Unit-I : DC Network Theorems

1. Basic Laws
2. Analysis with dependent current and voltage sources.
3. Node and Mesh Analysis.
4. Concept of duality and dual networks
5. Superposition theorem
6. Thevenin Theorem
7. Norton Theorem
8. Conversion of Voltage and Current Sources

Fundamental Quantities

Voltage — potential difference between 2 points

"across" quantity analogous to 'pressure' between two points

Current — flow of charge through a material

"through" quantity analogous to fluid flowing along a pipe

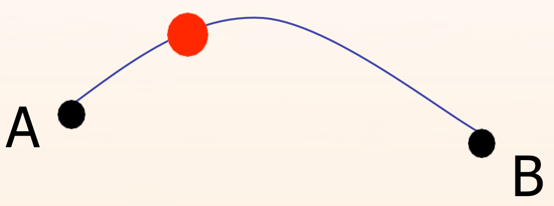
Units of measurement

- Voltage: volt (V)
- Current: ampere (A)

Power and Energy

Work done in moving a charge *δq* from A to B having a potential difference of *V* is

$$W = V\,\delta q$$

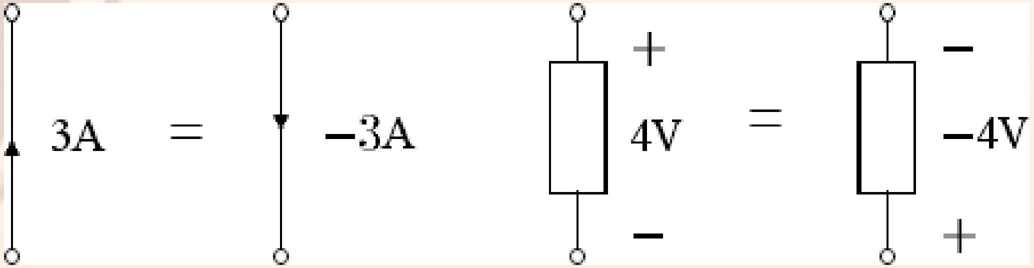


Power is work done per unit time, i.e.,

$$P = \lim_{\delta t \to 0} V \frac{\delta q}{\delta t} = V \frac{dq}{dt} = V I$$

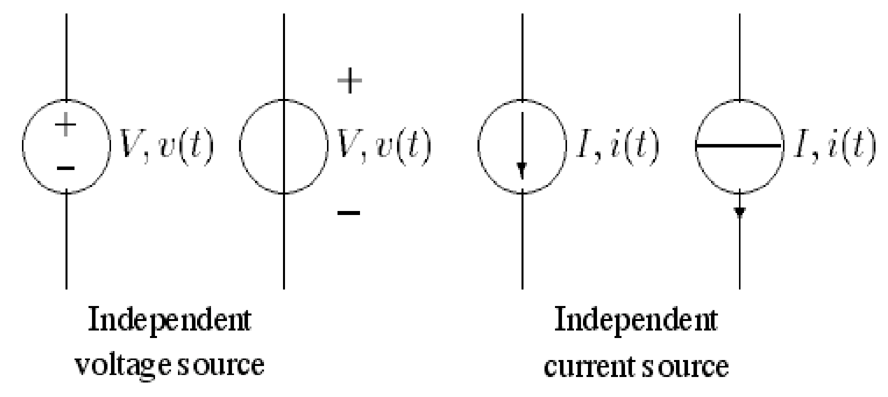
Direction and Polarity

- Current direction indicates the direction of flow of positive charge
- Voltage polarity indicates the relative potential between 2 points:
- **+** assigned to a higher potential point; and **–** assigned to a lower potential point.
- Direction and polarity are arbitrarily assigned on circuit diagrams.
- Actual direction and polarity will be governed by the sign of the value.



Independent Sources

Independent — Stubborn! never change!

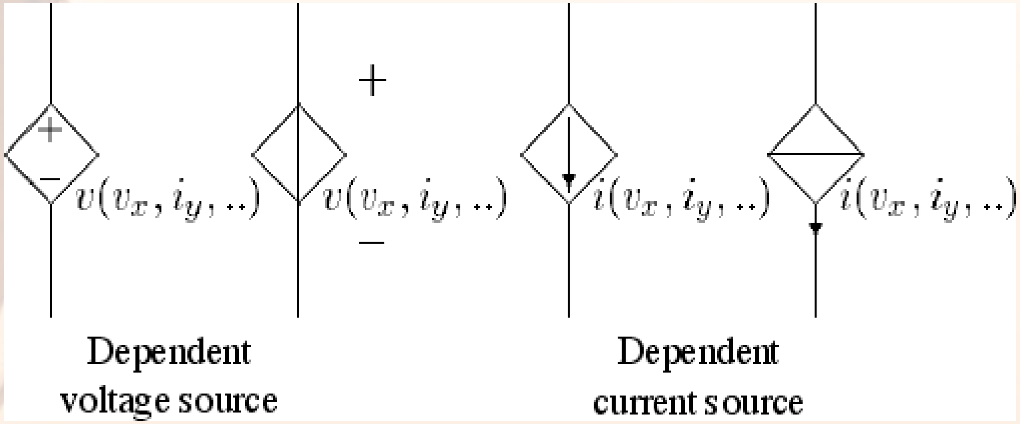


Maintains a voltage/current (fixed or varying) which is not affected by any other quantities.

An independent voltage source can never be shorted. An independent current source can never be opened.

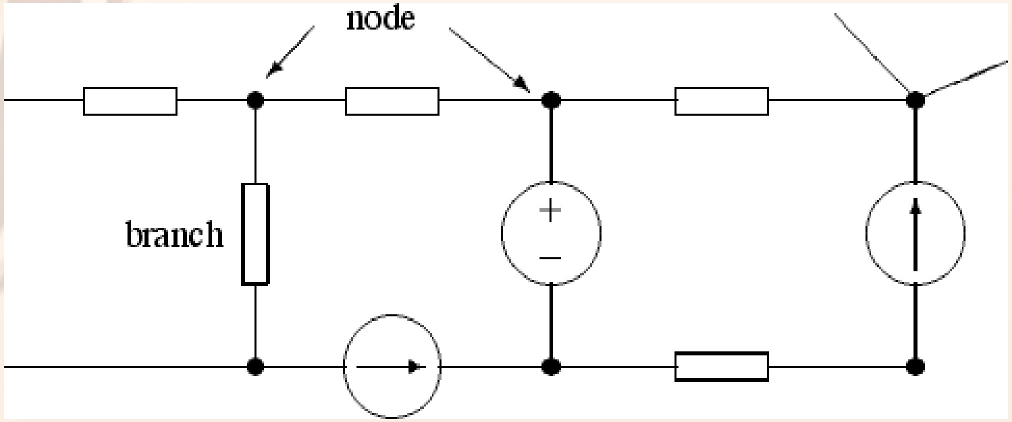
Dependent Sources

Dependent Sources: Values depend on some other voltage and current sources



Circuit

Collection of devices such as sources and resistors in which terminals are
connected together by conducting wires.
These wires converge in NODES
The devices are called BRANCHES of the circuit



Ohm’s Law

Ohm's Law states that the current flowing in a circuit is directly proportional to the applied potential difference at constant physical conditions.

$$V \propto I$$

Simply, $V = IR$

Resistors in Series and Parallel Circuits

Resistors in circuits

- To determine the current or voltage in a circuit that contains multiple resistors, the total resistance must first be calculated.
- Resistors can be combined in series or parallel.

Resistors in Series

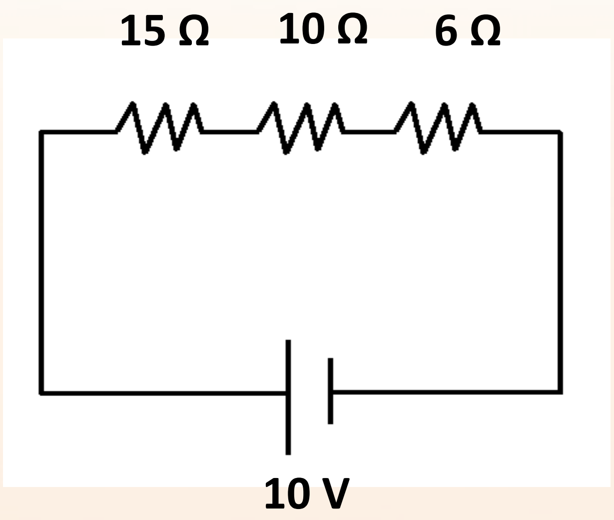
When connected in series, the total resistance (Rt) is equal to:

$$Rt = R1 + R2 + R3 + \ldots$$

The total resistance is always larger than any individual resistance.

