Jaipur Engineering College & Research Centre, Jaipur

Session 2020-21

Subject Notes

Unit-II

Basic Electrical Engineering (2FY3-08)

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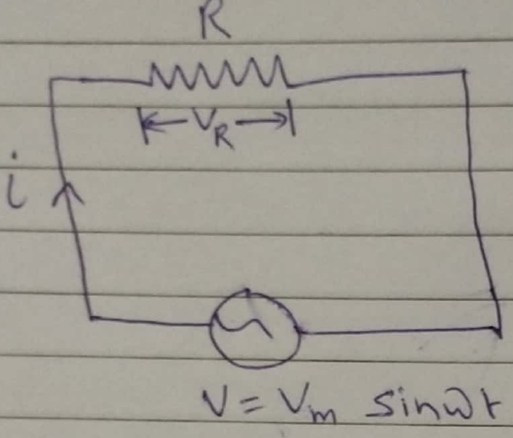
Department Electrical Engineering

Behavior of a Pure resistor in an AC Circuit ⇒

Consider a Pure resistor R connected across an alternating Voltage source V as shown in



Purely resistive circuit

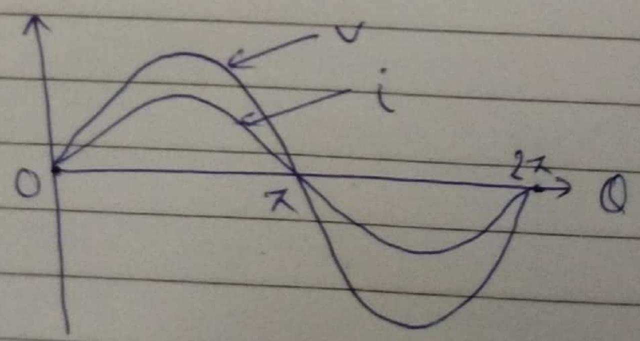
fig. Let alternating Voltage be $V = V_m \sin\omega t$ — (I)

Current – $i = \frac{V}{R} = \frac{V_m \sin\omega t}{R} = \frac{V_m}{R} \sin\omega t$

$i = I_m \sin\omega t$ — (II) $\left(\because I_m = \frac{V_m}{R} \right)$

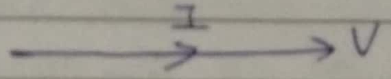
from eq. (I) & (II), it is clear that the current is in phase with the Voltage in a Purely resistive circuit

waveforms –



waveforms

Phasor diagram =>



Voltage and currents are in phase and there is no phase difference

Impedance —

It is the resistance offered to the flow of current in an ac circuit. In a purely resistive circuit

$$Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/R} = R$$

Phase difference — Voltage and current are in phase with each other, the phase diff. is zero. $\phi = 0^\circ$

Power factor —

It is defined as the cosine angle b/w the voltage and current phasors.

$$\text{Power factor} = \cos\phi = \cos 0^\circ = 1$$

Power —

Instantaneous power P is given by

$$P = vi$$

$$= V_m \sin\omega t \; I_m \sin\omega t$$

$$= V_m I_m \sin^2 \omega t = \frac{V_m I_m}{2}(1 - \cos 2\omega t)$$

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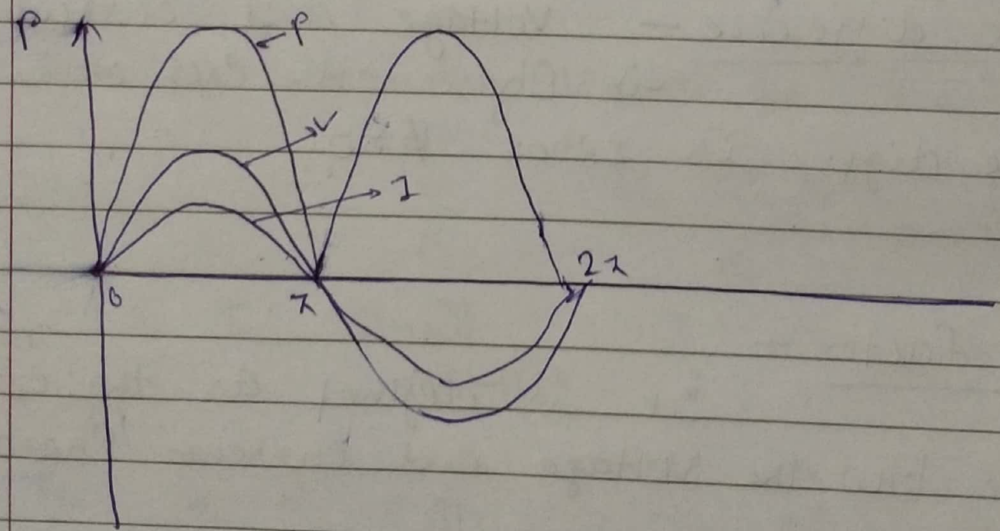
$$P = \frac{V_m I_m}{2} - \frac{V_m I_m}{2} \cos 2\omega t$$

Power consist of a constant part $\frac{V_m I_m}{2}$ and a fluctuating part $\frac{V_m I_m}{2} \cos 2\omega t$. The freq. of the fluctuating part is twice the applied voltage freq. and it's average value over one complete cycle is zero.

$$\text{Average Power } P = \frac{V_m I_m}{2} = \frac{V_m}{\sqrt{2}} \frac{I_m}{\sqrt{2}} = VI$$

Thus the power in a purely resistive circuit is equal to the product of rms values of voltage and current.



