Information Theory & Coding
(5CS3-01)

Unit-5 Notes

Vision of the Institute

To become a renowned center of outcome based learning and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of the Institute

M1- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
M2- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
M3- Offer opportunities for interaction between academia and industry.
M4- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

Vision of the Department

To become renowned Centre of excellence in computer science and engineering and make competent engineers & professionals with high ethical values prepared for lifelong learning.

Mission of the Department

M1-To impart outcome based education for emerging technologies in the field of computer science and engineering.
M2-To provide opportunities for interaction between academia and industry.
M3- To provide platform for lifelong learning by accepting the change in technologies
M4- To develop aptitude of fulfilling social responsibilities.

Program Outcomes (PO)

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problemsand design system components or processes that meet thespecified needs with appropriate consideration for the public health and safety, andthe cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issuesand the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Educational Objectives (PEO)

1. To provide students with the fundamentals of Engineering Sciences with more emphasis in **Computer Science &Engineering** by way of analyzing and exploiting engineering challenges.
2. To train students with good scientific and engineering knowledge so as to comprehend, analyze, design, and create novel products and solutions for the real life problems.
3. To inculcate professional and ethical attitude, effective communication skills, teamwork skills, multidisciplinary approach, entrepreneurial thinking and an ability to relate engineering issues with social issues.
4. To provide students with an academic environment aware of excellence, leadership, written ethical codes and guidelines, and the self-motivated life-long learning needed for a successful professional career.
5. To prepare students to excel in Industry and Higher education by Educating Students along with High moral values and Knowledge

Program Specific Outcomes (PSO)

PSO1: Ability to interpret and analyze network specific and cyber security issues, automation in real word environment.

PSO2: Ability to Design and Develop Mobile and Web-based applications under realistic constraints.

Course Outcome:

CO1: Apply the fundamental concepts of information theory viz. entropy, mutual information and channel capacity in communication system.

CO2: Examine the principles of source coding and data transmission.

CO3: Analyze linear block code, cyclic code and Convolution code.

CO4: Evaluate information theoretic methods to novel settings of encoding and decoding techniques.

CO-PO Mapping:

CO	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO1 0	PO1 1	PO1 2
Apply the fundamental concepts of information theory viz. entropy, mutual information and channel capacity in communication system.	3	2	2	2	1	1	1	1	1	1	1	3
Examine the principles of source coding and data transmission.	3	3	3	3	2	1	1	1	1	1	1	3
Analyze linear block code, cyclic code and Convolution code.	3	3	3	2	1	1	1	1	1	1	1	3
Evaluate information theoretic methods to novel settings of encoding and decoding techniques.	3	3	3	2	1	1	1	1	1	1	1	3

SYLLABUS:

RAJASTHAN TECHNICAL UNIVERSITY, KOTA

Syllabus

III Year-V Semester: B.Tech. Computer Science and Engineering

5CS3-01: Information Theory & Coding

Credit: 2
2L+0T+0P

Max. Marks: 100(IA:20, ETE:80)
End Term Exam: 2 Hours

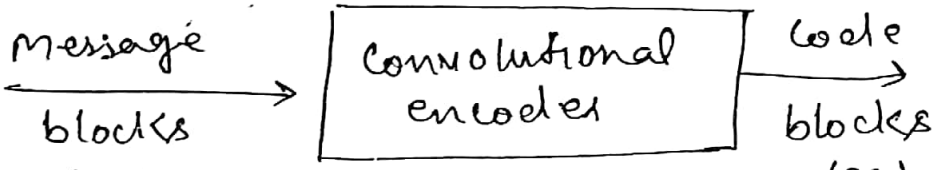
SN	Contents	Hours
1	**Introduction:**Objective, scope and outcome of the course.	01
2	**Introduction to information theory:** Uncertainty, Information and Entropy, Information measures for continuous random variables, source coding theorem. Discrete Memory less channels, Mutual information, Conditional entropy.	05
3	**Source coding schemes for data compaction:** Prefix code, Huffman code, Shanon-Fane code &Hempel-Ziv coding channel capacity. Channel coding theorem. Shannon limit.	05
4	**Linear Block Code:** Introduction to error connecting codes, coding & decoding of linear block code, minimum distance consideration, conversion of non-systematic form of matrices into systematic form.	05
5	**Cyclic Code:** Code Algebra, Basic properties of Galois fields (GF) polynomial operations over Galois fields, generating cyclic code by generating polynomial, parity check polynomial. Encoder & decoder for cyclic codes.	06
6	**Convolutional Code:** Convolutional encoders of different rates. Code Tree, Trllis and state diagram. Maximum likelihood decoding of convolutional code: The viterbi Algorithm fee distance of a convolutional code.	06
	Total	28

