JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Jaipur Engineering College and Research Center

- Year & Sem. – B. Tech. I-Year, Semester - I
- Subject – Engineering Physics (1FY2-02)
- Chapter - Electromagnetism (Part-I)
- Department - Applied Science (Physics)

Vision and Mission

- **Vision:**

➢ To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

- **Mission:**

➢ Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.

➢ Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.

➢ Offer opportunities for interaction between academia and industry.

➢ Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

Syllabus and Course Outcome of Introduction to Electromagnetism

Syllabus: Divergence and curl of electrostatic field, Laplace's and Poisson's equations for electrostatic potential, Bio-Savart law, Divergence and curl of static magnetic field, Faraday's law, Displacement current and magnetic field arising from time dependent electric field, Maxwell's equations, Flow of energy and Poynting vector.

CO4: Students will be able to describe key concepts and acquire basics of electrostatics and electromagnetism (e.g., Maxwell's equations) to explain electromagnetic waves propagation and generation in free space, dielectrics and conducting media.

Lecture Plan of Introduction to Electromagnetism

S. No	Topics	Lectures required	Lect. No.
1	General introduction to Electromagnetism.	1	33
2	Divergence and curl of electrostatic & magnet field	1	34
3	Laplace's and Poisson's equations for electrostatic potential.	1	35
4	Bio-Savart law and Faraday's law.	1	36
5	Displacement current and magnetic field arising from time dependent electric field	1	37
6	Maxwell's Equation first and second	1	38
7	Maxwell's Equation third and fourth	1	39
8	Flow of energy and Poynting vector.	1	40

Content

- Introduction and some definations
- Gradient, divergence and curl of electrostatic field
- Gauss Divergence Theorem
- Stoke's Theorem
- Biot Savart Law.
- Divergence and curl of static magnetic field
- Faradays Laws of electromagnetic induction
- Suggested reference books & links from NPTEL/IIT/RTU Plateforms
- Important Questions.

Introduction to Electromagnetism

- Earlier electric and magnetic fields are considered independent parameters but experimentally it was found that they are interlinked with each-other

- It deals with relationship between electricity and magnetism. Electricity makes magnetism (as per Ampere's law) and vice - versa also true (Faraday’s law).

- In this chapter we can understand Electromagnetic waves generation, propagation in different media which is useful for Communication Electronics.

Some important Definitions

- Electrostatics :- When charges are at rest.
- Electrodynamics :- When charges are in motion.
- Electromagnetic :- It generates when charges waves get accelerated.
- Electric Flux :- When electric lines of forces passing by a small surface area.
- Magnetic Flux :- When magnetic lines of forces passing by a small area.
- <u>Conduction Current</u> :- It flows through an electrical circuit due to flow of free electrons via connection wires.
- <u>Displacement Current</u> :- It arises due to rate of change of electric flux between the two plates of the capacitor.

- Stokes's Theorem :- It deals with relation between line integral and surface integral.

- Gauss Divergence theorem:- It deals with relationship between surface integral and volume integral.

- Biot – Savart Law:- This states that the magnetic field (dB) at point P due to an element of length (dl) carrying current (I)

$$|dB| = (\mu\mathbf{0} / 4\pi)\ (I\ dl\ \sin\Theta / r^2)$$

- Ampere’s Law:- It states that line integral of a magnetic field in a closed path (or loop) is μ times the net current (I) flowing through the area enclosed by the path.

$$\int B.dl = \mu\ \Sigma I$$

Gradient, divergence and curl of electrostatic field

Gradient of a Scalar Field

In a scalar field, the maximum rate of change of the scalar function in space is known as the gradient of the scalar field. It is a vector quantity and its direction is normal to the equiscalar surface i.e. the direction in which rate of change is maximum.