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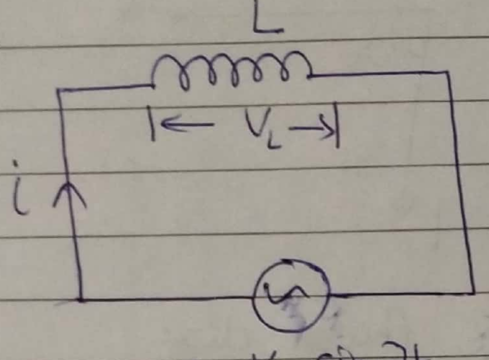
Behavior of a Pure inductor in an AC circuit -

Consider a Pure inductor L connected across an alternating voltage V as shown in fig.



$V = V_m \sin\omega t$

$$V = V_m \sin\omega t \quad -(1)$$

<u>current</u> -

$$i = \frac{1}{L}\int v\,dt$$

$$i = \frac{1}{L}\int V_m \sin\omega t = \frac{V_m}{\omega L}(-\cos\omega t)$$

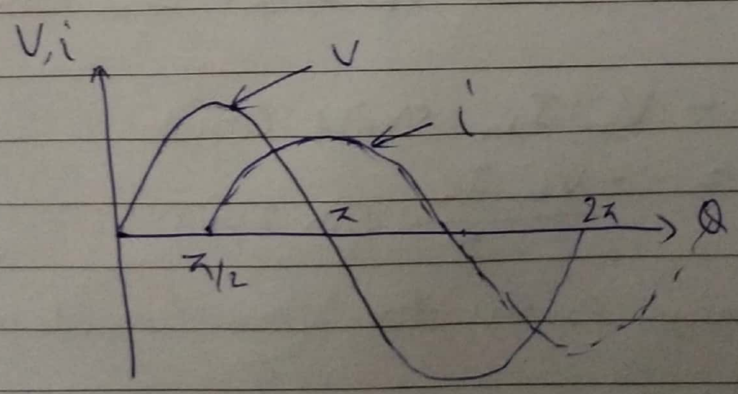
$$i = \frac{V_m}{\omega L}\sin(\omega t - \pi/2)$$

$$i = I_m \sin(\omega t - \pi/2) \quad \cdots \left(I_m = \frac{V_m}{\omega L}\right)$$

$-(11)$

(1) & from eq.ⁿ (11), it is clear that <u>the current lags behind the voltage by 90°</u> in a purely inductive circuit.

<u>waveform</u> -



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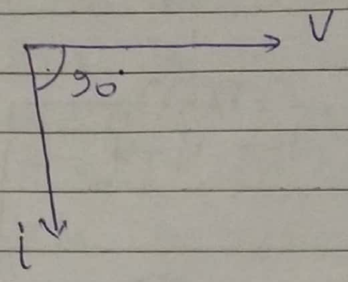
Phasor diagram -



Phasor dia.

Impedance - $Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{V_m/\omega L} = \omega L$

The quantity ωL is called inductive reactance, is denoted by X_L and is measured in Ohms
for a dc supply, $f=0$ $\therefore X_L = 0$.
Thus, an inductor acts as a short circuit for a dc supply.

Phase difference - It is the angle b/w voltage and current Phasors.

$$\phi = 90^\circ$$

Power factor $\Rightarrow$ $P.f = \cos\phi = \cos 90 = 0$

Power - $P = Vi = V_m \sin\omega t \; I_m \sin(\omega t - \pi/2)$

$$P = -V_m I_m \sin\omega t \cos\omega t$$
$$= \frac{-V_m I_m}{2} \sin 2\omega t$$

The average power for one complete cycle,

$$P = 0$$

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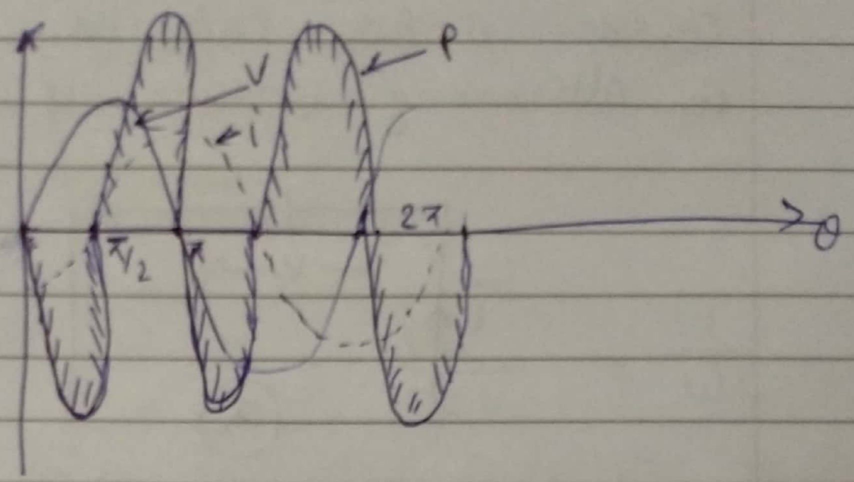
Hence, Power consumed by a Purely inductive circuit is zero.



