domain answer:



IVT

$$I(s) = \frac{s}{(s^2 + 1.2s + 1)^2}$$

$$\lim_{t\to 0} i(t) = \lim_{s\to\infty} sI(s)$$

$$= \lim_{s\to\infty} \frac{s^2}{(s^2 + 1.2s + 1)^2}$$

$$= \lim \frac{1/s^2}{\quad} = 0$$

FVT

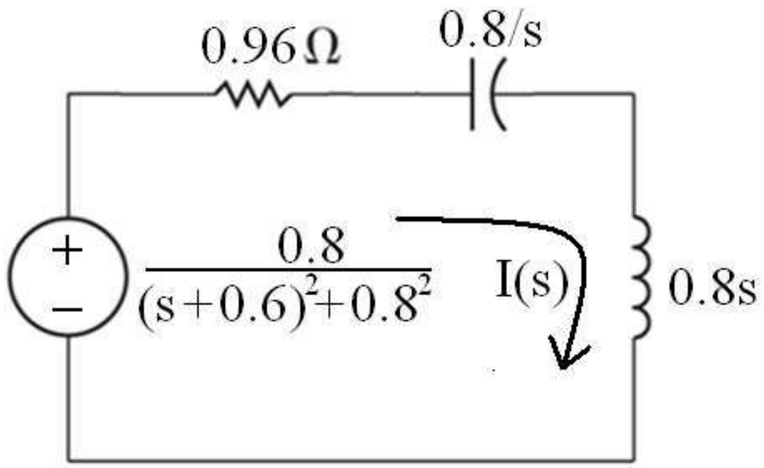
$$I(s) = \frac{s}{(s^2 + 1.2s + 1)^2}$$

$$\lim_{t\to\infty} i(t) = \lim_{s\to 0} sI(s)$$

$$= \lim_{s\to 0} \frac{s^2}{(s^2 + 1.2s + 1)^2} = 0$$

LaPlace Transform in Circuit Analysis

Example:
Partial fraction expansion:

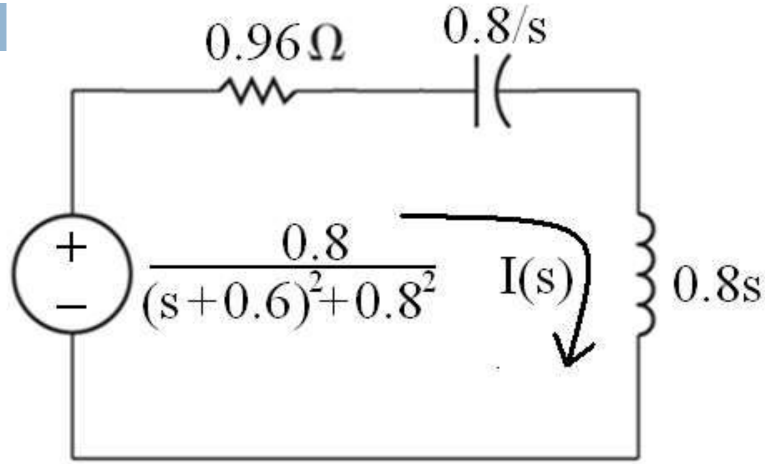


$$I(s)=\frac{s}{(s^2+1.2s+1)^2}=\frac{K_1}{(s+0.6-j0.8)^2}+\frac{K_2}{(s+0.6-j0.8)}$$

$$+\frac{K_1^*}{(s+0.6+j0.8)^2}+\frac{K_2^*}{(s+0.6+j0.8)}$$

LaPlace Transform in Circuit Analysis

Partial fraction expansion, continued:



$$I(s) = \frac{K_1}{(s+0.6-j0.8)^2} + \frac{K_2}{(s+0.6-j0.8)} + \ldots$$

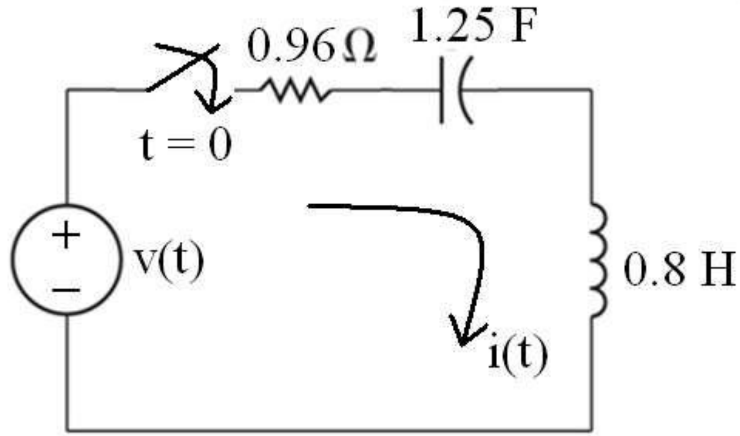
$$K_1 = \left.\frac{s}{(s+0.6+j0.8)^2}\right|_{s=-0.6+j0.8} = \frac{-0.6+j0.8}{(-0.6+j0.8+0.6+j0.8)^2} = 0.39\angle -53.13^\circ$$

$$K_2 = \frac{d}{ds}\left[\frac{s}{(s+0.6+j0.8)^2}\right] = \left[\frac{1}{(s+0.6+j0.8)^2} - \left[\frac{2s}{(s+0.6+j0.8)^3}\right]\right]_{s=-0.6+j0.8}$$

$$= \frac{1}{\quad} - \frac{2(-0.6+j0.8)}{\quad} = 0.29\angle -90^\circ$$

LaPlace Transform in Circuit Analysis

Example:
There is no initial energy stored in this circuit. Find i(t) if $v(t) = e^{-0.6t}\sin 0.8t$ V.



Inverse Laplace transform the result in the s-domain to get the time-domain result:

$$I(s) = \frac{0.39\angle -53.13^\circ}{(s+0.6-j0.8)^2} + \frac{0.29\angle 90^\circ}{(s+0.6-j0.8)} + \ldots$$

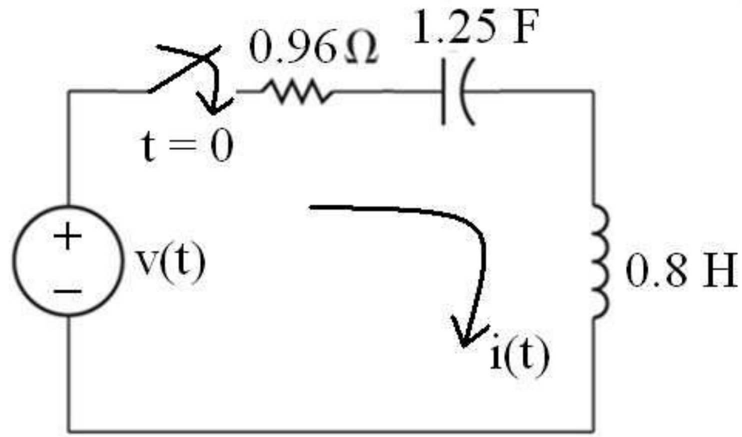
$$i(t) = 2(0.39)te^{-0.6t}\cos(0.8t-53.13^\circ) + 2(0.29)e^{-0.6t}\cos(0.8t+90^\circ)$$

$$= \left[0.78te^{-0.6t}\cos(0.8t-53.13^\circ) + 0.58e^{-0.6t}\cos(0.8t+90^\circ)\right]u(t) \text{ A}$$

Which term of the solution represents the forced response?

