9

CPM : Cost Model

9.1. INTRODUCTION

In CPM, time is related to cost and the object is to develop an optimum time-cost relationship. Many times it becomes necessary to complete the project earlier than the normal time (latest allowable time). In such situations, the cost of expediting the operations or activities has to be considered. *CPM makes use of the cost estimate along with the time estimate and provides a schedule for completing the activities at the minimum total cost.*

The ultimate object of the network techniques is not only to bring improvement in planning, scheduling and control of project but also to assess the possibility of arriving at a feasible and desirable time-cost relationship. The policy of every organisation is to reduce the target time so that the time saved may be utilised for additional production or otherwise. The overall project duration can be reduced by reducing the duration of only the critical activities in the project network. The durations of such activities may be reduced in two ways :

(*a*) by deploying more resources for the early completion of such activities.

(*b*) by relaxing the technical specifications for such activities.

The latter method mostly depends on engineering considerations, and is not being discussed here. In the whole of CPM Cost Model, we will be assuming that project duration is reduced by deploying more resources on critical activities.

For a given project, there is a certain range of time during which the project may be completed depending upon the resources employed on various critical operations. If the duration is made larger, cost will be reduced. On the other hand, reducing the project duration would increase the cost. A decision on this will depend on whether the committment of additional resources and expenses is worthwhile. *The optimum duration will be one which gives the most economic cost for completing the project.*

In CPM, there are two time and cost estimates for each activity : '*normal estimate*' and '*crash estimate*'. In the normal estimate, the emphasis is on *cost* with time being associated with minimum cost. The '*crash*' estimate involves the absolute minimum time required for the job and the cost necessary to achieve it. Here the emphasis is on '*time*'.

9.2. PROJECT COST

For any project, the relationship between total cost and overall duration is shown in the Fig. 9.1. It is clear from the figure that

(*a*) if a project goes on indefinitely, the cost will increase,

(*b*) similarly, cost will increase if project is expedited, and

(*c*) cost is minimum at some optimum project duration.

Our main concern, naturally, is to find the project duration which will keep the total project cost at a minimum.

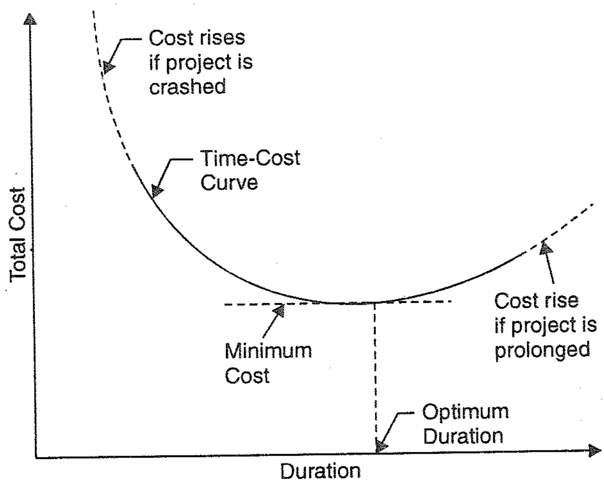


FIG. 9.1. VARIATION OF TOTAL PROJECT COST WITH DURATION.

Cost is considered to be a common parameter of the resources expenditure on a project. In other words, the application and use of man, money, machines, materials and time for the performance of various activities are all related to this common measure of cost.

Total Project Cost is the sum of two separate costs :

(*a*) *the direct cost* for accomplishing the work, and

(*b*) *the indirect cost* related to the control or duration of that work, financial overhead, lost production, and the like.

The components of the total cost are depicted in Fig. 9.2.

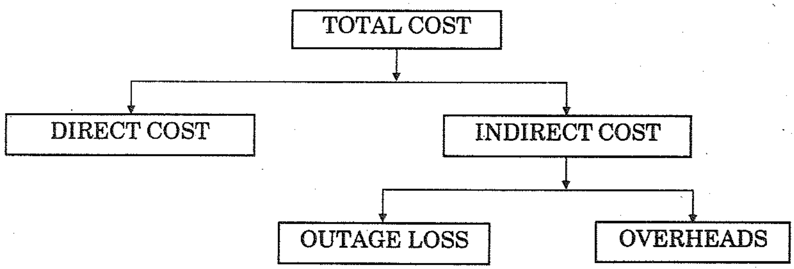
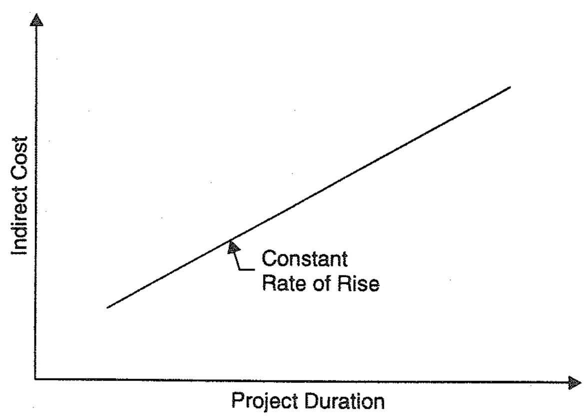


FIG. 9.2. COMPONENTS OF PROJECT COST.

