

RESEARCH CENTRE

Shri Ram ki Nangal, via Sitapura RIICO Jaipur- 302 022.

Weak Students ASSIGNMENT Year: B. Tech. I Year Semester: II Subject: Engineering Mathematics - II Session: 2020-21

CO1. To understand the concept of rank of matrix, inverse, Eigen values & amp; vectors along with solution of linear simultaneous equation determine inverse of a matrix using Cayley Hamilton Theorem.

1	Define the rank of a matrix.	
2	Determine the rank of the following matrix $\begin{bmatrix} 1 & 2 & 3 \\ 1 & 4 & 2 \\ 2 & 6 & 5 \end{bmatrix}$	(RTU 2008)
3	Find the rank of the following matrix $A = \begin{bmatrix} 1 & 1 & 1 \\ b+c & c+a & a+b \\ bc & ca & ab \end{bmatrix}$	
4	Find the rank of the following matrix $A = \begin{bmatrix} a & b & c \\ a^2 & b^2 & c^2 \end{bmatrix}$	[Gate CS 2018]
5	Using the Gauss method find the inverse of the matrix (i) $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ -2 & -4 & -4 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & -1 & 2 \\ -6 & 2 & 4 \\ -3 & 1 & 2 \end{bmatrix}$ (iii) $\begin{bmatrix} -1 & 2 & 2 \\ 2 & -1 & 2 \\ 2 & 2 & -1 \end{bmatrix}$	
6	Reduce the following matrix into its normal form and hence find its Rank (i) $\begin{bmatrix} 2 & 1 & -1-6 \\ 3 & -3 & 1 & 2 \\ 1 & 1 & 1 & 2 \end{bmatrix}$ (ii) $\begin{bmatrix} 8 & 1 & 3 & 6 \\ 0 & 3 & 2 & 2 \\ -8 & -1 & -34 \end{bmatrix}$ (iii) $\begin{bmatrix} 1 & 2 & 3 & 4 \\ 2 & 1 & 4 & 3 \\ 3 & 0 & 5-10 \end{bmatrix}$	
7.	Investigate the values of λ and μ so that the equations $2x + 3y + 5z = 9$, $7x + 3y - 2z = 8$, $2x + 3y + \lambda z = \mu$ have (i) No solution (ii) unique solution (iii) many solution	[Gate CE 2019]
8.	For what values of k the equation $x + y + z = 6$, $x + 2y + 3z = k$, $4x - 4x + 3x + 3z = k$, $4x - 4x + 3x +$	$+y + 10z = k^2$
9.	Investigate the values of λ and μ so that the equations $x + y + z = 6$, $x + 2y + 3z = 10$, $x + 2y + \lambda z = \mu$ have No solution (ii) unique solution (iii) many solution	DTU (2002, 2006)
10.	T est for consistency and solve (i) $2x - 3y + 7z = 5, 3x + y - 3z = 13, 2x + 19y - 47z = 32$ (ii) $5x + 3y + 7z = 4, 3x + 26y + 2z = 9, 7x + 2y + 10z = 5$ (iii) $x + 2y + 3z = 14, 3x + y + 2z = 11, 2x + 3y + z = 11$	KTU (2002, 2000)
12.	(iv) $2x + 3y + 4z = 11$, $x + 5y + 7z = 15$, $3x + 11y + 13z = 25$ Find the value of a and b for which the equation x + ay + z = 3, $x + 2y + 2z = b$, $x + 5y + 3z = 9$ are consistent.	(RTU 2007)
13.	Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 5 & 4 \\ 1 & 2 \end{bmatrix}$.	
14.	Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$	
15.	Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} 2 & 2 & -5 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$.	



