JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – III
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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Particular Integral case -3

P.I when $X = x^m$; $m \in N$

If $f(D) = (D - \alpha)$ then

$$P.I = \frac{1}{f(D)} x^m = \frac{1}{(D-\alpha)} x^m = \frac{-1}{\alpha\left(1 - \frac{D}{\alpha}\right)} x^m$$

$$= -\frac{1}{\alpha}\left(1 - \frac{D}{\alpha}\right)^{-1} x^m$$

$$= -\frac{1}{\alpha}\left(1 + \frac{D}{\alpha} + \frac{D^2}{\alpha^2} + \cdots + \frac{D^m}{\alpha^m}\right) x^m$$

$$= -\frac{1}{\alpha}\left(x^m + \frac{m x^{m-1}}{\alpha} + \cdots + \frac{\lfloor m}{\alpha^m}\right)$$

If $f(D) = (D-\alpha_1)(D-\alpha_2)\cdots\cdots(D-\alpha_n)$

$$P.I = \frac{1}{(D-\alpha_1)(D-\alpha_2)\cdots(D-\alpha_2)}\, x^m$$

By factorizing Partially

$$= \left(\frac{A_1}{(D-\alpha_1)} + \frac{A_2}{(D-\alpha_2)} + \cdots\cdots + \frac{A_n}{(D-\alpha_n)}\right) x^m$$

then we can use the Process similar to above article to find $\frac{A_1}{D-\alpha_1} x^m, \frac{A_2}{D-\alpha_2} x^m, \cdots, \frac{A_n}{D-\alpha_n} x^m$ separately.

Ex: $\frac{d^3y}{dx^3} - \frac{d^2y}{dx^2} - 6\frac{dy}{dx} = 1+x^2$

Sol: given eqn can be written as

$$(D^3 - D^2 - 6D)y = 1+x^2$$

Auxilliary eqn is : $m^3 - m^2 - 6m = 0$

$\Rightarrow m(m+2)(m-3) = 0, \quad \Rightarrow m = 0, -2, 3$

$$C.F = c_1 + c_2 e^{-2x} + c_3 e^{3x}$$

$$P.I = \frac{1}{D(D+2)(D-3)}(1+x^2)$$

$$= \frac{1}{D(D^2-D-6)}(1+x^2) = -\frac{1}{6D}\left[1+\frac{(D-D^2)}{6}\right]^{-1}(1+x^2)$$

$$= -\frac{1}{6D}\left[1-\frac{(D-D^2)}{6}+\frac{(D-D^2)^2}{36}+\cdots\right](1+x^2)$$

$$= -\frac{1}{6D}\left[1 - \frac{D}{6} + \frac{D^2}{6} + \frac{D^2}{36} + \frac{D^4}{36} - \frac{D^3}{18} + \cdots\right](1+x^2)$$

$$= -\frac{1}{6}\left[\frac{1}{D} - \frac{1}{6} + \frac{D}{6} + \frac{D}{36} - \frac{D^2}{18} + \cdots\right](1+x^2)$$

$$= -\frac{1}{6}\left[\frac{1}{D} - \frac{1}{6} + \frac{7D}{36} - \frac{D^2}{18} + \cdots\right](1+x^2)$$

$$= -\frac{1}{6}\left[x + \frac{x^3}{3} - \frac{1}{6} - \frac{x^2}{6} + \frac{7}{18}x - \frac{1}{9}\right]$$

$$= \frac{-25}{108}x + \frac{1}{36}x^2 - \frac{1}{18}x^3 + C$$

hence the Complete sol. is

$$y = C.F + P.I$$

$$y = C_1 + C_2 e^{-2x} + C_3 e^{3x} - \frac{25}{108}x + \frac{1}{36}x^2 - \frac{1}{18}x^3.$$

Ex: $\frac{d^3y}{dx^3} + 2\frac{d^2y}{dx^2} + \frac{dy}{dx} = e^{2x} + x^2 + x$

Sol: given equ can be written as

$m^3 + 2m^2 + m = 0 \Rightarrow m(m+1)^2 = 0 \Rightarrow m = 0, -1, -1$

$C.F = c_1 + (c_2 + c_3 x)e^{-x}$

$$P.I. = \frac{1}{D(D+1)^2}(e^{2x}+x^2+x)$$

$$= \frac{1}{D(D+1)^2}e^{2x} + \frac{1}{D}(1+D)^{-2}(x^2+x)$$

$$= \frac{1}{2\cdot(2+1)^2}e^{2x} + \frac{1}{D}(1-2D+3D^2-4D^3+\cdots)(x^2+x)$$

$$= \frac{1}{18}e^{2x} + \left(\frac{1}{D} - 2 + 3D - 4D^2 + \cdots\right)(x^2+x)$$

