Probability

1. Basic concepts of probability

Any process that allows us to ~~of~~ obtain Observations is known as an Experiment or Trial.

If an experiment is repeated ~~to~~ under similar conditions, then it can be classified into two categories -

a) Deterministic experiment.
b) Random experiment

a) **Deterministic experiment** → Experiment which have a single outcomes are called ~~det~~ deterministic.

Ex * If there are ten one rupee coin in a box and we draw a coin then it is bound to be a rupee coin.

* Pump air in a tube and it will expand.

other Def. → These are exp. whose outcomes are unique and certain.

b) **Random experiment** → The experiment whose future outcome cannot be predicted in advance known as Random experiment

Ex * In tossing of a coin we are not sure whether the result will be ~~held~~ head or tail

* In rolling a dice we do not know which number will turn up.

2. Some Definition

1) ~~A~~ Event :- An event is any collection of results or outcomes of an experiment.

Ex In drawing two cards from a pack of well shuffled cards, getting a Jack and an ace is an event.

2) Sample space :- The set of all possible outcomes of a random experiment is called its sample space.

Ex Sample space $S = \{H, T\}$ in tossing of a coin

Sample space $S = \{1, 2, 3, 4, 5, 6\}$ in rolling a die.

3) Sample point :- Every element of the sample space is called a sample point.

Ex in above example sample point are H & T

4) Finite sample space :- A sample space S is said to be finite sample space if the number of elements in S is finite otherwise it is said to be infinite sample space.

5) Trial :- Performing a random experiment is called a trial.

Ex - tossing a coin, drawing a card from a pack of 52 cards.

6) Exhaustive events :- The total number of possible outcomes in any trial are known as Exhaustive events or Exhaustive cases.

or

The event $A_1, A_2, ---A_n$ of the sample space S are said to be exhaustive events of the sample space S if they include all possible events i.e.

$$A_1 \cup A_2 \cup -- \cup A_n = S$$

Ex In tossing a coin, there are two exhaustive events, "head and tail"

$S = \{H, T\}$
$E_1 = \{H\}$; $E_2 = \{T\}$ are exhaustive event.

Ex consider the rolling of a dice
then $S = \{1, 2, 3, 4, 5, 6\}$
and $E_1 = \{$getting an even number$\} = \{2, 4, 6\}$

$E_2 = \{$getting a number $\geq 4\} = \{1, 2, 3, 4\}$

$E_3 = \{$getting an odd number$\} = \{1, 3, 5\}$

$\therefore$ exhaustive events $E_1 \cup E_2 \cup E_3 = S$

Ex. In throwing two dice, the exhaustive cases are $6 \times 6 = 6^2$ for any one of the 6 numbers ~~fr~~ from 1 to 6 on one dice can associated with any of the 6 number on the other dice.

In general, in throwing n dice, the exhaustive cases are 6^n

7. Mutually exclusive events :- Two or more than ~~tto~~ two events are called mutually exclusive if there is no element common to these events. In other words events are called mutually exclusive if the happening of one of them prevents the happening of the other events.

If E_1 & E_2 are said to mutually exclusive events if $\boxed{E_1 \cap E_2 = \phi}$

EX. If a dice is rolled then $S = \{1, 2, 3, 4, 5, 6\}$ then the events $E_1 = \{2, 4\}$ $E_2 = \{1\}$ $E_3 = \{5, 6\}$ are mutually exclusive events because $E_1 \cap E_2 \cap E_3 = \phi = E_1 \cap E_2 = E_2 \cap E_3 = E_1 \cap E_3$

8 **Equally Likely Event** :- If one of the event cannot ~~~~ be expected to happen in preference to another than such events are said to be equally likely event.

EX when a card is drawn from a well shuffled pack, any card may appear in the draw so that the 52 different cases are equally likely.

Ex In throwing a dice, all six faces are equally likely to come.

Ex Tossing a coin, getting a head (H) or a Tail (T) are equally likely events.