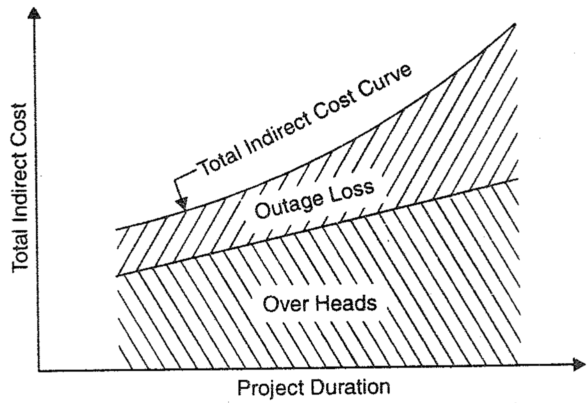
9.3. INDIRECT PROJECT COST

Indirect costs on a project are those expenditures which cannot be apportioned or clearly allocated to the individual activities of a project, but are assessed as a whole. The indirect cost includes the expenditure related to administrative and establishment charges, overhead, supervision, expenditure on a central store organisation, loss of revenue, lost profit, penalty etc. etc.

Indirect cost rises with increased duration. For any project, the relationship between Indirect Cost and Project Duration is shown in Fig. 9.3 (*a*) considering only overhead and supervision. It



(a) Indirect Cost (Supervisory & Overheads)



(b) Indirect Cost (Overhead + Outage Loss)

FIG. 9.3. VARIATION OF INDIRECT COSTS WITH PROJECT DURATION.

would be represented by a straight line, with a slope equal to daily overhead.

But when there is a loss in profits, due to inability to meet demand or due to some penalty due to delay, a corresponding cost increase must be added to the cost of overheads, producing the curve as shown in Fig. 9.3 (*b*). Such a loss is called the *outage loss*.

The total *indirect cost curve* will thus be curved.

9.4. DIRECT PROJECT COST

Direct project costs are those expenditures which are directly chargeable to and can be identified specifically with the activities of the project. These include labour cost, material cost, equipment cost etc.

For direct cost versus time relationship, consider the excavation of a trench for laying a pipe-line. It may be assumed that the nature of this job is such that only one man at a time would be able to work on it, taking a total of 40 manhours of work. The various ways of completing the work is illustrated in Table 9.1 assuming that the labour charges are at the rates of Rs. 12, 16 and 20 for the first, second and the third shifts respectively.

Table 9.1

S. No.	*Man power employed*	*Shift system*	*Time to complete*	*Charges*			*Total*
				Ist shift @ Rs. 12	*IInd shift @ Rs. 16*	*IIIrd shift @ Rs. 20*	
1.	One man	Single shift	5 days	60	—	—	60
2.	Two men	Double shift system	3 days (Three first shifts and 2 second shifts)	36	32	—	68
3.	Three men	Three shift system	2 days (Two first shifts, two second shifts and one third shift)	24	32	20	76

Fig. 9.4 shows the variation of direct cost with time for the case of Table 9.1.

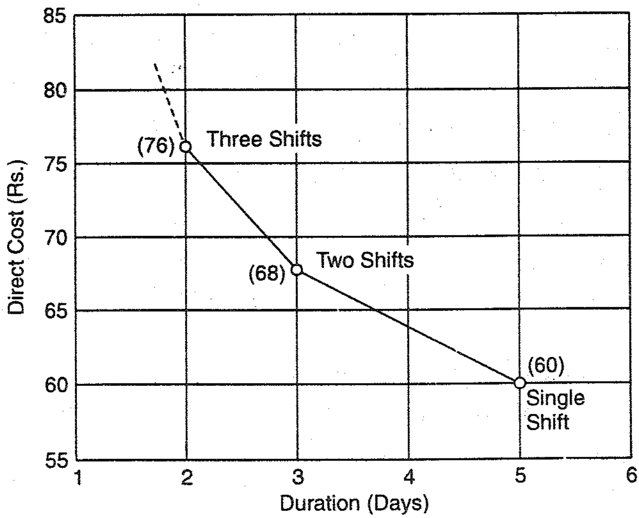


FIG. 9.4. VARIATION OF DIRECT COST WITH TIME.

The *direct cost curve*, having many segments, thus falls with increase in duration. However, the total *indirect cost curve* [Fig. 9.3 (*b*)] rises with increase in duration.

Fig. 9.5 shows a generalised curve between direct cost and project duration. The project has the *highest cost* corresponding to

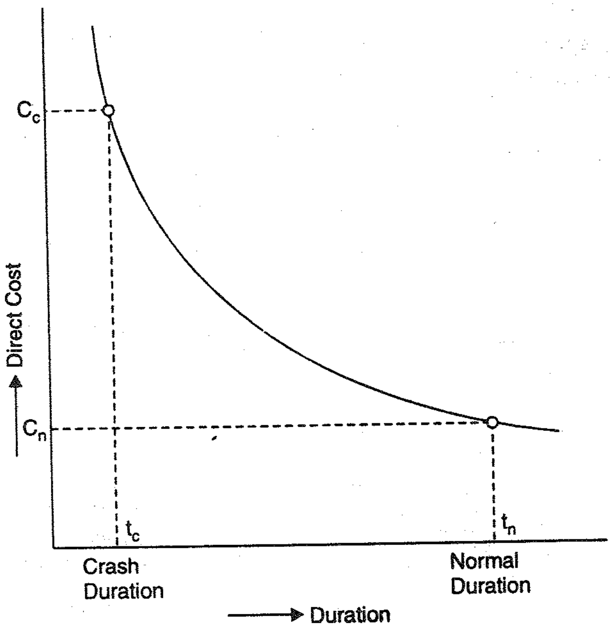


FIG. 9.5. GENERALISED DIRECT COST-TIME CURVE.

the *crash duration*, and has *normal cost* corresponding to the *normal duration*. Thus, we have two types of *costs* and two types of *times* defined below :

Normal time (t_n). Normal time is the standard time that an estimator would usually allow for an activity.