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 $\begin{bmatrix} 7 & -4 \\ -4 & 3 \\ -2 & 2 \\ 3 & -1 \\ -1 & 3 \end{bmatrix}$ Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} -6\\ 2\\ 6\\ -2 \end{bmatrix}$ Find the Eigen value and Eigen vector of the matrix $\begin{bmatrix} -6\\ -2\\ -2 \end{bmatrix}$ 16. (RTU 2014) 17. Find the Eigen value and Eigen vector of the final $\begin{bmatrix} 2 & -1 & 3 \end{bmatrix}$ Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$ and find its inverse. Also express 18. $A^5 - 4A^4 - 7A^3 + 11A^2 - A - 10I$ as linear polynomial in A. [2 1 1] Find the characteristic equation of the matrix $A = \begin{bmatrix} 0 & 1 & 0 \end{bmatrix}$ and hence compute A^{-1} also find 19. 2 1 1 the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$. Verify Cayley-Hamilton theorem for the matrix A and find its inverse 20. 2 -1 11 2 7 -212 -1(ii) -6 -12 (i) -1(RTU 2015) 1 -1 2 6 2 -1Using the Cayley-Hamilton theorem, find the inverse of 21. [1 1 $\begin{bmatrix} 0 & 3 \\ 1 & -1 \\ -1 & 1 \end{bmatrix} (\text{iii}) \begin{bmatrix} 1 & 1 & 3 \\ 1 & 3 & -3 \\ 2 & -4 & -4 \end{bmatrix} (\text{iv}) \begin{bmatrix} 1 & 2 & -2 \\ 1 & 1 & 1 \\ 1 & 3 & 1 \end{bmatrix}$ $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \end{bmatrix} \text{ to the diagonal form.}$ (ii) 2 1 22. Reduce the matrix A =(RTU 2000) 1 -1 0 23. State Cayley-Hamilton theorem. Find the characteristic equation of the matrix $A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$ and hence compute A^{-1} .also 24. find the matrix represented by $A^5 - 5A^4 + 3A^3 + 6A^2 - 6A + 2I$. State and explain the application of Cayley-Hamilton theorem. 25. Find the sum and product of Eigen values of a matrix 26. 1 - 31 5 A = |1|1 13 1 1 Find the sum and product of Eigen values of a matrix 27. 3 1 4] A= 0 2 6 L0 0 5J 28. Find the sum and product of Eigen values of a matrix 2 2 -1 2 A =2 -1 6 29. Test for consistency and solve: x + 2y + z = 3, 2x + 3y + 2z = 5, 3x - 5y + 5z = 2, 3x + 9y - z = 4Q30. Test for consistency and solve: 2x - 3y + 7z = 5, 3x + y - 3z = 13, 2x + 19y - 47z = 32Q31. Test for consistency and solve: x + 2y + 3z = 14, 3x + y + 2z = 11, 2x + 3y + z = 11 $\begin{array}{rrrrr} -2 & 3 & 0 & 4 \\ 0 & 0 & 6 & 1 \end{array}$ Q32. A 4 X 4 matrix is given below. Find its Eigen Values [Gate CE 2020]



Q46. Determine rank of the following matrices.

$$\begin{bmatrix} 1 & 2 & 3 & 2 \\ 2 & 3 & 5 & 1 \\ 1 & 3 & 4 & 5 \end{bmatrix}$$

Q47. Determine rank of the following matrices.

$$\begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & -2 & -4 \\ 3 & 1 & 3 & -2 \\ 6 & 3 & 0 & -7 \end{bmatrix}$$

Q48. Find the Eigen values and Eigen vectors of the Following matrices.

$$\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$$

Q49. Find the Eigen values and Eigen vectors of the Following matrices.

(RTU 2011)

(RTU 2016)



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$$\begin{bmatrix} 2 & -1 & 1 \\ 1 & -1 & 2 \end{bmatrix} (i) \begin{bmatrix} 2 & 3 & -1 & -1 \\ 1 & -1 & 2 & -2 \\ 3 & 3 & 0 & -7 \end{bmatrix}$$
Q50. Diagonalize the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & 1 \\ 0 & 0 & -3 \end{bmatrix}$
(RTU 2012)

