JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem

Subject –Engineering Mathematics-II

Unit – III

Presented by – (Dr.Vishal Saxena, Associate Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Particular Integral case -1

Rules for Finding Particular Integral (P.I)

Consider the eqn $f(D)y = X$

P.I is given as $P.I = \frac{1}{f(D)} \cdot X$

Now we will find P.I with different situations given for X as

1. e^{ax}

2. Sin ax or Cos ax

3. x^n ; $n \in N$

4. $e^{ax}.v$; v is any fn of x.

5. xV : v is any fn of x.

P.I. when $X = e^{ax}$

$$P.I = \frac{1}{f(D)} e^{ax} = \frac{1}{f(a)} e^{ax} \; ; \; f(a) \neq 0$$

If $f(a) = 0$, then $(D-a)^r$ exists as a factor of $f(D)$. i.e

if $f(D) = (D-a)^r \varphi(D)$

then $P.I = \frac{1}{f(D)} e^{ax} = \frac{1}{(D-a)^r \phi(D)} e^{ax}$

$$P.I = \frac{x^r}{\lfloor r} \frac{e^{ax}}{\phi(a)}.$$

Ex: Solve $\frac{d^2y}{dx^2} - 3\frac{dy}{dx} + 2y = e^{5x}$

Sol: given eqn can be written as

$$(D^2 - 3D + 2)y = e^{5x}$$

The auxilliary eqn is

$$m^2 - 3m + 2 = 0 \Rightarrow (m-1)(m-2) = 0$$

$$m = 1, 2$$

$$C.F = c_1 e^{x} + c_2 e^{2x}$$

$$P.I = \frac{1}{D^2 - 3D + 2} e^{5x} = \frac{1}{5^2 - 3\times 5 + 2} e^{5x}$$

$$= \frac{1}{12} e^{5x}.$$

Hence the Complete sol is

$$y = C.F + P.I$$

$$y = C_1 e^{x} + C_2 e^{2x} + \frac{1}{12} e^{5x}.$$

Ex: Solve $(D^2+D-2)y = e^x$

Sol: The auxilliary eqn is

$m^2+m-2=0 \Rightarrow (m-1)(m+2)=0 \Rightarrow m = 1, -2$

$C.F = c_1 e^x + c_2 e^{-2x}$

$P.I = \frac{1}{(D^2+D-2)} e^x \quad (\because f(a)=0)$

$$= \frac{1}{(D-1)(D+2)} e^{x} = \frac{1}{3(D-1)'} e^{x} = \frac{1}{3} \frac{x^{1}}{\lfloor 1} e^{x}$$

Hence the general / Complete soln is $y = C.F + P.I$

$$y = C_1 e^{x} + C_2 e^{-2x} + \frac{1}{3} x e^{x}.$$

Ex: $\frac{d^4y}{dx^4} - a^4 y = \cosh ax$

Sol: $(D^4 - a^4)y = \left(\frac{e^{ax} + e^{-ax}}{2}\right)$; $D = \frac{d}{dx}$

To find C.F, the auxiliary equ in m is

$$m^4 - a^4 = 0$$

$$(m^2 + a^2)(m^2 - a^2) = 0$$

$$m = a, -a, \pm ai$$

$$C.F = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos ax + c_4 \sin ax$$

$$P.I = \frac{1}{(D^4 - a^4)}\left(\frac{e^{ax} + e^{-ax}}{2}\right)$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{(D^4 - a^4)} e^{ax} + \frac{1}{(D^4 - a^4)} e^{-ax}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{(D^2 + a^2)(D^2 - a^2)} e^{ax} + \frac{1}{(D^2 + a^2)(D^2 - a^2)} e^{-ax}\right]$$

$$= \frac{1}{2}\left[\frac{1}{(D^2 + a^2)(D + a)(D - a)} e^{ax} + \frac{1}{(D^2 + a^2)(D + a)(D - a)} e^{-ax}\right]$$

$$\Rightarrow \frac{1}{2}\left[\frac{1}{(a^2+a^2)(a+a)}\cdot\frac{1}{(D-a)}e^{ax}+\frac{1}{(a^2+a^2)(-a-a)}\cdot\frac{1}{(D+a)}e^{-ax}\right]$$

$$= \frac{1}{2}\left[\frac{1}{4a^3}\cdot\frac{x^1}{\lfloor 1}e^{ax}-\frac{1}{4a^3}\frac{x^1}{\lfloor 1}e^{-ax}\right]$$

$$= \frac{1}{4a^3}x\left(\frac{e^{ax}-e^{-ax}}{2}\right)=\frac{x\sinh ax}{4a^3}$$