Crash time (t_c). Crash time is the *minimum possible* time in which an activity can be completed, by employing extra resources. Crash time is that time, beyond which the activity cannot be shortened by any amount of increase in resources.

Normal cost (C_n). This is direct cost required to complete the activity in normal time duration.

Crash cost (C_c). This is the direct cost corresponding to the completion of the activity within crash time.

9.5. SLOPE OF DIRECT COST CURVE

The direct cost curve is generally a curve, as shown in Fig. 9.5. However, this curve can be approximated by straight line or more than one straight line, depending upon the flatness of the curve. For example, the flat curve of Fig. 9.6 (a) can be approximated by a single straight line, while the curve of Fig. 9.6 (*b*) can be approximated by three straight lines.

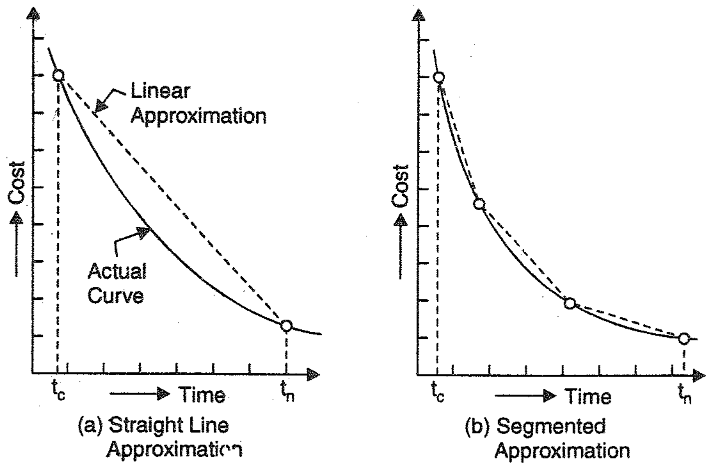


FIG. 9.6. DIRECT COST CURVE APPROXIMATION.

The straight line or segmented approximation of the direct cost curve is helpful in carrying out the project cost analysis. In such analysis, the *cost slope* is used.

Cost Slope

The cost slope is the slope of the direct cost curve, approximated as straight line. It is defined as follows :

$$\text{Cost slope} = \frac{\text{crash cost} - \text{normal cost}}{\text{normal time} - \text{crash time}}$$

or

$$CS = \frac{C_c - C_n}{t_n - t_c} = \frac{\Delta C}{\Delta t} \qquad ...(9.1)$$

where CS = cost slope

ΔC = increase in cost

Δt = decrease in time.

If the cost curve is approximated to a single straight line, it will have one cost slope. If, however, the cost curve has segmented approximation, it will have more than one cost slope. The single or multiple cost slopes will depend upon the non-linearity of the direct cost curve. The segmented approximation of cost curve, having multiple cost slopes, is more accurate but calculations are more involved. Generally, single cost slope is assumed.

9.6. TOTAL PROJECT COST AND OPTIMUM DURATION

The total project cost is the sum of the direct costs and indirect costs. Indirect cost curve is shown in Fig. 9.3 (*a*) which includes supervisory and over-head costs. However, if outage losses are also there, the curve for indirect costs will be as shown in Fig. 9.3 (*b*).

Fig. 9.7 shows the indirect cost curve, direct cost curve and the corresponding total curve.

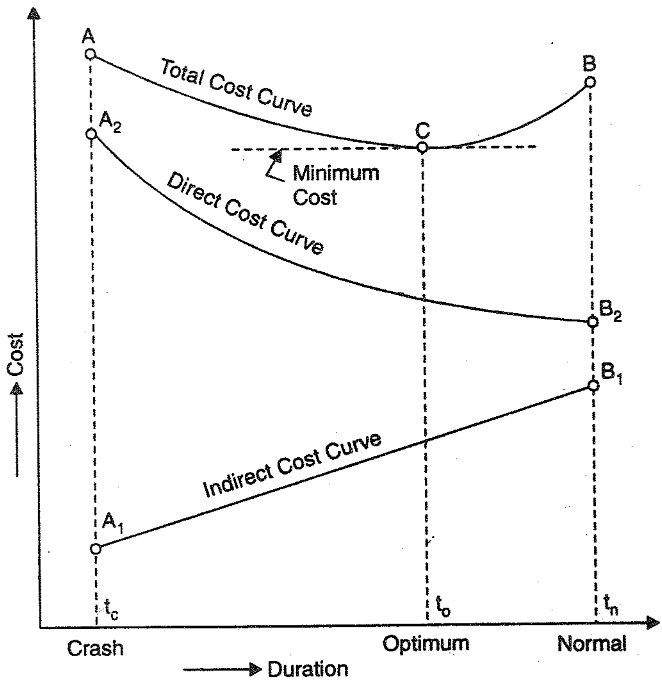


FIG. 9.7. TOTAL COST CURVE.