9. <u>Independent and dependent events</u> :— Two or more events are said to be independent if the happening or not happening or of any one does not depend (or is not affected) by the happening or non-happening of any other. otherwise they are said to be dependent.

Probability

Suppose there are 'n' exhaustive, mutually exclusive, equally likely outcomes and 'm' of them favourable to an event E, then the probability 'p' of the happening of 'E' is given by.

$$\boxed{p \text{ or } P(E) = \frac{\text{Favourable no. of cases}}{\text{Exhaustive no. of cases}} = \frac{m}{n}}$$

<u>Note</u> → 1. Since the number of cases favourable to happening of E is m and the exhaustive number of cases is n, therefore, the number of cases unfavourable to happening of E is <u>n−m</u>

~~9.~~ ~~q = P(Ē) =~~

Note 2: → The probability 'q' of non happening of the event E is given by

$$q = P(\bar{E}) = \frac{\text{unfavourable } \cancel{\text{case}} \text{ no of cases}}{\text{Exhaustive number of cases}} = \frac{n-m}{n}$$

$$q = 1 - \frac{m}{n} \Rightarrow 1 - p$$

$$\therefore \quad p + q = 1 \quad \text{i.e} \Rightarrow P(E) + P(\bar{E}) = 1$$

p and q are non negative and cannot exceed unity $0 \le p \le 1$; $0 \le q \le 1$

Note 3:– If an event E ~~can~~ can happen in 'm' ways and fails to happen in 'n' ways then

$P(E)$ = ~~Prop~~ Probability of happening of E $= \frac{m}{m+n}$

$P(\bar{E})$ = ~~Pob~~ Probability of ~~hap~~ not happening of E $= \frac{n}{m+n}$

m → favourable to E   n → unfavourable to E

n+m → Exhaustive number of cases.

Note 4 → If $P(E) = 1$; E is called a certain event i.e. chance of its happening is cent percent. If $P(E) = 0$ then E is called an impossible event.

Ques Find the probability of obtaining a total of 6 in a single throw of two dice.

Sol^n^ Total cases – $6 \times 6 = 36$.

Favourable cases –

Dice 1 : 5 4 3 2 1

Dice 2 : 1 2 3 4 5

Therefore required probability $= \frac{5}{36}$ Ans.

Ques If a year is selected at random, find the probability that February 30 is a Friday.

Sol^n^ The given event can never occur as no year contains a day which is February 30. Hence required probability is zero.

Ques Consider the random tossing of a coin, find the probability that the coin will stand erect on its edge.

Sol^n^ It is very rare to happen that the coin stands ~~the~~ erect on its edge. hence considering it as an impossible event the required probability is zero.

Addition Theorem of Probability

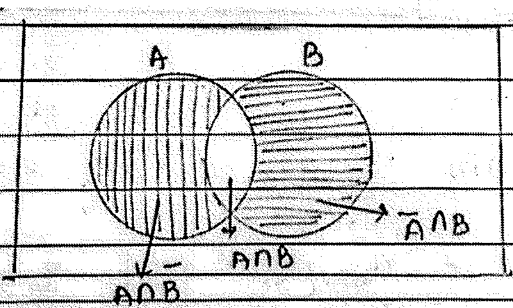
Statement :- If A and B are any two events, then