When Power is Positive, the energy is supplied from the source to build up the magnetic field in the inductor. When Power is negative the energy is returned to the source and magnetic field collapses. Hence Power circulates in a Purely inductive circuit. The circulating Power is called as reactive Power.

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Behavior of a Pure capacitor in an AC circuit

Consider a Pure capacitor 'C' connected across an alternating voltage V as shown in fig.

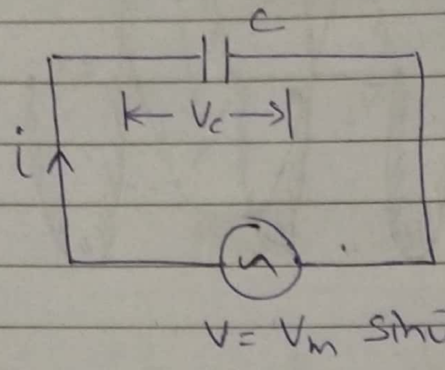


fig- Purely capacitive circuit

$$V = V_m \sin\omega t \quad —(I)$$

Current - $i = C\frac{dV}{dt} = C\frac{d}{dt}(V_m \sin\omega t)$

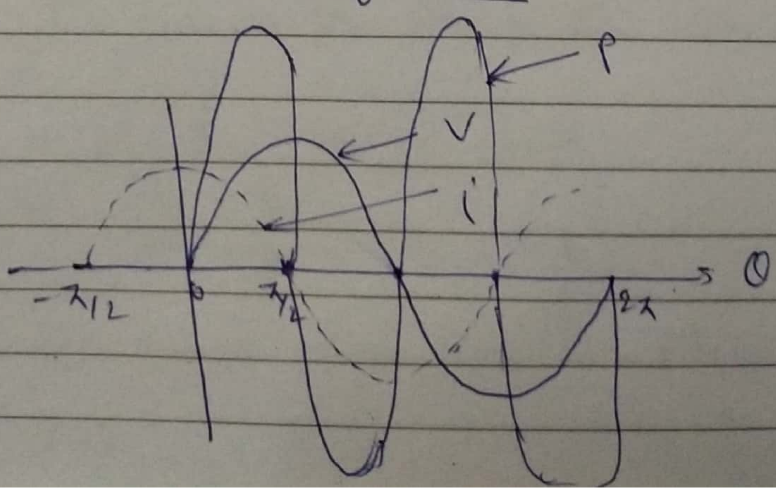
$$i = \omega C V_m \cos\omega t$$

$$= \omega C V_m \sin(\omega t + 90^\circ)$$

$$= I_m \sin(\omega t + \pi/2) \quad .... (I_m = \omega C V_m) \quad —(II)$$

from eq. (I) & (II) it is clear that the current leads the voltage by 90° in a Purely capacitive circuit

wave form -



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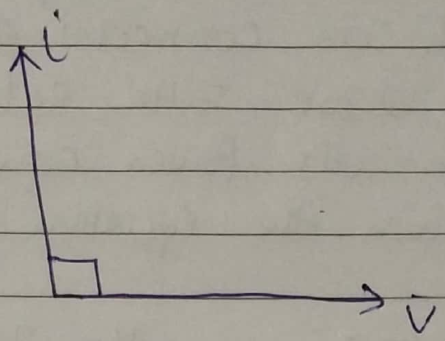
Phasor dia -



Impedance - $Z = \frac{V}{I} = \frac{V_m}{I_m} = \frac{V_m}{\omega C V_m} = \frac{1}{\omega C}$

The quantity $1/\omega C$ is called capacitive reactance is denoted by X_c and is measured in ohms.

for dc supply $f = 0$ $\therefore$ $X_c = \infty$

Thus, the capacitor acts as an open ckt for a dc supply.

Phase diff. - $\phi = 90°$

Power factor - $p.f. = \cos\phi = \cos 90° = 0$

Power $p = vi$

$p = V_m \sin\omega t \, I_m \sin(\omega t + 90°) = V_m I_m \sin\omega t \cos\omega t$

$= \frac{V_m I_m}{2} \sin 2\omega t$

average power for one complete cycle, $P = 0$

when power is positive i.e voltage increases across the plates of capacitors energy is supplied from source to build up the electrostatic field b/w the plates of capacitor and capacitor is energized. when power is negative, i.e voltage decreases, the collapsing electrostatic field returns the stored energy to the source. This circulating power is called as reactive power.