From the total cost curve *ACB* of Fig. 9.7, we find that the *minimum total cost* is obtained at some duration known as the *optimum duration*. The corresponding cost is known as the *minimum cost*. If the project duration is increased, total cost will increase, while if project duration is decreased to the crash value, project cost will be the highest.

If the direct cost curve is approximated to one or more straight lines, the corresponding total cost curve will be as shown in Fig. 9.8.

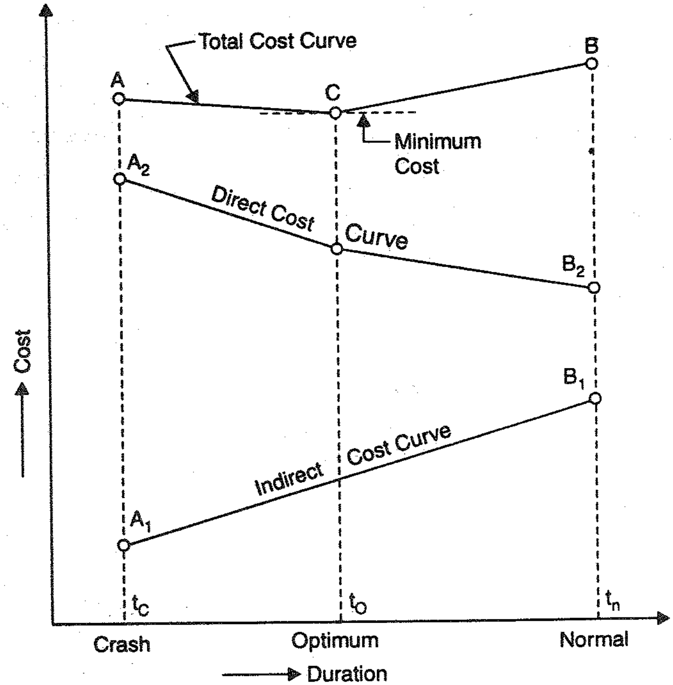


FIG. 9.8. TOTAL COST CURVE, LINEAR APPROXIMATION.

9.7. CONTRACTING THE NETWORK FOR COST OPTIMIZATION

After having established the *critical path* in a network (chapter 8) corresponding to the given normal durations of the activities, the next question that remains to be examined is 'what will be the cost structure of the project if some or all of the activities are crashed ?' For answering this question, we must have the normal direct cost data for each activity if it is to be completed in normal time duration and also the crashed direct cost data if that activity is crashed or hastened. For this data, the cost slope for each activity can be determined, *assuming* straight lines for the direct cost curve of each activity. The indirect cost rate per day should also be known, so that the total cost of the project can be found by adding the direct and indirect costs.

The *normal time* that the project will take for its completion will be the *sum of the normal time durations of each activity along the critical path*. Similarly, the *minimum time* that the project will take for its completion will be the sum of the *crashed time duration of each activity along the critical path*. If all the activities (critical as well as non-critical) are crashed, the cost will be very high without any additional advantage over and above the one obtained by crashing only the critical activities. The non-critical activities need not be speeded up, since their crashing is not going to decrease the project duration further.

However, it may happen that certain non-critical activities may become *critical* in the process of crashing the critical activities. It is, therefore, essential to proceed step by step in crashing one critical activity at a time and examining whether any other non-critical activity has also become critical in that process or not. For this, it is better to start with crashing *first* that critical activity which has the lowest cost slope. Then we take another critical activity which is having next higher cost slope. While crashing an activity fully (*i.e.* by Δt duration), it should be examined whether this crashing affects any other non-critical activity or not. For example, suppose activities A, B and D are critical, while activity C is not critical (Fig. 9.9).

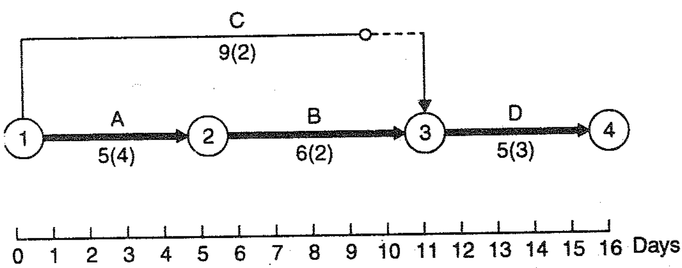


FIG. 9.9. ORIGINAL NETWORK BEFORE CRASHING.

The normal time for each activity are given below the activity arrow while the crash times are also given in the bracket. Now if critical activity B is fully crashed from 6 days (normal time) to 2 days (crashed time) such that Δt = 4 days, activity C will become critical. Hence activity B is first crashed by 2 days only, and extra cost is found. Then activity B is crashed by 2 days along with crashing activity C also by 2 days. The extra cost so involved will be equal to the combined cost slope of the activities B and C multiplied by 2. For doing this, it is better to draw the time-scaled version of the network shown in Fig. 9.9 based on the assumption that all the activities start at their earliest start times. The corresponding network, after crashing activity B partly by 2 days only is shown in

Fig. 9.10 (*a*) in which all the activities are critical, while Fig. 9.9 (*b*) shows the network after further crashing *B* by $\Delta t = 2$ days, and crashing *C* also by $\Delta t = 2$ days.