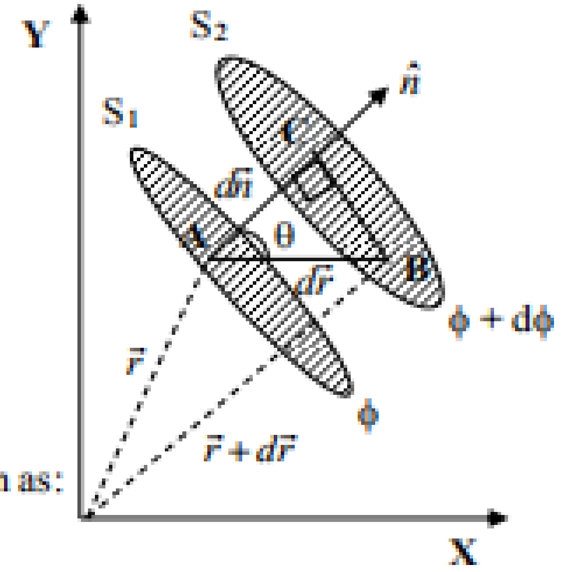
If two scalar field surfaces S_1 and S_2 having coefficient scalar quantities ϕ and $\phi + d\phi$ (as shown in figure), then the rate of change of scalar quantity in direction AB is $\frac{d\phi}{dr}$ and maximum rate of change of scalar quantity will be

at $d\vec{r} = d\vec{n}$:

$$\left(\frac{d\phi}{dr}\right)\max = \frac{d\phi}{dn} \qquad \{\because d\vec{n} \text{ is minimum of } d\vec{r}\}$$

is called gradient of ϕ, i.e.

$$grad\,\phi = \vec{\nabla}\phi = \left(\frac{d\phi}{dn}\right)\hat{n}$$



Gradient of a Scalar in Cartesian Coordinates will be written as:

$$\nabla\phi = \hat{i}\frac{\partial\phi}{\partial x} + \hat{j}\frac{\partial\phi}{\partial y} + \hat{k}\frac{\partial\phi}{\partial z}$$

Example of Gradient of a scaler

If $V = x^2y^2 + xyz$ find Gradient of V (∇V) at a point P(2,1,1)

$$\nabla V = \frac{\partial V}{\partial x}\vec{a_x} + \frac{\partial V}{\partial y}\vec{a_y} + \frac{\partial V}{\partial z}\vec{a_z}$$

$$\nabla V = \frac{\partial(x^2y^2 + xyz)}{\partial x}\vec{a_x} + \frac{\partial(x^2y^2 + xyz)}{\partial y}\vec{a_y} + \frac{\partial(x^2y^2 + xyz)}{\partial z}\vec{a_z}$$

$$\nabla V = (2x\,y^2 + yz)\vec{a_x} + (2x^2y + xz)\vec{a_y} + (xyz)\,\vec{a_z}$$

At point P

$$\nabla V_p = \{(2 \times 2 \times 1^2) + (1 \times 1)\}\vec{a_x} + \{(2 \times 2^2 \times 1) + (2 \times 1)\}\vec{a_y} + \{2 \times 1 \times 1\}\vec{a_z}$$

$$\nabla V_p = 5\vec{a_x} + 10\vec{a_y} + 2\vec{a_z}$$

Divergence of Vector field

The vector flux per unit volume emerging out from the closed surface enclosing a volume element placed in a vector field is known as the divergence of vector field.

If a closed surface enclosing a volume element dV placed in a vector field $\vec{A}$ (x, y, z), then the flux emerges out from the surface, will be:

$$\phi = \oint_S \vec{A} d\vec{S}$$

and divergence of a vector $\vec{A}$ at volume element

$$div\,\vec{A} = \lim_{dV \to 0} \left[\frac{1}{dV} \oint_S \vec{A} d\vec{S} \right]$$

The divergence of a vector field is a scalar quantity. The divergence of a vector field $\vec{A}$ in Cartesian coordinate is written as:

$$div\,\vec{A} = \vec{\nabla}.\vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

Physical meaning of divergence of a vector field & Example

The divergence of vector field is the flowing of vector flux, per sec. per unit volume element. The divergence of a vector field may be positive, negative or zero.