Hence the Complete sol is

$$y = C.F + P.I$$

$$y = c_1 e^{ax} + c_2 e^{-ax} + c_3 \cos ax + c_4 \sin ax + \frac{x \sinh ax}{4a^3}.$$

Ex: $\left[(D-1)^2(D^2+1)^2\right]y = e^x$, ; $D = \frac{d}{dx}$

Sol: The auxilliary eqn is

$$(m-1)^2(m^2+1)^2 = 0$$

$$m = 1, 1, \pm i, \pm i$$

$$C.F = (c_1 + c_2 x)e^x + (c_3 + c_4 x)\cos x + (c_5 + c_6 x)\sin x.$$

$$P.I = \frac{1}{(D-1)^2(D^2+1)^2} e^x$$

$$= \frac{1}{(D-1)^2[1^2+1]^2} e^x = \frac{1}{4(D-1)^2} e^x$$

$$= \frac{1}{4} \cdot \frac{x^2}{\lfloor 2} e^x = \frac{1}{8} x^2 e^x.$$

So the required sol is

$$y = C.F + P.I.$$

Special Case.

when $Q(x)$ is $\tan ax / \sec ax$.

Then we find P.I by

$$\frac{1}{D-\alpha} \tan ax = e^{\alpha x} \int e^{-\alpha x} \tan ax \, dx$$

Ex: Solve $\frac{d^2y}{dx^2} + 4y = \tan 2x$

Sol: The auxilliary eqn is

$m^2 + 4 = 0 \Rightarrow m^2 = -4$

$m = \pm 2i$

$C.F = C_1 \cos 2x + C_2 \sin 2x$

$$P.I = \frac{1}{D^2+4} \tan 2x$$

$$= \frac{1}{(D+2i)(D-2i)} \tan 2x$$

$$= \frac{1}{4i}\left[\frac{1}{D-2i} - \frac{1}{D+2i}\right] \tan 2x$$

$$\frac{1}{4i}\left[\frac{1}{(D-2i)} \tan 2x - \frac{1}{D-(-2i)} \tan 2x\right]$$

Now using $\frac{1}{D-\alpha}\tan ax = e^{\alpha x}\int e^{-\alpha x}\tan ax\, dx$

$$= \frac{1}{4i}\left[e^{2ix}\int e^{-2ix}\tan 2x\, dx - e^{-2ix}\int e^{2ix}\tan 2x\, dx\right]$$

$$= \frac{1}{4i}\left[e^{2ix}\int(\cos 2x - i\sin 2x)\tan 2x\, dx\right.$$

$$\left. - e^{-2ix}\int(\cos 2x + i\sin 2x)\tan 2x\, dx\right]$$

$$= \frac{1}{4i}\left[e^{2ix}\int \left(\sin 2x - i\,\frac{\sin^2 2x}{\cos 2x}\right) dx \right.$$

$$\left. - e^{-2ix}\int \left(\sin 2x + i\,\frac{\sin^2 2x}{\cos 2x}\right) dx \right]$$

$$= \frac{1}{4i}\left[e^{2ix}\left(-\frac{\cos 2x}{2}\right) - ie^{2ix}\int \frac{(1-\cos^2 2x)}{\cos 2x}\, dx \right.$$

$$\left. + e^{-2ix}\left(\frac{\cos 2x}{2}\right) - ie^{-2ix}\int \frac{(1-\cos^2 2x)}{\cos 2x}\, dx \right]$$

