CONTETS OF UNIT -I

- INTRODUCTION
- HISTORY AND DEVELOPMENT OF OPTIMIZATION
- METHODS OF OPTIMIZATION
- ENGINEERING APPLICATION OF OPTIMIZATION
- CLASSFICATION OF OPTIMIZATION PROBLEMS
- FORMULATION OF DESIGN PROBLEMS AS A MATHEMATICAL PROGRAMMING
- CONCLUSION

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

1. **Introduction:**

- Optimization is the act of obtaining the best result under given circumstances.
- Optimization can be defined as the process of finding the conditions that give the maximum or minimum of a function.
- The optimum seeking methods are also known as *mathematical programming techniques* and are generally studied as a part of operations research.
- *Operations research* is a branch of mathematics concerned with the application of scientific methods and techniques to decision making problems and with establishing the best or optimal solutions.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

history of optimization

- The main origin is during the second world war . At that time the scientist of England were asked to study the strategic and tactical problems related to air and land defense of the country .
- Due to the limited resources (military etc.), it was necessary to make the most utilization of them.
- During world –war II , the military commands of U.S.A. and U.K. from a inter disciplinary terms of scientists for scientific research into strategic and tactical military operations .
- Their mission was to formulate a plan for military commands so that they can make the best use of their scarce military resources , food and affords and also to implement the decision effectively .

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

- These scientists were not actually engaged in fighting the war of military operations but their strategically initiatives of military commands and other intellectual support helped them to win the war.

After the end of the war , because of the success of the military team in war , the industrial managers were attracted towards this team of scientists. They wanted that this team of scientists should help them , to find a way to minimize the cost and maximize the profit.

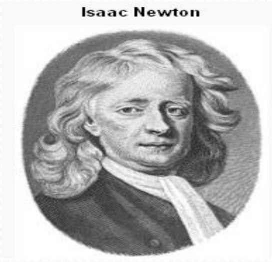
Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

- The first mathematical technique in this field was developed as "Simplex Method" in linear programming in 1947 . Since then a lot of new techniques and application have been developed in this field.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Historical development

- ***Isaac Newton (1642-1727)***
  (The development of differential calculus methods of optimization)



- ***Joseph-Louis Lagrange (1736-1813)***
  (Calculus of variations, minimization of functionals, method of optimization for constrained problems)

- ***Augustin-Louis Cauchy (1789-1857)***
  (Solution by direct substitution, steepest descent method for unconstrained optimization)

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Methods of operations research

Methods of operations research can mainly be divided into three parts . This division , of course , is not unique.

(a)Mathematical programming techniques

(b)Statistical process techniques

(c)Statistical methods

(a)Mathematical programming techniques

(1)Linear programming: this deals with the problems in which the objective function along with the constraints are linear . Also all the decision variables should be positive.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

(2) **<u>Integer programming</u> :** the linear programming problem in which some or all the decision variables are integers.

(3)**<u>Quadratic programming</u> :** in this objective function is quadratic but the constraints are linear .

(4)**<u>Non –linear programming</u> :** objective and constraints , both the functions can be non linear , but at least one of the two (objective or constraints) must be non linear .

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

(5)<u>Dynamic programming</u>: break up the given problem in different stages and solve it.

(6)<u>Game theory</u> : deals with the strategies of a game.

(7)Geometric programming

(8)Stochastic programming

(9)Separable programming

(10)Multi objective programming

(11)Calculus of variation

(12)Network methods : CPM and PERT etc

.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

(b) Stochastic process techniques

(1)Statistical decision theory

(2)Markov process

(3)Queuing theory

(4)Renewal theory

(5)Simulation methods

(6)Reliability theory

(c) Statistical methods

(1)Regression analysis

(2)Cluster analysis pattern recognition

(3)Design of experiments

(4)Discriminate analysis etc

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Engineering application of optimization

Optimization can solve almost all the problems in engineering . Some of the applications from different branches of engineering are given below:

(1)In bringing out new design (more efficient) of a vehicles.

(2)Finding the optimal trajectories of space vehicles.

(3)Design of water resources system for maximum benefit .

(4)Minimum weight design of structures for earth quake, wind etc.

(5)Optimum designs of gears , machine tools and other mechanical equipments.