(1) If the divergence of vector field is positive, then it means that a source is situated in the enclosed volume element and vector field are emerging out from it.

(2) If the divergence of vector field is negative, then it means that a sink is situated in the enclosed volume element and vector field are flowing into enclosed volume and is being collected there.

(3) If the divergence of the vector field is zero, then there is neither an increase nor a decrease in the amount of vector field in the enclosed volume, i.e. neither a source nor a sink of vector field inside the enclosed volume.

If $D = (2xyz - y^2)\vec{a_x} + (x^2z - 2xy)\vec{a_y} + x^2y\,\vec{a_z}$, find divergence ($\nabla \cdot \vec{D}$) of $\vec{D}$ at a point P(2,3-1).

We know that $\nabla \cdot \vec{D} = \left\{\frac{\partial D_x}{\partial x} + \frac{\partial D_y}{\partial y} + \frac{\partial D_z}{\partial z}\right\}$

$$\nabla \cdot \vec{D} = (2yz) + (2x)$$

At point 'P'

$$\nabla \cdot \vec{D} = (2 \times 3 \times -1) + (-2 \times 2) = -10$$

Gauss's Divergence of a Vector field

Let in vector field $\vec{A}$, a volume element V enclosed by a surface S. If surface S divide into small areas dS_1, dS_2, … then the total emerging flux from surface is the sum of emerging flux from each small areas i.e.

$$\phi = \vec{A}.d\vec{S}_1 + \vec{A}.d\vec{S}_2 + \vec{A}.d\vec{S}_3 + \ldots$$

$$\phi = \oiint_S \vec{A}.d\vec{S} = \sum \vec{A}.d\vec{S}_i \qquad \ldots (1)$$

But from divergence of vector field $\vec{A}$ for small area ds_i:

$$\vec{\nabla}.\vec{A} = \frac{1}{dV}\left(\vec{A}.d\vec{S}_i\right)$$

$$\Rightarrow \quad \vec{A}.d\vec{S}_i = \left(\vec{\nabla}.\vec{A}\right)dV \qquad \ldots (2)$$

from equation (1) & (2)

$$\oiint_S \vec{A}.d\vec{S} = \sum\left(\vec{\nabla}.\vec{A}\right)dV$$

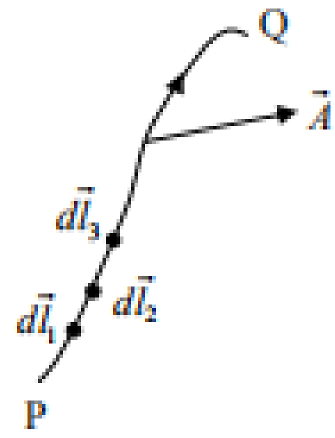
if $\quad dV \to 0 \quad \Rightarrow \quad \Sigma = \iiint_V$

$$\Rightarrow \quad \boxed{\oiint_S \vec{A}.d\vec{S} = \iiint_V \left(\vec{\nabla}.\vec{A}\right)dV} \qquad \ldots (3)$$

That is the surface integral of vector $\vec{A}$ taken over the closed surface enclosed volume V is equal to the volume integral of the divergence of vector field $\vec{A}$ taken over volume V. This state is called **Gauss Divergence Theorem**.

Line Integral of a Vector Field

Consider PQ be the path in the vector field $\vec{A}$ and divided into small line elements $d\vec{l}_1, d\vec{l}_2, d\vec{l}_3, ..., d\vec{l}_n$, then the sum of the scalar products of vector field $\vec{A}$ and line elements i.e.



$$W = \vec{A}.d\vec{l}_1 + \vec{A}.d\vec{l}_2 + ... + \vec{A}.d\vec{l}_n$$

$$\Rightarrow \quad W = \sum_P^Q \vec{A}.d\vec{l}_i$$

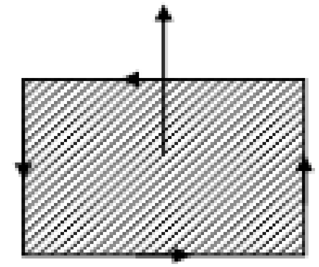
But $\quad dl \to 0$, then $\quad \sum_P^Q = \int_P^Q$

$$\Rightarrow \quad W = \int_P^Q \vec{A}.d\vec{l}$$

This integral is known as the line integral of a vector field from a point P to a point Q.