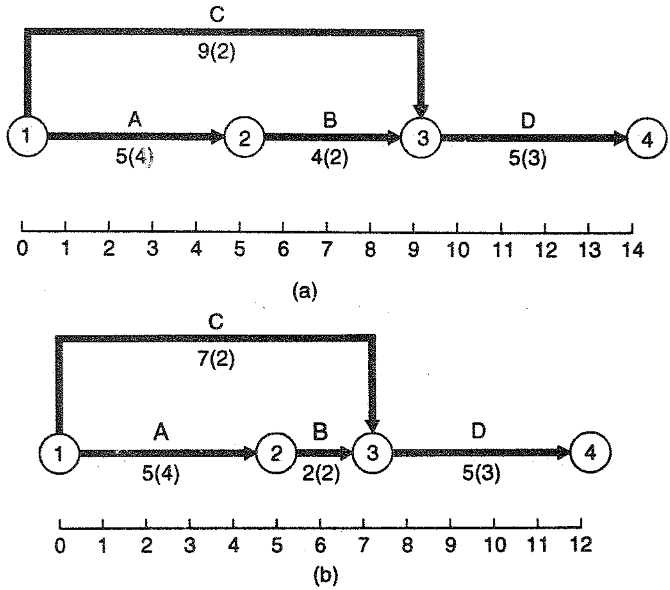
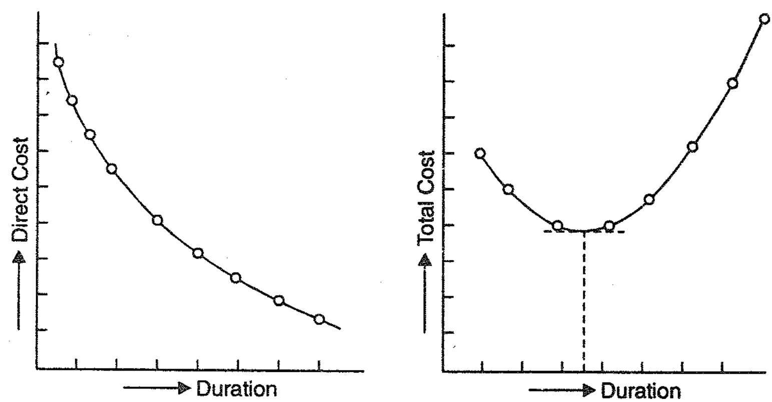


FIG. 9.10. CRASHING ACTIVITY *B* IN STEPS.

During each step of crashing procedure, the direct cost is calculated ; this direct cost will naturally be higher than the one corresponding to the normal time duration. However, the indirect cost will *decrease*, because the project duration has been decreased.



(a) Direct Cost Curve (b) Total Cost Curve

FIG. 9.11. TIME COST RELATIONS.

Hence the total cost is found in each step. A curve is then plotted between the total cost and time. It will be seen that while the direct cost goes no increasing as we reduce the duration, the total cost may decrease till an optimum duration is reached when the cost has a minimum value. If the time duration decreases further, the total cost may increase again, as shown in Fig. 9.11 (*b*). *Thus, it may not always be profitable to crash all the critical activities to their fullest duration.*

9.8. STEPS IN TIME COST OPTIMIZATION

The time-cost optimization is done in the following steps :

1. ESTABLISH	:	direct cost-time relationships for various activities of the project, by analysing past cost records.
2. DETERMINE	:	cost slopes for various activities and arrange them in the ascending order of cost slope.
3. COMPUTE	:	direct cost for the network with normal duration of activities.
4. CRASH	:	the activities in the critical path as per ranking, *i.e.* starting with the critical activity having the lowest slope.
5. CONTINUE	:	crashing the critical activities in the ascending order of the slope.
6. CRASH	:	parallel non-critical activities which have become critical by the reduction of critical path duration due to crashing in steps 4 and 5.
7. CONTINUE	:	crashing process through steps 4 to 6, till a stage is reached beyond which no further crashing is possible.
8. FIND	:	total cost of project at every stage by adding indirect costs to the direct costs determind above.
9. PLOT	:	total cost-duration curve.

10. PICK UP : the optimum duration corresponding to which least total project cost is obtained.

The process of cost optimization is illustrated with the following examples.

9.9. ILLUSTRATIVE EXAMPLES

Example 9.1. *Table 9.2 gives the information about various activities of network shown in Fig. 9.12.*

Table 9.2

Activity	*Normal duration (days)*	*Normal cost (Rs.)*	*Crash duration (days)*	*Crash cost (Rs.)*
1—2	*9*	*8000*	*6*	*9500*
2—3	*5*	*5000*	*3*	*5500*

The project overhead costs are @ Rs. 300.0 per day. Determine (a) direct cost-duration relationship, (b) total cost-duration relationship and the corresponding least cost plan (network).

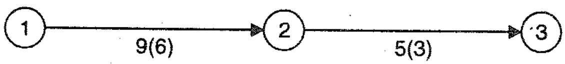


FIG. 9.12

Solution.

Step 1. *Cost-slopes.*

Fig. 9.12 shows the network, with the normal duration of each activity entered below its activity arrow, while the crash duration entered in the bracket.