$$P(A\cup B) = P(A) + P(B) - P(A\cap B)$$

i.e. $P(A \text{ OR } B) = P(A) + P(B) - P(A \text{ And } B)$

Proof A and $\bar{A}\cap B$ are disjoint sets and their union is $A\cup B$

$$A\cup B = A\cup(\bar{A}\cap B)$$



$\therefore$ $P(A\cup B) = P[A\cup(\bar{A}\cap B)]$

$= P(A) + P(\bar{A}\cap B)$

$= P(A) + P(\bar{A}\cap B) + P(A\cap B) - P(A\cap B)$

$= P(A) + [P(\bar{A}\cap B) + P(A\cap B)] - P(A\cap B)$

$= P(A) + [P(\bar{A}\cap B)\cup P(A\cap B)] - P(A\cap B)$

$= P(A) + [P(\bar{A}\cap B) + P(A\cap B)] - P(A\cap B)$

$= P(A) + P[(\bar{A} \cap B) \cup (A \cap B)] - P(A \cap B)$

$= P(A) + P(B) - P(A \cap B)$

$\therefore \boxed{P(A \cup B) = P(A) + P(B) - P(A \cap B)}$

Note – If A and B are two mutually disjoint events then $(A \cap B) = \phi$ so tha

$P(A \cap B) = P(\phi) = 0$

$\therefore \quad P(A \cup B) = P(A) + P(B) - 0$

$\boxed{P(A \cup B) = P(A) + P(B)}$

Note – $P(A \cup B)$ is written as $P(A+B)$ and $P(A \cap B)$ is written as $P(AB)$.

Extension to general law of addition of probability –

* $P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$

* If $A_1, A_2, \ldots A_n$ are mutually exclusive events then the probability of the happening

of one of them is —

$$P(A_1 \cup A_2 \cup \cdots \cup A_n) = P(A_1 + A_2 + \cdots A_n)$$

$$= P(A_1) + P(A_2) + \cdots P(A_n)$$

$\left[\because P(A_1 \cap A_2) \cdots P(A_{n-1} \cap A_n) = 0\right.$ because mutually exclusive$\left.\right]$

Note → $P(A) + P(B) \geq P(A \cup B)$

$$P(\bar{A} \cup \bar{B}) = 1 - P(A \cap B)$$

or $$P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$$

Ques If the probabilities are 0.87, 0.36 and 0.29 that, while under warranty a new car will require repairs on the engine, drive train, or both, what is the probability that a car will require one or the other or both kinds of repairs under the warranty?

Sol^n^ Let A and B be the events denoting repair on the engine and drive train respectively

Given $P(A) = 0.87$ $\quad P(B) = 0.36$

$P(A \cap B) = 0.29$ $\quad P(A \cup B) = ?$

By addition law —

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$
$$= 0.87 + 0.36 - 0.29$$
$$= 0.94 \text{ Ans.}$$

Ques. In a group of 160 graduate engineering students, 92 are enrolled in an advanced course in statistics, 63 are enrolled in a course in operations research, and 40 are enrolled in both. How many of these students are not enrolled in either course?

Sol^n Let A denote the event that a student is enrolled in advanced course in statistics and B be the event that he is enrolled in a course in operation research.

Given $P(A) = \frac{92}{160}$ $\quad P(B) = \frac{63}{160}$

$$P(A \cap B) = \frac{40}{160}$$

$$\therefore P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$$= \frac{92}{160} + \frac{63}{160} - \frac{40}{160}$$

$$= \frac{155 - 40}{160} = \frac{115}{160}$$

Required Probability $= P(\bar{A} \cap \bar{B}) = 1 - P(A \cup B)$

$$= 1 - \frac{115}{160} = \frac{45}{160} \text{ Ans}$$

Conditional Probability

If A and B be two events in a sample space and let $P(A) \neq 0$; then the conditional probability of B, denoted by $P(B/A)$ is the probability of B, assuming that A has already ~~happend~~ happened.
It is given by –

$$P\left(\frac{B}{A}\right) = \frac{P(A \cap B)}{P(A)} \quad \text{——} \; (1)$$

Similarly the conditional probability $P(A/B)$ is given by –

$$P(A/B) = \frac{P(A \cap B)}{P(B)} \quad \text{if } P(B) \neq 0 \quad \text{——} \; (2)$$

• Law of multiplication of Probability

from (1) & (2) $P(A \cap B) = P(A/B) \cdot P(B)$

$$= P(B/A) * P(A)$$

is known as law of multiplication of probability

[redacted]

This law can be generalized to more than two events.