Curl of a Vector Field

The line integral depends on the orientation of the surface enclosed by the closed path. If the surface enclosed by the closed path is kept in such a way that the line integral or circulation becomes maximum, than in this orientation the ratio of the line integral to the area enclosed by the path (with area tending to zero) is equal to the magnitude of curl of the vector field at the position of area element, i.e.



da

$$\left|curl\ \vec{A}\right| = \lim_{dr \to 0} \frac{W_{max.}}{dS}$$

$$\left|\vec{\nabla} \times \vec{A}\right| = \lim_{dS \to 0} \left[\frac{1}{dS} \left\{ \oint_C \vec{A}.d\vec{l} \right\}_{max} \right]$$

if $\hat{n}$ is the unit vector in the direction of vector area element, then

$$\left(\vec{\nabla} \times \vec{A}\right).\hat{n} = \lim_{dS \to 0} \left[\frac{1}{dS} \oint_C \vec{A}.d\vec{l} \right]$$

It's direction is perpendicular to the plane which contains the area element with maximum circulation around its boundary. The curl of a vector field is a vector quantity.

The curl of a vector field $\vec{A}$ in Cartesian coordinate system is written as:

$$curl\ \vec{A} = \vec{\nabla} \times \vec{A} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ A_x & A_y & A_z \end{vmatrix}$$

Stoke's Curl Theorem

Let in a vector field $\vec{A}$, an area enclosed by a close path C, if this area divided into small areas, then remains outer boundary C the total line integral of common path of all areas become zero. So the sum of line integrals along boundary of areas is equal to the line integral along boundary C.

i.e. $$\oint_C \vec{A}.d\vec{l} = \sum \vec{A}.d\vec{l}_i \qquad \dots (1)$$

By the definition of curl for small area at boundary

$$\left(\vec{\nabla}\times\vec{A}\right).\hat{n} = \frac{1}{dS}\left(\vec{A}.d\vec{l}_i\right) \quad \Rightarrow \quad \vec{A}.d\vec{l}_i = \left(\vec{\nabla}\times\vec{A}\right)\hat{n}\,dS$$

$$\Rightarrow \quad \vec{A}.d\vec{l}_i = \left(\vec{\nabla}\times\vec{A}\right).d\vec{S} \qquad \left(\because \hat{n}\,dS = d\vec{S}\right) \qquad \dots (2)$$

from equation (1) and (2)

$$\oint_C \vec{A}.d\vec{l} = \sum \left(\vec{\nabla}\times\vec{A}\right).d\vec{S}$$

But $\quad dS \to 0 \qquad \sum = \int$

$$\Rightarrow \quad \boxed{\oint_C \vec{A}.d\vec{l} = \iint_s \left(\vec{\nabla}\times\vec{A}\right)d\vec{S}}$$

i.e. the line integral of vector $\vec{A}$ around the boundary of surface is equal to the surface integral of the curl of vector $\vec{A}$ taken over any surface S. This state is called **Stoke's Curl Theorem.**

Biot Savart Law

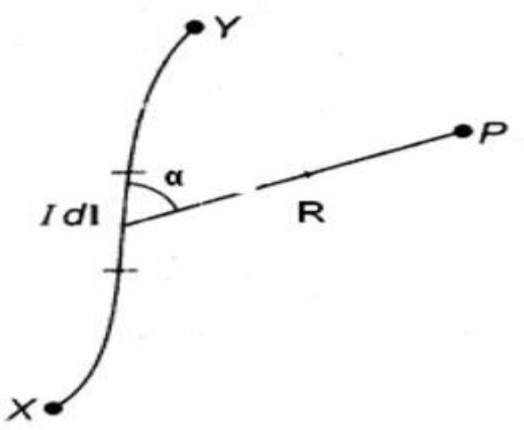
According to Biot-Savart law the magnetic field intensity proceed at point P due to differential current element IdL is proportional to the product IdL and the sine of the angle 'α' between the element and line joining point P to the element and is inversely proportional to the square of the distance R between point P and the element.

i.e.