LECTURE PLAN:

Unit No./ Total lec. Req.	Topics	Lect. Req.
	Objective, Scope & Outcome of the Course	1
Unit-1	Introduction to information theory, Uncertainty, Entropy	1
	Information measures for continuous random variables	1
	Numerical problem on entropy	1
	Source coding theorem, Discrete memory less channels	1
	Mutual information, Conditional entropy	1
Unit-2	Prefix code, Huffman coding	1
	Shannon – fanon coding	1
	Numerical on haffman and shanon fano coding	1
	Hempel-Ziv coding	1
	Channel capacity, Channel coding theorem, Shannon limit	1
Unit-3	Introduction to error correcting codes	1
	Coding and decoding of linear block code	1
	Numerical problem on Linear block code	1
	Error correcting codes, Minimum distance consideration	1
	Conversion of non symmetric form of matrix into symmetric form	1
Unit-4	Code algebra	1
	Basic properties of Galois Field(GF)	1
	Polynomial operation over Galois field	1
	Generating cyclic code by generating polynomial	1
	Numerical Problems on generator polynomial	1
	Parity check polynomial , Encoder and decoder for cyclic codes	1
Unit-5	Convolutional encoders of different rates	1
	Code tree	1
	Trellis diagram	1
	state diagram	1
	Maximum likelihood decoding of convolution code	1
	Viterbi algorithm, Free distance of convolution codes	1

CONVOLUTIONAL CODES! A convolutional codes is a type of error correcting code in which each K bit information symbol to be encoded, is transformed into an n-bit symbol where K/n is called the code rate($n \geq K$).

Convolutional codes are used to improve the performance of the digital radio, mobile phone, satellit links and bluetooth implementations.



General Convolutional Encoder

Code Parameters!-

n = no of output bits
K = no of input bits
M = no of memory registes.

Commonly K and n parameters range from 1 to 8, and M from 2 to 10 and the code rate from $\frac{1}{8}$ to $\frac{7}{8}$.

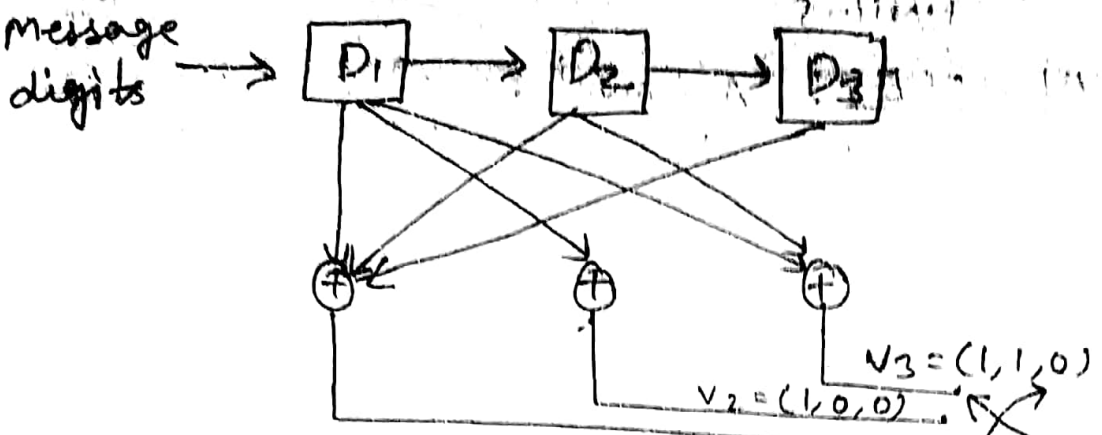
L = Constraint length of the code and defined by

$$L = K(M-1)$$

The Constraint length L Represents the number of bits in the encoder memory that affects the generation of the ~~AA~~ n-output bits.