Sample Problem

Calculate the total current through the circuit.



$Rt = 15\ \Omega + 10\ \Omega + 6\ \Omega$

$Rt = 31\ \Omega$

$I = V/Rt = 10\ V / 31\ \Omega = 0.32\ A$

Resistors in Parallel

When connected in parallel, the total resistance (Rt) is equal to:

$$1/Rt = 1/R1 + 1/R2 + 1/R3 + \ldots$$

Due to this reciprocal relationship, the total resistance is always smaller than any individual resistance.

Sample Problem

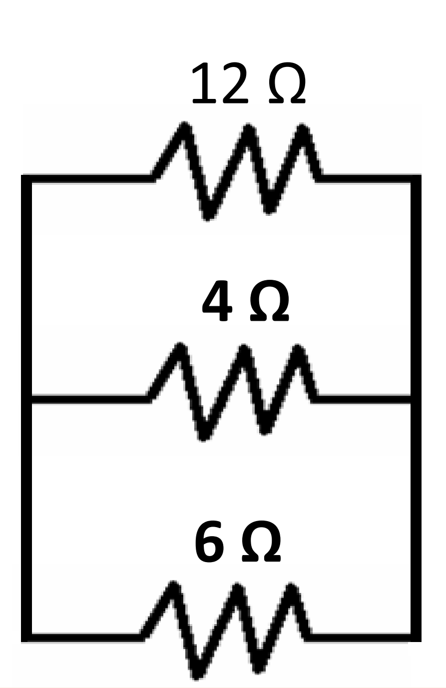
Calculate the total resistance through this segment of a circuit.

$1/Rt = 1/12\ \Omega + 1/4\ \Omega + 1/6\ \Omega$

$= 1/12\ \Omega + 3/12\ \Omega + 2/12\ \Omega$

$1/Rt = 6/12\ \Omega = ½\ \Omega$

$Rt = 2\ \Omega$

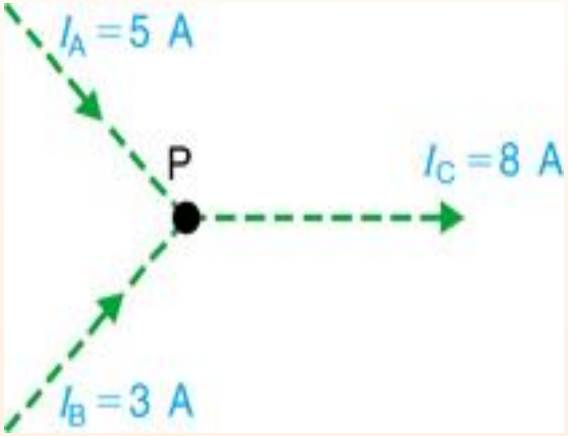


Kirchhoff’s Laws

Kirchhoff’s Current Law (KCL)
Kirchhoff’s Voltage Law (KVL)

Kirchhoff’s Current Law (KCL)

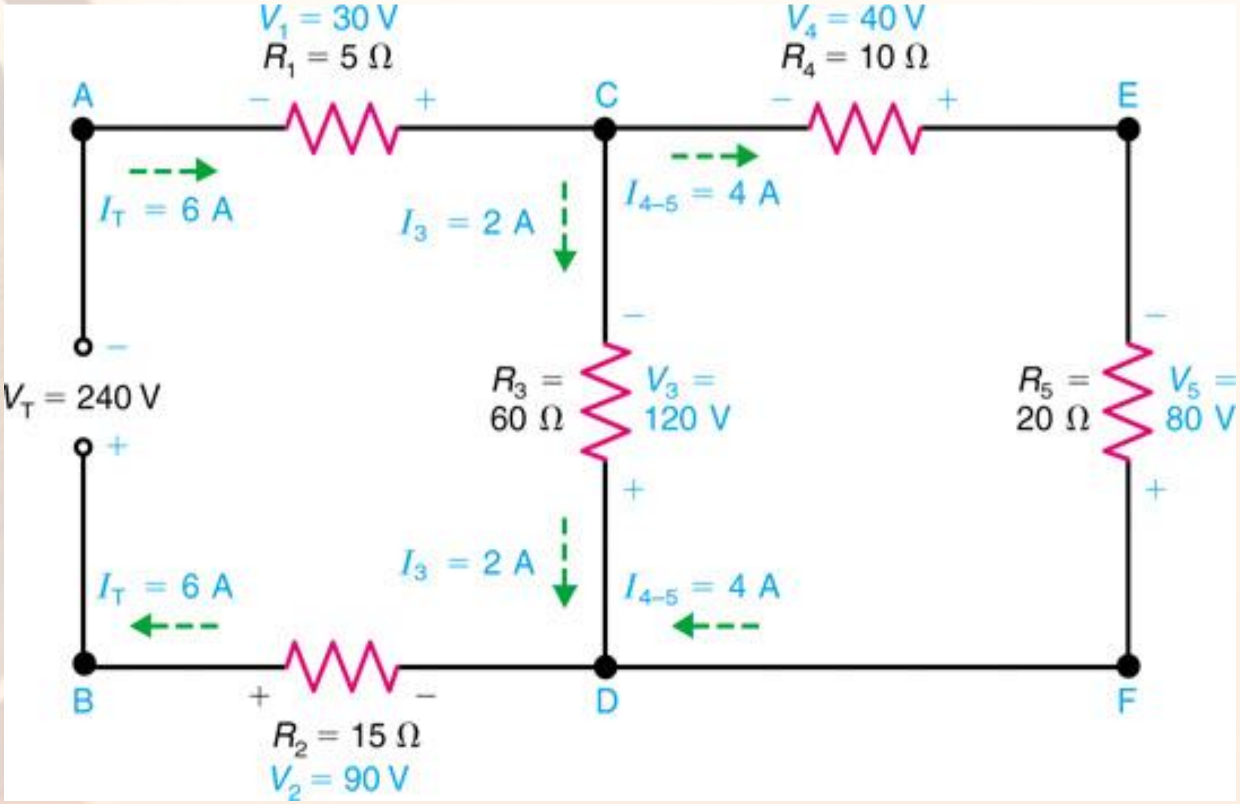
The sum of currents entering any node in a circuit is equal to the sum of currents leaving that node.



Kirchhoff’s Current Law may also be stated as

$$I_{IN} = I_{OUT}$$

Kirchhoff’s Current Law (KCL)



Series-parallel circuit illustrating Kirchhoff’s laws.

Kirchhoff's Current Law (KCL)

The 6-A I_T into point C divides into the 2-A I_3 and 4-A $I_{4\text{-}5}$

$I_{4\text{-}5}$ is the current through R_4 and R_5

$$I_T - I_3 - I_{4\text{-}5} = 0$$
$$6A - 2A - 4A = 0$$

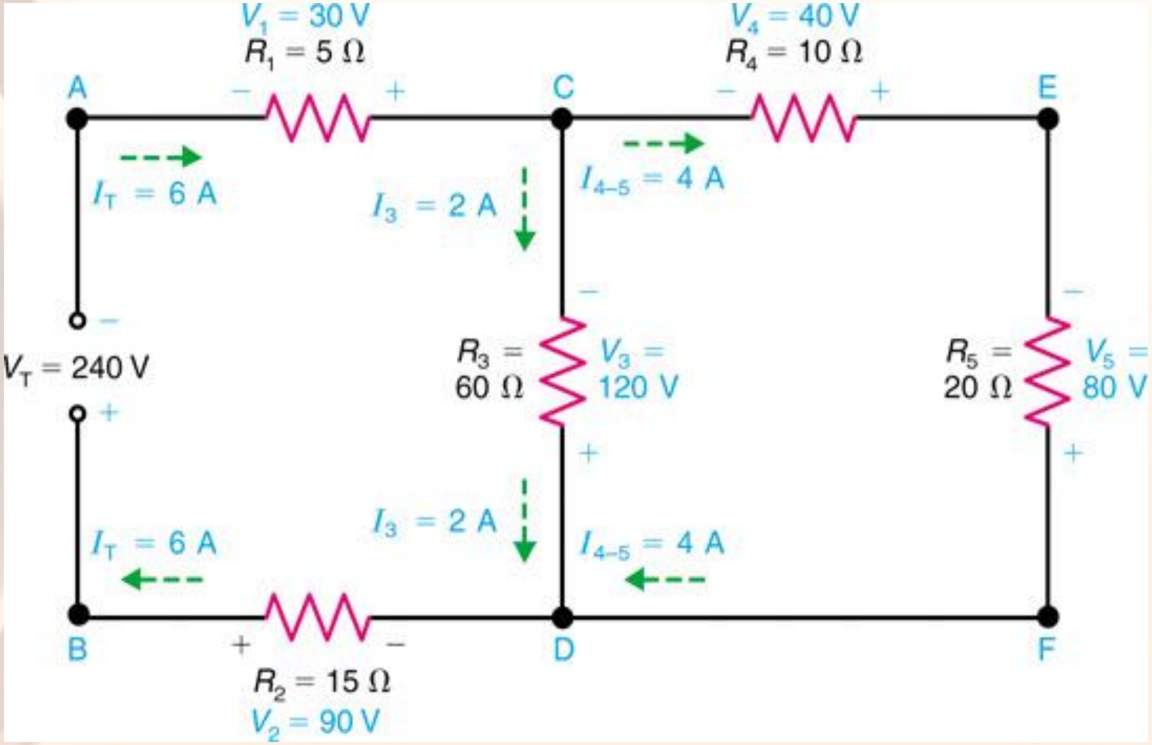
At either point C or point D, the sum of the 2-A and the 4-A branch currents must equal the 6A line current. Therefore

