Information Theory & Coding
(5CS3-01)

Unit-3 Notes

Vision of the Institute

To become a renowned center of outcome based learning and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission of the Institute

M1- Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.
M2- Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.
M3- Offer opportunities for interaction between academia and industry.
M4- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

Vision of the Department

To become renowned Centre of excellence in computer science and engineering and make competent engineers & professionals with high ethical values prepared for lifelong learning.

Mission of the Department

M1-To impart outcome based education for emerging technologies in the field of computer science and engineering.
M2-To provide opportunities for interaction between academia and industry.
M3- To provide platform for lifelong learning by accepting the change in technologies
M4- To develop aptitude of fulfilling social responsibilities.

Program Outcomes (PO)

1. **Engineering knowledge:** Apply the knowledge of mathematics, science, engineering fundamentals, and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problemsand design system components or processes that meet thespecified needs with appropriate consideration for the public health and safety, andthe cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.
6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issuesand the consequent responsibilities relevant to the professional engineering practice.
7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.
8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.
9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.
10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.
11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.
12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Program Educational Objectives (PEO)

1. To provide students with the fundamentals of Engineering Sciences with more emphasis in **Computer Science &Engineering** by way of analyzing and exploiting engineering challenges.
2. To train students with good scientific and engineering knowledge so as to comprehend, analyze, design, and create novel products and solutions for the real life problems.
3. To inculcate professional and ethical attitude, effective communication skills, teamwork skills, multidisciplinary approach, entrepreneurial thinking and an ability to relate engineering issues with social issues.
4. To provide students with an academic environment aware of excellence, leadership, written ethical codes and guidelines, and the self-motivated life-long learning needed for a successful professional career.
5. To prepare students to excel in Industry and Higher education by Educating Students along with High moral values and Knowledge

Program Specific Outcomes (PSO)

PSO1: Ability to interpret and analyze network specific and cyber security issues, automation in real word environment.

PSO2: Ability to Design and Develop Mobile and Web-based applications under realistic constraints.

Course Outcome:

CO1: Apply the fundamental concepts of information theory viz. entropy, mutual information and channel capacity in communication system.

CO2: Examine the principles of source coding and data transmission.

CO3: Analyze linear block code, cyclic code and Convolution code.

CO4: Evaluate information theoretic methods to novel settings of encoding and decoding techniques.

CO-PO Mapping:

CO	PO 1	PO 2	PO 3	PO 4	PO 5	PO 6	PO 7	PO 8	PO 9	PO10	PO11	PO12
Apply the fundamental concepts of information theory viz. entropy, mutual information and channel capacity in communication system.	3	2	2	2	1	1	1	1	1	1	1	3
Examine the principles of source coding and data transmission.	3	3	3	3	2	1	1	1	1	1	1	3
Analyze linear block code, cyclic code and Convolution code.	3	3	3	2	1	1	1	1	1	1	1	3
Evaluate information theoretic methods to novel settings of encoding and decoding techniques.	3	3	3	2	1	1	1	1	1	1	1	3

SYLLABUS:

RAJASTHAN TECHNICAL UNIVERSITY, KOTA

Syllabus

III Year-V Semester: B.Tech. Computer Science and Engineering

5CS3-01: Information Theory & Coding

Credit: 2
2L+0T+0P

Max. Marks: 100(IA:20, ETE:80)
End Term Exam: 2 Hours

SN	Contents	Hours
1	**Introduction:**Objective, scope and outcome of the course.	01
2	**Introduction to information theory:** Uncertainty, Information and Entropy, Information measures for continuous random variables, source coding theorem. Discrete Memory less channels, Mutual information, Conditional entropy.	05
3	**Source coding schemes for data compaction:** Prefix code, Huffman code, Shanon-Fane code &Hempel-Ziv coding channel capacity. Channel coding theorem. Shannon limit.	05
4	**Linear Block Code:** Introduction to error connecting codes, coding & decoding of linear block code, minimum distance consideration, conversion of non-systematic form of matrices into systematic form.	05
5	**Cyclic Code:** Code Algebra, Basic properties of Galois fields (GF) polynomial operations over Galois fields, generating cyclic code by generating polynomial, parity check polynomial. Encoder & decoder for cyclic codes.	06
6	**Convolutional Code:** Convolutional encoders of different rates. Code Tree, Trllis and state diagram. Maximum likelihood decoding of convolutional code: The viterbi Algorithm fee distance of a convolutional code.	06
	Total	**28**