Example:

There is no initial energy stored in this circuit. Find i(t) if $v(t) = e^{-0.6t}\sin 0.8t$ V.



$$i(t) = [0.78te^{-0.6t}\cos(0.8t - 53.13°) + 0.58e^{-0.6t}\cos(0.8t + 90°)]u(t) \text{ A}$$

A. First term

B. Second term

C. Neither

LaPlace Transform in Circuit Analysis

Recipe for Laplace transform circuit analysis:

1. Redraw the circuit – note that you need to find the initial conditions and decide how to represent them in the circuit.
2. Any voltages or currents with values given are Laplace-transformed using the functional and operational tables.
3. Any voltages or currents represented symbolically, using i(t) and v(t), are replaced with the symbols I(s) and V(s).
4. All component values are replaced with the corresponding complex impedance, Z(s), and the appropriate source representing initial conditions.
5. Use DC circuit analysis techniques to write the s-domain equations and solve them. Check your solutions with IVT and FVT.
6. Inverse-Laplace transform s-domain solutions (using the partial fraction expansion technique and the Laplace tables) to get time-domain solutions. Check your solutions at $t = 0$ and $t = \infty$.

LaPlace Transform in Circuit Analysis

Aside – How do you inverse Laplace transform F(s) if it is an improper rational function? (Note – this won't happen in linear circuits, but can happen in other systems modeled with differential equations!)

Example:

$$\mathcal{L}^{-1}\left\{\frac{s^2+6s+7}{(s+1)(s+2)}\right\} \qquad (\text{Note:}\quad \mathrm{O}\{\mathrm{D}(s)\} > \mathrm{O}\{\mathrm{N}(s)\}\ \text{does not hold!})$$

See next slide!

LaPlace Transform in Circuit Analysis

$$\mathcal{L}^{-1}\left\{\frac{s^2+6s+7}{(s+1)(s+2)}\right\} \qquad \text{(Note: } O\{D(s)\} > O\{N(s)\} \text{ does not hold!)}$$

$$\begin{array}{r} 1 \\ s^2+3s+2 \overline{\big) s^2+6s+7} \\ \underline{-\,s^2+3s+2} \\ 3s+5 \end{array}$$

$$\Rightarrow \frac{s^2+6s+7}{(s+1)(s+2)} = 1 + \frac{3s+5}{(s+1)(s+2)} = 1 + \frac{K_1}{(s+1)} + \frac{K_2}{(s+2)}$$

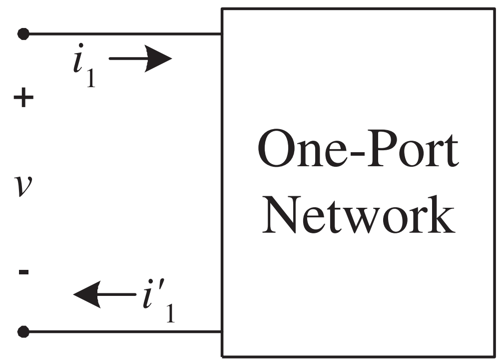
$$K_1 = \left.\frac{3s+5}{(s+2)}\right|_{s=-1} = 2; \qquad K_2 = \left.\frac{3s+5}{(s+1)}\right|_{s=-2} = 1$$

$$\mathcal{L}^{-1}\left\{1 + \frac{2}{(s+1)} + \frac{1}{(s+2)}\right\} = \delta(t) + \left[2e^{-t} + e^{-2t}\right]u(t)$$

Two-Port Networks

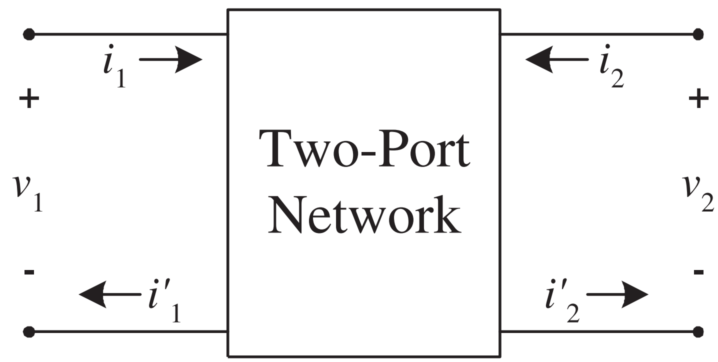
- Definitions
- Impedance Parameters
- Admittance Parameters
- Hybrid Parameters
- Transmission Parameters
- Cascaded Two-Port Networks
- Examples
- Applications

One-Port Networks



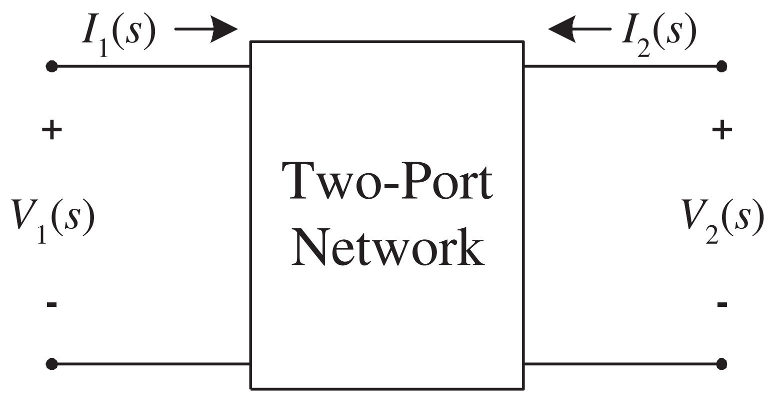
- A pair of terminals at which a signal (voltage or current) may enter or leave is called a **port**
- A network having only one such pair of terminals is called a **one-port network**
- No connections may be made to any other nodes internal to the network
- By KCL, we therefore have $i_1 = i'_1$
- We discussed in ECE 221 how one-port networks may be modeled by their Thévenin or Norton equivalents

Two-Port Networks: Definitions & Requirements



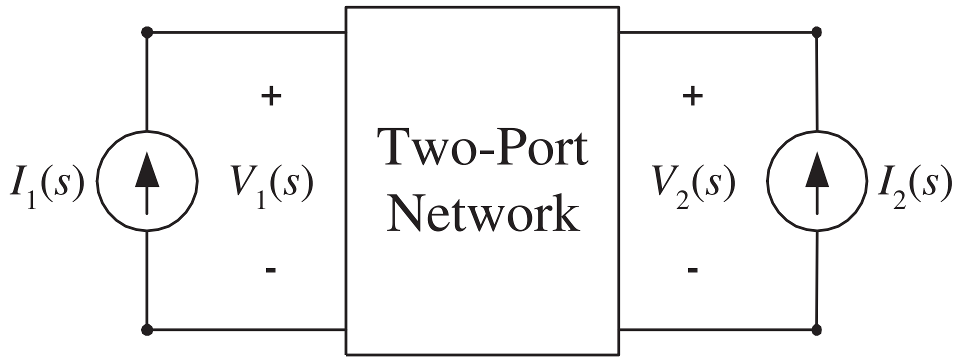
- Two-port networks are used to describe the relationship between a pair of terminals
- The analysis methods we will discuss require the following conditions be met
  1. Linearity
  2. No independent sources inside the network
  3. No stored energy inside the network (zero initial conditions)
  4. $i_1 = i'_1$ and $i_2 = i'_2$