(6)Optimum design of electrical equipments

(7)Electrical networking.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Engineering application of optimization

8. Design of aircraft and aerospace structure for minimum weight.
9. Design of structures : frames , bridges , towers etc. for minimum cost.
10. Production planning , scheduling and controlling .
11. Travelling salesman problem.
12. Design of chemical processing equipments and plants.
13. Planning of maintenance and replacement of equipment to reduce operating costs.
14. Inventory control system.
15. Design of control system.
16. controlling the idle and waiting times and queuing in production lines to reduce the cost etc.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Classification of optimization problems

Optimization problem can be classified in several ways . Some of them are discussed below .

- **Classification based on the existence / non – existence of constraints**
- **Constrained optimization problem:** if the given problem has constraints, it is a constrained optimization problem.

Minimize f(x)

subject to $g_j(x) \le 0, j = 1,2......m$

$$h_k(x) = 0, k = 1,2,3....p$$

$$X = (x_{1,} x_{2,....} x_n)^T$$

Is a most general constrained optimization problem.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

- ***Unconstrained optimization problem:***– a problem with no constraints attached to it is an unconstrained problem . The problem
- Minimize f(x) for $X = (x_1, x_2, \ldots\ldots x_n)^T$
- X is a general unconstrained problem.
- Classification based on the nature of the functions involved non linear $f(x); g_j(x); j = 1,2,\ldots.m; h_k(x), k = 1,2\ldots..p$
- Programming problem: if any of the function
- Is non –linear then the programming is called a non-linear programming Problem.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

- ***Quadratic programming problem –***

A non- linear programming problem with a quadratic objective function and linear constrained is called a quadratic programming problem .constrained must satisfy the non – negativity conditions.

The statement of the general quadratic programming problem is :

$$f(x) = \sum_{i=1}^{n}\sum_{j=1}^{n} Q_{ij} x_i x_j + \sum_{i=1}^{n} q_i x_i + C$$

$$s.to \sum_{i=1}^{n} a_{ij} x_i = b_{j,\, j=1,2,3\ldots\ldots m}$$

Where Q_{ij}, a_i, c, a_{ij} are all constants.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Linear programming problem:

In the programming if all the function involved are linear in nature with the non-negativity condition , then it is called a linear programming problem. The general linear programming problem is :

$$Minmize,\ f(x) = \sum_{i=1}^{n} c_i x_i$$

$$s.to,\ \sum_{i=1}^{n} a_{ij} x_i \le b_j,\ j = 1,2.....m$$

$$x_i \ge 0, i = 1,2,......n$$

Where c_i, a_{ij}, b_j are all constants

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Classification based on the nature of the decision variables:

- In this , depending on the values permitted for the design variables , problems can be classified as real valued and integer valued programming problem.
- **Integer programming problem**: integer programming problem is a linear programming problem in which some or all the variables take integer values.
- All the programming problems we have discussed above are real valued programming problems.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Formulation of problems using mathematical (symbolic) models:

Examples 1:- A company is making two products .A and B . The cost of producing one unit of product A and B is Rs 60 and Rs 80 respectively. As per agreement, the company has to supply at least 200 units of product B to its regular customers. One unit of the product A requires one machine hour whereas product B has machine hours available abundantly within the company. Total machine hours available for product A are 400 hours . One unit of each product A and B requires one labor hour each and total cost of production by satisfying the given requirements . Formulate the problem as a linear programming problem.

Solution: let x_1, and, x_2 be the number of units of product A and B to be manufactured respectively , then the L.P model is given by :

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Minimize

$$z = 60x_1 + 80x_2$$
$$s.to,\ x_2 \geq 200$$
$$x_1 \leq 400$$
$$x_1 + x_2 \leq 500$$
$$x_1, x_2 \geq 0$$

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Exercise:-

- Q.1 How do you solve a maximization problem as minimization problem?
- Q.2 what is the difference between linear and non – linear programming problem?
- Q.3 what is integer programming problem?
- Q.4 state the engineering application of optimization ?
- Q.5 what is optimization? Give five application in engineering disciplines.
- Q.6 write a short notes on optimization techniques.

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

conclusion

- The conclusion of this unit is that , the term of "optimization" refers to the process of searching for the optimum solution from a set of candidates to the problem of interest based on certain performance criteria, which has found broad applications in manufacturing, engineering finance and logistics etc.
- The goal of optimization is to find the best value for each variable in order to achieve satisfactory performance .

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

- https://www.youtube.com/watch?v=ovlaJxZjmiE
- https://www.youtube.com/watch?v=YSewtaL3tYY

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur

Thank you

Prof.(Dr.) Ashok Singh Shekhawat
Department of Mathematics,
JECRC, Jaipur