LECTURE PLAN:

Unit No./ Total lec. Req.	Topics	Lect. Req.
	Objective, Scope & Outcome of the Course	1
Unit-1	Introduction to information theory, Uncertainty, Entropy	1
	Information measures for continuous random variables	1
	Numerical problem on entropy	1
	Source coding theorem, Discrete memory less channels	1
	Mutual information, Conditional entropy	1
Unit-2	Prefix code, Huffman coding	1
	Shannon – fanon coding	1
	Numerical on haffman and shanon fano coding	1
	Hempel-Ziv coding	1
	Channel capacity, Channel coding theorem, Shannon limit	1
Unit-3	Introduction to error correcting codes	1
	Coding and decoding of linear block code	1
	Numerical problem on Linear block code	1
	Error correcting codes, Minimum distance consideration	1
	Conversion of non symmetric form of matrix into symmetric form	1
Unit-4	Code algebra	1
	Basic properties of Galois Field(GF)	1
	Polynomial operation over Galois field	1
	Generating cyclic code by generating polynomial	1
	Numerical Problems on generator polynomial	1
	Parity check polynomial , Encoder and decoder for cyclic codes	1
Unit-5	Convolutional encoders of different rates	1
	Code tree	1
	Trellis diagram	1
	state diagram	1
	Maximum likelihood decoding of convolution code	1
	Viterbi algorithm, Free distance of convolution codes	1

Linear block codes!-

for the purpose of encoding message for error protection, the long message are broken into message blocks consisting of K bits of information.

than parity bits (reudundant bits) are added to these K bits according to certain rules of coding. and the codewords of length n bits, inclusive of (n-K) parity bits are transmitted through noisy channel. at the receiver the K message bits are decoded from the received message blocks.

with K bits of information per block, there are 2^K possible messages, out of the 2^n words that may be generated with n bits. This set of 2^K words is called a block code.

in a linear block codes, each block of K messages bits is encoded into block of n bits by adding (n-K) check bits derived from K message bits

Message block → $\boxed{\text{channel encoder}}$ → Code block

$\boxed{\text{Message}}$
|← K bits →|

$\boxed{\text{Message} \quad \text{bits}}$
← K →|← h →|

$K + h = n$

The n-bit block of channel encoder is called the codeword & codes in which message appear at the beginning of codewords are systematic codes.

Matrix description of linear block codes!-

The encoding operation of linear block code consisting of 2 steps:-

(a) The information sequence is segmented into message (broken) blocks, each ~~bit~~ block containing K-successive information bits

(b) The encoder transforms each message block into a large block of n-bits according to pre determine set of rules. The n-K additional bits are general from linear combination of message bits and these are called parity check bits.

Encoding operation using Matrix:-

Encoding means form codeword using concept of Matrix. Matrix represent message block as Row vector or K-tuple $D = (d_1, d_2$ where message bit can be 0 or 1.

we have 2^K distinct message block. each message block is transformed into a codeword C of length n, which is represented as $C = (C_1, C_2, \ldots C_n$

There are K bits in a message block so 2^K Combination of message & 2^K distinct codewords. This set of 2^K codewords are called code vectors. This is said to be (n, K) block code.

in Systemetic LBC, the first K bits of code words are message bits $[C_1, C_2, \ldots C_K] = [d_1 d_2 \ldots d_K]$ and remaining n-K bits in codeword are the check bits

$$C = DG$$

$$[C_1, C_2, \ldots C_n] = [d_1, d_2 \ldots d_K] \begin{bmatrix} 1 & 0 & 0 & \cdots & 0 & P_{11} & P_{12} & \cdots & P_{1(n-K)} \\ 0 & 1 & 0 & \cdots & 0 & P_{21} & P_{22} & \cdots & P_{2(n-K)} \\ \vdots & \vdots & \vdots & \vdots & & & & & \\ \vdots & \vdots & \vdots & \vdots & & & & & \\ 0 & 0 & 0 & \cdots & 1 & P_{K1} & P_{K2} & \cdots & P_{K(n-K)} \end{bmatrix}$$