$$I_{in} = I_{out}$$

Kirchhoff's Voltage Law (KVL)

The algebraic sum of the voltage rises and voltage drops in any closed path (loop) must equal to zero.

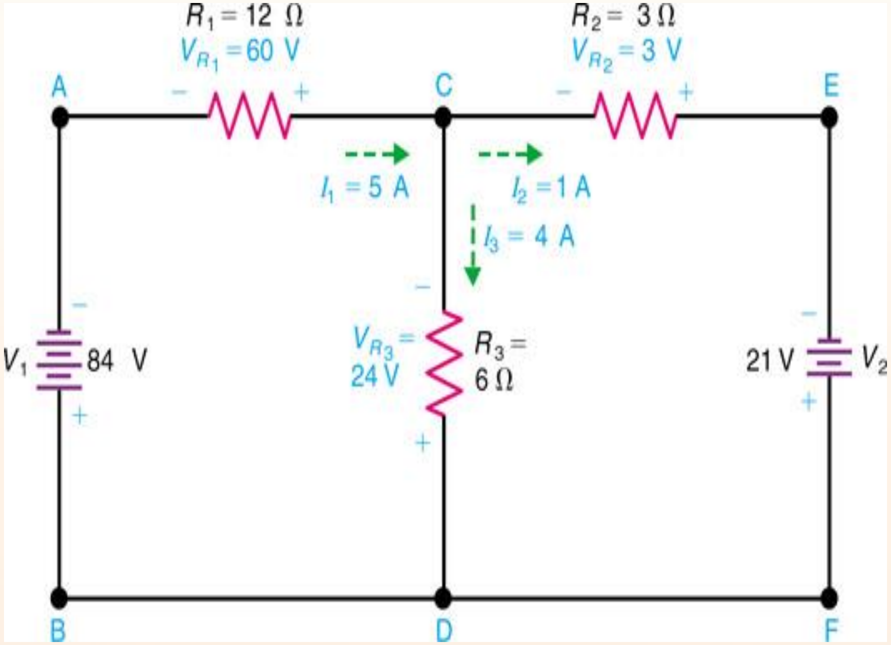
Kirchhoff’s Voltage Law (KVL)



Series-parallel circuit illustrating Kirchhoff’s laws.

Applying KCL in loop ACDB,
84V − 60V − 24V = 0

Applying KCL in loop CEFD,
21V + 3V − 24V = 0

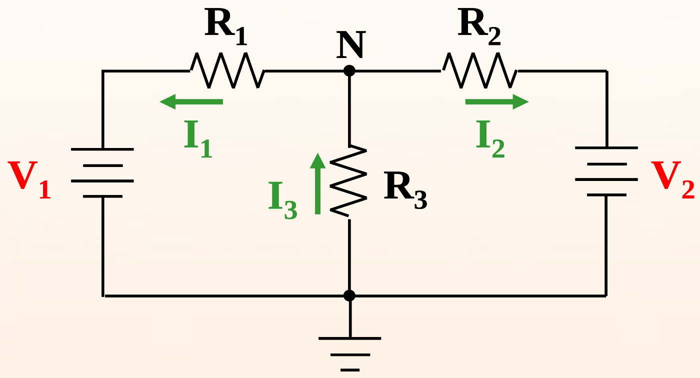


Node-Voltage Analysis

- A principal node is a point where three or more currents divide or combine, other than ground.
- The method of node voltage analysis uses algebraic equations for the node currents to determine each node voltage.
- Use KCL to determine node currents
- Use Ohm’s Law to calculate the voltages.
- The number of current equations required to solve a circuit is one less than the number of principal nodes.
- One node must be the reference point for specifying the voltage at any other node.