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Q. An ac circuit consist of a pure resistance of $10\,\Omega$ and is connected ~~acro~~ across an ac supply of 230V, 50Hz calculate
(i) current (ii) Power consumed (iii) Power factor
(iv) Write down the equation for voltage and current

Solution:- $R = 10\Omega$, $V = 230V$, $f = 50Hz$

(i) $i = V/R = \frac{230}{10} = 23A$

(ii) $P = VI = 230 \times 23 = 5290\ W$

(iii) Power factor $= \cos\phi = \cos 0' = 1$

(iv)

$V = V_m \sin\omega t$

$V = (\sqrt{2}\,V) \sin(2\pi f t)$

$= (\sqrt{2} \times 230) \sin(2\pi \times 50 t)$

$= 325.27 \sin 314.16 t$

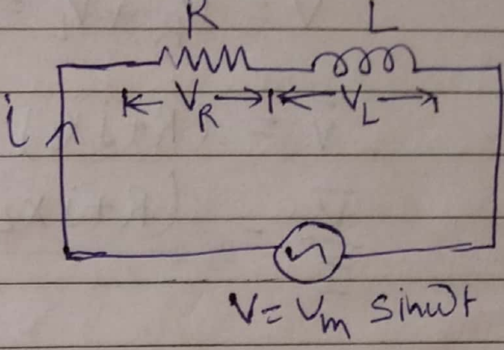
Similarly $i = I_m \sin\omega t$

$i = \sqrt{2}\, I \sin(2\pi \times 50 \times t)$

$i = 32.53 \sin 314.16 t$

Series R-L circuit ⇒

Fig shows a Pure resistor R connected in series with a Pure inductor L across an alternating Voltage V.



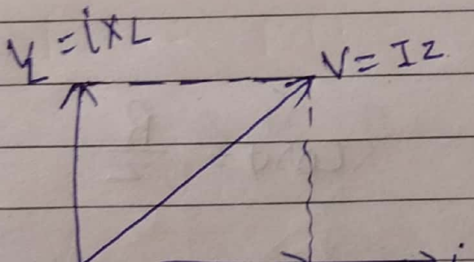
Series R-L circuit

$V_R = IR$

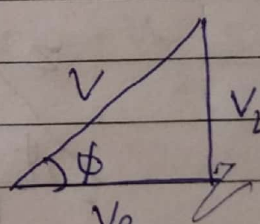
$V_L = I X_L$

The voltage V_R is in phase with current i where as the voltage V_L leads the current i by 90°

The applied voltage, being equal to the Phasor sum of V_R and V_L, will be given by the diagonal of the parallelogram



Phasor dia.



$\cos\phi = \frac{V_R}{V}$

Voltage triangle

it is clear from phasor dia. that current i lags behind the applied voltage by an angle ϕ ($0° < \phi < 90°$).

Impedance -

$$\bar{V} = \bar{V}_R + \bar{V}_L = R\bar{I} + jX_L\bar{I}$$

$$\bar{V} = (R + jX_L)\,I$$

$$\frac{\bar{V}}{\bar{I}} = (R + jX_L) = \bar{Z}$$

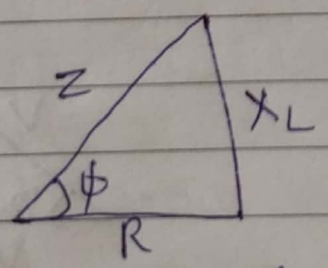
$$\bar{Z} = Z\angle\phi$$

$$Z = \sqrt{R^2 + X_L^2} = \sqrt{R^2 + \omega^2 L^2}$$

$$\phi = \tan^{-1}\left(\frac{X_L}{R}\right) = \tan^{-1}\left(\frac{\omega L}{R}\right)$$

The quantity Z is called the complex impedance of R-L circuit.

Impedance triangle -



impedance triangle.

$$\cos\phi = \frac{R}{Z}$$

Current - from Phasor dia, I lags behind the voltage by an angle ϕ.

if $V = V_m \sin\omega t$

then $i = I_m \sin(\omega t - \phi)$

where $I_m = \frac{V_m}{Z}$ and $\phi = \tan^{-1}\left(\frac{\omega L}{R}\right)$

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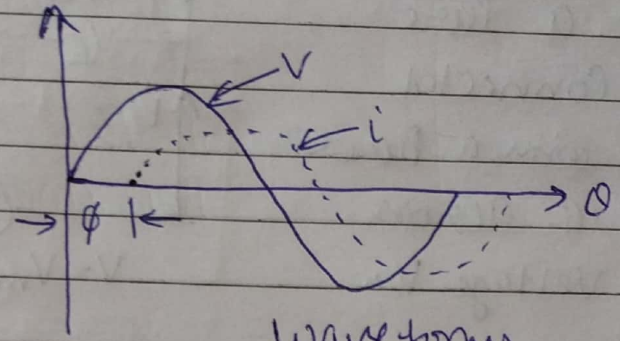
Waveforms – The Voltage and current waveforms are shown below



Waveforms

Power – Instantaneous Power P is given by

$$P = Vi$$

$$P = V_m \sin\omega t \; I_m \sin(\omega t - \phi)$$

$$= V_m I_m \sin\omega t \sin(\omega t - \phi)$$

$$= V_m I_m \left[\frac{\cos\phi - \cos(2\omega t - \phi)}{2}\right]$$

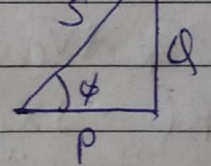
$$P = \frac{V_m I_m}{2}\cos\phi - \frac{V_m I_m}{2}\cos(2\omega t - \phi)$$