where G is the generator matrix

nehamamtalucky

Generator Matrix

$$G = \begin{bmatrix} 1 & 0 & 0 & - & 0 & P_{11} & P_{12} & \cdots & P_{1(n-K)} \\ 0 & 1 & 0 & - & 0 & P_{21} & P_{22} & \cdots & P_{2(n-K)} \\ \vdots & \vdots & \vdots & - & \vdots & \vdots & \vdots & & \vdots \\ 0 & 0 & 0 & - & 1 & P_{K1} & P_{K2} & \cdots & P_{K(n-K)} \end{bmatrix}$$

$$G = [I_K : P]_{K \times n}$$

Here G is the combination of identity matrix I_K of order K & P is the arbitrary matrix of order $(K \times n-K)$.

Q// Given

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

findout all the possible code vectors.

Sol^n

$$G = [I_K : P]_{K \times n}$$

So $K = 3$ are message bits and

$2^3 = 8$ possible combination of codewords which is given as

(000) (001) (010) (011) (100) (101) (110) (111).

First method

$$C = DG$$

for $D = 111$

$$C = (111) \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

$$= (111000)$$

Likewise

D(Message)	Code	Vectors
000	000	000
001	001	110
010	010	101
011	011	011
100	100	011
101	101	101
110	110	110
111	111	000

Second method

Given $G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$
$\qquad I_K \qquad\qquad P$

for D = 000 C = 000000

D = 001 C = 001 110

Because in G matrix corresponding to 001 we have 110

∴ C = 001110

D = 010 C = 010 101

D = 011

this is the sum of second and third rows ∴ correspond check bits are also added

∴ C = 011011

Q. the Generator matrix for a (6, 3) block code is shown below. obtain all code vector for this code.

$(n, K) = (6, 3)$

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

Sol^n: $G = [I_K \vdots P]_{K\times n}$ — ①

comparing eq^n ① with given matrix we note that

$$I_K = I_{3\times 3} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$P_{3\times 3} = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

Here $K=3$ and $n=6$

This means that block size of the message vector is 3 bits. thus there will be 8 possible message vectors as shown in table

Bits of message vector in one block

SNO	m_1	m_2	m_3
1	0	0	0
2	0	0	1
3	0	1	0
4	0	1	1
5	1	0	0
6	1	0	1
7	1	1	0
8	1	1	1

The P sub matrix is given as

$$P = \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

for the check bits vector there will be three bits. They may be obtained as

$$[c_4 c_5 c_6] = [m_1\ m_2\ m_3] \begin{bmatrix} 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 1 & 0 \end{bmatrix}$$

from the above matrix multiplications. we get.

$c_4 = (0\times m_1) \oplus m_2 \oplus m_3$ — ①

$c_5 = (m_1) \oplus (0 \times m_2) \oplus m_3$ — (2)

$c_6 = (m_1) \oplus (m_2) \oplus (0 \times m_3)$ — (3)

53/72

from last three eqn we get

$c_4 = m_2 \oplus m_3$ — (4)

$c_5 = m_1 \oplus m_3$ — (5)

$c_6 = m_1 \oplus m_2$ — (6)

The eqn no (4) (5) and (6) give check bits for each block of m_1, m_2, m_3 message bits for example for first $(m_1, m_2, m_3) = 000$. we have

$c_4 = 0 \oplus 0 = 0$

$c_5 = 0 \oplus 0 = 0$

$c_6 = 0 \oplus 0 = 0$ ie $(c_1, c_2, c_3) = 000$

for Second block of $(m_1, m_2, m_3) = 001$

$c_4 = 0 \oplus 1 = 1$

$c_5 = 0 \oplus 1 = 1$

$c_6 = 0 \oplus 0 = 0$

ie $(c_4, c_5, c_6) = 110$

S.No	m_1	m_2	m_3	$c_4 = m_2 \oplus m_3$	$c_5 = m_1 \oplus m_3$	$c_6 = m_1 \oplus m_2$	m_1	m_2	m_3	m_4	m_5	m_6
1	0	0	0	0	0	0	0	0	0	0	0	0
2	0	0	1	1	1	0	0	0	1	1	1	0
3	0	1	0	1	0	1	0	1	0	1	0	1
4	0	1	1	0	1	1	0	1	1	0	1	1
5	1	0	0	0	1	1	1	0	0	0	1	1
6	1	0	1	1	0	1	1	0	1	1	0	1
7	1	1	0	1	1	0	1	1	0	1	1	0
8	1	1	1	0	0	0	1	1	1	0	0	0