CO2: To solve Ordinary D.E of first order, first degree and first order higher degree using various methods
(1. Solve: $(x^4y^4 + x^2x^2 + xy)ydx + (x^4y^4 - x^2x^2 + xy)xdy = 0$
(RTU 2016)
(2. Solve: $(D^2 + a^2)y = \sec cax$
(3. Solve: $(D^2 + 3D + 2)y = \sec^{2x} \sin x$
(5. Solve: $(D^2 + 3D + 2)y = e^{2x} \sin x$
(6. Solve: $(D^2 + 3D + 2)y = e^{2x} \sin x$
(7. Solve: $(D^2 + 3D + 2)y = e^{2x} \sin x$
(7. Solve: $(D^2 + 3D + 2)y = \sin x$
(8. Solve: $(D^2 + 3D + 2)y = \sin x$
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(8. Solve: $(D^2 + 3D + 2)y = \sin x$
(8. Solve: $(D^2 + 3D + 2)y = \sin x$
(9. Solve: $(D^2 + 3D + 2)y = \sin x$
(9. Solve: $(D^2 + 3D + 2)y = \sin x$
(9. Solve: $(D^2 + 3D + 2)y = \sin x$
(9. Solve: $(D^2 + 3D + 2)y = \sin x$
(9. Solve: $(2^2 + 3^2)dx + (x^2y^3 - y)dx = 0$
(9. Solve $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$
(9. Solve $(x^3y^2 + x)dy + (x^2y^3 - y)dx = 0$
(9. Solve $(x^3y^2 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
(9. Solve $(xy^2 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
(9. Solve $(xy^4 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
(9. Solve $(xy^4 + y)dx + 2(x^2y^2 + x + y^4)dy = 0$
(9. Solve the differential equation:
a. $(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdx = 0$
(9. Solve the differential equation:
a. $(xy \sin xy + \cos xy)ydx + (xy \sin xy - \cos xy)xdx = 0$
(9. Solve the $\frac{d}{dx} = \frac{2y - x - 4}{2y - x^2} = 0$
(9. Find the general solution of the differential equation $y^2 - (2^4 + 2x) y'$.
(9. Solve the $\frac{d}{dx} = \frac{2y - x - 4}{2y - x^2} = 0$
(9. Solve the differential equation $\frac{d^3y}{dx^2} + a^2y = \csc x$.
(23. Solve the differential equation $\frac{d^3y}{dx^2} + a^2y = \csc x$.
(24. Solve equation $\frac{d^3y}{dx^2} - 4y = \cosh(2x - 1) + 3^x$.
(25. Solve $x^2 \left(\frac{dy}{dx} \right)^2 + 2x \frac{dy}{dx} - y = 0$.
(26. Solve $x^2 \left(\frac{dy}{dx} \right)^2 - 2xy \left(\frac{dy}{dx} + 2y^2 - x^2 = 0$

Q27. Solve
$$\frac{dy}{dx} + \frac{1}{y \log y} x =$$

Q27. Solve $\frac{dy}{dx} + \frac{1}{y \log y} x = \frac{1}{y}$ Q28. Solve $(x^2y - 2xy^2)dx - (x^3 - 3x^2y) dy = 0$



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CO3 To find the complete solution of D.E of higher order with constant coefficient & variable coefficients

- $x\frac{d^2y}{dx^2}$ +(x+n) $\frac{dy}{dx}$ + (n+1)y=0, where n is not an integer. Q1.
- Q2. Solve y"+y=0 by power series method
- Find the series solution of $(x-x^2) y'' + (1-5x)y' 4y = 0$ about x = 0. Q3. (RTU2016)
- Find the series solution of $(x-x^2)y'' + (1-5x)y' 4y = 0$ about x = 0. Q4. X $(1-x^2)\frac{d^2y}{dx^2} + (1-3x^2)\frac{dy}{dx} + xy = 0,$
- Find the series solution of $(1-x^2)\frac{d^2y}{dx^2} + 2x\frac{dy}{dx} + y = 0$ Q5.
- Find the series solution of $x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + x^2 y = 0$. Q6. (RTU2015)
- Find the series solution of $2x^2 \frac{d^2y}{dx^2} + (2x^2 x)\frac{dy}{dx} + y = 0$. Q7.
- Find the series solution of $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + (x^2 1) y = 0$. Q8.
- Q9. Solve the following Legendre's Differential Equation $(1-x^{2})\frac{d^{2}y}{dx^{2}} - 2x\frac{dy}{dx} + n (n+1) y = 0$
- Find the series solution of $(1-x)\frac{d^2y}{dx^2} 3x\frac{dy}{dx} y = 0$, Q10.
- Q11. Solve the differential equation: $\frac{d^2y}{dx^2} 4y = \sin h (2x+1) + 4^x$
- Q12. Solve the differential eq. (D²+3D+2) y = e^{e^x}
- Q13. Ordinary differential equation is given below. Find its general solution. [Gate (CE) 2020] $6\frac{d^2y}{dx^2} + \frac{dy}{dx} - y = 0$
- Q14. Complementary function(C.F) of $(D^4 + 2D^3 3D^2)y = x^2$ is
- Q15. The particular integral (P.I) of $(D^2 + 4)y = \cos 2x$ is :
- Check whether the given equation is exact or not. Q16.

$$x^{2}\frac{d^{2}y}{dx^{2}} + 3x\frac{dy}{dx} + y = \frac{1}{(1-x)^{2}}$$

Q17. Solve the equation $\frac{d^3y}{dx^3} + 4y = 4tan2x$

- Q18. Solve in series $2x^2y'' + xy' (x + 1)y = 0(5)$
- Q19. Solve $(D^2 2D + 4)y = e^x \cos x$.
- Q20. Solve $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = \sin(\log x^2)$
- Q21. Solve $\cos x \frac{d^2 y}{dx^2} + \sin x \frac{dy}{dx} 2y\cos^3 x = 2\cos^5 x$