Node-Voltage Analysis

Node Voltage Method



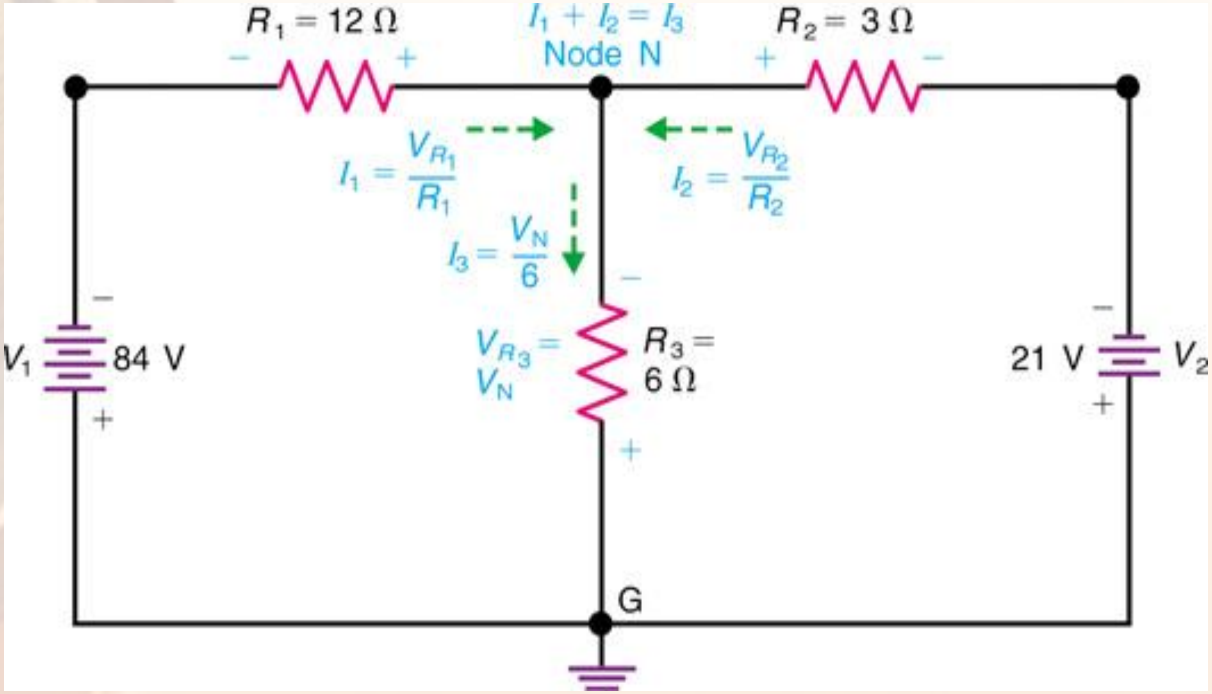
At node N: $I_1 + I_2 = I_3$

or

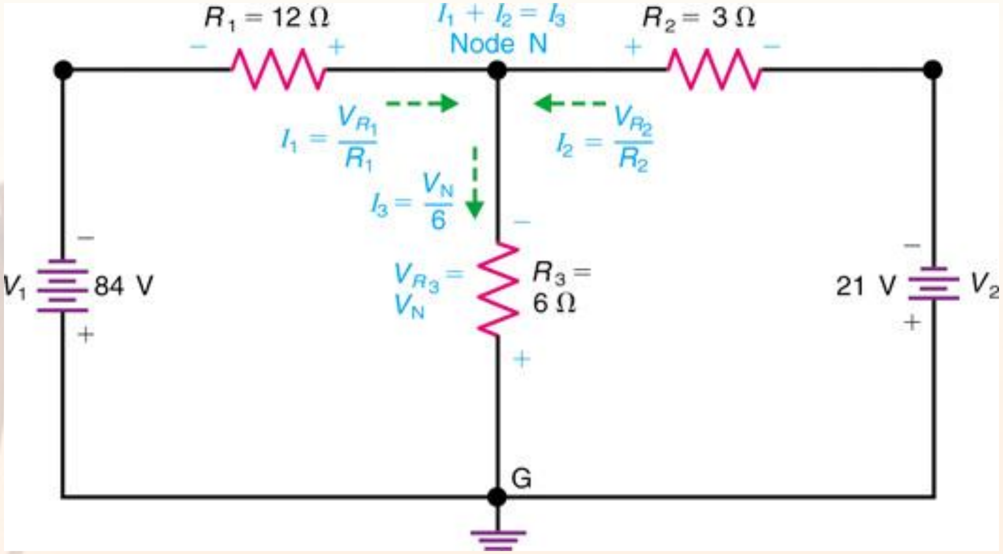
$$\frac{V_{R_1}}{R_1} + \frac{V_{R_2}}{R_2} = \frac{V_N}{R_3}$$

Node-Voltage Analysis

Problem: Find Voltage across node Vn.



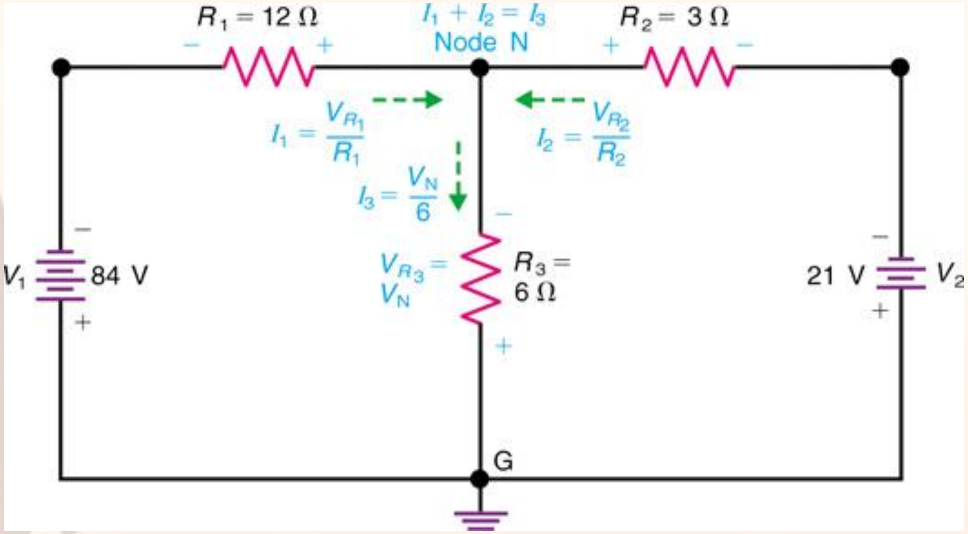
Node-Voltage Analysis



$$V_{R1}/R_1 + V_{R2}/R_2 = V_N/R_3$$

$$V_{R1}/12 + V_{R2}/3 = V_N/6$$

Node-Voltage Analysis



$V_{R1} + V_N = 84$ or $V_{R1} = 84 - V_N$

For the loop with V_2 of 21V,

$V_{R2} + V_N = 21$ or $V_{R2} = 21 - V_N$

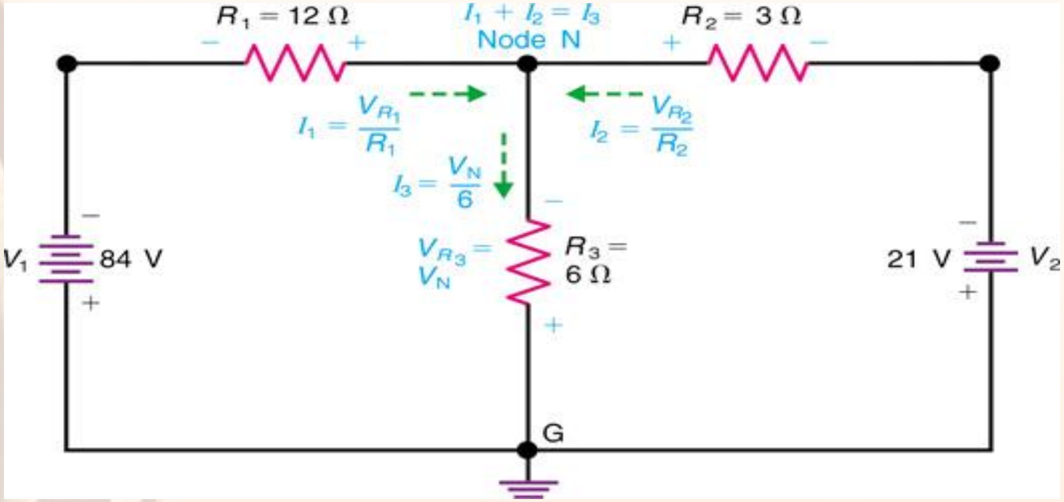
Substituting values

$I_1 + I_2 = I_3$

Using the value of each V in terms of V_N

$84 - V_N/12 + 21 - V_N/3 = V_N/6$

Node-Voltage Analysis



This equation has only one unknown, V_N. Clearing fractions by multiplying each term by 12, the equation is

$$(84 - V_N) + 4(21 - V_N) = 2V_N$$

$$84 - V_N + 84 - 4V_N = 2V_N$$

$$-7V_N = -168$$

$$V_N = 24V$$

Node-Voltage Analysis

Node Equations

Applies KCL to currents in and out of a node point. Currents are specified as *V/R* so the equation of currents can be solved to find a node voltage.

Loop Equations

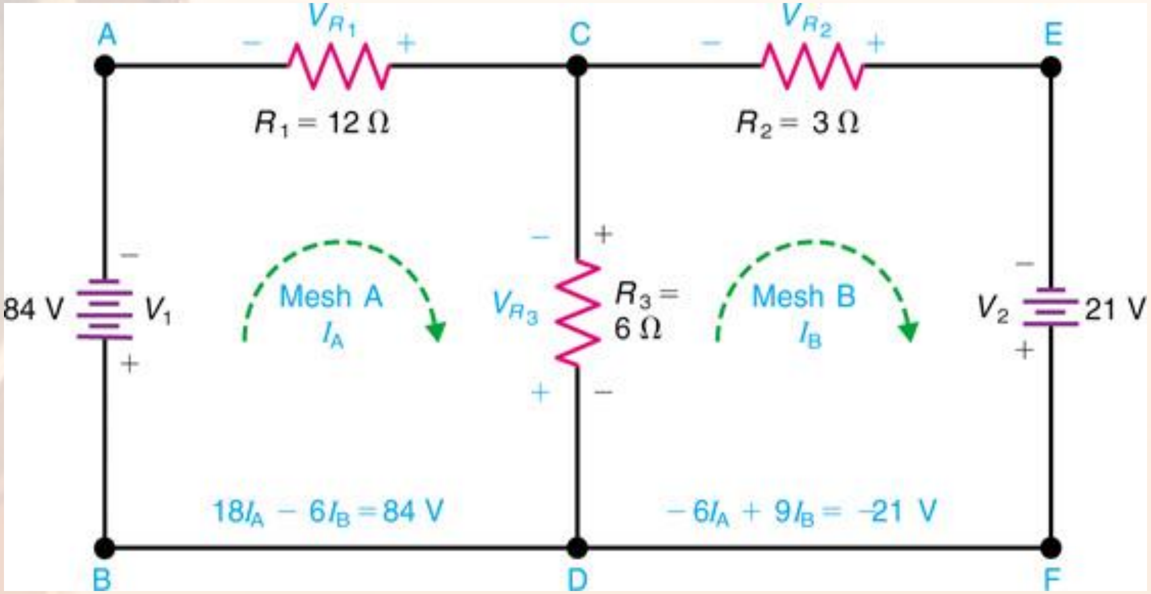
Applies KVL to the voltages in a closed path. Voltages are specified as *IR* so the equation of voltages can be solved to find a loop current.