Average Power $P = \frac{V_m I_m}{2}\cos\phi = \frac{V_m}{\sqrt{2}}\frac{I_m}{\sqrt{2}}\cos\phi$

$$P = VI\cos\phi$$

{note → average of $\frac{V_m I_m}{2}\cos(2\omega t - \phi)$ will be zero}

Power triangle –



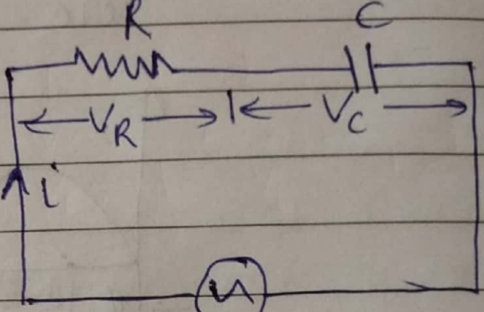
$\cos\phi = \frac{P}{S}$

Power factor –

$$P.f = \cos\phi$$

R-C Series Circuit ⇒

Fig shows a pure resistor R connected in series with a pure capacitance C across an alternating voltage V.



$V = V_m \sin\omega t$

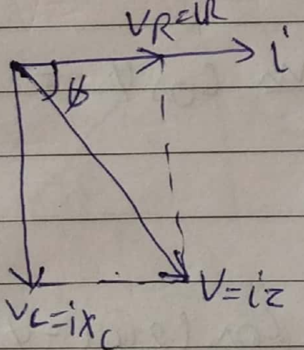
$V_R = iR$

$V_C = iX_C$

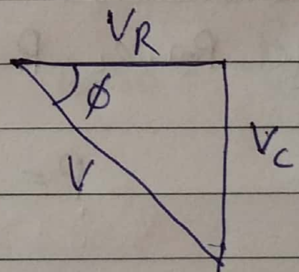
V_R is in phase with current i, whereas voltage V_c lags behind the current i by 90°

$$V = \bar{V}_R + V_C$$

Phasor dia-



Phasor dia



$\cos\phi = \frac{V_R}{V}$

Voltage triangle

Impedance — $\bar{V} = \bar{V}_R + \bar{V}_C$

$\bar{V} = R\bar{i} - jX_C\bar{i}$

$\bar{V} = (R - jX_C)\bar{i}$

$\frac{\bar{V}}{\bar{i}} = R - jX_C = \bar{Z}$

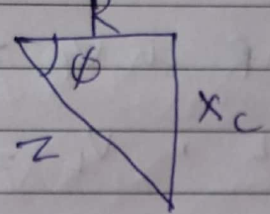
$\bar{Z} = Z\angle{-\phi}$

$$Z = \sqrt{R^2 + X_C^2} = \sqrt{R^2 + \frac{1}{\omega^2 C^2}}$$

$$\phi = \tan^{-1}\left(\frac{X_C}{R}\right) = \tan^{-1}\left(\frac{1}{\omega C R}\right)$$

The quantity $\bar{Z}$ is called the complex impedance of R-C circuit.

impedance triangle -



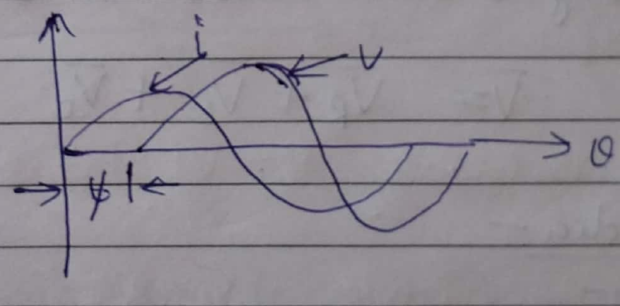
$$\cos\phi = \frac{R}{Z}$$

Current -

$$i = I_m \sin(\omega t + \phi)$$

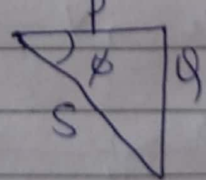
where $I_m = \frac{V_m}{Z}$

waveform-



Power - $P = VI\cos\phi = I^2 R$

Power triangle -



$$\cos\phi = \frac{P}{S}$$

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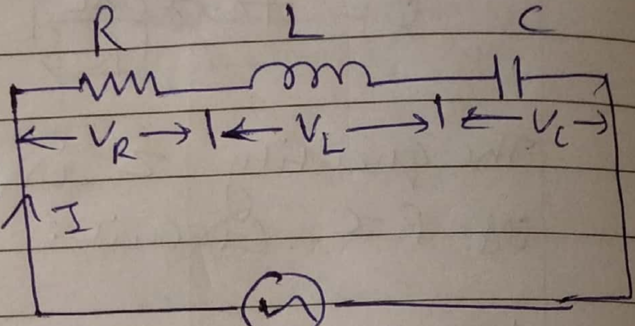
R-L-C series circuit ⇒

fig shows pure resistor, pure inductor and pure capacitor is connected in series across an alternating voltage V.