$$= \frac{1}{4i}\left[-\cos 2x\left(\frac{e^{2ix}-e^{-2ix}}{2}\right) - i\left(e^{2ix}+e^{-2ix}\right)\int\left(\sec 2x - \cos 2x\right)dx\right]$$

$$= \frac{1}{4i}\left[-i\cos 2x \sin 2x - 2i\cos 2x\left\{\frac{\log(\sec 2x+\tan 2x)}{2} - \frac{\sin 2x}{2}\right\}\right]$$

$$= -\frac{1}{4}\cancel{\cos 2x \sin 2x} - \frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x) + \frac{1}{4}\cancel{\cos 2x \sin 2x}$$

$$= -\frac{1}{4}\cos 2x \log(\sec 2x + \tan 2x)$$

hence the general sol is $y = C.F + P.I$

$$y = c_1 \cos 2x + c_2 \sin 2x - \frac{1}{4} \cos 2x \log(\sec 2x + \tan 2x).$$

Ex: $\dfrac{d^2y}{dx^2} + a^2y = \sec ax$

Sol: the auxilliary eqn is

$(m^2+a^2)=0 \Rightarrow m = \pm ai$

$C.F = c_1 \cos ax + c_2 \sin ax$

$$P.I = \frac{1}{D^2+a^2} \sec ax$$

$$= \frac{1}{2ai}\left[\frac{1}{(D-ai)} - \frac{1}{(D+ai)}\right] \sec ax$$

Now using $\frac{1}{(D-\alpha i)}\sec ax = e^{i\alpha x}\int e^{-i\alpha x}\sec ax\,dx$

Now $\frac{1}{(D-ai)}\sec ax = e^{iax}\int e^{-iax}\sec ax\,dx$

$= e^{iax}\int \frac{(\cos ax - i\sin ax)}{\cos ax}\,dx$

$= e^{iax}\int (1 - i\tan ax)\,dx = e^{iax}\left[x + \frac{i}{a}\log(\cos ax)\right]$

Similarly $\frac{1}{(D+ai)}\sec ax = e^{-aix}\left[x - \frac{i}{a}\log(\cos ax)\right]$

Thus $P.I = \frac{1}{2ia}\left[e^{iax}\left\{x + \frac{i}{a}\log(\cos ax)\right\}\right.$

$$\left. - e^{-iax}\left\{x - \frac{i}{a}\log(\cos ax)\right\}\right]$$

$$= \frac{x}{2ia}\left(e^{iax} - e^{-iax}\right) + \frac{1}{a^2}\log(\cos ax)\left(\frac{e^{iax} + e^{-iax}}{2}\right)$$

$$= \frac{x}{a} \sin ax + \frac{1}{a^2} \cos ax \cdot \log(\cos ax)$$

hence the complete soln is

$$y = C.F + P.I.$$

$$y = C_1 \cos ax + C_2 \sin ax + \frac{1}{a^2} \cos ax \cdot \log(\cos ax).$$

Practice Problems

1. $$\frac{d^3y}{dx^3} - y = (1+e^x)^2$$

Ans: $y = c_1 e^x + e^{-\frac{x}{2}}\left[c_2 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_3 \sin\left(\frac{\sqrt{3}x}{2}\right)\right] - 1 + \frac{2xe^x}{3} + \frac{e^{2x}}{7}$

2. $$\frac{d^2y}{dx^2} + \frac{dy}{dx} + y = e^{-x}$$

Ans: $y = e^{-x/2}\left[c_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right] + e^{-x}$

3. $(D^3 + D^2 - D - 1)y = \cosh x, \quad D = \frac{d}{dx}$

Ans: $y = c_1 e^x + (c_2 + c_3 x) e^{-x} + \frac{1}{8} x e^x - \frac{1}{8} x^2 e^{-x}$

4. $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + 2y = \sinh x$

Ans: $y = e^{-x}[c_1 \cos x + c_2 \sin x] + \frac{e^x}{10} - \frac{e^{-x}}{2}$

5. $\frac{d^2y}{dx^2} + y = \operatorname{cosec} x$

Ans: $y = c_1 \cos x + c_2 \sin x - x \cos x + \sin x \cdot \log(\sin x)$

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Thank you!