The cost slope for each activity will be as under :

Table 9.3

Activity (i—j)	*ΔC (Rs.)*	*Δt (days)*	*Cost slope = $\frac{\Delta C}{\Delta t}$ (Rs./days)*
1—2	1500	3	500
2—3	500	2	250

Step 2. *Normal duration direct cost.*

The normal duration for the project = 9 + 5 = 14 days.

∴ Normal duration cost = 8000 + 5000 = 13000.

Step 3. Activity 2-3 has the least slope. Let us therefore crash it first, though in serial-activities network, any activity can be crashed first, or even all the activities can be crashed simultaneously to their corresponding crash durations. Duration by which activity 2-3 can be crashed = 2 days.

Extra cost of crashing activity = 2-3 by 2 days

$= 250 \times 2 = 500$

Project duration $= 9 + 3 = 12$ days.

$\therefore$ Direct cost for 12 days project duration

$= 13000 + 500 = 13500.$

Step 4. After having fully crashed activity 2-3, let us crash activity 1-2 from its normal duration of 9 days to its crash duration of 6 days.

$\Delta t = 9 - 6 = 3$ days.

Extra cost of crashing $= 3 \times 500 = 1500$

Project duration $= 6 + 3 = 9$ days.

$\therefore$ Direct cost for 9 days project duration $= 13500 + 1500$

$= 15000$

The corresponding network with all the activities crashed is shown in Fig. 9.13.

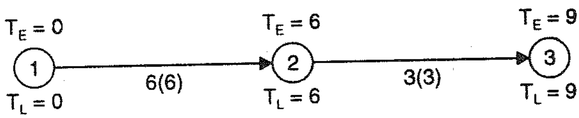


FIG. 9.13

Step 5. ***Total cost of project.***

The total cost of the project, for any duration, is obtained by adding the indirect costs overheads to the corresponding direct costs. The values are tabulated in Table 9.4.

Table 9.4

Duration → *(days)*	*14 Normal*	*12*	*9*
Direct cost	13000	13500	15000
Indirect cost	4200	3600	2700
Total cost	17200	17100	17700

Step 6. ***Cost duration on curves.***

Fig. 9.14 shows the cost-time curves for direct cost, indirect cost and total cost. From Table 9.3 as well as Fig. 9.14, it is evident that total cost is *minimum* for a project duration of 12 days. Thus the *optimum duration* of the project is **12 days** and *minimum cost* corresponding to it is **Rs. 17100.**

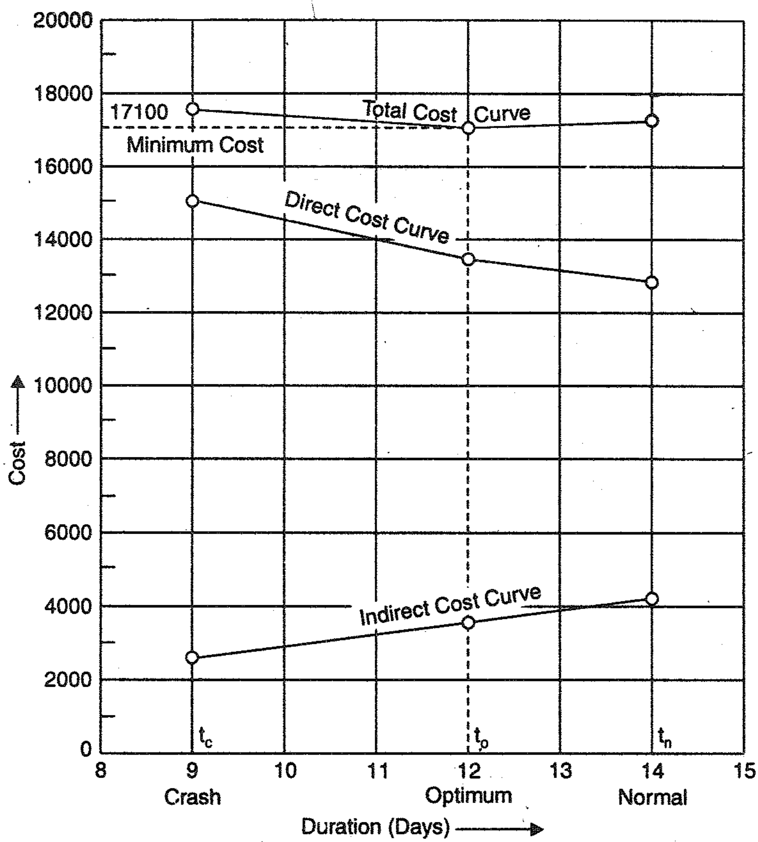


FIG. 9.14. COST-DURATION CURVES.

Example 9.2. *If in example 9.1, in addition to the overheads there is an outage loss of Rs.100 per day upto and including 11th day and Rs. 200 per day thereafter, find the total cost-duration relationship.*

Solution. The indirect costs consist of cost of overheads and the outage loss. Hence the indirect cost curve will not be a single straight line, but will be segmented. In order to get the total cost, we should have the data of the direct costs for all durations. In Table

9.4, the direct cost data is available for 14, 12 and 9 days durations only. The direct costs for intermediate durations (*i.e.*, 13, 11 and 10 days) can be calculated as under :

(*i*) ***For 13 days duration***

Activity 2-3 crashed by Δt = 1 day.