$V = V_m \sin\omega t$

Voltage drop across R is $V_R = IR$

" " L " $V_L = I X_L$

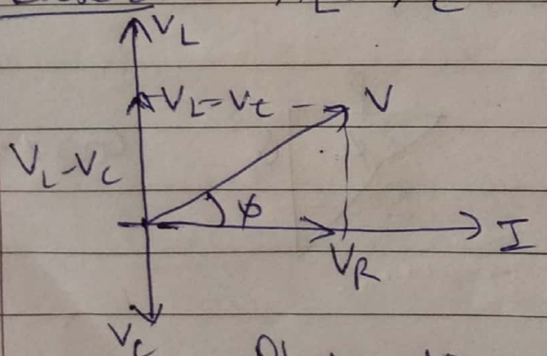
$V_C = I X_C$

The voltage V_R is in phase with I, voltage V_L leads the current I by 90° and V_C lags ~~behind~~ behind current I by 90°

and $\bar{V} = \bar{V_R} + \bar{V_L} + \bar{V_c}$

Phasor dia –

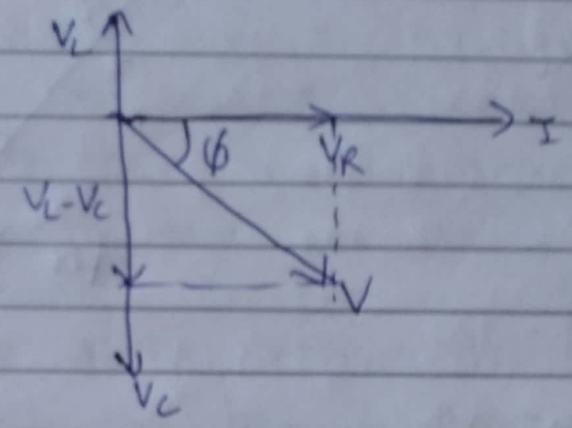
Case I – $X_L > X_C$



Phasor dia

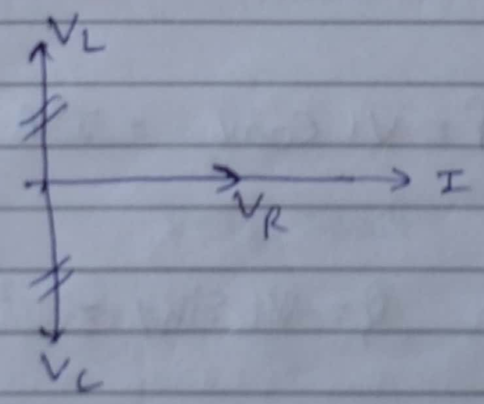
The reactance X will be inductive in nature and the circuit will behaves like an R-L circuit.

Case-2 - $X_C > X_L$



The reactance X will be capacitive in nature and the circuit will behave like an R-C circuit

Case-3 - $X_L = X_C$



$V_L = V_C$, will cancle out eachother circuit will behaves like a pure resistance.

Impedance - $\bar{V} = \bar{V}_R + \bar{V}_L + \bar{V}_C = R\bar{I} + jX_L\bar{I} - jX_C\bar{I}$

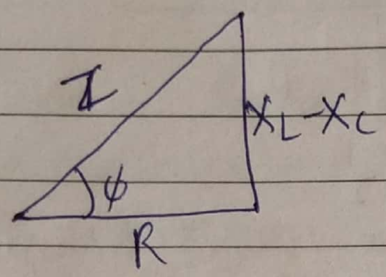
$$\bar{V} = [R + j(X_L - X_C)]\bar{I}$$

$$\frac{\bar{V}}{\bar{I}} = R + j(X_L - X_C) = \bar{Z}$$

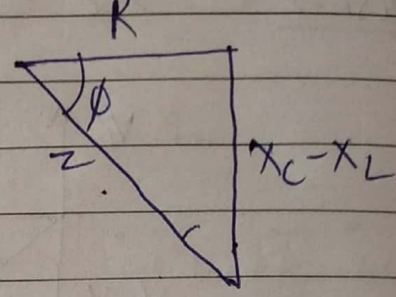
$\bar{Z} = Z\angle\phi$; $Z = \sqrt{R^2 + (X_L - X_C)^2}$

