

## ASSIGNMENT

Year: B. Tech. I Year

Semester: I

Subject: Engineering Mathematics - I

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**CO1. Understand fundamental concepts of improper integrals, beta and gamma functions and their properties. Evaluation of Multiple Integrals in finding the areas, volume enclosed by several curves after its tracing and its application in proving certain theorems.**

Q.1 Give the trigonometric representation of beta function.

Q2. Solve  $\int_0^{\frac{\pi}{2}} \sin^3 \theta \cos^4 \theta d\theta$ .

Q.3 Evaluate  $\int_0^1 \frac{1}{\sqrt{1-x^4}} dx$

Q.4 Show that  $\int_0^2 (8 - x^3)^{-\frac{1}{3}} dx = \frac{2\pi}{3\sqrt{3}}$ .

Q.5 Evaluate  $\int_0^{\infty} \frac{1}{1+x^4} dx$

Q.6 Show that  $\int_0^1 \sqrt{1-x^4} dx = \frac{\left(\sqrt{\frac{1}{4}}\right)^2}{6\sqrt{2\pi}}$

Q.7 Prove that  $\int_0^{\infty} \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi}{128}$

Q.8 Prove that the surface area and volume of the solid generated by revolution of the loop

Of the curve  $x = t^2, y = \left(t - \frac{t^3}{3}\right)$  about the X axes are  $3\pi$  and  $\frac{3\pi}{4}$ .

Q.9 Find the volume of the solid generated by revolution of the Cissoid  $y^2(2a - x) = x^3$

About its asymptote.

Q.10 Find the volume and surface area of the solid generated by revolution of the curve

cardioid  $r = a(1 + \cos\theta)$  about the initial line and about the line  $\theta = \frac{\pi}{2}$ .

**CO2. Interpret the concept of a series as the sum of a sequence, and use the sequence of partial sums to determine convergence of a series. Understand derivatives of power, trigonometric, exponential, hyperbolic, logarithmic series.**

Q.1 Test the Convergence of the series  $x + \frac{2^2 \cdot x^2}{!2} + \frac{3^3 \cdot x^3}{!3} + \frac{4^4 \cdot x^4}{!4} + \dots \infty$

Q.2 Test the Convergence of the series  $1^p + \left(\frac{1}{2}\right)^p + \left(\frac{1 \times 3}{2 \times 4}\right)^p + \left(\frac{1 \times 3 \times 5}{2 \times 4 \times 6}\right)^p + \dots$

Q.3 Test the Convergence of the series  $\sum_{n=1}^{\infty} \frac{\sqrt[3]{3n^2+1}}{\sqrt[4]{4n^3+2n+7}}$ .

Q.4 Test the Convergence of the series whose  $n^{th}$  term is  $\frac{2^n}{n^3}$

Q.5 Test the Convergence of the series  $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n} + \sqrt{n+1}}$

Q.6 Test the Convergence of the series  $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8} \dots$

Q.7 Test the Convergence of the series for positive values of x;

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots + \frac{2^n - 2}{2^{n+1}} + \dots$$

Q.8 Test the Convergence of the series  $\frac{x^2}{2 \log 2} + \frac{x^3}{3 \log 3} + \frac{x^4}{4 \log 4} + \dots$

**CO3. Recognize odd, even and periodic function and express them in Fourier series using Euler's formulae.**

Q.1 Find the Fourier series to represent  $f(x) = x - x^2$  from  $x = -1$  to  $x = 1$ .

Q.2 Find the Fourier series to represent  $f(x) = x^3$ , in  $0 < x < 4$ .

Q.3 If the function  $f(x)$  is defined by  $f(x) = |\cos x|$ ,  $-\pi < x < \pi$ .

Q.4 If the function  $f(x)$  is defined by  $f(x) = |\sin x|$ ,  $-\pi < x < \pi$ .

Q.5 Find the Half Range Sine series of  $f(x) = x(\pi - x)$ ,  $0 < x < \pi$

Q.6 Find a series of sines and cosines of multiples of x which will represent the function  $f(x) = x + x^2$  in the interval  $-\pi < x < \pi$ . Hence show that  $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \dots = \frac{\pi^2}{6}$

Q.7 Find the Fourier series expansion for  $f(x)$  if  $f(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$ . Deduce that  $\frac{1}{1^2} +$

$$\frac{1}{3^2} + \frac{1}{5^2} + \dots \infty = \frac{\pi^2}{8}$$

Q.8 If the function  $f(x)$  is defined by  $f(x) = |x|$ ,  $-\pi < x < \pi$ . Obtain a Fourier series of  $f(x)$ . Deduce that  $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \dots = \frac{\pi^2}{8}$

Q.9 Expand  $f(x) = x \sin x$  in the range  $0 < x < 2\pi$  as a Fourier series.

**CO4. Understand the concept of limits, continuity and differentiability of functions of several variables. Analytical definition of partial derivative. Maxima and minima of functions of several variables Define gradient, divergence and curl of scalar and vector functions.**

Q.1 Find the directional derivative of  $f(x, y) = \sqrt{xy}$  at  $P(2, 8)$  in the direction of  $Q(5, 4)$ .

Q.2 In a  $\Delta ABC$ , find the maxima and minima of  $u = \sin A \sin B \sin C$  where  $A + B + C = \pi$

Q.3 Find the tangent plane to the elliptic parabolic  $z = 2x^2 + y^2$  at the point  $P(1, 1, 3)$ .

Q.4 Discuss the continuity of the function  $f(x, y) = \begin{cases} \frac{x}{\sqrt{x^2 + y^2}}, & (x, y) \neq (0, 0) \\ 2, & (x, y) = (0, 0) \end{cases}$ ,

when  $(x, y) = (0, 0)$ .

Q.5 If  $(x, y) = \begin{cases} \frac{x^3 - y^3}{x^2 + y^2}, & \text{when } x \neq 0, y \neq 0 \\ 0, & \text{when } x = 0, y = 0 \end{cases}$ , Show that  $f$  is discontinuous at

origin.

Q.6 In a  $\Delta ABC$ , find the maxima and minima of  $u = \sin A \sin B \sin C$  where  $A + B + C = \pi$



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Q.7 Show that the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid  $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$  is  $\frac{8abc}{3\sqrt{3}}$

Q.8 Find the tangent plane to the elliptic paraboloid  $z = 2x^2 + y^2$  at the point P(1, 1, 3).