$$= \frac{1}{18} e^{2x} + \frac{x^3}{3} + \frac{x^2}{2} - 2x^2 - 2x + 6x + 3 - 8$$

$$= \frac{1}{18} e^{2x} + \frac{x^3}{3} - \frac{3}{2} x^2 + 4x - 5$$

hence the complete sol is:

$$y = C.F + P.I$$

$$y = c_1 + (c_2 + c_3 x) e^{-x} + \frac{1}{18} e^{2x} + \frac{x^3}{3} - \frac{3}{2} x^2 + 4x - 5.$$

Ex: $\dfrac{d^5y}{dx^5} - \dfrac{dy}{dx} = 12e^x + 8\sin x - 2x$

Sol: given eqn can be written as

$$(D^5 - D)\,y = 12e^x + 8\sin x - 2x$$

auxilliary eqn is : $m^5 - m = 0$

$$m(m-1)(m+1)(m^2+1) = 0$$

$$m = 0,\ 1,\ -1 \pm i$$

$$C.F = C_1 + C_2 e^x + C_3 e^{-x} + (C_4 \cos x + C_5 \sin x).$$

$$P.I = \frac{1}{D(D-1)(D+1)(D^2+1)} (12e^x + 8\sin x - 2x)$$

$$= 12 \cdot \frac{1}{D(D-1)(D+1)(D^2+1)} e^x + 8 \cdot \frac{1}{D(D-1)(D+1)(D^2+1)} \sin x$$

$$- 2 \cdot \frac{1}{D(D^4-1)} x$$

$$= 12 \cdot \frac{1}{(D-1) \cdot 1 \cdot (1+1)(1+1)} e^x + \frac{8 \cdot 1}{D(-1^2-1)(D^2+1)} \sin x$$

$$+ 2 \cdot \frac{1}{D} (1-D^4)^{-1} x$$

$$= \frac{12}{4} \cdot e^{x} \frac{1}{(D-1)} + \frac{4}{D^2+1}\left(\frac{1}{D}\sin x\right) + \frac{2}{D}\left(1+D^4+\dots\right)x$$

$$= 3xe^{x} - \frac{4}{(D^2+1)}(-\cos x) + \frac{2}{D}(x)$$

$$= 3xe^{x} + \frac{4}{2}x\sin x + x^2$$

hence the required sol is

$$y = C.F + P.I$$

$$y = c_1 + c_2 e^{x} + c_3 e^{-x} + c_4 \cos x + c_5 \sin x + 3xe^{x} + 2x\sin x + x^2$$

Ex: $\dfrac{d^4y}{dx^4} + 2\dfrac{d^3y}{dx^3} - 3\dfrac{d^2y}{dx^2} = x^3 + 3e^{2x} + 4\sin x$

Sol: given eqn can be written as

$$(D^4 + 2D^3 - 3D^2)y = x^3 + 3e^{2x} + 4\sin x$$

whose Auxilliary eqn is:

$$m^4 + 2m^3 - 3m^2 = 0$$

$$m^2(m^2+2m-3)=0$$

$$\Rightarrow m^2(m-1)(m+3)=0 \Rightarrow m=0,0,1,-3$$

$$C.F = (C_1+C_2x) + C_3e^x + C_4e^{-3x}$$

$$P.I = \frac{1}{D^2(D-1)(D+3)}(x^2+3e^{2x}+4\sin x)$$

$$= \frac{1}{D^2(D^2+2D-3)}x^2 + \frac{1}{D^4+2D^3-3D^2}(3e^{2x}+4\sin x)$$

$$= \frac{1}{-3D^2}\left[1 - \frac{(2D+D^2)}{3}\right]^{-1} x^2 + \frac{3e^{2x}}{2^4 + 2.2^3 - 3\cdot 2^2}$$

$$+ 4\cdot \frac{1}{(-1)^2 + 2D(-1) - 3(-1)} \sin x$$

$$= -\frac{1}{3D^2}\left(1 + \frac{(2D+D^2)}{3} + \frac{(2D+D^2)^2}{9} + \cdots\right) x^2 + \frac{3e^{2x}}{20}$$

$$+ \frac{2\cdot(2+D)}{4-D^2} \sin x$$

$$= \frac{-1}{3D^2}\left(x^2 + \frac{4x}{3} + \frac{2}{3} + \frac{8}{9}\right) + \frac{3}{20} e^{2x} + \frac{2(2+D)}{4-(-1)} \sin x$$