CONVOLUTIONAL ENCODING!-

1) **Encoder Representation!-** First draw M boxes representing the M memory registes. Then draw n modulo adders to represent the n output bits. Now connect the memory registes to the adders using the generator Polynomials.



This (3,1,3) Convolutional Encoder has 3 memory registers, 1 input bit and 3 output bits.

operation of encoder :- the above Encoder is a rate 1/3 code. each input bit is coded into 3 output bits. The constraint length of the code is 2. The 3 output bits are Produced by the 3 modulo-2 adders by adding up certain bits in the memory registers. The selection of which bits are to be added to produce the output bit is called the generator polynomial (g) for that output bit.

for example :- The first output bit has a generator polynomial of (1,1,1). The output bit 2 has a generator polynomial of (1,0,0) and the third output bit has a polynomial of (1,1,0). The output bits just the sum of these bits.

$$V_1 = \text{Mod } 2 (D_1 + D_2 + D_3)$$

$$V_2 = \text{Mod } 2 (D_1)$$

$$V_3 = \text{Mod } 2 (D_1 + D_2)$$

The steps of operation :-

1. initially shift registers are cleared. The first bit of input data entered into D_1.
2. During this message bit interval the commutator samples modulo-2 adder output's V_1, V_2, V_3 Single

message bit yields three output bits.

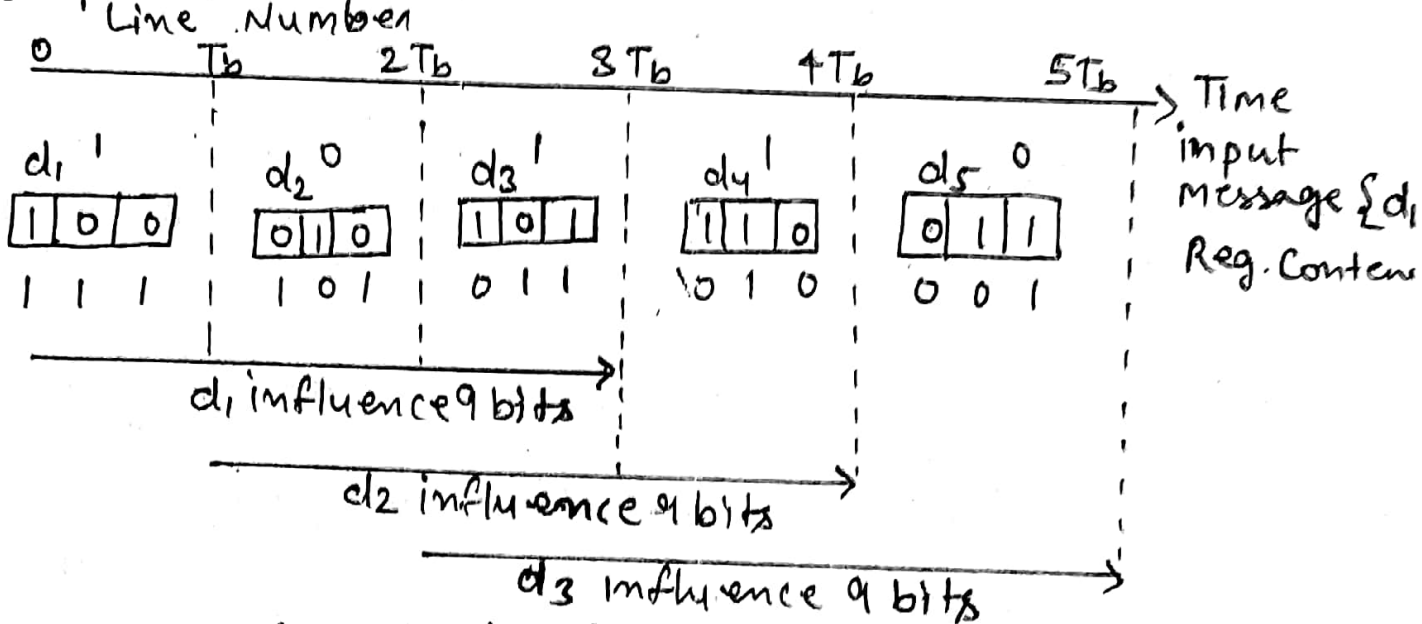
3. Now, Next bit of message entered into D_1, while cont of D_1 is shifted to D_2 and again commutator sample three outputs.