Two-Port Networks: Defining Equations



- If the network contains dependent sources, one or more of the equivalent resistors may be negative
- Generally, the network is analyzed in the s domain
- Each two-port has exactly two governing equations that can be written in terms of any pair of network variables
- Like Thévenin and Norton equivalents of one-ports, once we know a set of governing equations we no longer need to know what is inside the box

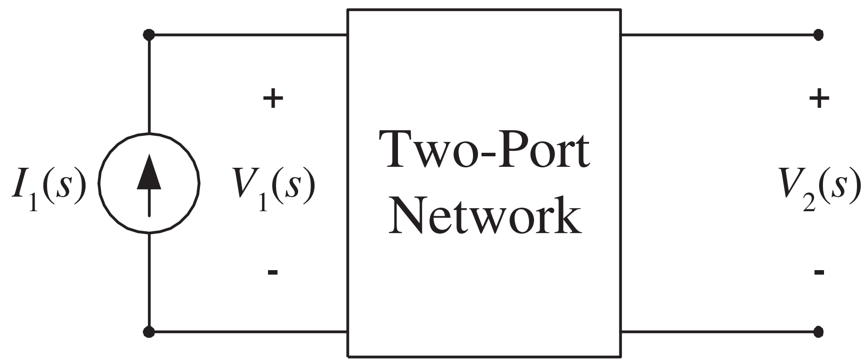
Impedance Parameters



$$
\begin{aligned}
V_1 &= z_{11}I_1 + z_{12}I_2 \\
V_2 &= z_{21}I_1 + z_{22}I_2
\end{aligned}
\qquad
\begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}
$$

- Suppose the currents and voltages can be measured
- Alternatively, if the circuit in the box is known, V_1 and V_2 can be calculated based on circuit analysis
- Relationship can be written in terms of the **impedance** parameters
- We can also calculate the impedance parameters after making two sets of measurements

Impedance Parameter Measurements

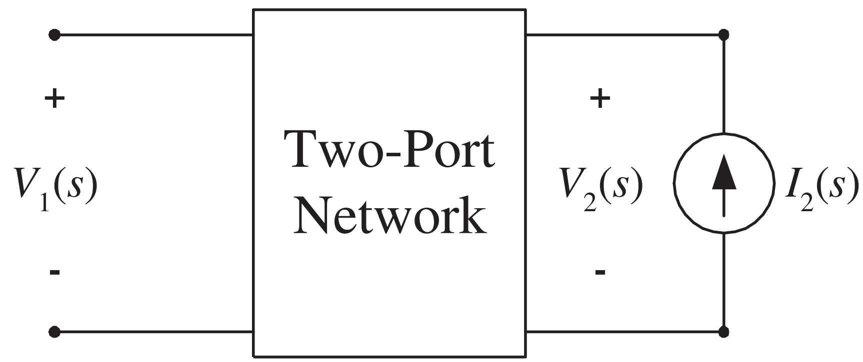


$$\begin{aligned} V_1 &= z_{11} I_1 + z_{12} I_2 \\ V_2 &= z_{21} I_1 + z_{22} I_2 \end{aligned}$$

If the right port is an open circuit $(I_2 = 0)$, then we can easily solve for two of the impedance parameters:

$$z_{11} = \left. \frac{V_1}{I_1} \right|_{I_2=0} \qquad z_{21} = \left. \frac{V_2}{I_1} \right|_{I_2=0}$$

Impedance Parameter Measurements Continued

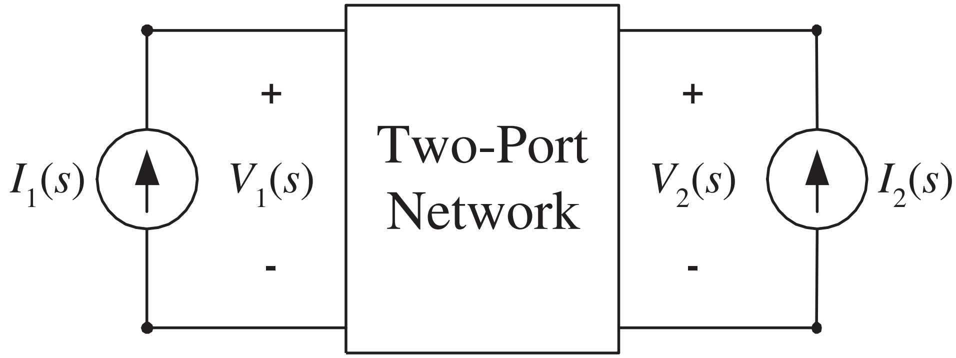


$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned}$$

If the left port is an open circuit $(I_1 = 0)$, then we can easily solve for the other two impedance parameters:

$$z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

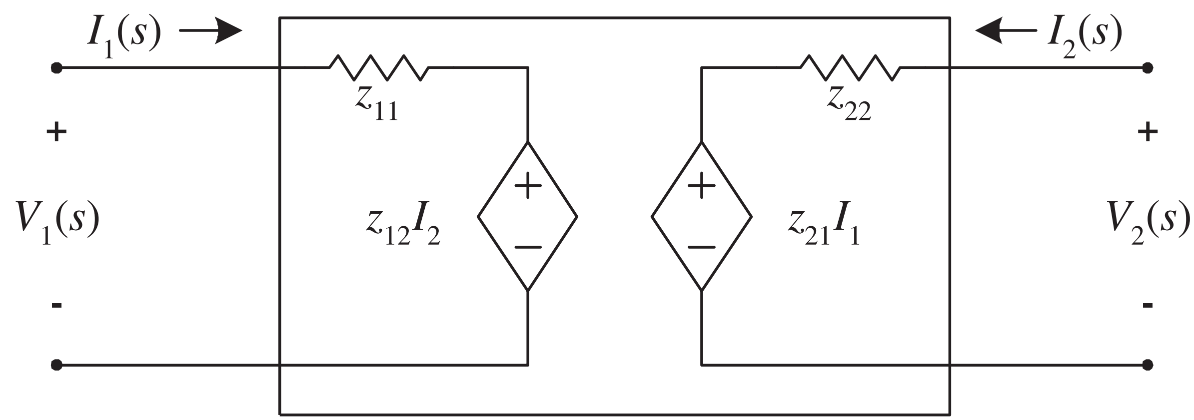
Impedance Parameter Measurements Summarized



$$z_{11} = \left.\frac{V_1}{I_1}\right|_{I_2=0} \qquad z_{12} = \left.\frac{V_1}{I_2}\right|_{I_1=0}$$

$$z_{21} = \left.\frac{V_2}{I_1}\right|_{I_2=0} \qquad z_{22} = \left.\frac{V_2}{I_2}\right|_{I_1=0}$$