If A, B, C are any three events in a sample space S such that $P(A \cap B) \neq 0$ then

$$P(A \cap B \cap C) = P(A) \cdot P(B/A)\, P\left(\frac{C}{A \cap B}\right)$$

Ques If the probability that a research project will be well planned is 0.80 and the probability that it is well planned and well executed is 0.72, what is the probability that a research project that is well planned will also be well executed?

Solⁿ. Let A be the event that the project is well planned and B be the event that it is well executed. Then given

$P(A) = 0.80$ $\quad P(A \cap B) = 0.72$

Required probability $P(B/A) = \dfrac{P(A \cap B)}{P(A)}$

$$= \frac{0.72}{0.80}$$

$$= 0.90 \quad \text{Ans.}$$

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Que. The ~~sy~~ supervisor of a group of 20 construction workers wants to get the opinion of 2 of them (to be selected at random) about certain new safety regulation. If 12 of them favour the new regulations and the other 8 are against it, what is the probability that both of the worker chosen by the supervisor will be ~~agaig~~ against ~~#~~ new ~~saftet~~ safety regulations?

Sol^n Let A and B denote the events of selection of first and second worker which are against the safety regulations respectively.

$$P(A) = \frac{8}{20} \qquad P\left(\frac{B}{A}\right) = \frac{7}{19}$$

Therefore Required Probability

$$P(AB) = P(A) \cdot P(B/A)$$

$$= \frac{8}{20} \cdot \frac{7}{19} = \frac{14}{95} \text{ Ans.}$$

Independent Events

Two events A and B are independent if the occurrence of one does not affect the probability of occurrence of the other. If A and B are independent events, then.

$$\boxed{P(A \cap B) = P(A) \cdot P(B)}$$

Note → If the events A and B are independent then the events $\bar{A}$ and $\bar{B}$ are also independent.

Note → Two possible mutually exclusive events are always dependent. Hence two independent event cannot be mutually exclusive. But dependent events may be or may not be mutually exclusive.

Note → If $A_1, A_2, \dots A_n$ are n of independent events then

$$P(A_1 A_2 \dots A_n) = P(A_1) \times P(A_2) \times \dots \times P(A_n)$$

Note → Let p be the probability that an event will occure in one trial, then the probability that it will occure in succession of r trials is

$$p \cdot p \cdot p \dots r \text{ times} = p^r$$

Note :— If $p_1, p_2, p_3 \dots p_n$ be the probabilities of n independent events, then the probability of all of these ~~falling is~~ failing is — (17)

$$= (1-p_1)\cdot(1-p_2) \dots (1-p_n)$$

Note :— The probability that at least one of these must happen is

$$= 1 - [(1-p_1)(1-p_2) \dots (1-p_n)]$$

Ques Two cards are drawn at random from an ordinary deck of 52 playing cards. What is the probability of getting two aces if

i) the first card is replaced before the second card is drawn?

ii) The first card is not replaced before the second card is drawn?

Solⁿ i) As there are four aces in a pack of 52 cards and the event of drawing first and ~~secto~~ second card are independent (as card is being replaced)

i) ∴ Required probability $= \cancel{{}^4C_{52} \times {}^4C_{52}}$ $\frac{{}^4C_1}{{}^{52}C_1} \times \frac{{}^4C_1}{{}^{52}C_1}$ $\left(\frac{F}{T}\right)$

$$= \frac{4}{52} \times \frac{4}{52} = \frac{1}{169}$$

ii) Here the favourable cases for drawing of second card depends on the event of drawing first card, as the card is not being replaced

$\therefore$ Required Probability $= \frac{4}{52} \times \frac{3}{51} = \frac{1}{221}$

Ans

Ques A problem is in statistics is given to three students whose probabilities of solving it are $\frac{1}{2}$, $\frac{1}{3}$ and $\frac{1}{4}$ respectively. what is the chance that the problem will be solved if all of them they independently?