Method of Mesh Currents

A mesh is the simplest possible loop.
Mesh currents flow around each mesh without branching.
The difference between a mesh current and a branch current is that a mesh current does not divide at a branch point.
A mesh current is an assumed current; a branch current is the actual current.
IR drops and KVL are used for determining mesh currents.

Method of Mesh Currents

The number of meshes is the number of mesh currents. This is also the number of equations required to solve the circuit.



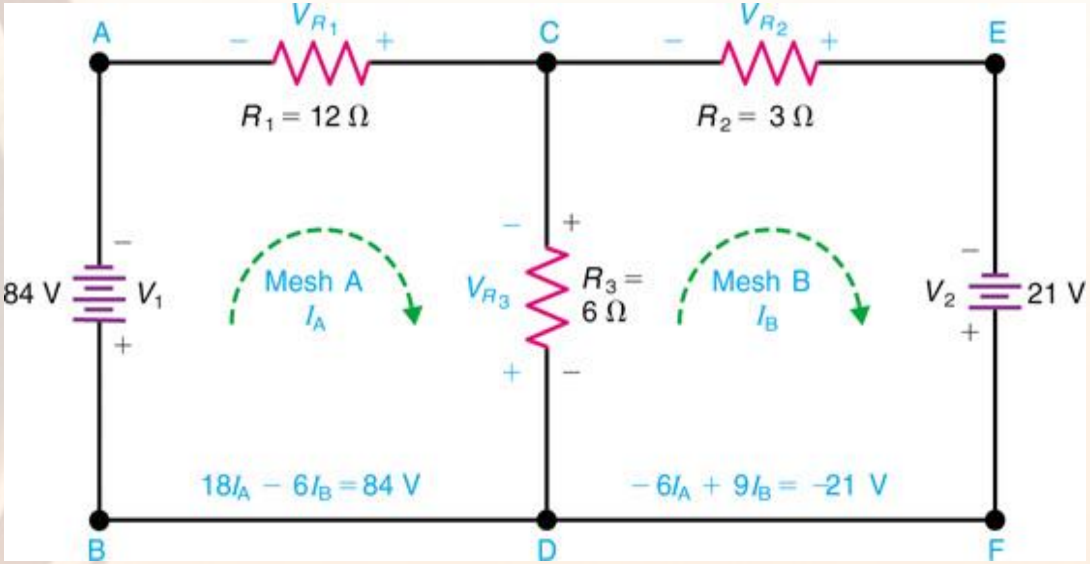
Method of Mesh Currents

A clockwise assumption is standard. Any drop in a mesh produced by its own mesh current is considered positive because it is added in the direction of the current.

Mesh A: $18I_A - 6I_B = 84V$
Mesh B: $6I_A + 9I_B = -21V$

Method of Mesh Currents

The mesh drops are written collectively here:



Mesh A: $18I_A - 6I_B = 84$

Mesh B: $-6I_A + 9I_B = -21$

Method of Mesh Currents

Use either the rules for meshes with mesh currents or the rules for loops with branch currents, but do not mix the two methods.

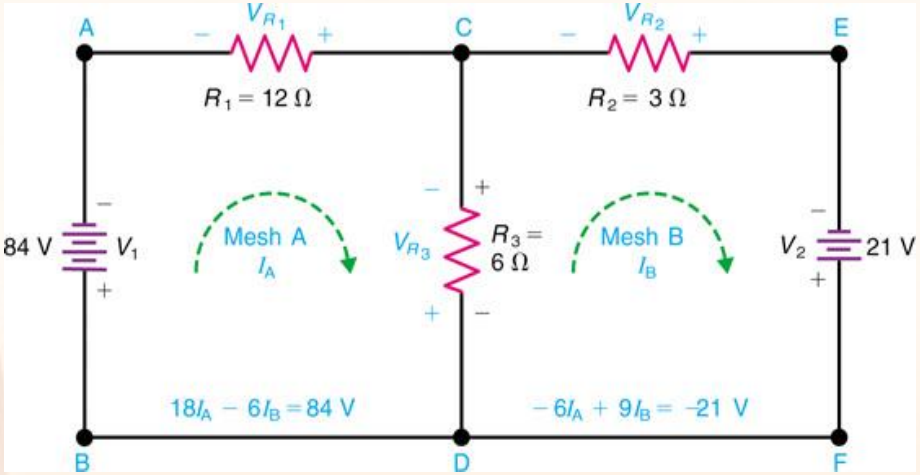
To eliminate I_B and solve for I_A, divide the first equation by 2 and the second by 3. then

$$9I_A - 3I_B = 42$$
$$-2I_A + 3I_B = -7$$

Add the equations, term by term, to eliminate I_B. Then

$$7I_A = 35$$
$$I_A = 5A$$

Method of Mesh Currents



To calculate I_B, substitute 5 for I_A in the second equation:

$$-2(5) + 3I_B = -7$$

$$3I_B = -7 + 10 = 3$$

$$I_B = 1A$$

The positive solutions mean that the electron flow for both I_A and I_B is actually clockwise, as assumed.

Superposition Theorem

In any linear circuit containing multiple independent sources, the current or voltage at any point in the network may be calculated as algebraic sum of the individual contributions of each source acting alone

The process of using Superposition Theorem

- Superposition theorem is applicable only to linear circuits and responses.
- Select any one source and short all other voltage sources and open all current sources if internal impedance is not known. If known replace them by their impedance.
- Find out the current or voltage across the required element, due to the source under consideration.
- Repeat the above steps for all other sources.
- Add all the individual effects produced by individual sources to obtain the total current in or across the voltage element

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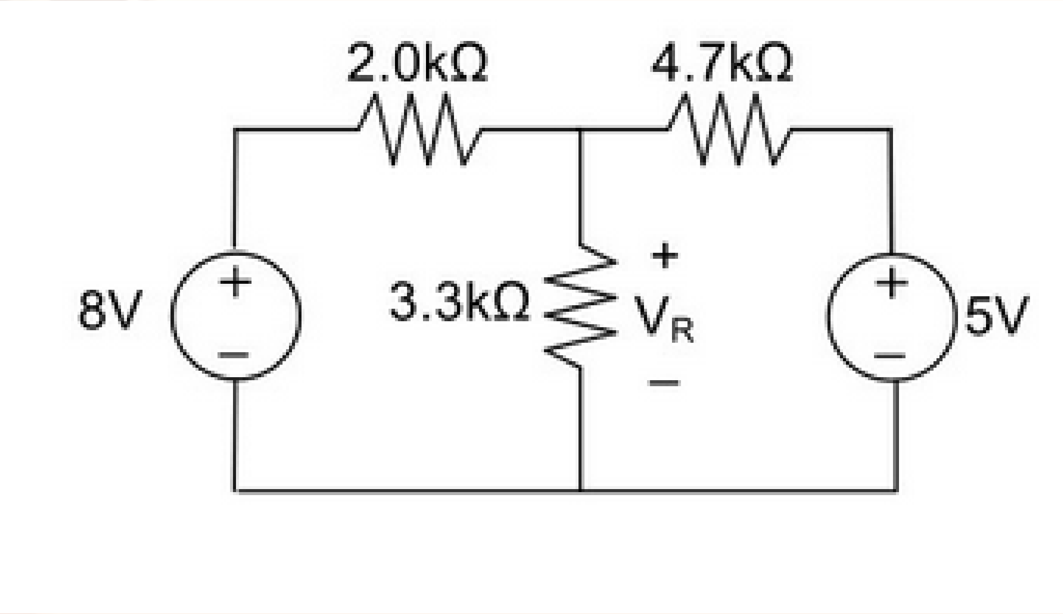
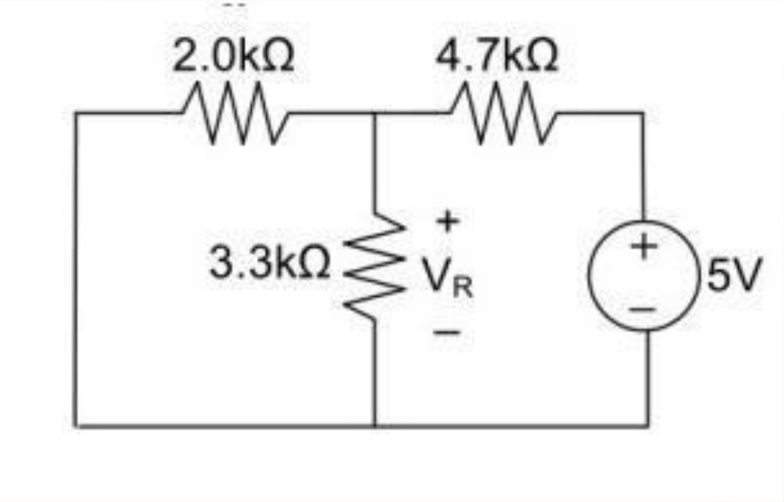
Problem: Using the superposition theorem, determine the voltage drop and current across the resistor 3.3K as shown in figure below