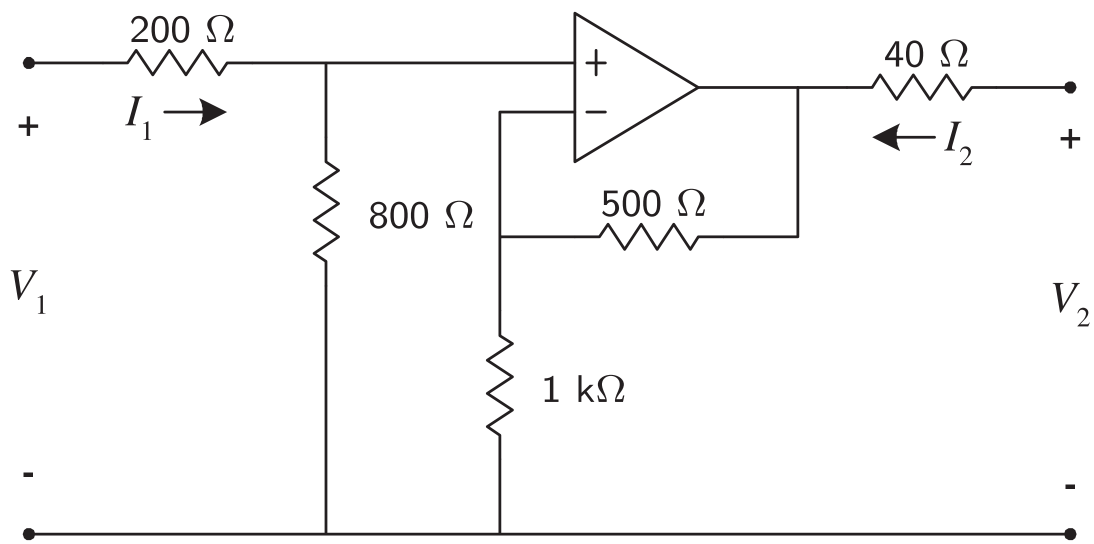
Impedance Parameter Equivalent



$$
\begin{aligned}
V_1 &= z_{11}I_1 + z_{12}I_2 \\
V_2 &= z_{21}I_1 + z_{22}I_2
\end{aligned}
$$

- Once we know what the impedance parameters are, we can model the behavior of the two-port with an equivalent circuit.
- Notice the similarity to Thévenin and Norton equivalents

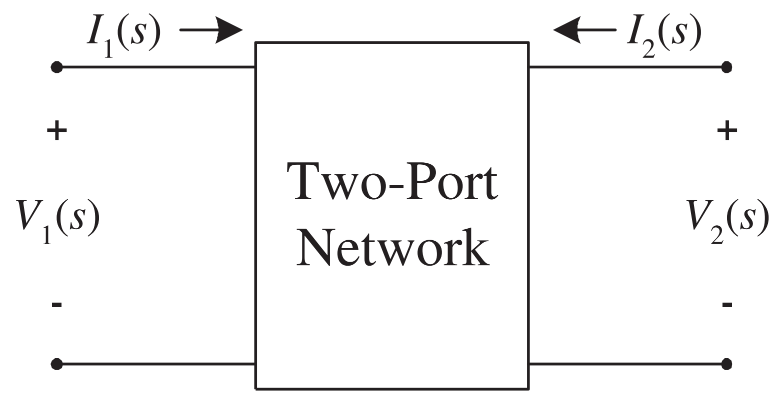
Example 1: Impedance Parameters



Find the z parameters of the circuit.

Example 1: Workspace

Example 2: Parameter Conversion



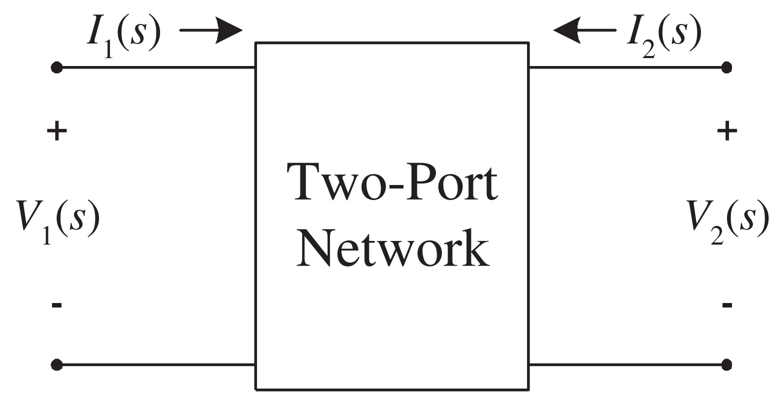
$$V_1 = z_{11}I_1 + z_{12}I_2$$
$$V_2 = z_{21}I_1 + z_{22}I_2$$

In general, the two defining equations can be written in terms of any pair of variables. For example, rewrite the defining equations in terms of the voltages V_1 and V_2.

Example 2: Workspace

Example 2: Workspace Continued

Impedance & Admittance Parameters



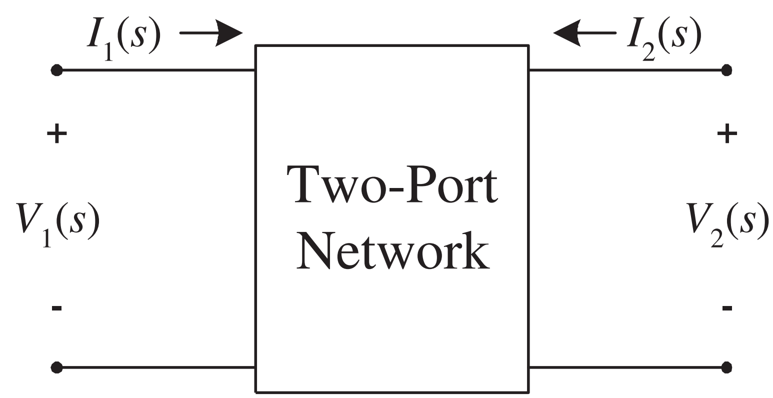
Impedance Parameters

$$\begin{aligned} V_1 &= z_{11}I_1 + z_{12}I_2 \\ V_2 &= z_{21}I_1 + z_{22}I_2 \end{aligned} \qquad \begin{bmatrix} V_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} z_{11} & z_{12} \\ z_{21} & z_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ I_2 \end{bmatrix}$$

Admittance Parameters

$$\begin{aligned} I_1 &= y_{11}V_1 + y_{12}V_2 \\ I_2 &= y_{21}V_1 + y_{22}V_2 \end{aligned} \qquad \begin{bmatrix} I_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} y_{11} & y_{12} \\ y_{21} & y_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ V_2 \end{bmatrix}$$

