JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & I Sem

Subject –Engineering Mathematics-I

Unit – 3

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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

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- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
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- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

Fourier Series for odd and even function:

- **Even Function:** A function $f(x)$ is said to be even function , if
- $f(-x) = f(x)$ for all values of x
- For example , $x^2, \cos x, \sec x$, are even function.
- **Properties of even function:**
- The product of two even function is even function is even.
- The sum of two even function is even function is even
- The graph of even function is symmetric about the y-axis.
- For even function $f(x)$, $\int_{-a}^{a} f(x)\,dx = 2\int_0^a f(x)dx$.
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- **Odd Function:** A function $f(x)$ is said to be even function , if
- $f(-x) = -f(x)$ for all values of x
- For example , $x, x^3, \sin x,\ \operatorname{cosec} x$, $\tan x$ are odd function.
- **Properties of odd function:**
- The product of two odd function is even function is even.
- The sum(or difference) of two odd function is odd function is even.
- The product of one even function and one odd function is odd function.
- The graph of even function is symmetric about origin.
- For odd function $f(x)$, $\int_{-a}^{a} f(x)\, dx = 0$.

Fourier Series of Even and odd Function:

- The Fourier series expansion of the function $f(x)$ in $(-\pi, \pi)$ is given by
- $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n coxnx + \sum_{n=1}^{\infty} b_n \sin nx \qquad (1)$$
- where $$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x)dx$$
- $$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).cosnx.dx$$
- $$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(x).\sin nx\,.dx$$

- Now if $f(x)$ is even function in $(-\pi, \pi)$ then all $b_n's$ will be zero. therefore the Fourier series of an even function contains only cosines terms and is given by
- $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n coxnx \qquad (2)$$
- where $$a_0 = \frac{2}{\pi}\int_{0}^{\pi} f(x)dx$$
- $$a_n = \frac{2}{\pi}\int_{0}^{\pi} f(x).cosnx.dx$$

- And if Now if $f(x)$ is odd function in $(-\pi, \pi)$ then a_0 and all $a_n{}'s$ will be zero. Therefore the Fourier series of an odd function contains only sine terms and is given by
- $$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \qquad (3)$$
- where $$b_n = \frac{2}{\pi}\int_0^{\pi} f(x).\sin nx \,.\, dx$$

Example: Find the Fourier series expansion of the function $f(x) = xsin\,x$, in the interval $(-\pi, \pi)$. Hence show that

$$\frac{\pi-1}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots \ldots \ldots .$$

- **Solution:** Since the given function satisfies
- $$f(-x) = (-x)sin\,(-x) = xsin\,x = f(x),$$
- therefore $f(x) = xsin\,x$ is even function so $b_n = 0$.
- Let Fourier series expansion be
- $$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n coxnx \qquad (1)$$
- where $a_0 = \frac{2}{\pi}\int_0^{\pi} f(x)dx$
- $= \frac{2}{\pi}\int_0^{\pi} xsin\,x.dx$
- $= \frac{2}{\pi}[x(-\cos x) - (\sin x)]_0^{\pi} = 1$
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- $$a_n = \frac{2}{\pi}\int_0^{\pi} f(x).cosnx.dx = \frac{2}{\pi}\int_0^{\pi} xsin\,x.\cos nx\,.dx$$
- $= \frac{1}{\pi}\int_0^{\pi} 2.xsin\,x.\cos nx\,.dx$
- $= \frac{1}{\pi}\int_0^{\pi} x.\{\sin(n+1)x - \sin(n-1)x\,.dx$

- Integrating by parts taking $\{\sin(n+1)x - \sin(n-1)x$ as first function
- $$= \frac{1}{\pi}\left[x\left(\frac{\cos(n+1)x}{n+1} - \frac{\cos(n-1)x}{n-1}\right) - (1)\left(\frac{\sin(n+1)x}{(n+1)^2} + \frac{\sin(n-1)x}{(n-1)^2}\right)\right]_0^\pi$$
- $$= \frac{1}{\pi}\left[-\pi\frac{\cos(n+1\pi)}{n+1} + \pi\frac{\cos(n-1)\pi}{n-1}\right], \qquad n \neq 1$$
- $$= \frac{2(-1)^{n+1}}{n^2-1} \qquad , n \neq 1.$$
- For $n = 1$, $\quad a_1 = \frac{2}{\pi}\int_0^\pi f(x).cosx.dx = \frac{2}{\pi}\int_0^\pi xsin\ x.\cos x\,.dx$
- $$= \frac{1}{\pi}\int_0^\pi x.sin\ 2x.dx$$
- $$\frac{1}{\pi}\left[x\left(-\frac{\cos 2x}{2}\right) - (1)\left(-\frac{\sin 2x}{4}\right)\right]_0^\pi = \frac{1}{2}$$

- Therefore putting the value of a_0, a_1 and a_n in Equation (1), we have
- $$f(x) = x\sin x = 1 - \frac{1}{2} + 2\sum_{n=2}^{\infty} \frac{(-1)^{n+1}}{n^2-1} . \cos nx \qquad (2)$$
- Equation (2) is required Fourier Series expansion.
- Now , putting $x = \frac{\pi}{2}$ in equation (2), we have
- $$\frac{\pi-1}{4} = \frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \frac{1}{7.9} + \cdots \ldots \ldots.$$ Proved.

Example: Find the Fourier series expansion of the function

$$f(x) = x^3, -\pi < x < \pi.$$

Solution: Since the given function satisfies $f(-x) = (-x)^3 = -x^3 = -f(x)$, therefore $f(x) = x^3$ is odd function so $a_0 = 0, a_n = 0.$

- $$f(x) = \sum_{n=1}^{\infty} b_n \sin nx \qquad (1)$$
- where $$b_n = \frac{2}{\pi}\int_0^{\pi} f(x).\sin nx\,.dx$$
- $$\frac{2}{\pi}\int_0^{\pi} x^3.\sin nx\,.dx$$
- Integrating by parts taking x^3 as first function
- $$= \frac{2}{\pi}\left[x^3\left(-\frac{\cos nx}{n}\right) - 3x^2\left(-\frac{\sin nx}{n^2}\right) + 6x\left(-\frac{\cos nx}{n^3}\right) - 6\left(-\frac{\sin nx}{n^4}\right)\right]_0^{\pi}$$

$$= \frac{2}{\pi}\left[-\pi^3\left(\frac{\cos n\pi}{n}\right) + 6\pi\left(\frac{\cos n\pi}{n^3}\right)\right]$$

- $= 2(-1)^n\left[\frac{\pi^3}{n} + \frac{6}{n^3}\right]$
- Therefore putting the value of b_n in Equation (1), we have
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$$f(x) = x^3 = 2\sum_{i=1}^{\infty}(-1)^n\left[\frac{\pi^3}{n} + \frac{6}{n^3}\right]\sin nx$$

(2)

- Equation (2) is required Fourier Series expansion.

Refrences

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Thank you!