4. This process continue until all the Shift registers are cleared.

for example message bits are 101100

	D_1	D_2	D_3	V_1	V_2	V_3
1	1	0	0	1	1	1
0	0	1	0	1	0	1
1	1	0	1	0	1	1
1	1	1	0	0	1	0
0	0	1	1	0	0	1

Now all the Shift registers are not cleared so data bi. 0 will be entered until all the Shift registers are cleared. so only one extra bit is required in this example. these bits are called "flush bits" and thei output is called "tail bits".

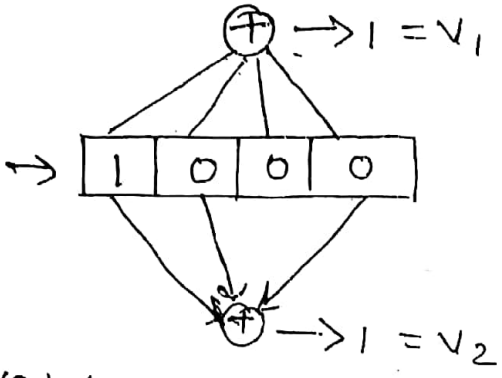


Convolutional Encoder

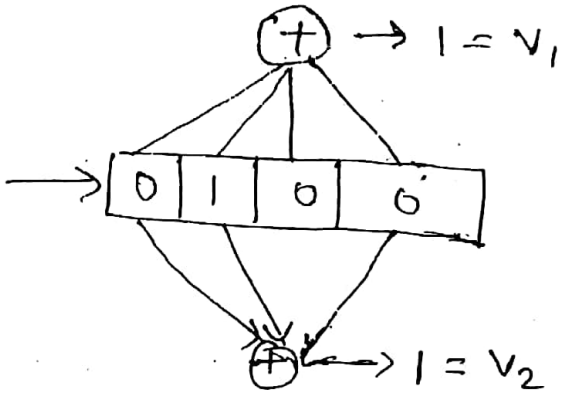
Convolving the input/data sequence with the generator Poly nomial gives the output sequence, that is why these codes are called Convolutional codes.

State Representation and state Diagram:

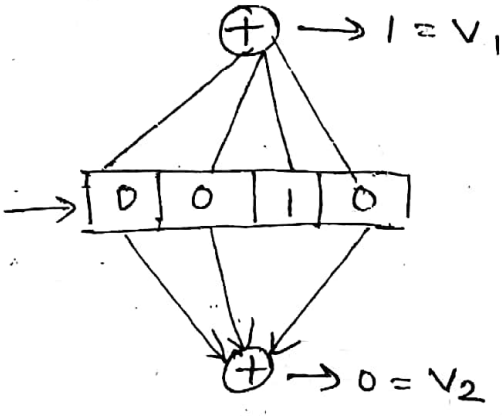
a two bit sequence of 10 with the (2,1,4) code and see how this process work with convolutional encodes.



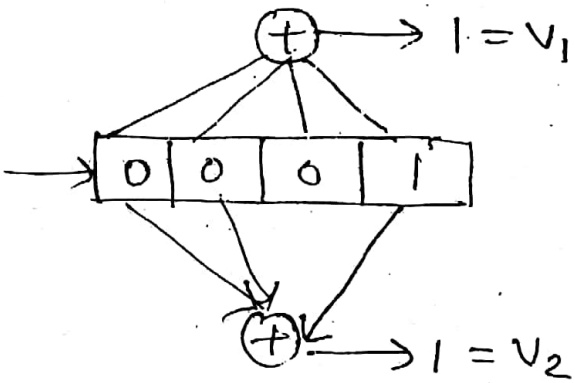
(a) t = 0, I/P State = 000
input bit = 1, O/P bits = 11
O/P state = 100



(b) t = 1, i/p state = 100
input bit = 0, o/p bit =



(c) t = 2, i/p State = 010
input bit = 0, O/P bits = 10



(d) t = 3, i/p state = on
input bit = 0, o/p bits = 11

The numbers of combinations of bits in the last (m-1) register are called the states of the code and are defined by

Number of states = 2^L

where L is the constraint length of the code and is equal to K(m-1).

the eight states of this (2,1,4) code are: 000, 001, 010, 011, 100, 101, 110, 111.