Hybrid Parameters



Hybrid Parameters

$$V_1 = h_{11}I_1 + h_{12}V_2$$
$$I_2 = h_{21}I_1 + h_{22}V_2$$

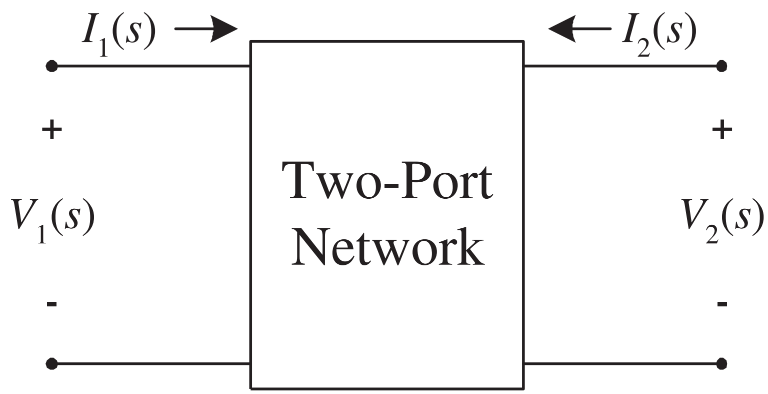
$$\begin{bmatrix} V_1 \\ I_2 \end{bmatrix} = \begin{bmatrix} h_{11} & h_{12} \\ h_{21} & h_{22} \end{bmatrix} \begin{bmatrix} I_1 \\ V_2 \end{bmatrix}$$

Inverse Hybrid Parameters

$$I_1 = g_{11}V_1 + g_{12}I_2$$
$$V_2 = g_{21}V_1 + g_{22}I_2$$

$$\begin{bmatrix} I_1 \\ V_2 \end{bmatrix} = \begin{bmatrix} g_{11} & g_{12} \\ g_{21} & g_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ I_2 \end{bmatrix}$$

Transmission Parameters



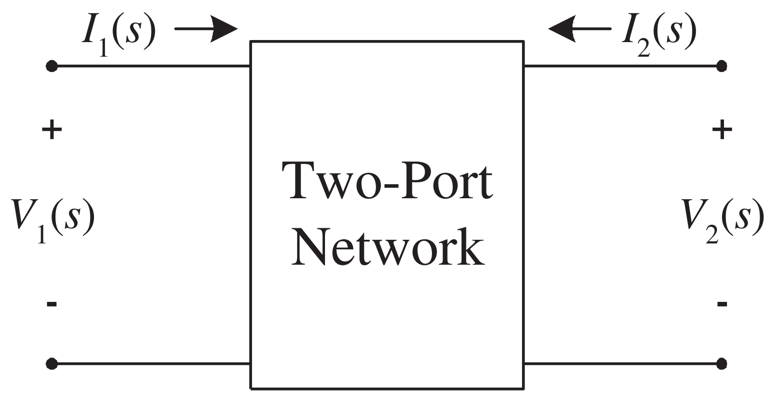
Transmission Parameters

$$V_1 = a_{11}V_2 - a_{12}I_2 \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & b_{12} \\ a_{21} & a_{22} \end{bmatrix} \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix} = A \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$
$$I_1 = a_{21}V_2 - a_{22}I_2$$

Inverse Transmission Parameters

$$V_2 = b_{11}V_1 - b_{12}I_1 \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix} \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix} = B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$
$$I_2 = b_{21}V_1 - b_{22}I_1$$

Transmission Parameter Conversion



- Altogether there are 6 sets of parameters
- Each set completely describes the two-port network
- Any set of parameters can be converted to any other set
- We have seen one example of a conversion
- A complete table of conversions is listed in the text (Pg. 933)
- You should have a copy of this in your notes for the final

Example 3: Two-Port Measurements

The following measurements were taken from a two-port network. Find the transmission parameters.

Port 2 Open

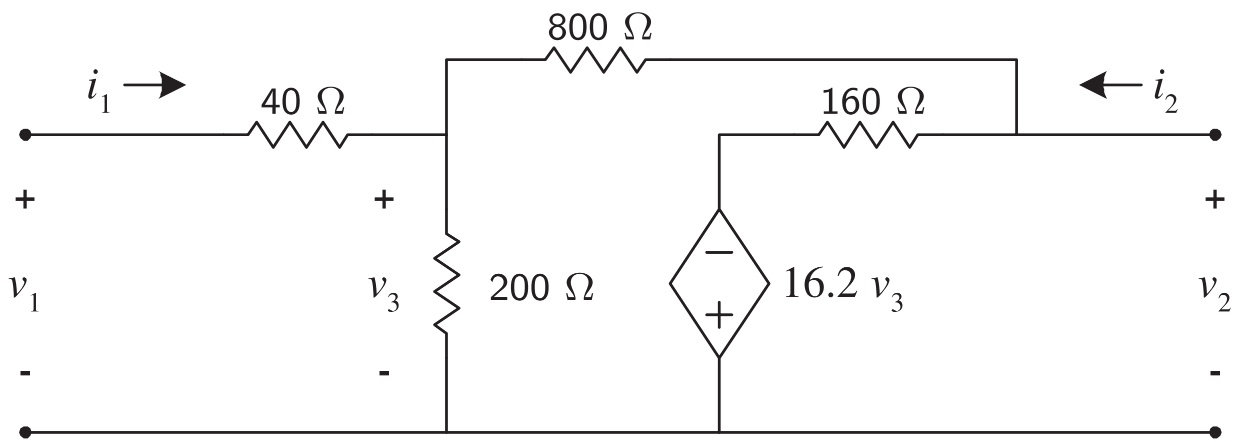
$$\begin{aligned} V_1 &= 150\cos(4000t) \text{ V applied} \\ I_1 &= 25\cos(4000t - 45^\circ) \text{ A measured} \\ V_2 &= 1000\cos(4000t + 15^\circ) \text{ V measured} \end{aligned}$$

Port 2 Shorted

$$\begin{aligned} V_1 &= 30\cos(4000t) \text{ V applied} \\ I_1 &= 1.5\cos(4000t + 30^\circ) \text{ A measured} \\ I_2 &= 0.25\cos(4000t + 150^\circ) \text{ A measured} \end{aligned}$$

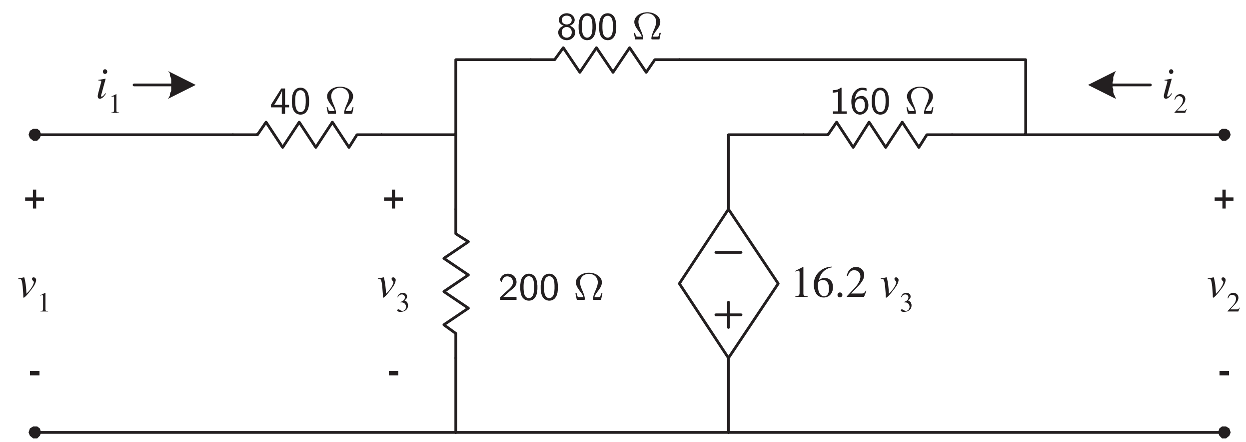
Example 3: Workspace

Example 4: Two-Port Analysis



Find the hybrid parameters for the circuit shown above.