$$= -\frac{1}{3}\left(\frac{x^4}{12} + \frac{2}{9}x^3 + \frac{14}{9}\cdot\frac{x^2}{2}\right) + \frac{3}{20}e^{2x} + \frac{2}{5}(2\sin x + \cos x)$$

hence the general soln is

$$y = C.F + P.I$$

$$y = c_1 + c_2 x + c_3 e^{x} + c_4 e^{-3x} + \frac{3}{20}e^{2x} + \frac{1}{5}(2\sin x + \cos x)$$

$$-\frac{1}{3}\left(\frac{x^4}{12} + \frac{2}{9}x^3 + \frac{7}{9}x^2\right).$$

Ex: $(D^3+2D^2+D)y = e^{-x} + \cos x + x^2$

Sol: A.E is $m^3+2m^2+m=0 \Rightarrow m=0,-1,-1$

$$C.F = c_1 e^{0x} + (c_2+c_3x)e^{-x}$$

$$P.I = \frac{1}{(D^3+2D^2+D)}(e^{-x}+\cos x+x^2)$$

$$= \frac{1}{(D^3+2D^2+D)} e^{-x} + \frac{1}{(D^3+2D^2+D)} \cos x + \frac{1}{(D^3+2D^2+D)} x^2$$

$$= \frac{1}{D(D+1)^2} e^{-x} + \frac{1}{(-4)D+2(-4)+D} \cos x + \frac{1}{D(D+1)^2} x^2$$

$$= \frac{1}{-(D+1)^2} e^{-x} + \frac{1}{-3D-8} \cos x + \frac{1}{D} (D+1)^{-2} x^2$$

$$= \frac{x^2}{-2} e^{-x} - \frac{1}{(3D+8)(3D-8)}(3D-8) \cos x + \frac{1}{D} (1-2D+3D^2-4D^3 \cdots) x^2$$

$$= -\frac{x^2}{2}e^{-x} - \frac{(3D-8)}{9D^2-64}\cos x + \frac{1}{D}(x^2-4x+6)$$

$$= -\frac{x^2}{2}e^{-x} - \frac{(3D-8)}{-9-64}\cos x + \frac{x^3}{3} - 2x^2 + 6x$$

$$= -\frac{x^2}{2}e^{-x} + \frac{1}{73}(3D-8)\cos x + \frac{x^3}{3} - 2x^2 + 6x$$

$$= -\frac{x^2}{2}e^{-x} - \frac{3\sin x}{73} - \frac{8}{73}\cos x + \frac{x^3}{3} - 2x^2 + 6x$$

hence the required sol is

$$y = C.F + P.I$$

$$y = C_1 + (C_2 + C_3 x)e^{-x} - \frac{x^2}{2}e^{-x} - \frac{3}{73}\sin x - \frac{8}{73}\cos x$$

$$+ \frac{x^3}{3} - 2x^2 + 6x.$$

Practice Problems

1. $(D^2-3D+2)y = \sin 3x + x^2 + x + e^{4x}$

Ans: $y = c_1 e^{x} + c_2 e^{2x} + \frac{9}{130}\cos 3x - \frac{7}{130}\sin 3x + \frac{1}{2}(x^2+4x+5) + \frac{e^{4x}}{6}$

2. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = x^2$

Ans: $y = e^{-x}(c_1\cos x + c_2 \sin x) + \frac{1}{2}(x^2-2x+1)$

3. $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = x^2 + e^x + \cos 2x$

Ans: $y = (C_1 + C_2 x)e^{2x} + \frac{1}{4}\left(x^2 + 2x + \frac{3}{2}\right) + e^x - \frac{1}{8}\sin 2x$

4. $(D^2+1)y = e^{-x} + \cos x + x^3$

Ans: $y = C_1\cos x + C_2\sin x + \frac{1}{2}e^{-x} + \frac{1}{2}x\sin x + x^3 - 6x$

5. $(D^2-2D+3)y = \cos x + x^2$

Ans: $y = e^{x}(C_1\cos\sqrt{2}x + C_2\sin\sqrt{2}x) + \frac{1}{4}(\cos x - \sin x)$

$+\frac{1}{3}\left(x^2+\frac{4x}{3}+\frac{2}{9}\right)$

6. $(D^2-5D+6)y = x$

Ans: $y = C_1e^{2x} + C_2e^{3x} + \frac{x}{6} + \frac{5}{36}$

7. $\frac{d^2y}{dx^2} + 4y = \sin^2 x$

Ans: $y = C_1 \cos 2x + C_2 \sin 2x - \frac{1}{8}(x \sin 2x - 1)$.

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Thank you!