Soln. Required Probability $=$ P (at least one of the student solve the problem)

$= 1 - P$ (none of the student solve the problem)

$= 1 - [(1-p_1)(1-p_2)(1-p_3)]$

$= 1 - \left[\left(1-\frac{1}{2}\right)\left(1-\frac{1}{3}\right)\left(1-\frac{1}{4}\right)\right]$

$= 1 - \left[\left(\frac{1}{2}\right)\left(\frac{2}{3}\right)\left(\frac{3}{4}\right)\right]$

$= 1 - \frac{1}{4} = 3/4$ Ans.

Ques A bag contains 10 gold and 8 silver coins. Two successive drawing of 4 coins are made such that i) coins are replaced before the second trial ii) coins are not replaced before the second trail. Determine the probability that the first drawing will give 4 gold and the second four silver coins.

Sol^n. Let A be the event that first drawing gives 4 gold coins and B be the event that the second drawing will give four silver ~~8~~ coins.

i) Events are independent

$$\therefore \quad P(A\cap B) = P(A)\cdot P(B)$$

$$= \frac{{}^{10}C_4}{{}^{18}C_4} \cdot \frac{{}^{8}C_4}{{}^{18}C_4}$$

$$= \frac{\dfrac{10\times9\times8\times7}{4\times3\times2\times1} \times \dfrac{8\times7\times6\times5}{4\times3\times2\times1}}{\dfrac{18\times17\times16\times15}{4\times3\times2\times1} \times \dfrac{18\times17\times16\times15}{4\times3\times2\times1}}$$

$$= \frac{\overset{5}{\cancel{10}}\times\cancel{9}\times\cancel{8}\times7\times\cancel{8}\times7\times\overset{3}{\cancel{6}}\times\cancel{5}}{\underset{2}{\cancel{18}}\times17\times\underset{2}{\cancel{16}}\times\underset{3}{\cancel{15}}\times18\times17\times\underset{2}{\cancel{16}}\times15}$$

$$= \frac{7\times7\times\cancel{6}\;1}{17\times18\times17\times2\times\cancel{15}\;3} = .00156 \cong .002$$

ii) Events are independent.

$$P(A \cap B) = P(A) \cdot P(B/A)$$

$$= \frac{^{10}C_4}{^{18}C_4} \times \frac{^{8}C_4}{^{14}C_4}$$

$$= \frac{\cancel{10}^{5} \times \cancel{9} \times \cancel{8} \times \cancel{7}}{\cancel{18}_{2} \times 17 \times \cancel{16}_{2} \times \cancel{15}_{3}} \times \frac{\cancel{8}^{2} \times 7 \times \cancel{6}^{3} \times 5}{\cancel{14}_{2} \times 13 \times \cancel{12}_{3} \times 11}$$

$$= \frac{\cancel{2} \times 7 \times 5}{17 \times \cancel{2} \times 13 \times 3 \times 11} = \frac{35}{7293} = .00479$$

$$\simeq .005 \text{ Ans.}$$

~~I.~~ Illustrative Examples

Ques A fair ~~dice~~ coin is tossed 4 times. Define the sample space corresponding to the random experiment. Also give the subsets corresponding to the following events and their respective probabilities

(i) more heads than trials are obtained

(ii) Tails occur on even numbered tossed.

Solⁿ Sample space is

S = { HTTT, THTT, TTHT, TTTH, HHTT, THTH, HTHT, THHT, ~~TT~~ HTTH, HHHT, HHTH, HTHH, THHH, HHHH, ~~TTTT~~ TTTT, TTHH }

(i) $E_1 = \{HHHT, HHTH, HTHH, HHHH\}$ $THHH\}$

(21)

$P(E_1) = \frac{\text{Favourable cases}}{\text{Total cases}} = \frac{5}{16}$ An.

ii) $E_2 = \{HTTT, TTHT, \cancel{TTTH}, TTTT, HTHT$

$P(E_2) = \frac{4}{16} = 1/4$ An.