Q22. Using the method of variation of parameter, solve $\frac{d^2y}{dx^2} - y = \frac{2}{1+e^x}$ (RTU 2014)

- Q23. Solve $x^2 \frac{d^2y}{dx^2} + 5x \frac{dy}{dx} + 2y = 2x^2 + 10$
- Q24. Solve $(D^2 2D + 4)y = e^x \sin x$.
- Q25. Solve: $(y^2 ay)dy + (ax x^2)dx =$
- Q26. Solve $(x^2 ay)dy + (ax y^2)dx = \emptyset$
- Q27. Solve: $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$
- Q28. Solve: $x^2 \frac{d^2 y}{dx^2} 3x \frac{dy}{dx} + 4y = 2x^2$ Q29. Solve: $(D^2 4D + 4)y = 8x^2 e^{2x} sin2x$



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Q30. Solve :
$$(D^2 - 5D + 4)y = x^2 e^{2x} sinx$$

Q31. The Differential Equation [GATE (CE) 2014]
 $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} + y = e^{2x}$ is a
Q32. The general solution of differential equation
 $x^2 \frac{d^2y}{dx^2} + x \frac{dy}{dx} + y = 0$ is a
Q33. Q33. Consider the differential equation
 $x^2 \frac{d^2y}{dx^2} + 3x \frac{dy}{dx} - y = 0$. Which of the solution of this equation for x>0? [GATE (EE) 2014]
Q34. If y=f(x) is solution of $\frac{d^2y}{dx^2} = 0$ with the boundary condition y=5 at x=0,
and $\frac{dy}{dx} = 2 at x = 10, f(15) = \cdots \dots \dots$...
[GATE (ME) 2014]
Q35. Q35. The Differential equation
 $\frac{d^2y}{dx^2} + (x^2 + 4x) \frac{dy}{dx} + y = x^8 - 8 is \dots \dots \dots$.
[GATE (ME) 2014]
Q36. Solve $\frac{d^2y}{dx^2} - \cot x \frac{dy}{dx} - \sin^2 x. y = \cos x - \cos^3 x$
Q37. Solve $x^2 \frac{d^2y}{dx^2} - x \frac{dy}{dx} - 3y = x^2 \log x$.
Q38. Complementary function(CF) of $(D^2 + 2D^3 - 3D^2)y = x^2 is \dots$ [GATE (CE) 1999]
Q39. The particular integral(P.I) of $(D^2 + 4y) = \cos 4x is :$
Q40. Solve $d^2y \frac{dx}{dx} - 4x \frac{dy}{dx} + 3y = (1 + x)^2 Using Method of Variation of Parameter.
Q41. Solve $x^2 \frac{d^2y}{dx^2} - 4x \frac{dy}{dx} + 4y = (1 + x)^2$
Q45. Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dx}{dx} + 4y = (1 + x)^2$
Q45. Solve $x^2 \frac{d^2y}{dx^2} - 3x \frac{dx}{dx} + 4y = (1 + x)^2$
Q45. Solve $(1 + x)^2 \frac{d^2y}{dx^2} + 3(2 + 3x) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$ (RTU 2014)
Q47. Solve $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$
Q49. Solve $(1 + x)^2 \frac{d^2y}{dx^2} + (1 + x) \frac{dy}{dx} - 36y = 3x^2 + 4x + 1$
Q51. Solve $(x^2 \frac{d^2y}{dx^2} - 3x \frac{dy}{dx} + 4y = (1 + x)^4$$



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Co4: To solve partial differential equations with its applications in Laplace equation, Heat & amp; Wave equation