Parity check Matrix (H)

$$H = \left[\begin{array}{cccc|cccc} P_{11} & P_{21} & \cdots & P_{K1} & 1 & 0 & \cdots & 0 \\ P_{12} & P_{22} & \cdots & P_{K2} & 0 & 1 & \cdots & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots & & \vdots \\ P_{1(n-K)} & P_{2(n-K)} & \cdots & P_{K(n-K)} & 0 & 0 & \cdots & 1 \end{array}\right]_{(n-K)\times n}$$

$$H = \left[P^T_{(n-K)\times K} \,\vdots\, I_{n-K}\right]_{(n-K)\times n} \quad \text{—— (1)}$$

The Parity check Matrix is used to varify the codewo c generated by generator matrix $G = [I_K \vdots P]$. This varification can be done by following steps.

Step1 : c is a correct codeword in (n, K) block code, gene-rated by G, if and only if

$$cH^T = 0 \quad \text{—— (2)}$$

where H^T is the transpose of Parity check Matrix H. The generator matrix is used in encoding operation where as ~~generator~~ Parity check matrix is used for decoding operation.

step 2 : if c be the codeword that was transmitted over a noisy channel and let R be the noise corrupted vector that was received. The vector R is the combination of code vector c and error vector E

$$R = C + E \quad \text{—— (3)}$$

Step3 : The receiver does the decoding operation by determining an (n-K) vector S. The vector S is called error syndrome.

$$S = RH^T \quad \text{—— (4)}$$

from eqn (3) and (4)

$S = (C+E)H^T$

$S = CH^T + EH^T \quad [\because CH^T = 0]$

$S = EH^T$

we can say, Syndrome of received vector is zero, if R is a valid code word.

Step 4 if error occurs then S of received codevector R will be non zero. if $S \neq 0$, then S is compared with H^T. The row with this syndrome matches indicates the position where error is present.

Q. Consider a (7,4) block code generated by

$$G_1 = \left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

find out the error vector?

Step 1:-

$H = [P^T \mid I_{n-k}]_{(n-k)\times n}$

$$H = \left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]$$

$\underbrace{}_{P^T} \quad \underbrace{}_{I_3}$

Step 2. we have $K = 4$

$n = 7$

$\therefore 2^K = 2^4$ codewords for 2^4 messages (0000) ..... (1111).

Scanned by CamScanner

Step3: Choose a specific value of D from the 16 Compinations for example 1011.

$$C = DG$$

$$DG = [1011]\left[\begin{array}{cccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 1 & 0 & 1 & 1 \end{array}\right]$$

$$C = [1011\underline{001}]$$

Step4 Calculate Syndrome

$$S = CH^T$$

$$S = (1011001)\left[\begin{array}{cccc|ccc} 1 & 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 1 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 1 & 0 & 0 & 1 \end{array}\right]^T$$

$$S = (1011001)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Syndrome is zero, it means it is a valid code word.

Step5 if R = 1001001 is given, find

$$S = RH^T$$

$$S = (1001001)\begin{bmatrix} 1 & 1 & 1 \\ 1 & 1 & 0 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = [101]$$

$S = 101$

~~step~~

Step 6 Compare value of S by H^T. Now 101 is equal to third row of H^T. it means third bit from the left is in error. make it one if zero and 0 if it is 1.

So the transmitted word is

$$C = 1011001$$

Error vector $E = R - C$

$$E = 0010000$$

Q. A parity check code has the parity check matrix

$$H = \begin{bmatrix} \underbrace{\begin{matrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 0 & 1 & 1 \end{matrix}}_{P^T} & \underbrace{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{matrix}}_{I_{n-K}} \end{bmatrix}_{(n-K)\times n}$$

(a) Determine the generator matrix G.