Qus. A bag has 2 white and 1 black ball, while the other has 2 white and 2 black balls. A ball is drawn from each bag. Find the chance that there is atleast one white ball drawn.

Solⁿ. P (at least one white ball drawn) = 1 − P (no white ball drawn)

= 1 − P (both balls are black)

= 1 − P (ball drawn from one bag is black) × P(ball drawn from second is black)

$= 1 - \frac{1}{3} \times \frac{2}{4}$

$= 1 - \frac{1}{6} = 5/6$ An

Ques. From a pack of 52 cards, one card is drawn at random. ~~find~~ find the probability that the card drawn is either a spade, or a court, or a Jack.

Soln. Let ~~a~~ A be the ~~card~~ event that card drawn is spade, B be the event that card drawn is court card (King, Queen and Jack) and C be the event that card drawn is Jack.

Now $P(A) = \frac{13}{52}$ $P(A \cap B) = \frac{3}{52}$ $P(A \cap B \cap C) = \frac{1}{52}$

$P(B) = \frac{12}{52}$ $P(B \cap C)$ $\frac{4}{52}$

$P(C) = \frac{4}{52}$ $P(A \cap C) = \frac{1}{52}$

Required probability $P(A \cup B \cup C)$

$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(B \cap C) - P(C \cap A) + P(A \cap B \cap C)$$

$$P(A \cup B \cup C) = \frac{13}{52} + \frac{12}{52} + \cancel{\frac{4}{52}} - \frac{3}{52} - \cancel{\frac{4}{52}} - \cancel{\frac{1}{52}} + \cancel{\frac{1}{52}}$$

$$= \frac{25-3}{52} = \frac{22}{52} = \frac{11}{26} \text{ Ans.}$$

Ques An urn contain 2 white, 3 red and 4 black balls. Three balls are drawn from the urn. Find the chance that :-

i) all are ~~the~~ of the same colour.

ii) all are of different colours.

iii) Two are of same colour and third of different colour.

Soln. i) The balls drawn are either three red or three black hence

Required probability $= \dfrac{{}^3C_3 + {}^4C_3}{{}^9C_3}$

$$\frac{\left(\frac{3\times2\times1}{3\times2\times1}\right) + \left(\frac{4\times3\times2}{3\times2\times1}\right)}{\frac{9\times8\times7}{3\times2\times1}} = \frac{1+4}{84}$$

$$= \frac{5}{84}$$

ii) One white ~~and~~ one Red ~~Red~~ and one black ball is drawn from the urn.

Required Probability $= \dfrac{{}^2C_1 \times {}^3C_1 \times {}^4C_1}{{}^9C_3}$

$$= \frac{2\times3\times4}{\frac{9\times8\times7}{3\times2\times1}} = \frac{2\times3\times4\times3\times2\times1}{9\times8\times7}$$

$$= 2/7$$

iii) The balls drawn are 2 white and one from rest of 7 balls or 2 red and one from rest of 6 balls or ~~and one~~ 2 black and one from rest of 5

Required probability $= \dfrac{{}^2C_2 \times {}^7C_1 + {}^3C_2 \times {}^6C_1 + {}^4C_2 \times {}^5C_1}{{}^9C_3}$

$$= \frac{1 \times 7 + \frac{3 \times \cancel{2}}{\cancel{2} \times 1} \times 6 + \frac{\overset{2}{\cancel{4}} \times 3}{\cancel{2} \times 1} \times 5}{{}^3\frac{\cancel{9} \times \overset{4}{\cancel{8}} \times 7}{\cancel{3} \times \cancel{2} \times 1}}$$

$$= \frac{7 + 18 + 30}{84} = \frac{55}{84} \text{ Ans.}$$

Ques A, B and C toss a coin in succession on the understanding that the first one to throw a head wins. Find their ~~to~~ respective chances of winning.