$$\phi = \tan^{-1}\left(\frac{X_L - X_C}{R}\right)$$

Impedance triangle -



for $X_L > X_C$



for $X_C > X_L$

Current equation -

$$i = I_m \sin(\omega t \mp \phi)$$

\- sign used when $X_L > X_C$

\+ sign used when $X_C > X_L$

Power - $P = VI \cos\phi = I^2 R$ Watt
Active Power

Reactive Power $Q = VI \sin\phi = I^2 X$ VAR

Apparent Power $S = VI = I^2 Z$

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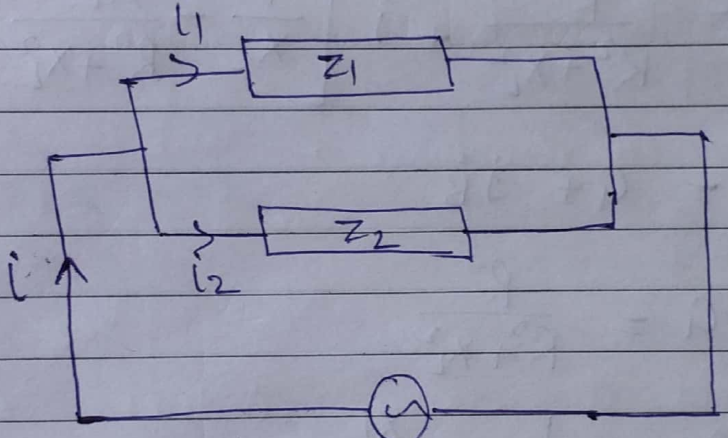
Parallel AC circuits -

In Parallel circuit, resistor, inductor and capacitor or any combination of ~~there~~ these elements are connected across same supply. hence the voltage is same across each branch of the Parallel ac circuit. The total current supplied to the circuit is equal to the phasor sum of the branch currents.



$V = V_m \sin \omega t$

$$\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$$

$$\bar{Y} = \bar{Y}_1 + \bar{Y}_2$$

where Y represents the admittance of the circuit and is define as the reciprocal of impedance. The real part of admittance is called conductance (G) and imaginary part is called susceptance (B), and these are measured in mho (℧) or siemens (S)

if $\bar{Z}_1 = R + jX_L$ and $\bar{Z}_2 = -jX_C$

then, $\frac{1}{Z} = \frac{1}{Z_1} + \frac{1}{Z_2}$

$$= \frac{1}{R+jX_L} + \frac{1}{-jX_C}$$

$$= \frac{1}{R+jX_L} \times \frac{R-jX_L}{R-jX_L} + \frac{1}{-jX_C} \times \cancel{\frac{jX_C}{jX_C}}$$

$$= \frac{R-jX_L}{R^2+X_L^2} + j\frac{1}{X_C}$$

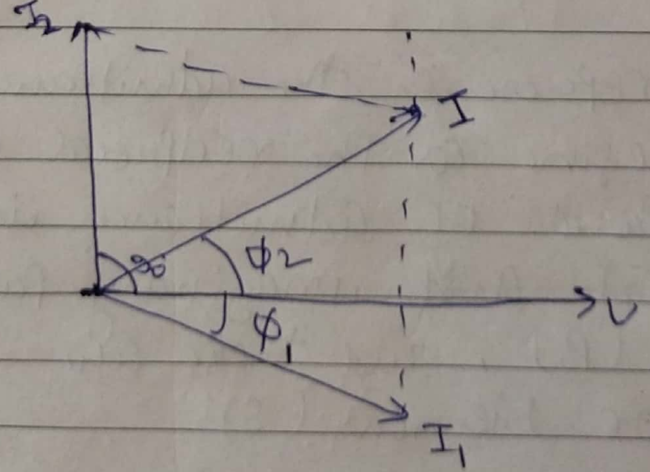
$$= \frac{R}{R^2+X_L^2} + j\left[\frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}\right]$$

$$= G + jB$$

where $G = \frac{R}{R^2+X_L^2}$

$$B = \frac{1}{X_C} - \frac{X_L}{R^2+X_L^2}$$

The current in the Parallel ac circuit can be found as the Phasor sum of the branch currents $\bar{I} = \bar{I}_1 + \bar{I}_2$



Phasor dia.

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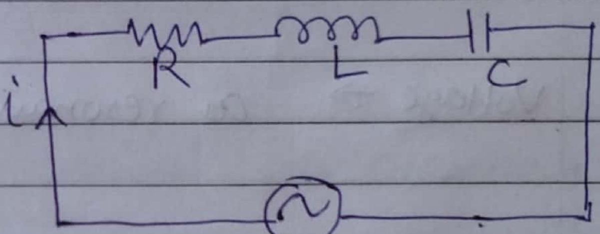
Series Resonance ⇒

A circuit containing reactance is said to be in resonance if the voltage across the circuit is in phase with current through it. At resonance, the circuit thus behaves as a pure resistor and the net reactance is zero.

Consider the series R-L-C circuit as shown in fig.



$V = V_m \sin\omega t$

$$\bar{Z} = R + jX_L - jX_C$$

$$= R + j\omega L - \frac{j}{\omega C}$$

$$Z = R + j\left(\omega L - \frac{1}{\omega C}\right)$$

at resonance Z must be resistive, therefore the condition for resonance is

$$\omega L - \frac{1}{\omega C} = 0$$

$$\omega L = \frac{1}{\omega C}$$

$$\omega^2 = \frac{1}{LC}$$

$$\omega_0 = \omega = \sqrt{\frac{1}{LC}}$$

$$f_0 = f = \frac{1}{2\pi\sqrt{LC}}$$

where fo is the resonance frequency of ckt.