(b) find the code ~~vector~~ word that ~~begings~~ Start with 101....

(c) Suppose that the received codeword is 110110

Decode this received word

Sol^n

(a) Since H is a (6,3) matrix $n = 6$ and $K = 3$

$$P = \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

then the generator matrix G is

$$G = [I_k | P] = \begin{array}{c} \begin{matrix} c_1 & c_2 & c_3 & c_4 & c_5 & c_6 \end{matrix} \\ \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix} \end{array} \begin{matrix} R_1 \\ R_2 \\ R_3 \end{matrix}$$

(b)

$C = DG$

$$C = [101] \begin{bmatrix} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{bmatrix}$$

$$C = [101011]$$

(c) $R = [110110]$

$$S = RH^T$$

$$= [110110] \begin{bmatrix} 1 & 1 & 0 \\ 0 & 1 & 1 \\ 1 & 0 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow [011]$$

Since S is equal to the second row of H^T, an error is at the second bit, the correct code word is 100110 and the data bits are 100.

* Error Detection & Correction Capabilities of Linear block Codes:-

1) Weight of Codeword (C):- it is defined as the number of nonzero bits in C.

2) Hamming Distance:- it is the distance between two code vector C_1 and C_2 by the no of components in which they differ.

3) Minimum Distance:- it is the smallest distance between any pair of codeword in code. it is represented by dmin. minimum distance dmin is equal to the minimum weight of any nonzero code vector.

Q: Consider (6,3) block code with

$$G = \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 0 & 1 & 1 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 1 & 1 & 0 \end{array}\right]$$

find the weight of codeword, and min distance of cod

D	C	weight
000	000000	0
001	001110	3
010	010101	3
011	011011	4
100	100011	3
101	101101	4
110	110110	4
111	111000	3

$d_{min} = 3$

Repetition Codes :- Repetition codes represents the simplest type of linear block codes. A single message bit is encoded into a block of n identical bits, producing an (n,1) block code. There are only two code words in this code: an all zero code word and an all one code word.

for example, a repetition code with $k=1$ and $n=5$. in this case we have four parity bits that are the same as the message bit.

Hence the identity matrix $I_k = 1$ and Pa matrix P consist of a 1-by-4 vector that has 1 for its all its elements. So

$$G = [1 \mid 1\,1\,1\,1]_{k \times n}$$

so H will be

$$H = [P^T \mid I_{n-k}]_{n-k \times n}$$

2

Consider (5, 1) Repetition code. 42(1)

(a) Evaluate the syndrome S for all possible single error patterns.

$S = eH^T$

Possible values of e are

e	S
1 0 0 0 0	1 1 1 1
0 1 0 0 0	1 0 0 0
0 0 1 0 0	0 1 0 0
0 0 0 1 0	0 0 1 0
0 0 0 0 1	0 0 0 1

$G = [1\ 1\ 1\ 1\ 1]$

$$H = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{bmatrix}$$

$$H^T = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

(b) Repeat (a) for all ten possible double error patterns

$e = [1\,1\,0\,0\,0]$

$S = eH^T$

$= [0\,1\,1\,1]$

$e_1 = 1\,1\,0\,0\,0$

$e_2 = 1\,0\,1\,0\,0$

$e_3 = 1\,0\,0\,1\,0$

$e_4 = 1\,0\,0\,0\,1$

$e_5 = 0\,1\,1\,0\,0$

$e_6 = 0\,1\,0\,1\,0$

$e_7 = 0\,1\,0\,0\,1$

$e_8 = 0\,0\,1\,1\,0$

$e_9 = 0\,0\,1\,0\,1$

$e_{10} = 0\,0\,0\,1\,1$

$$H = \left[\begin{array}{c|cccc} 1 & 1 & 0 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 0 & 1 \end{array}\right]$$

Since the message vector consists of a single message symbol 0 or 1 there are only two code words 00000 and 11111 in the (5, 1) repetition code.

⊛ Single error correcting code:

Hamming code:

i) Linear block code is capable of correcting single errors if $d_{min} = 3$.

ii) Now error occurred is in jth bit of the code word, if the syndrome of received vector is equal to jth row of H^T.