Example 4: Workspace



Example 4: Workspace Continued

Example 5: Two-Port Measurements

The following measurements were taken from a two-port network. Find the transmission parameters.

Port 1 Open

$$V_1 = 1 \text{ mV}$$
$$V_2 = 10 \text{ V}$$
$$I_2 = 200 \ \mu\text{A}$$

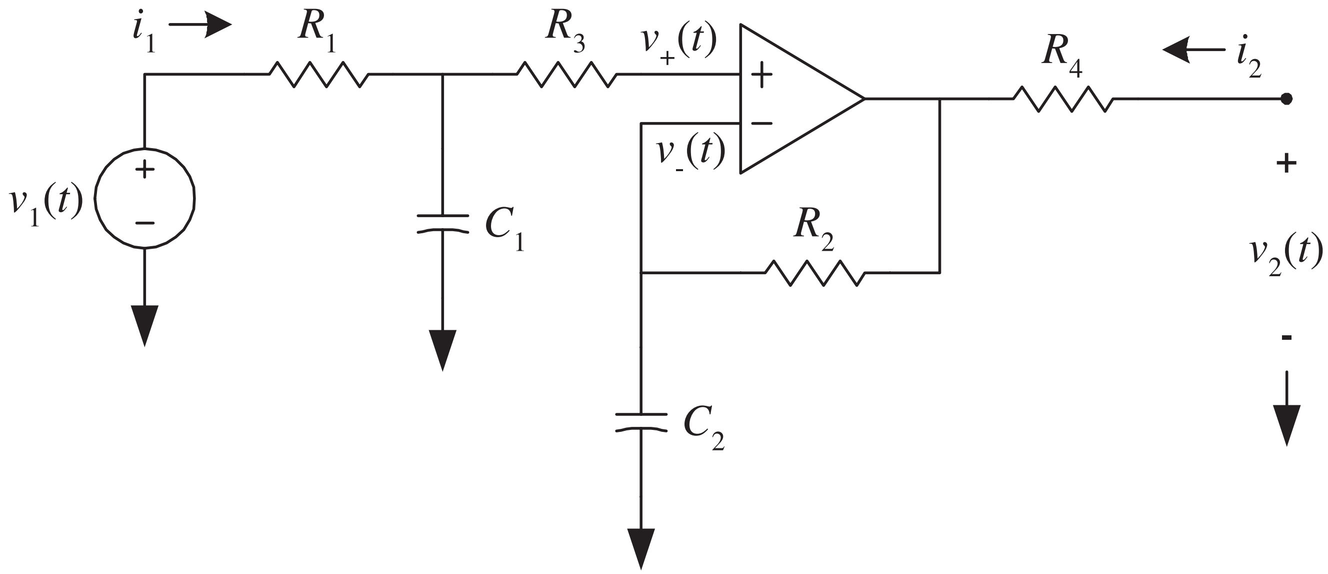
Port 1 Shorted

$$I_1 = -0.5 \ \mu\text{A}$$
$$I_2 = 80 \ \mu\text{A}$$
$$V_2 = 5 \text{ V}$$

Hint: $\triangle_b = b_{11}b_{22} - b_{12}b_{21}$, $a_{11} = \frac{b_{22}}{\triangle_b}$, $a_{12} = \frac{b_{12}}{\triangle_b}$, $a_{21} = \frac{b_{21}}{\triangle_b}$, and $a_{22} = \frac{b_{11}}{\triangle_b}$.