Step 1: Remove the 8V power supply from the original circuit, such that the new circuit becomes as the following and then measure voltage across resistor

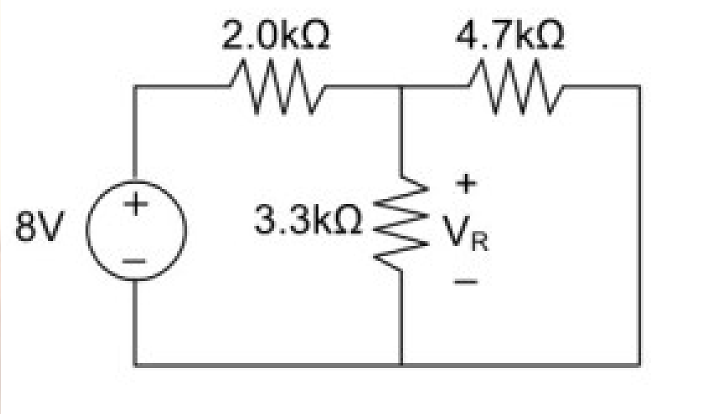


Here 3.3K and 2K are in parallel, therefore resultant resistance will be 1.245K.

Using voltage divider rule voltage across 1.245K will be

V1= [1.245/(1.245+4.7)]*5 = 1.047V

Step 2: Remove the 5V power supply from the original circuit such that the new circuit becomes as the following and then measure voltage across resistor.



Here 3.3K and 4.7K are in parallel, therefore resultant resistance will be 1.938K. Using voltage divider rule voltage across 1.938K will be

V2= [1.938/(1.938+2)]*8 = 3.9377V

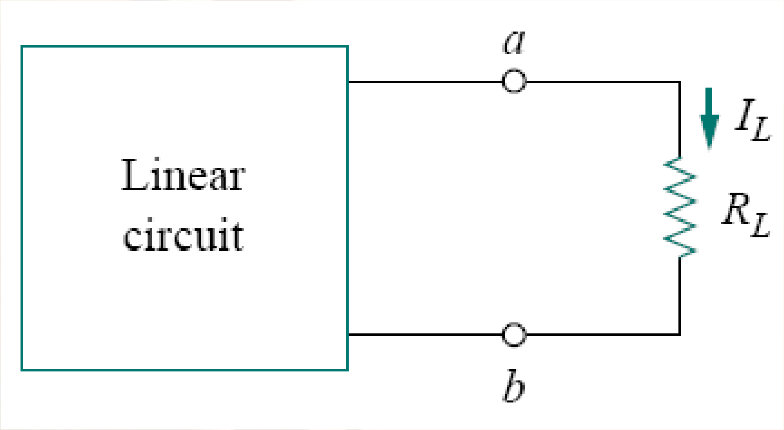
Step 3: Therefore voltage drop across 3.3K resistor is

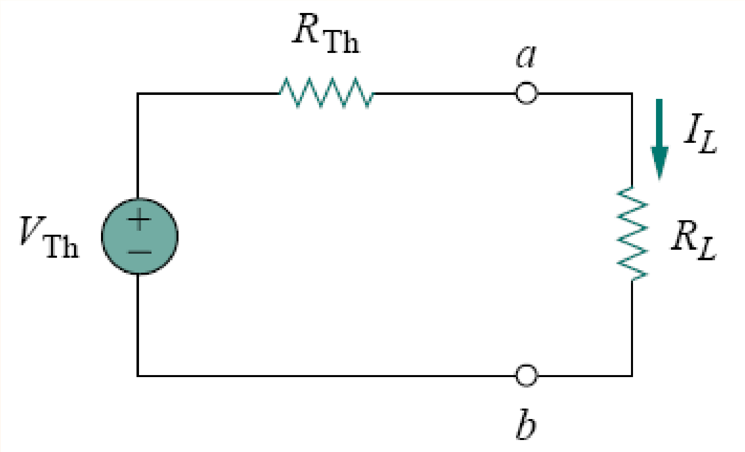
V1+V2 = 1.047+3.9377=4.9847

Thevenin’s Theorem

Thevenin’s theorem states that any two terminal linear network or circuit can be represented with an equivalent network or circuit, which consists of a voltage source V_{th} in series with a resistor R_{th}. It is known as Thevenin’s equivalent circuit. A linear circuit may contain independent sources, dependent sources, and resistors.

Thevenin's Theorem



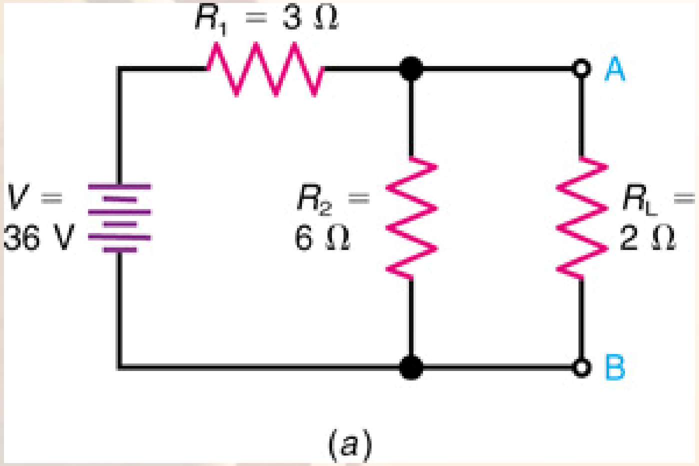


$$I_L = \frac{V_{Th}}{R_{Th} + R_L} \qquad V_L = R_L I_L = \frac{R_L}{R_{Th} + R_L} V_{Th}$$

The process of using Thevenin's Theorem

- Remove load resistance from the circuit.
- Determining Thevenin's Resistance $R_{TH.}$
- R_{TH} is determined by shorting the voltage source and calculating
- the circuit's total resistance as seen from open terminals A and B.
- Determining Thevenin's Voltage
- V_{TH} is determined by calculating the voltage between open
- terminals A and B.
- Find load current by applying loop equation.4

Thevenin's Theorem



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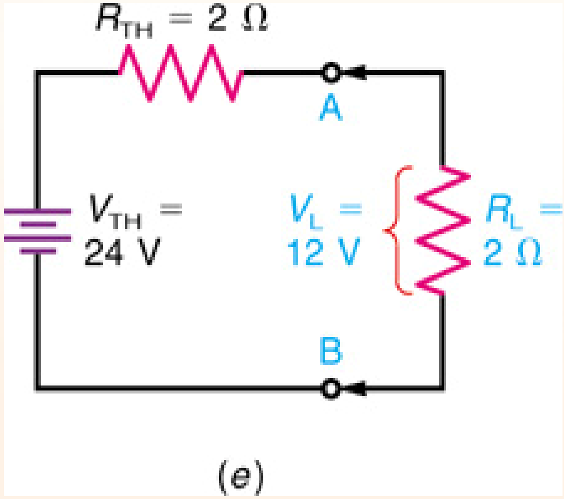
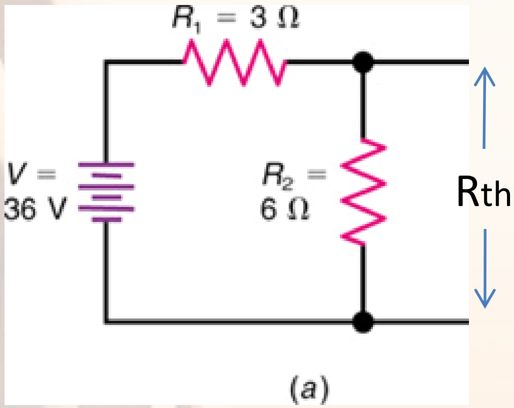
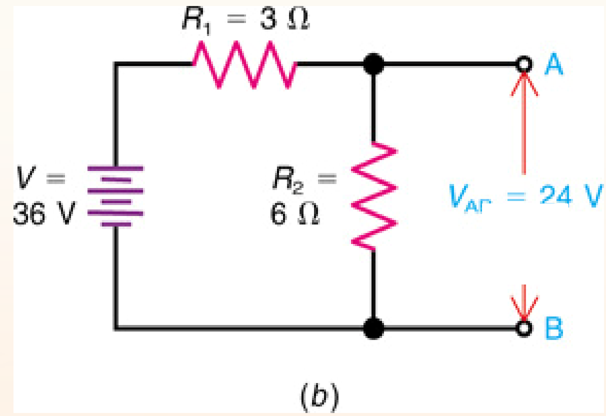


Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin's Theorem

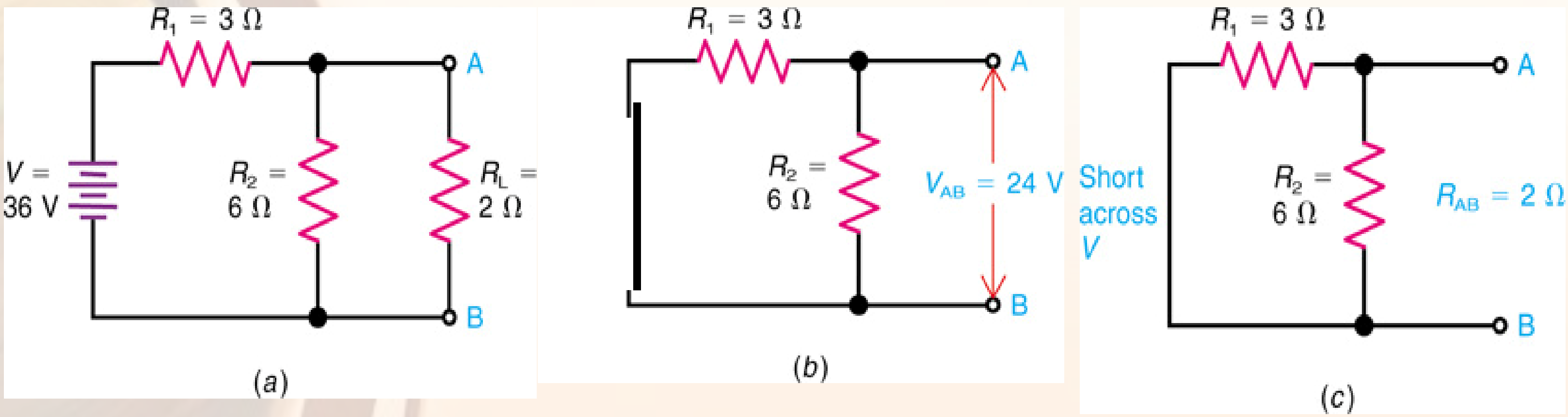




$$V_{R2} := 36V \cdot \frac{6\Omega}{3\Omega + 6\Omega} \qquad V_{R2} = 24\,V \qquad V_{AB} := V_{R2}$$

Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit V to find that R_{AB} is 2Ω.

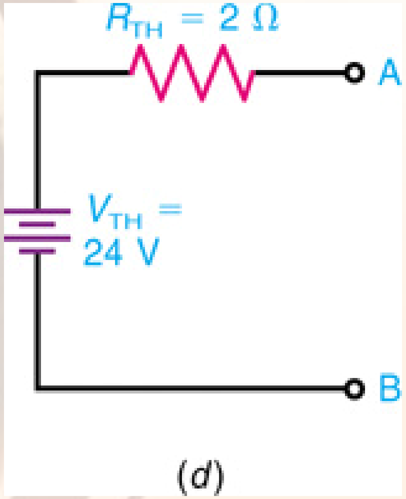
Thevenin's Theorem

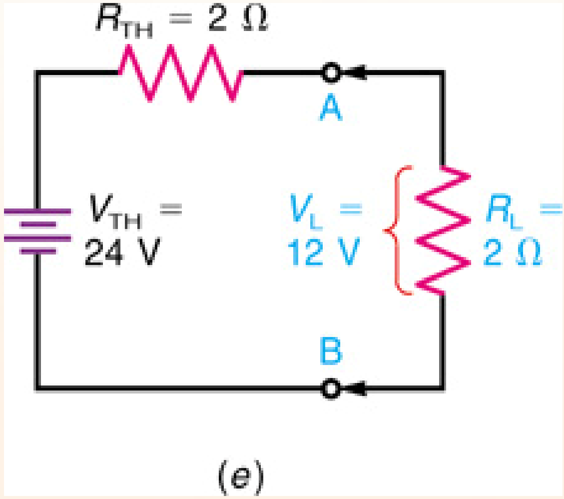


$$R_{TH} = \frac{3\Omega \cdot 6\Omega}{3\Omega + 6\Omega} \qquad R_{TH} = 2\Omega$$

Application of Thevenin's theorem. (*a*) Actual circuit with terminals A and B across R_L. (*b*) Disconnect R_L to find that V_{AB} is 24V. (*c*) Short-circuit *V* to find that R_{AB} is 2Ω.

Thevenin's Theorem





Thevenin equivalent circuit. (*e*) Reconnect R_L at terminals A and B to find that V_L is 12V.

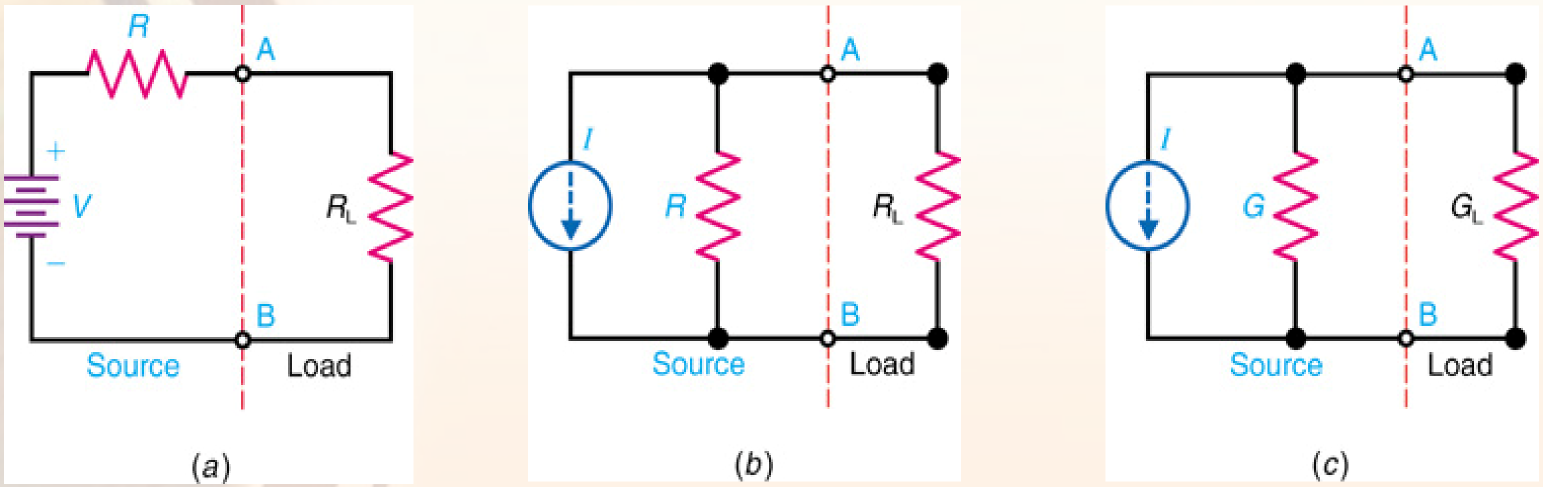
Norton's Theorem

Any linear electric network or complex circuit with current and voltage sources can be replaced by an equivalent circuit containing a single independent current source I_N and a parallel resistance R_N.

Norton’s Theorem

- Norton’s theorem is used to simplify a network in terms of currents instead of voltages.
- It reduces a network to a simple parallel circuit with a current source (comparable to a voltage source).
- Norton’s theorem states that any network with two terminals can be replaced by a single current source and parallel resistance connected across the terminals.