∴ Cost of activity 1-2 for 9 days duration = 8000

Cost of activity 2-3 for 3 days duration

$= 5000 + 1 \times 250$

$= 5250$

∴ Total direct cost

$= 8000 + 5250$

$= 13250.$

(*ii*) ***For 11 days duration***

Activity 2-3 is fully crashed to its duration of 3 days, while activity 1-2 has been crashed from its 9 days duration to 8 days duration.

Cost of activity 2-3 = 5500

Cost of activity 1-2 for 8 days duration

$= 8000 + 1 \times 500$

$= 8500$

∴ Total direct cost = 5500 + 8500 = 14000.

(*iii*) ***For 10 days duration***

Activity 1-2 further crashed by 1 day, at an extra cost of 1 × 500 = 500.

∴ Total direct cost = 14000 + 500 = 14500.

The total cost cannot be found by adding the indirect costs to the direct cost, as shown in Table 9.5

Table 9.5

Duration (days) →	*14 Normal*	*13*	*12*	*11*	*10*	*9 (Fully crashed)*
Direct cost	13000	13250	13500	14000	14500	15000
Overheads	4200	3900	3600	3300	3000	2700
Outage loss	1700	1500	1300	1100	1000	900
Total cost	18900	18650	18400	18400	18500	18600

The time-cost relationships are shown in Fig. 9.15. From Fig. 9.15, it is clear that the minimum total cost is obtained corresponding to the project duration of 11 days or 12 days. Hence the *optimum* project duration is **11 days** and the corresponding *minimum cost* is **Rs. 18400.**

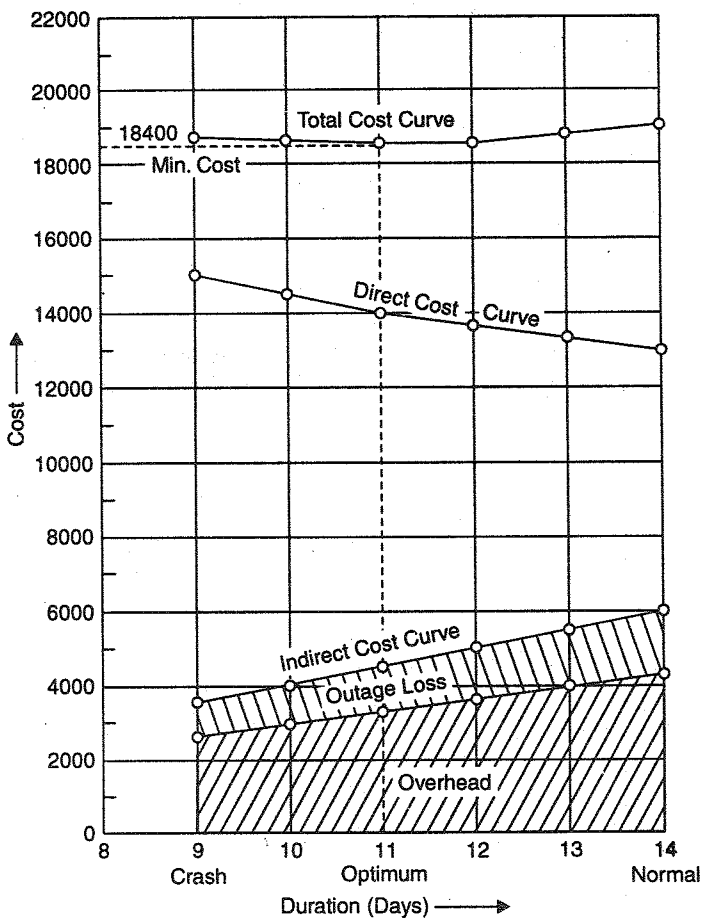


FIG. 9.15

Example 9.3. *Table 9.6 gives the data about durations and costs if various activities of the network shown in Fig. 9.16.*

Table 9.6

Activity	*Normal duration (weeks)*	*Normal cost (Rs.)*	*Crash duration (weeks)*	*Crash cost (Rs.)*
1—2	4	4000	2	12000
2—3	5	3000	2	7500
2—4	7	3600	5	6000
3—4	4	5000	2	10000

The project overhead costs are Rs. 2000 per week. Find the optimum duration and the cost associated with it. Also, draw the least cost network.

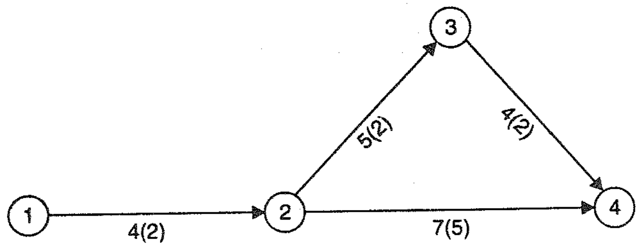
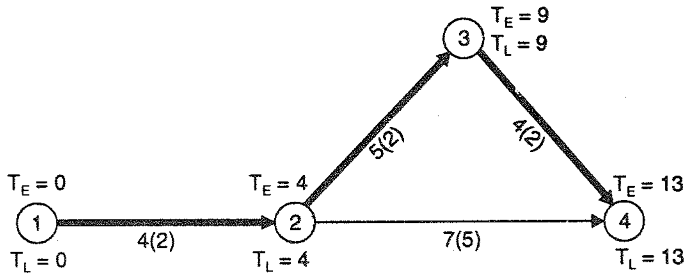