7. The Parity check bits of a (8,4) block code are generated by

$$c_5 = d_1 + d_2 + d_4$$
$$c_6 = d_1 + d_2 + d_3$$
$$c_7 = d_1 + d_3 + d_4$$
$$c_8 = d_2 + d_3 + d_4$$

where d_1, d_2, d_3, d_4 are the message digits. find out generator matrix and Parity check matrix for this code.

$n=8$, $k=4$

$P_{11}=1$ $P_{21}=1$ $P_{31}=0$ $P_{41}=1$

$P_{12}=1$ $P_{22}=1$ $P_{32}=1$ $P_{42}=0$

$P_{13}=1$ $P_{23}=0$ $P_{33}=1$ $P_{43}=1$

$P_{14}=0$ $P_{24}=1$ $P_{34}=1$ $P_{44}=1$

$$G = [I_k \mid P]$$

$$P = \begin{bmatrix} P_{11} & P_{12} & \cdots & P_{1(n-k)} \\ P_{21} & P_{22} & \cdots & P_{2(n-k)} \\ \vdots & \vdots & & \vdots \\ P_{k1} & P_{k2} & \cdots & P_{k(n-k)} \end{bmatrix}$$

$$P = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 1 & 1 & 0 & 1 \\ 0 & 1 & 1 & 1 \\ 1 & 0 & 1 & 1 \end{bmatrix}$$

$$G = \begin{bmatrix} & & & \\ & & & \\ & & & \end{bmatrix}$$

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8. The repetition code is an (n,1) block code there are only two code words in repetition code, an all 0 codeword and all 1 codeword. Consider case of repetition codeword $n=5$?

(a) Construct generator matrix G for (5,1) Block code.

$k=1$ $n=5$

$P = [1\ 1\ 1\ 1]$

$G = [1 \mid 1\ 1\ 1\ 1]$

$d_1 = 0$ $c_1 = [0][1\ 1\ 1\ 1] = [0\ 0\ 0\ 0]$

$d_2 = 1$ $c_2 = [1][1\ 1\ 1\ 1] = [1\ 1\ 1\ 1]$

(b) Find Parity check matrix H

$$H = [P^T | I_4]$$

$$\gg \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

(c) Show $GH^T = 0$

$$\gg [1\,1\,1\,1] \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$\gg (0\,0\,0\,0\,0) = 0$$

6. A (15, 5) linear cyclic code has a generator polynomial

$$g(x) = 1+x+x^2+x^4+x^5+x^8+x^{10}$$

(a) Draw block diagrams of an encoder and syndrome calculator for this code.

(b) Find the code polynomial for the message polynomial $D(X) = 1+X^2+X^4$ (in a systematic form).