Norton's Theorem

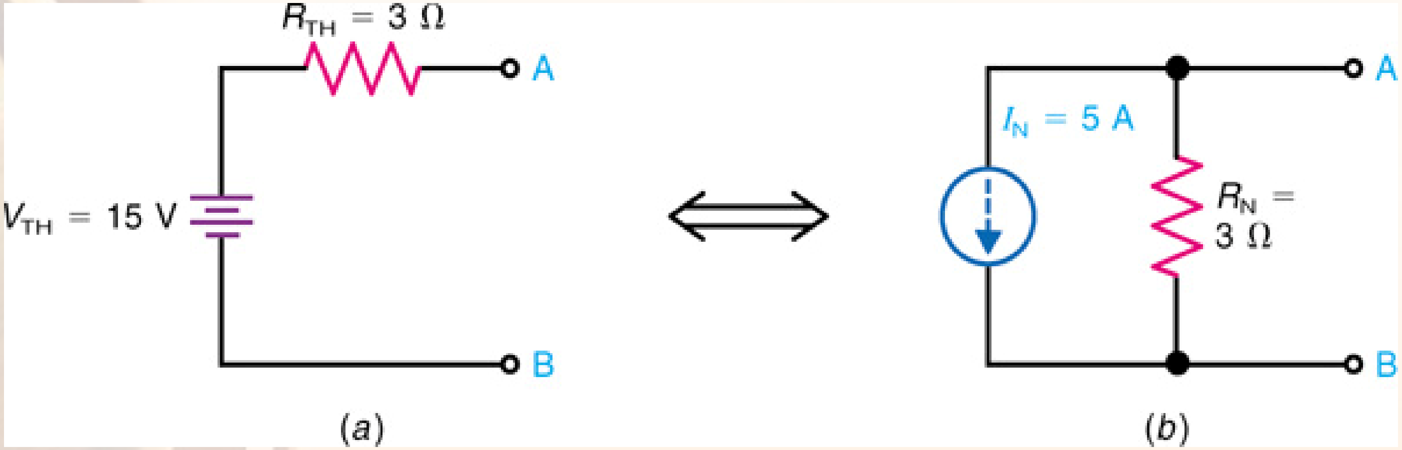


General forms for a voltage source or current source connected to a load R_L across terminals A and B. (*a*) Voltage source *V* with series *R*. (*b*) Current source *I* with parallel *R*. (*c*) Current source *I* with parallel conductance *G*.

Thevenin-Norton Conversions

- **Thevenin's theorem** says that any network can be represented by a voltage source and series resistance.
- **Norton's theorem** says that the same network can be represented by a current source and shunt resistance.
- Therefore, it is possible to convert directly from a Thevenin form to a Norton form and vice versa.
- Thevenin-Norton conversions are often useful.

Thevenin-Norton Conversions

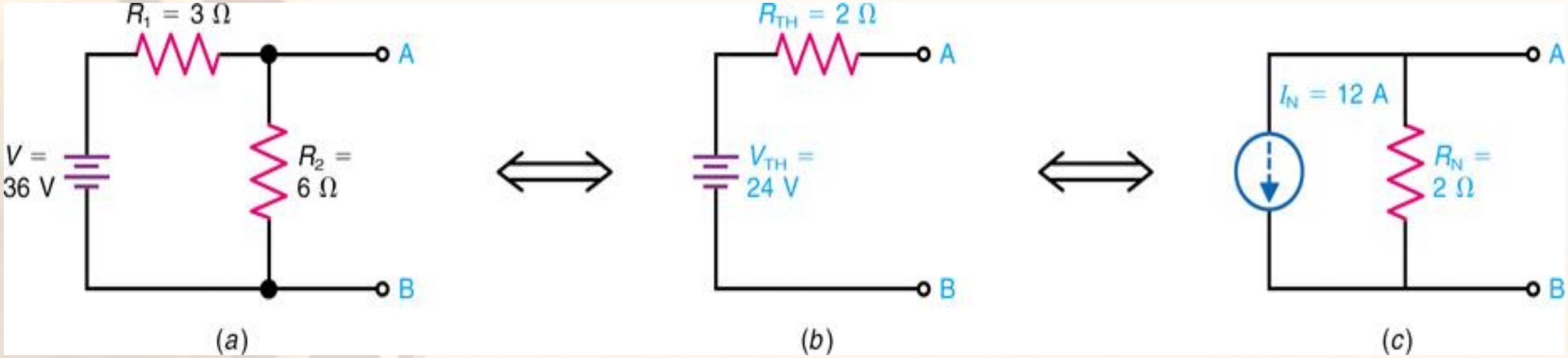


Thevenin **Norton**

$$V_{TH} := 15V \qquad R_{TH} := 3\Omega \qquad I_N := \frac{V_{TH}}{R_{TH}} \qquad I_N = 5\,\mathrm{A}$$

Thevenin equivalent circuit in (*a*) corresponds to the Norton equivalent in (*b*).

Thevenin-Norton Conversions



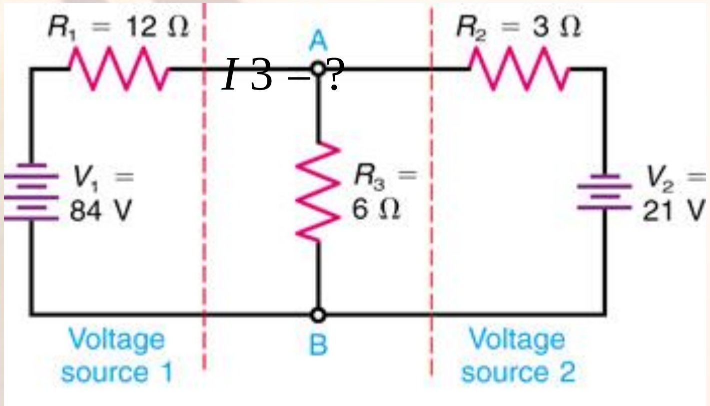
$$V_{TH} := 24V \qquad R_{TH} := 2\Omega \qquad I_N := \frac{V_{TH}}{R_{TH}} \qquad I_N = 12\,\mathrm{A}$$

Example of Thevenin-Norton conversions. (*a*) Original circuit, the same as in and (*b*) Thevenin equivalent. (*c*) Norton equivalent.

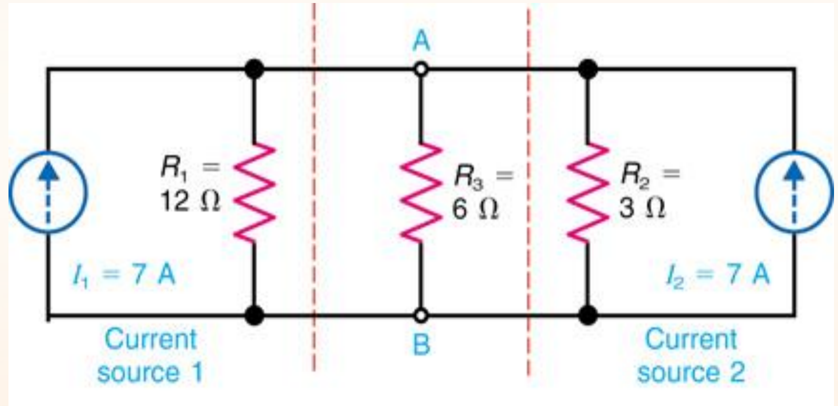
Conversion of Voltage and Current Sources

- Converting voltage and current sources can simplify circuits, especially those with multiple sources.
- Current sources are easier for parallel connections, where currents can be added or divided.
- Voltage sources are easier for series connections, where voltages can be added or divided.

Conversion of Voltage and Current Sources



(a)

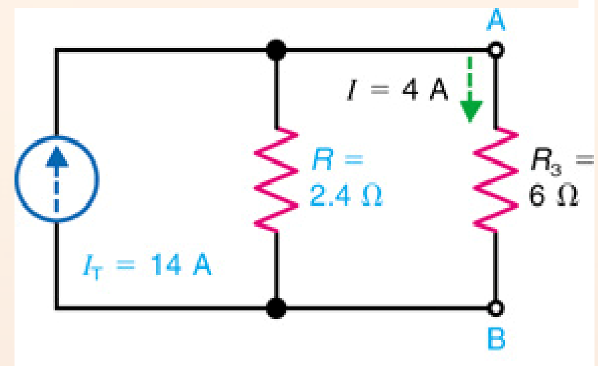


(b)

$V1 := 84V$ $\quad R1 := 12\Omega$ $\quad I1 := \dfrac{V1}{R1}$ $\quad I1 = 7\ \text{A}$

$V2 := 21V$ $\quad R2 := 3\Omega$ $\quad I2 := \dfrac{V2}{R2}$ $\quad I2 = 7\ \text{A}$

$$\frac{1}{\dfrac{1}{R1} + \dfrac{1}{R2}} = 2.4\,\Omega \qquad I3 := \frac{2.4\Omega}{2.4\Omega + 6\Omega} \cdot 14A \qquad I3 = 4\ \text{A}$$



(c)