JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – I
Presented by – (Dr.Vishal Saxena, Associate Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Eigen values and Eigen vectors(examples)

Q.3 Find the eigen values and eigen vectors of matrix

$$A = \begin{bmatrix} 2 & -1 & 1 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Sol^n $|A - \lambda I| = 0$

$$\begin{vmatrix} 2-\lambda & -1 & 1 \\ -1 & 2-\lambda & -1 \\ 1 & -1 & 2-\lambda \end{vmatrix} = 0$$

$$(2-\lambda)\left[(2-\lambda)^2-1\right]+1\left[-2+\lambda+1\right]+1\left[1-2+\lambda\right]=0$$

$$\lambda^3-6\lambda^2+9\lambda-4=0$$

$$\lambda=1,1,4$$

For $\lambda=1$

$$\begin{bmatrix} +1 & -1 & 1 \\ -1 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1-x_2+x_3=0$$

Let $x_3=0$, $x_2=1$ $x_1=1$

or let $x_3=1$, $x_2=0 \Rightarrow x_1=-1$

so eigen vectors corresponding to $\lambda_1 = 1$

$$\begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix} \quad \& \quad \begin{bmatrix} -1 \\ 0 \\ 1 \end{bmatrix}$$

for $\lambda = 4$

$$\begin{bmatrix} -2 & -1 & 1 \\ -1 & -2 & -1 \\ 1 & -1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$\frac{x_1}{1} = \frac{x_2}{-1} = \frac{x_3}{1} = k_3 \text{ (let)}$$

Eigen vector corresponding to $\lambda = 4$ is

$$X_2 = \begin{bmatrix} 1 \\ -1 \\ 1 \end{bmatrix}$$

Q.5 Find the eigen values and eigen vectors of the following matrices

$$\begin{bmatrix} -2 & 1 & 1 \\ -11 & 4 & 5 \\ -1 & 1 & 0 \end{bmatrix}$$

Solⁿ The characteristic eqⁿ of A is

$$|A-\lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 1 & 1 \\ -11 & 4-\lambda & 5 \\ -1 & 1 & 0-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)[-\lambda(4-\lambda)-5]-1[11\lambda+5]+1[-11+4-\lambda]=0$$

$$\lambda^3-2\lambda^2-\lambda+2=0$$

$$\lambda=1,-1,2$$

(i) for $\lambda=1$

$$|A-\lambda I|=0$$

$$\begin{bmatrix}-3 & 1 & 1\\ -11 & 3 & 5\\ -1 & 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

Applying $R_2\rightarrow R_2-3R_1$

$$\begin{bmatrix}-3 & 1 & 1\\ -2 & 0 & 2\\ -1 & 1 & -1\end{bmatrix}\begin{bmatrix}x_1\\ x_2\\ x_3\end{bmatrix}=\begin{bmatrix}0\\0\\0\end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} -3 & 1 & 1 \\ -2 & 0 & 2 \\ 2 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -3 & 1 & 1 \\ -2 & 0 & 2 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-3x_1 + x_2 + x_3 = 0$$

$$-2x_1 + 2x_3 = 0 \Rightarrow x_1 = x_3$$

Let $x_3 = k = x_1 \Rightarrow x_2 = 2k$

$$X_1 = \begin{bmatrix} k \\ 2k \\ k \end{bmatrix} \quad \text{or} \quad k = 1 \quad X_1 = \begin{bmatrix} 1 \\ 2 \\ 1 \end{bmatrix}$$

(ii) for $\lambda = -1$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} -1 & 1 & 1 \\ -11 & 5 & 5 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \to R_2 - 5R_1$

$$\begin{bmatrix} -1 & 1 & 1 \\ -6 & 0 & 0 \\ -1 & 1 & 1 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} -1 & 1 & 1 \\ -6 & 0 & 0 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-x_1 + x_2 + x_3 = 0$$

$$-6x_1 = 0 \quad \Rightarrow \quad x_1 = 0$$

Let $x_2 = -x_3 = k_1$

$$X_2 = \begin{bmatrix} 0 \\ k_1 \\ -k_1 \end{bmatrix} \quad \text{or} \quad k_1 = 1 \quad X_2 = \begin{bmatrix} 0 \\ 1 \\ -1 \end{bmatrix}$$