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Power factor → $P.F = \cos\phi = \frac{R}{Z}$

at resonance - $Z = R$; $P.F = \cos\phi = \frac{R}{R} = 1$

Current → Since the impedance is minimum, the current is maximum at resonance. Thus, the circuit accepts more current and as such an R-L-C circuit under resonance is called an acceptor circuit.

$$I_0 = \frac{V}{Z} = \frac{V}{R}$$

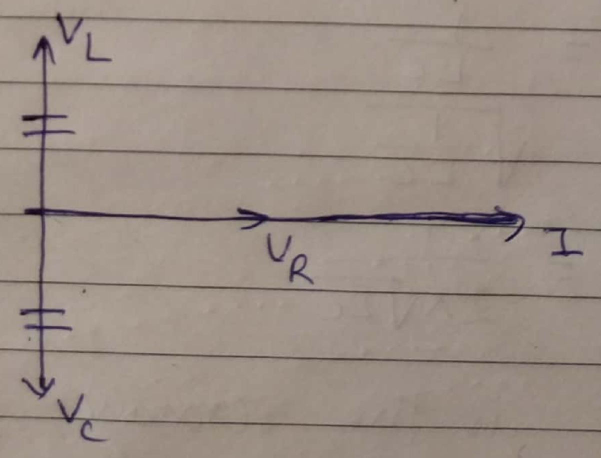
Voltage → at resonance, $\omega_0 L = \frac{1}{\omega_0 C}$

$$\omega_0 L I_0 = \frac{1}{\omega_0 C} I_0$$

$$V_{L_0} = V_{C_0}$$

Thus Potential diff. across inductor equal to Potential diff across capacitor being equal and opposite. Cancel each other. also I_0 max., V_{L0} and V_{C0} will also be max. Thus voltage magnification takes place during resonance. Hence it is also reffered to as voltage magnification circuit

Phasor dia →

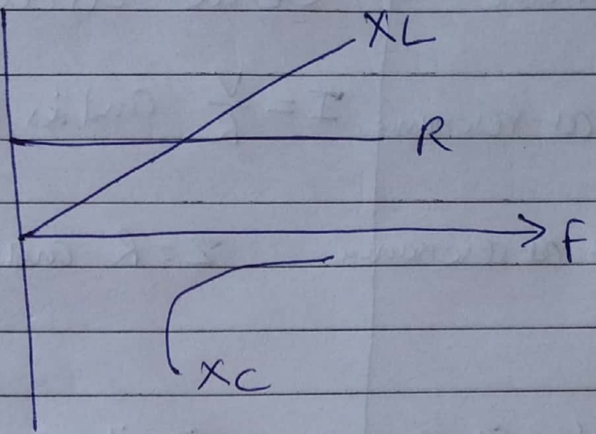


Phasor dia.

Behavior of R,L and C with Change in frequency →

Resistance remains constant with freq.

X_L is directly proportional to f.



and X_C is inversely proportional to f

fig Behaviour of RLC with change in freq.

$$z = R + j(X_L - X_C)$$

(a) when $f < f_0$ impedance is capacitive
(b) when $f = f_0$ impedance is resistive
(c) when $f > f_0$ impedance is inductive

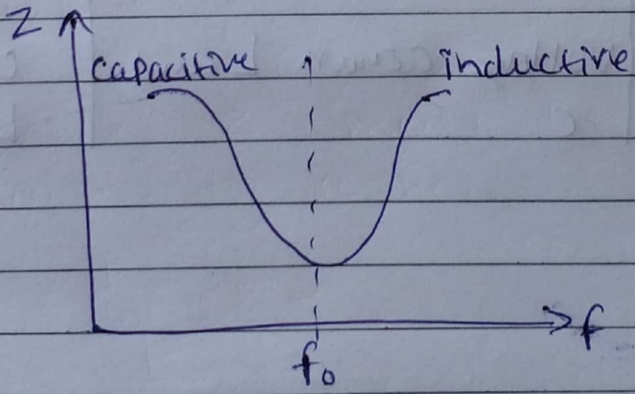


Fig - impedance.

quality factor → it is a measure of voltage magnification in the series resonance circuit.

$$Q_0 = \frac{\text{Voltage across inductor or capacitor}}{\text{Voltage at resonance}}$$

$$= \frac{V_{L0}}{V} = \frac{V_{C0}}{V} \quad \Bigg| \quad Q_0 = \frac{1}{R}\sqrt{\frac{L}{C}}$$

Comparison of series and Parallel resonance ⇒

Parameter	Series circuit	Parallel circuit
Current at resonance	$I = \frac{V}{R}$ and is max.	$I = \frac{VCR}{L}$ and is min
impedance at resonance	$Z = R$ and is min	$Z = \frac{L}{CR}$ and is max.
Power factor at resonance	unity	unity
Resonant freq.	$f_0 = \frac{1}{2\pi\sqrt{LC}}$	$f_0 = \frac{1}{2\pi}\sqrt{\frac{1}{LC} - \frac{R^2}{L^2}}$
Q factor	$Q = \frac{2\pi f_0 L}{R}$	$Q = \frac{2\pi f_0 L}{R}$
it magnifies	Voltage across L and C	current through L and C.