(*i*) $d\vec{H} \propto IdL$

(*ii*) $d\vec{H} \propto \sin\alpha$

(*iii*) $d\vec{H} \propto \frac{1}{R^2}$



$$\rightarrow \quad d\vec{H} \propto \frac{IdL\ \sin\alpha}{R^2}$$

$$\rightarrow \quad d\vec{H} = K\frac{IdL\ \sin\alpha}{R^2}$$

$$\rightarrow \quad d\vec{H} = \frac{IdL\ \sin\alpha}{4\pi R^2}$$

$$\boxed{\rightarrow \quad d\vec{H} = \frac{IdL\ \sin\alpha}{4\pi R^2}} \qquad \left\{\therefore K = \frac{1}{4\pi}\right\} \qquad \ldots\ldots\ldots\ldots(1)$$

If $\hat{r}$ be the unit vector in direction from $d\vec{L}$ to P

Then $\quad d\vec{L} \times \hat{r} = dL \sin \propto \qquad \ldots\ldots\ldots\ldots(2)$

$$\boxed{\rightarrow \quad d\vec{H} = \frac{Id\vec{L} \times \hat{r}}{4\pi R^2}} \frac{Ampere}{meter} \qquad \ldots\ldots\ldots\ldots(3)$$

$Eq^n(3)$ gives the differential magnetic field intensity at point P.

Total magnetic field intensity can be obtained by integrating equation (3)

i.e. $$\vec{H} = \oint d\vec{H}$$

$\rightarrow$ $$\vec{H} = \oint \frac{I d\vec{L} \times \hat{r}}{4\pi R^2}$$

$\rightarrow$ $$\vec{H} = \oint \frac{I d\vec{L}}{4\pi R^2} \times \frac{\vec{R}}{|\vec{R}|}$$

$\rightarrow$ $$\vec{H} = \oint \frac{I d\vec{L} \times \vec{R}}{4\pi R^3}$$

$\rightarrow$ $$\vec{H} = \frac{1}{4\pi} \oint I \frac{d\vec{L} \times \vec{R}}{R^3} \qquad \text{... ... ... (4)}$$

Eq^n (4) gives the integral form of Biot-savart Law.

Curl of Static Magnetic field

1.Curl of a vector function($\vec{H}$) is equal to cross product of ∇ and $\vec{H}$ ($\nabla \times \vec{H}$)

2.(The physical interpretation of curl of vector field :)

Curl of a vector field gives the maximum rotation of vector field per unit area as the area tends to zero.

$$\nabla \times \vec{H} = \lim_{ds \to 0} \frac{\oint \vec{H} \cdot \vec{dl}}{ds}$$

3.If the curl of the vector field is zero then it is called as irrotational field

3. If $\vec{H} = H_x \vec{a_x} + H_y \vec{a_y} + H_z \vec{a_z}$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

If $\vec{H} = x^2yz\,\vec{a_x} + xz\vec{a_z}$ find $\nabla \times H$

$$\nabla \times \vec{H} = \begin{vmatrix} \vec{a_x} & \vec{a_y} & \vec{a_y} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ H_x & H_y & H_z \end{vmatrix}$$

$$\nabla \times \vec{H} = \left(\frac{\partial H_z}{\partial y} - \frac{\partial H_y}{\partial z}\right)\vec{a_x} + \left(\frac{\partial H_x}{\partial z} - \frac{\partial H_z}{\partial x}\right)\vec{a_y} + \left(\frac{\partial H_y}{\partial x} - \frac{\partial H_x}{\partial y}\right)\vec{a_z}$$