(iii) H^T is a matrix of order $[n \times (n-K)]$ meaning n rows and each of n-K bits.

The Hamming code are single error correcting block code whose parameters are $\underset{(6,3)}{n, K}$

Code length $n = 2^r - 1$ $= 2^6 - 1 = n = 63$

Number of parity check bits $r = (n-K)$ $= 3$

Number of Information bits $K = (2^n - n - 1)$

Hamming distance $d_{min} = 3$

Error correcting capability $= 1$ Error correcting $= \frac{d_{min} - 1}{2}$

so for any positive integer r, a Hamming code is a SEC code.

Minimum Size (n) for LBC:- H^T is a matrix of order $n \times n-K$, meaning n row and each of n-K bits. each row in H^T has n-K entries which could be 0 or 1. Hence 2^{n-K} distinct row out of which we select $(2^{n-k}-1)$ row. we must not use a row of 0's since a syndrome of 0's corresponds to no error.

$$\therefore 2^{n-k}-1 \geq n$$

$$n-k \geq \log_2(n+1)$$

$$\boxed{n \geq k+\log_2(n+1)}$$

so we can determine minimum size ~~of~~ n for codeword

Q: design a generator matrix for LBC with minimum distance of three and message block size of 8 bits.

Given $d_{min} = 3$

$k = 8$

$$n \geq k + \log_2(n+1)$$

$$n \geq 8 + \log_2(n+1)$$

Smallest value of n that satisfies this inequality is 12. $n=12$

So it is a (12, 8) LBC (n, k)

$\therefore$ H^T is of order $(n \times n-k) = (12 \times 4)$

$$H^T = \begin{bmatrix} P \\ \hline I_{n-k} \end{bmatrix}_{n \times n-k}$$

So

$$H^T = \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 1 & 1 & 0 \\ 0 & 1 & 1 & 1 \\ \hline & \underbrace{\qquad\qquad}_{I_4} & & \end{bmatrix}$$

$$\begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

and $G = [I_k | P]$

$$G = \left[\begin{array}{cccccccc|cccc} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 1 & 0 & 1 & 1 & 1 \end{array}\right]$$

⊛ Systemetic linear block code :-

A Systemetic (n, k) linear block code is a mapp. from a k dimensional message vector to n dimensional codeword that is divide into k message digit and $(n-k)$ parity digits.

A Systemetic linear block code will have a generator matrix of the form

$$G = [P | I_k]$$

$$G = \left[\begin{array}{ccc|ccc} P_{00} & P_{01} & P_{0,n-k-1} & 1 & 0 & 0 \\ P_{10} & P_{11} & P_{1,n-k-1} & 0 & 1 & 0 \\ \vdots & \vdots & & \vdots & \vdots & \vdots \\ P_{k-1,0} & P_{k-1,1} & P_{k-1,n-k-1} & 0 & 0 & 1 \end{array}\right]$$

where P is the parity array portion of G.

we know that $C = DG$

$$[c_1, c_2 \ldots c_n] = [d_0, d_1 \ldots d_k] \left[\begin{array}{ccc|ccc} P_{00} & P_{01} & P_{0,n-k-1} & 1 & 0 & 0 \\ P_{10} & P_{11} & P_{1,n-k-1} & 0 & 1 & 0 \\ \vdots & \vdots & \vdots & \vdots & \vdots & \vdots \\ P_{k-1,0} & P_{k-1,1} & P_{k-1,n-k-1} & 0 & 0 & 1 \end{array}\right]$$

and the general code vector n type

$$C = \underbrace{P_1 \; P_2 \ldots P_{n-k}}_{\text{Parity bits}} \; \underbrace{d_1, d_2, \ldots d_k}_{\text{Message bits}}$$

$P_0 = d_0 P_{00} + d_1 P_{10} \ldots + d_k P_{k-1,0}$

$P_1 = d_0 P_{01} + d_1 P_{11} \ldots + d_k P_{k-1,1}$

$P_2 = d_0 P_{02} + d_1 P_{12} \ldots + d_k P_{k-1,2}$

$P_{n-k} = d_0 P_{0(n-k-1)} + d_1 P_{1(n-k-1)} + \dots \, d_k P_{k(n-k-1)}$

for example

$$c = [d_0\ d_1\ d_2] \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \end{bmatrix}$$

$$c = \underbrace{d_0 + d_2}, \underbrace{d_0 + d_1}, \underbrace{d_1 + d_2}, \underbrace{d_0}, \underbrace{d_1}, \underbrace{d_2}$$

the above eqn show that the first parity bit is the sum of the first and third message bits. the second parity bits is the sum of second and third message bits.