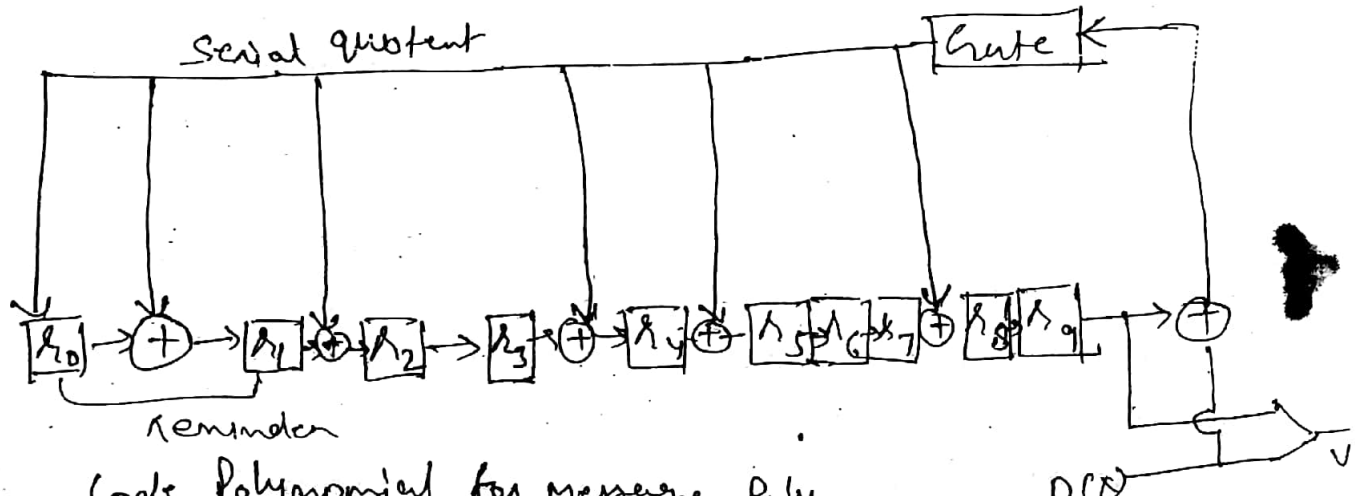
94 107 113, 114, 93, 141, 150, 156, 127, 109 122 98 142, 96, 97, 88

$n-K = 10$

no of shift reg. = 10

$g_0=1 \quad g_1=1 \quad g_2=1 \quad g_3=0 \quad g_4=1 \quad g_5=1 \quad g_6=0$

$g_7=0 \quad g_8=1 \quad g_9=0 \quad g_{10}=1$



(b) Code Polynomial for message Poly in Systematic form

$$D(X) = 1+X^2+X^4$$

$$\frac{X^{n-k}}{g(x)} \Rightarrow \frac{x^{10}(1+x^2+x^4)}{1+x+x^2+x^4+x^5+x^8+x^{10}}$$