Soln. $P(\text{getting head}) = p = \frac{1}{2}$

$\therefore \quad q = 1 - \frac{1}{2} = 1/2$

Let A tossing first

P (A wins) = Probability that A gets head at first throw or fourth throw or 7^{th} throw ..

$$= p + q^3 p + q^6 p + \cdots$$

$$= p(1 + q^3 + q^6 + \cdots) = p(1-q^3)^{-1} = \frac{p}{(1-q^3)}$$

$$P(A \text{ wins}) = \frac{1/2}{\{1-(1/2)^3\}} = \frac{1/2}{1-1/8} = \frac{1}{2} \times \frac{8}{7} = \frac{4}{7}$$

P(B wins) = Probability that B gets head at second or fifth or eight throw ...

$$= qp + q^4 p + q^7 p + \cdots$$

$$= qp(1 + q^3 + q^6 \cdots) = \frac{qp}{1-q^3}$$

$$= \frac{1/2 \times 1/2}{1 - 1/8} = \frac{1}{4} \times \frac{8}{7} = \frac{2}{7} \text{ Ans.}$$

Ques A and B take turns in throwing of two dice & the first to throw 9 will be awarded a prize. If A has the first turn, show that their chances of winning are in the ratio 9:8

Soln. Favourable cases for getting 9 as sum of number of ~~at~~ two dice are $= \{(6,3)\ (5,4)\ (4,5)\ (3,6)\}$

Hence P (getting sum of numbers on two dice as 9) $= \frac{4}{36}$

$$= 1/9 = p$$

$$q = 1 - p = 1 - 1/9 = 8/9$$

~~Thus~~ A is first done to throw dice

then A will win in 1^{st} or 3^{rd} or 5^{th} --- throws with respective probabilities as p, ~~pq~~ q^2p, q^4p ---

$$P(A\ \text{winning}) = p + q^2p + q^4p + \cdots$$

$$= p\,(1 + q^2 + q^4 + \cdots)$$

$$= \frac{p}{1-q^2} = \frac{1/9}{1 - 64/81}$$

$$= \frac{1}{\cancel{9}1} \times \frac{\cancel{81}^{9}}{17} = \frac{9}{17}$$

Similarly B will win in 2^{nd} or 4^{th} or 6^{th} --- throws with respective probabilities as qp, q^3p, q^5p ---

$$P(B\ \text{winning}) = qp + q^3p + q^5p + \cdots$$

$$= qp\,[1 + q^2 + q^4 + \cdots]$$

$$= \frac{qp}{(1-q^2)} = \frac{8/9 \times 1/9}{1 - 64/81} = \frac{8}{17}$$

Hence there chance of winning are - 9 : 8 Ans.

Ques These groups of children contain respectively 3 girls and 1 boy; 2 girls and 2 boys; 1 girl and 3 boys; One child is selected at random from each group. Find the probability of selecting 1 girl and 2 boys.

Sol^n. Let G denotes selection of a girl and
B " " " " boy
then various cases of selection are GBB, BGB, BBG.

Required Probability = P(GBB) + P(BGB) + P(BBG)

$P(GBB) = \frac{{}^3C_1}{{}^4C_1} \times \frac{{}^2C_1}{{}^4C_1} \times \frac{{}^3C_1}{{}^4C_1} = \frac{3}{4} \times \frac{\cancel{2}\,1}{\cancel{4}\,2} \times \frac{3}{4} = \frac{9}{32}$

$P(BGB) = \frac{1}{4} \times \frac{\cancel{2}}{\cancel{4}} \times \frac{3}{4} = \frac{3}{32}$

$P(BBG) = \frac{1}{4} \times \frac{\cancel{2}}{\cancel{4}_2} \times \frac{1}{4} = \frac{1}{32}$

Required Probability $= \frac{9}{32} + \frac{3}{32} + \frac{1}{32} = \frac{13}{32}$ Ans.