⊛ STANDARD ARRAY:-

Syndrome based Decoding Method:- This method describe syndrome based decoding for linear block codes. this method used a decoding table by this decoding table we can easily decode the received code word by less computations.

A standard array for an (n,k) linear block code is given by

$$\begin{matrix} C_1 & C_2 & \dots & C_{2^k} \\ E_2 & C_2 + E_2 & \dots & C_{2^k} + E_2 \\ E_3 & C_2 + E_3 & \dots & C_{2^k} + E_3 \\ \vdots & & & \\ E_{2^{n-k}} & C_2 + E_{2^{n-k}} & \dots & C_{2^k} + E_{2^{n-k}} \end{matrix}$$

where $C_1, C_2 \dots C_{2^k}$ denotes the 2^k code vector of an (n,k) linear block code. Let R denotes the received vector which may have one of 2^n possible values.

Steps for Creating Decoding table :-

code vector $C_1, C_2 \ldots C_{2^K}$ are Placed in first row.
The Second row is completed by adding E_2 to all Code words.

3. This Process continues untill all of 2^n n tuple are used
4. The rows of the standard array are cosets and first element in each row is a coset ~~lead~~ Leader.

Q: We consider a (6,3) group code whose generator Matrix is

$$G = \begin{array}{c} \left[\begin{array}{ccc|ccc} 1 & 0 & 0 & 1 & 1 & 0 \\ 0 & 1 & 0 & 0 & 1 & 1 \\ 0 & 0 & 1 & 1 & 0 & 1 \end{array}\right] \end{array} \begin{array}{c} r_1 \\ r_2 \\ r_3 \end{array}$$

The Codeword are given by

c = (000000), (001101), (010011), (011110), (100110), (101011), (110101), (111000)

Procedure for Constructing of decoding table :-

(a) first List in a row of the elements of the group code starting with identity.

$$\begin{array}{cccccc} (000000, & 100110, & 010011, & 001101, & 110101, & 101011, \\ I & r_1 & r_2 & r_3 & r_1+r_2 & r_1+r_3 \end{array}$$

$$\begin{array}{cc} 011110, & 111000) \\ r_2+r_3 & r_1+r_2+r_3 \end{array}$$

(b) Select an element x of binary sequence of length 6. then list the elements of the cost $x+c$ with directly below c, for example $x = 100000$, then we have

000000	100110	010011	001101	110101	101011	011110	111000
100000	000110	110011	101101	010101	001011	111110	011000

(c) Repeat step (b) for binary sequence of lengths 6.

DECODING TABLE (STANDARD ARRAY)

C

COSET LEADER							
000000	100110	010011	001101	110101	101011	011110	111000
100000	000110	110011	101101	010101	001011	111110	011000
010000	110110	000011	011101	100101	111011	001110	101000
001000	101110	011011	000101	111101	100011	010110	110000
000100	100010	010111	001001	110001	101111	~~011010~~	~~111100~~
000010	100100	010001	001111	110111	101001	011100	111010
000001	100111	010010	001100	110100	101010	011111	111001
100001	000111	110010	101100	010100	001010	111111	011001

E R

The enteries in the first column are called the coset leaders. Now in order to decode any receiv sequence R. we find the column containint R. Now the topmost element of that column is the decoded codeword.

Properties of standard array:-

1. each element in standard array is distinct and hence column of standard array T_j are distinct.
2. if error pattern caused by channel conciedes wi coset leader, then received word is correctly decoded otherwise not.

E_1 = 100000	100000
E_2 = 110000	010000
E_3 = 011000	001000
E_4 = 001100	000100
E_5 = 000110	000010
E_6 = 000011	000001
E_7 = 100001	100001