$$x^{10}+x^8+x^5+x^4+x^2+x+1 \;\overline{)\; x^{14}+x^{12}+x^{10}} \;(\; x^4+1$$

$$x^{14}+x^{12}+x^9+x^8+x^6+x^5+x^4$$

$$x^{10}+x^9+x^8+x^6+x^5+x^4$$

$$x^{10}+x^8+x^5+x^4+x^2+x+1$$

$$x^9+x^6+x^2+x+1$$

$h(x) = 1 + x + x^2 + x^6 + x^9$

$V = \underbrace{1110001001}_{A}\ \underbrace{10101}_{D}$

$$\frac{x^{n-k} m(x)}{h(x)} \Big| g(x) \quad q(x)$$

$$v(x) = q(x) g(x) = h(x) + x^{n-k} m(x)$$

$S = [s_0 \; s_1 \; \dots]^T$

$$m(x) + \frac{E(x)}{g(x)} = q(x) + \frac{S(x)}{g(x)}$$

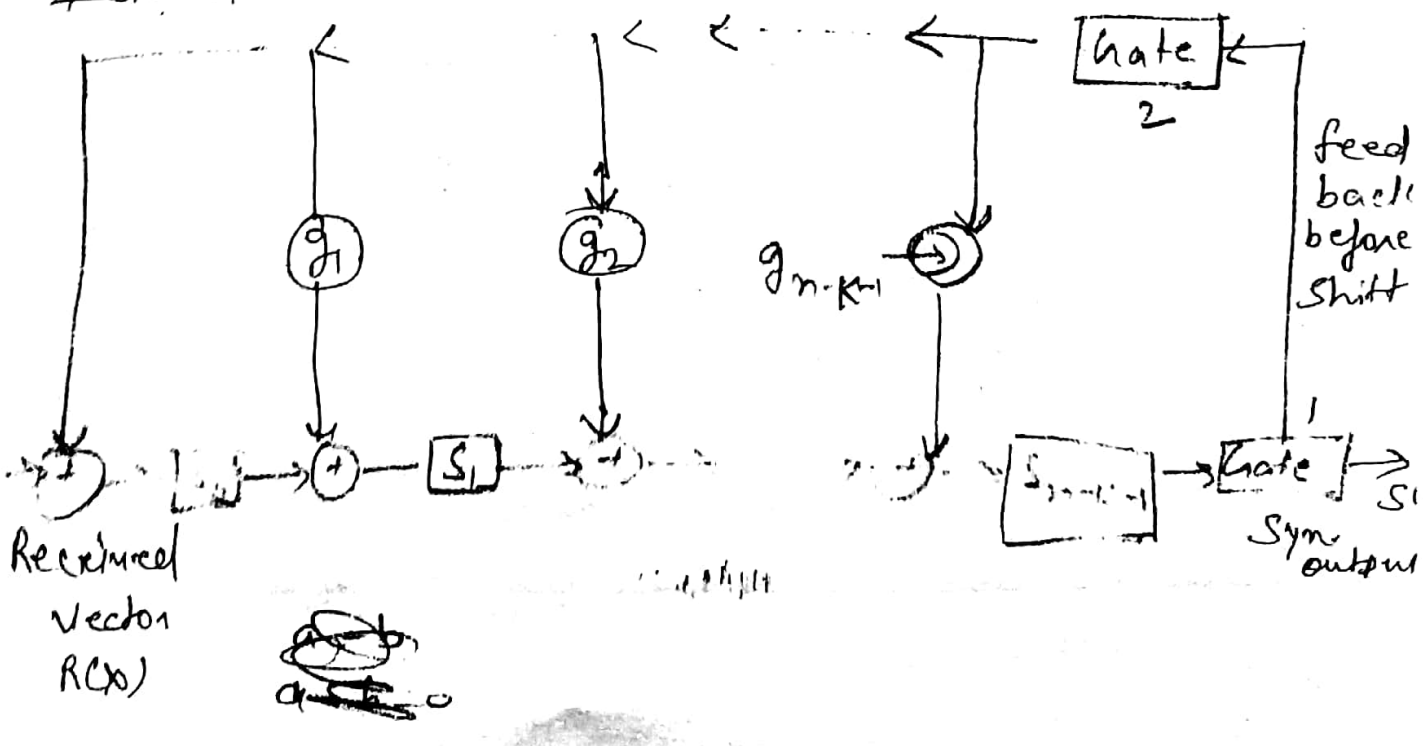
$\frac{E(x)}{g(x)}$

$$D \Rightarrow m(x) - q(x) + \frac{S(x)}{g(x)} + \frac{E(x)}{g(x)}$$

$$0 = m(x) g(x) - q(x) g(x) + S(x) - E(x)$$

$$g(x)[m(x) - q(x)] + S(x)$$

Syndrome ckt:



Q: For a (6,3) CBC the three Parity check bits c_4, c_5, and c_6 are formed from the following

$$c_4 = d_1 \oplus d_3$$
$$c_5 = d_1 \oplus d_2 \oplus d_3$$
$$c_6 = d_1 \oplus d_2$$

(a) write down the generator matrix?

$$c_4 = P_{11} d_1 \oplus P_{21} d_2 \oplus P_{31} d_3$$
$$c_5 = P_{12} d_1 \oplus P_{22} d_2 \oplus P_{32} d_3$$
$$c_6 = P_{13} d_1 \oplus P_{23} d_2 \oplus P_{33} d_3$$

$$[d_1\ d_2\ d_3] \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$P_{11}=1 \quad P_{21}=0 \quad P_{31}=1$

$P_{12}=1 \quad P_{22}=1 \quad P_{32}=1$

$P_{13}=1 \quad P_{23}=1 \quad P_{33}=0$

$$P = \begin{bmatrix} P_{11} & P_{12} & P_{13} \\ P_{21} & P_{22} & P_{23} \\ P_{31} & P_{32} & P_{33} \end{bmatrix}$$

$$\therefore P = \begin{bmatrix} 1 & 1 & 1 \\ 0 & 1 & 1 \\ 1 & 1 & 0 \end{bmatrix} \qquad G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

(b) Construct all possible codewords

$K=3 \quad 2^K = 8$

(c) Suppose $R = 010111$. find location of error.

$C = 010011$

error detecting capabilities $d_{min} - 1$

error correcting capabilities $\frac{d_{min}-1}{2}$