$$= \left(\frac{\partial\,(xz)}{\partial y} - \frac{\partial(0)}{\partial z}\right)\vec{a_x} + \left(\frac{\partial x^2yz}{\partial z} - \frac{\partial(xz)}{\partial x}\right)\vec{a_y} + \left(\frac{\partial\,(0)}{\partial x} - \frac{\partial x^2yz}{\partial y}\right)\vec{a_z}$$

$$= (0-0)\vec{a_x} + (x^2y - z)\vec{a_y} + (0 - x^2z)\vec{a_z}$$

$$\nabla \times \vec{H} = (x^2y - z)\vec{a_y} - x^2z\,\vec{a_z}$$

Stokes theorem

Stokes theorem states that the circulation of a vector field ,around a closed path, is equal to the surface integral of the curl of that vector field, over the open surface bounded by the closed path.

$$\nabla \times H = \lim_{ds \to 0} \frac{\oint H \cdot dl}{ds}$$

$$\oint_l \vec{H} \cdot \vec{dl} = \oiint_s (\nabla \times \vec{A}) \cdot \vec{ds}$$

$$\oiint_s (\nabla \times \vec{A}) \cdot \vec{ds} = \oint_l \vec{H} \cdot \vec{dl}$$

Faraday's Law of Electromagnetic Induction

It is seen that a current carrying conductor produces a magnetic field, So, Faraday said if a current could produce a magnetic field then the reverse effect should also be possible that is a changing magnetic field should also produce a current in the loop or closed circuit

Faraday's law states that "the electromotive force induced in a close path is equal to the negative of the time rate of change of magnetic flux enclosed by path" i.e.

$$\text{e.m.f.} = -N\frac{d\varphi B}{dt} \qquad \ldots\ldots(1)$$

Where $\frac{d\varphi B}{dt}$ is the time rate of change of magnetic flux (φ_B) N is number of turns of the coil and negative sign is according to Lenz's Law.

Suggested reference Books & links from NPTEL/IIT/RTU Plateforms

Reference Books:

1. Introduction to Electrodynamics: David J Griffiths (Prentice-Hall of India)
2. Engineering Physics: Malik and Singh (Tata McGraw Hill)
3. Engineering Physics: S. Mani Naidu (Pearson Education)
4. Concept of Modern Phyiscs: A. Baiser (Tata McGraw Hill)
5. Engineering Physics : Y. C. Bhatt (Ashirwad Publications)
6. Engineering Physics : S. K. Sharma (Genius Publication)
7. Engineering Physics: D. K. Bhattacharya (Oxford Higher Education)

Suggested Links:

1. https://nptel.ac.in/courses/115/106/115106122/(a series of video lectures by Prof. Nirmal Ganguli IISER Bhopal)
2. https://nptel.ac.in/courses/115/101/115101004/ (a series of lectures by Prof. Amol Dighe, IIT Bombay)
3. https://nptel.ac.in/courses/115/104/115104088/ (a series of video lectures by Prof. Manoj Harbola IIT Kanpur)
4. https://nptel.ac.in/courses/115/101/115101005/ (a series of video lectures by Prof. D. K. Ghosh, IIT Bombay)
5. https://youtu.be/Av-n6qrb-y8 (a video lecture by Prof. Nirmal Ganguli IISER Bhopal)

Important Questions

1. Explain the gradient and divergence with physical significance.
2. Explain the curl of a vector field.
3. Explain Stoke's curl theorem.
4. Explain Gauss divergence theorem.
5. State Faraday's law of electromagnetic induction.
6. State Biot-Savart's law. Obtain an expression for magnetic field intensity due to infinitely long straight line current.
7. A circular coil is of 10 turns and radius 01m. If a current of 5A flows through it, calculate the field in the coil from a distance of 2m.
8. Prove that div curl A = 0
9. If A= 20x2 i+10y2j+4k then find divergence at point (0,1,0).
10. If E = 20i+10x2 j + 4yk then find curl of E at (2,4,6).

Thank You

JECRC Foundation