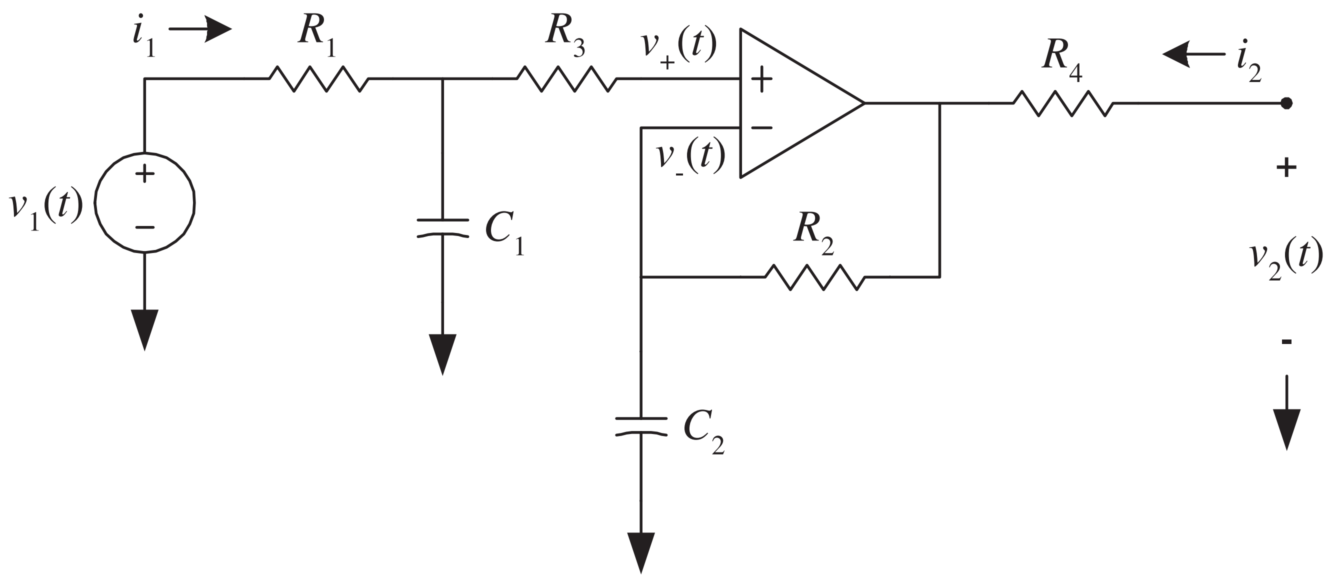
Example 5: Workspace

Example 6: Two-Port Analysis

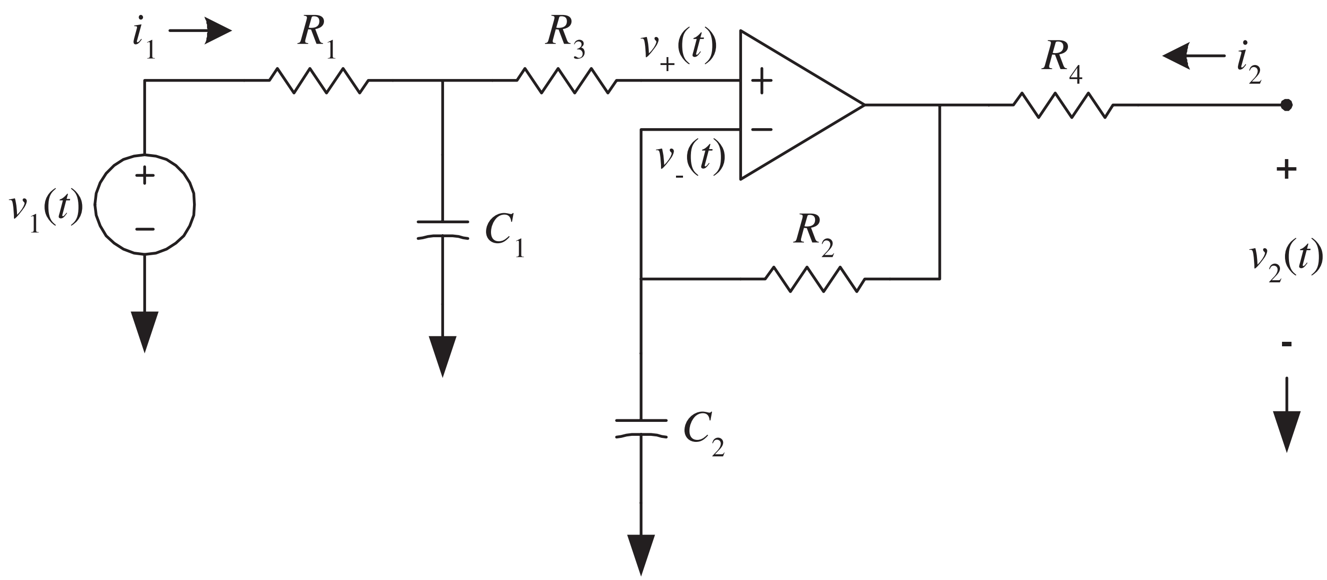


Find an expression for the transfer function, h_{11}, z_{11}, g_{12}, g_{22}, a_{11}, and y_{21}.

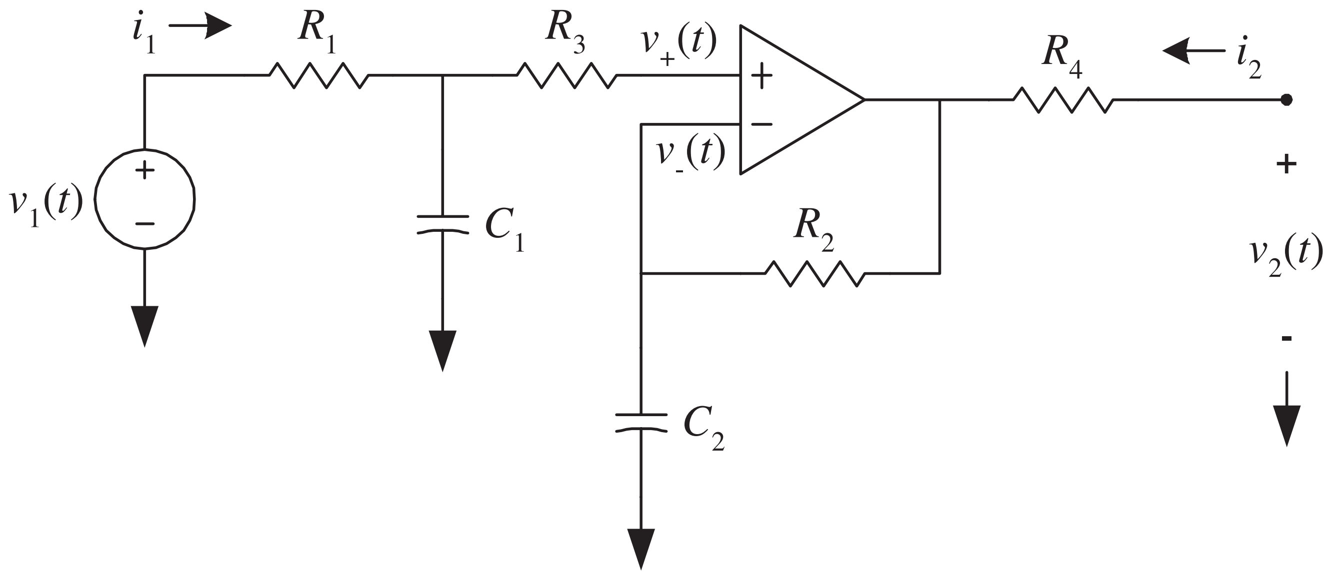
Example 6: Workspace



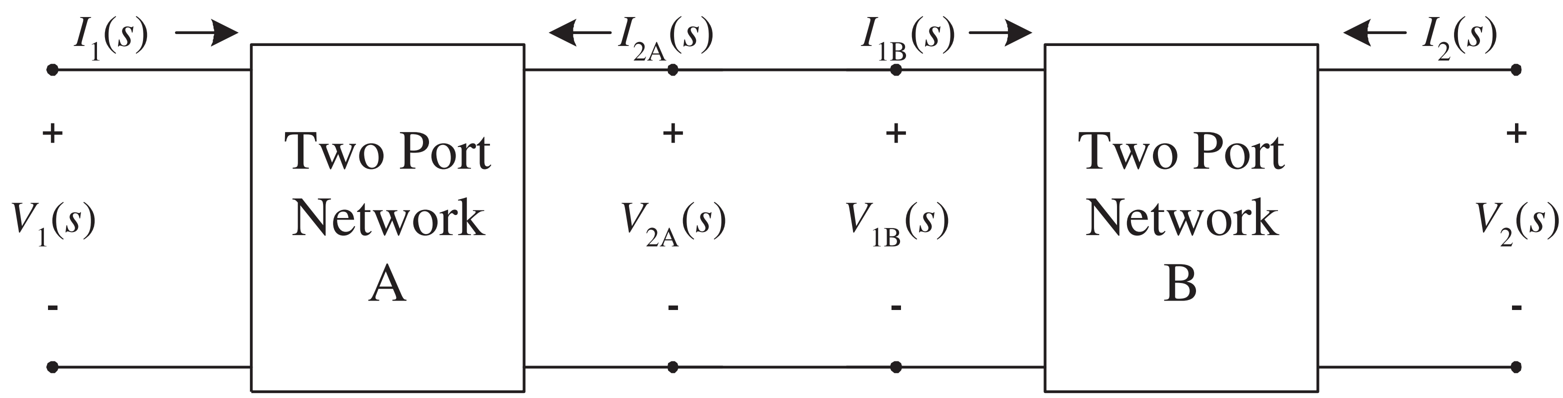
Example 6: Workspace Continued (1)



Example 6: Workspace Continued (2)