- 1. Using the method of separation of variable Solve $\frac{\partial^2 z}{\partial x^2} 2 \frac{\partial z}{\partial x} + \frac{\partial z}{\partial y} = 0$ (RTU 2018)
- 2. Solve the following equation by the method of separation of variable: $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u$, given $u = 3e^{-y} e^{-5y}$ when x = 0
- 3. Solve by the method of separation of variables: $3\frac{\partial u}{\partial x} + 2\frac{\partial u}{\partial y} = 0$, $u(x, 0) = 4e^{-x}$
- 4. Solve the equation by the method of separation of variables: $\frac{\partial^2 u}{\partial x^2} = \frac{\partial u}{\partial y} + 2u$, given that u = 0 and $\frac{\partial u}{\partial x} = 1 + e^{-3y}$ when x = 0 for all values of y. (RTU 2019)
- $\mathbf{u} = \mathbf{0}$ and $\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = \mathbf{1} + \mathbf{e}^{-3\mathbf{y}}$ when $\mathbf{x} = \mathbf{0}$ for all values of \mathbf{y} . (RTU 20) 5. Using the method of separation of variables Solve $\frac{\partial \mathbf{u}}{\partial \mathbf{x}} = 2\frac{\partial \mathbf{u}}{\partial \mathbf{t}} + \mathbf{u}$ where $\mathbf{u}(\mathbf{x}, \mathbf{0}) = 6e^{-3x}$
- 6. A tightly stretched string with fixed ends points x = 0 and x = l is initially in a position given by $y = y_0 sin^3 \left(\frac{\pi x}{l}\right)$. if is released from rest find the displacement y(x, t).
- 7. Write the mathematical form of one dimensional wave equation and discuss its solution.
- 8. An infinitely long plane uniform plate is bounded by two parallel edges and an end at right angle to them. The breadth is π . This end is maintained at temperature u_0 at all points and other edges are at zero temperature. Determine the temperature at any point of the plate in steady-state.
- 9. Write the mathematical form of one dimensional heat equation and discuss its solution.
- 10. Write the mathematical form of Laplace Equation and discuss its solution.
- 11. A bar 1000 cm long, with insulated sides, has its ends kept at 0C and 100C until steady state condition prevail. Two ends are suddenly insulated and kept so. Find the temp. Distribution.
- 12. Solve $\frac{\partial^2 \mathbf{u}}{\partial \mathbf{x}^2} + \frac{\partial^2 \mathbf{u}}{\partial \mathbf{y}^2} = \mathbf{0}$
- 13. Solve the equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2}$ with boundary conditions $u(x,0) = 3\sin \pi x$, u(1,0)=0, u(1,t)=0.
- 14. The point of trisection of a string are pulled aside through the same distance on opposite side of the position of equilibrium and the string is released from the rest . Derive tan expression for the displacement of the string at subsequent time and show that the midpoint of the string always remains at rest.
- 15. A string is stretched and fastened to two point's l apart. Motion is started
- 16. By displacing the string in the form $y y = a \sin(\frac{\pi x}{l})$ from it is released at t=0. Find the displacement at any time t.
- 17. Discuss the method of separation of variables to solve partial differential equations.
- 18. Discuss the solution of two dimensional heat equations.
- 19. Two ends A and B of a rod 20 cm long have temp 30C and 80C until steady state prevails. the temp of the ends are changed to 40C and 60C respectively .find the temp distribution in the rod at any time t.
- 20. Two ends A and B of a rod 10 cm long have temp 50C and 100C until steady state prevails. the temp of the ends are changed to 90C and 60C respectively .find the temp distribution in the rod at any time t.



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21. Solve
$$\sqrt{p} + \sqrt{q} = 1$$

22. Form the partial differential equation by eliminating the arbitrary function from $z = f(x^2 + y^2)$.
23. Solve $(y^2 + z^2 - x^2)p - 2xyq = -2xz$ [GATE (EC) 2014]
24. Solve $y = 2px + p^{2n}$
25. Solve $y = 2px + p^4$
26. Solve $y = 2px - xp^2$
27. Solve $y - 2px = tan^{-1}(xp^2)$
28. Solve $y = 2px + y^2 p^3$
30. Solve $y = 2px + y^2 p^3$
31. Solve $p = tan \left(x - \frac{p}{1+p^2}\right)$ [GATE (CS) 2014]
32. Solve $(x^2 - yz) p + (y^2 - zx)q = z^2 - xy$
34. Solve $(y^2 + z^2 - x^2)p - 2xyq = 2zx$
35. Find the complete integral of $p x + q y = p q$
36. Solve $p^2 + q^2 - 2xy - 2y^2p^2 - 2xy^2p = 0$
37. Solve $y = 2px + y^2p^3$
38. Solve $p = tan \left(x - \frac{p}{1+p^2}\right)$ where $p = \frac{dy}{dx}$ (RTU 2016)
39. Solve $p^3 + 2xp^2 - y^2p^2 - 2xy^2p = 0$
40. Solve $p = \cos(y-px)$
41. Solve sin $px \cos y = \cos px \sin y + p$ (RTU 2015)
42. Solve $(y^2+z^2-x^2) p - 2xyq = 2zx$
43. Solve $(y^2+z^2-x^2) p - 2xyq = 2zx$
44. Solve $y = pxy + x^4p^2$
45. Solve $p^2 + (x - e^x)p - xe^x = 0$
46. Solve $y = -px + x^4p^2$
47. Solve $9(y + xp \log p) = (2 + 3 \log p)p^3$
48. Solve $p^2 + q^2 = x + y$
49. Solve: $p^2 + q^2 = x + y$
49. Solve: $p = xy^2 + pq + qy = yz$ by Charpit's method (RTU2016)