(iii)

For $\lambda = 2$

$$|A - \lambda I| = 0$$

$$\begin{bmatrix} -4 & 1 & 1 \\ -11 & 2 & 5 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_2 \rightarrow R_2 - 2R_1$

$$\begin{bmatrix} -4 & 1 & 1 \\ -3 & 0 & 3 \\ -1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 - R_1$

$$\begin{bmatrix} -4 & 1 & 1 \\ -3 & 0 & 3 \\ 3 & 0 & -3 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\begin{bmatrix} -4 & 1 & 1 \\ -3 & 0 & 3 \\ 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-4x_1 + x_2 + x_3 = 0$$

$$-3x_4 + 3x_3 = 0 \Rightarrow x_3 = x_1 = k_2$$

$$x_2 = 3k_2$$

$$X_3 = \begin{bmatrix} k_2 \\ 3k_2 \\ k_2 \end{bmatrix} \qquad k_2 = 1 \qquad X_3 = \begin{bmatrix} 1 \\ 3 \\ 1 \end{bmatrix}$$

Q.6 Find the eigen values and eigen vectors of

$$A = \begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$$

Solⁿ

$$|A - \lambda I| = 0$$

$$\begin{vmatrix} -2-\lambda & 2 & -3 \\ 2 & 1-\lambda & -6 \\ -1 & -2 & 0-\lambda \end{vmatrix} = 0$$

$$(-2-\lambda)[-\lambda(1-\lambda)-12]-2[-2\lambda-6]-3[-4+(1-\lambda)]=0$$

$$\lambda^3+\lambda^2-21\lambda-45=0$$

~~$\lambda++$~~ $\lambda=-3,-3,5$

(i) For $\lambda=-3$ $\quad (A-\lambda I)X=0$

$$\begin{bmatrix} 1 & 2 & -3 \\ 2 & 4 & -6 \\ -1 & -2 & 3 \end{bmatrix}\begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}=\begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$x_1+2x_2-3x_3=0$$
$$2x_1+4x_2-6x_3=0$$
$$-x_1-2x_2+3x_3=0$$

This is a set of one eqn with three unknowns. Hence it has an infinite no. of soln

$x_1 + 2x_2 - 3x_3 = 0$

Choosing $x_2 = 0,\ x_1 = 3,\ x_3 = 1$

and

$x_3 = 0 \quad x_1 = -2,\ x_2 = 1$

vectors corresponding to $\lambda = -3$ is $\begin{bmatrix} 3 \\ 0 \\ 1 \end{bmatrix}$ and $\begin{bmatrix} -2 \\ 1 \\ 0 \end{bmatrix}$

$$\begin{bmatrix} 3k_1 - 2k_2 \\ k_2 \\ k_1 \end{bmatrix}$$

(iii) for $\lambda = 5$

$$\begin{bmatrix} -7 & 2 & -3 \\ 2 & -4 & -6 \\ -1 & -2 & -5 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$$

$$-7x_1 + 2x_2 - 3x_3 = 0$$
$$2x_1 - 4x_2 - 6x_3 = 0$$
$$-x_1 - 2x_2 - 5x_3 = 0$$

Taking first two eq^n

$$\frac{x_1}{-24} = \frac{x_2}{-48} = \frac{x_3}{24}$$

$$\frac{x_1}{-1} = \frac{x_2}{-2} = \frac{x_3}{1} = k_3$$

For $\lambda = 5$ $\quad X_3 = \begin{bmatrix} -k_3 \\ -2k_3 \\ k_3 \end{bmatrix}$ for $k_3 = 1$ $\quad \begin{bmatrix} 1 \\ 2 \\ -1 \end{bmatrix}$

Refrences

1.Advanced Engineering Mathematics by Prof.ERWIN KREYSZIG (Ch.10,page no.557-580)
2. Advanced Engineering Mathematics by Prof.H.K Dass (Ch.14,page no.851-875)
3.Advanced Engineering Mathematics by B.V RAMANA (Ch.20,pageno.20.1.20.5)
4.NPTEL Lectures available on

http://www.infocobuild.com/education/audio-video-courses/mathematics/TransformTechniquesForEngineers-IIT-Madras/lecture-47.html

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Thank you!