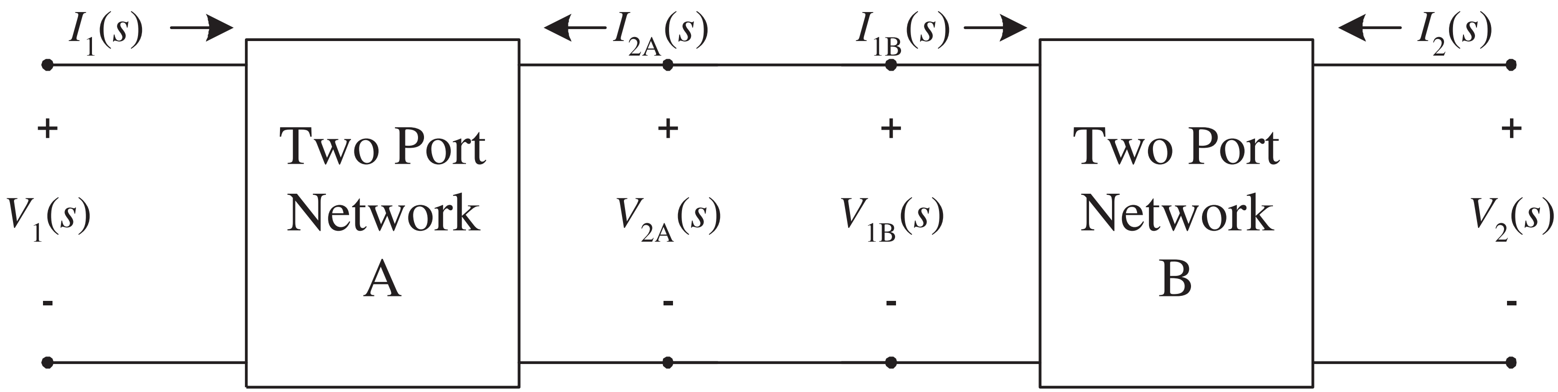


Cascaded Two-Port Networks



- Two networks are **cascaded** when the output of one is the input of the other
- Note that $V_{2A} = V_{1B}$ and $-I_{2A} = I_{1B}$
- The transmission parameters take advantage of these properties

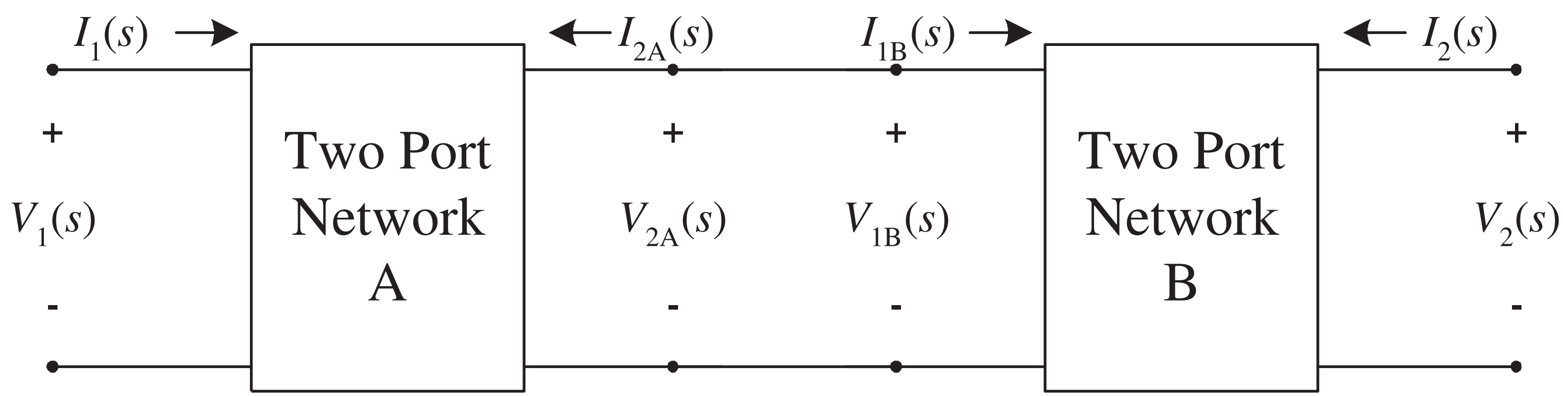
Cascaded Two-Port Networks



$$\begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_A \begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix} \qquad \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

$$\begin{bmatrix} V_{2A} \\ -I_{2A} \end{bmatrix} = \begin{bmatrix} V_{1B} \\ I_{1B} \end{bmatrix} \qquad \begin{bmatrix} V_1 \\ I_1 \end{bmatrix} = \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_A \begin{bmatrix} a_{11} & a_{12} \\ a_{21} & a_{22} \end{bmatrix}_B \begin{bmatrix} V_2 \\ -I_2 \end{bmatrix}$$

Cascaded Two-Port Networks Continued

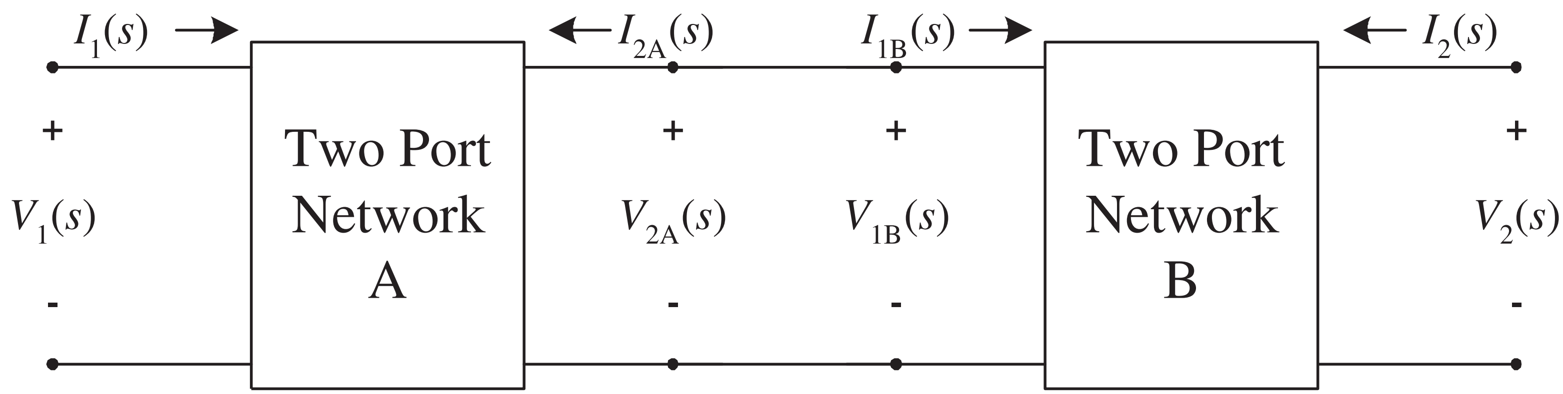


The inverse transmission parameters are also convenient for cascaded networks.

$$\begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_A \begin{bmatrix} V_{1B} \\ -I_{1B} \end{bmatrix} \qquad \begin{bmatrix} V_{2A} \\ I_{2A} \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_B \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

$$\begin{bmatrix} V_{1B} \\ -I_{1B} \end{bmatrix} = \begin{bmatrix} V_{2A} \\ I_{2A} \end{bmatrix} \qquad \begin{bmatrix} V_2 \\ I_2 \end{bmatrix} = \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_A \begin{bmatrix} b_{11} & b_{12} \\ b_{21} & b_{22} \end{bmatrix}_B \begin{bmatrix} V_1 \\ -I_1 \end{bmatrix}$$

Cascaded Systems: Two-Port Networks versus $H(s)$



- Two-ports and transfer functions $H(s)$ are closely related
- $H(s)$ only relates the input signal to the output signal
- Two-ports relate both voltages and currents at each port
- You cannot cascade $H(s)$ unless the circuits are active
- Two-port networks have no such restriction
- Two-ports are used to design passive filters
- However, two-ports are more complicated than $H(s)$