FIG. 9.16

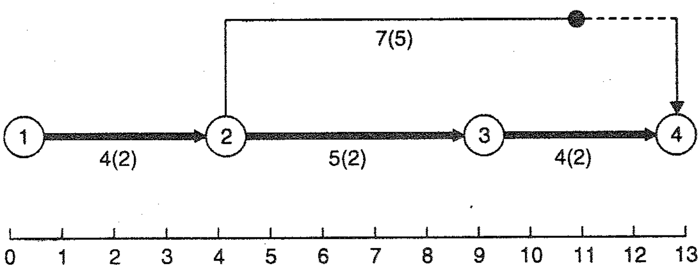
Solution.

Step 1. *Time-scaled version of network*

Fig. 9.17 (*a*) shows the original network while Fig. 9.17 (*b*) shows the time-scaled version of the network drawn on the assumption that all the activities start at their earliest times. The normal completion time of each activity is mentioned below its arrow, while the corresponding crash duration is mentioned in the bracket. The dotted portion of any activity arrow denotes the total float of that activity. The *critical path* is along the activities 1-2, 2-3, 3-4 (shown by thick lines). The critical activities are shown along the horizontal straight path in the time-scaled version of the network.



(a) Original Network



(b) Time Scaled Version

FIG. 9.17

Step 2. ***Cost slopes***

The cost slopes of various activities will be as shown in Table 9.7 :

Table 9.7

Activities	ΔC *(Rs.)*	Δt *(weeks)*	*Cost slope Rs./week*
1—2	12000 – 4000 = 8000	4 – 2 = 2	4000
2—3	7500 – 3000 = 4500	5 – 2 = 3	1500
2—4	6000 – 3600 = 2400	7 – 5 = 2	1200
3—4	10000 – 5000 = 5000	4 – 2 = 2	2500

Step 3. ***Direct cost of normal duration project***

The normal duration of the project is the sum of the normal durations of each activity on the critical path. (It is not the sum of normal durations of all the activities).

$\therefore$ Normal duration of the project

$$= 4 + 5 + 4 = 13 \text{ weeks.}$$

Direct cost of the project will be equal to the sum of the normal cost of *all* the activities.

$\therefore$ Direct cost = 4000 + 3000 + 3600 + 5000
= 15600.

Step 4. *First stage crashing*

While crashing the activities, we shall first select that critical activity which has the minimum cost slope. For the present case, critical activity 2-4 has the minimum cost slop of 1500 per week. Let us crash this first. Its crash period is 2 weeks, *i.e.*, $\Delta t = 5 - 2 = 3$ weeks. However, crashing it by 3 weeks will affect non-critical activity 2-4, which has a float of only 2 weeks. Hence let us restrict the crashing of 2-3 by 2 weeks only, in the *first stage*. New duration of the project = 13 – 2 = 11 weeks.

Extra cost of crashing activity 2-3 by 2 weeks
= 2 × 1500 = 3000

$\therefore$ Direct cost of project of 11 weeks duration
= 15600 + 3000 = 18600.

The time-scaled version of the network, after first stage crashing, for the project of 11 weeks duration is shown in Fig. 9.18.

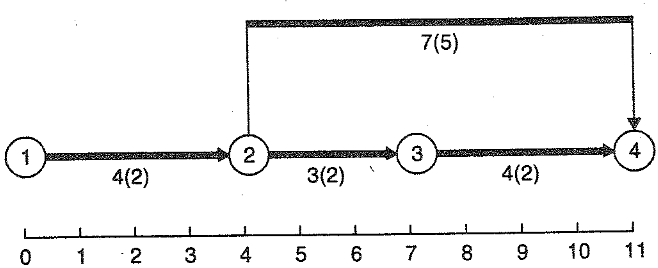


FIG. 9.18. NETWORK FOR 11 WEEKS DURATION.

Step 5. *Second stage crashing*

From Fig. 9.18, it is clear that activity 2-4, lying on the parallel path, has also become *critical*, though activity 2-3 has still 1 week crashing left. However, activity 2-3 cannot be crashed along, unless 2-4 is also crashed. Let us therefore crash 2-3 for 1 week, and also crash 2-4 simultaneously for 1 week. However, this combined crashing will be useful *only* if the *combined* cost slope of these two activities is *less* than the cost slope of any of the remaining critical activity or the combined cost slopes of critical activities on parallel path.

For the present case, further crashing can be done with three alternatives :

(*i*) Crashing activities 2-3 and 2-4 simultaneously, having a combined cost slope of 1500 + 1200 = 2700 per week.

(*ii*) Crashing activities 3-4 and 2-4 simultaneously, having a combined cost slope of 2500 + 1200 = 3700 per week.

(*iii*) Crashing activity 1-2 alone, having a cost slope of Rs. 4500 per week.

Out of these, the first alternative has the minimum cost slope.

Thus, the extra cost of crashing 2-3 and 2-4 by 1 week

$$= 2700 \times 1 = 2700$$

∴ Direct cost of project for 10 weeks duration

$$= 18600 + 2700 = 21300.$$

In this step, activity 2-3 has been crashed to its fullest extent. Fig. 9.19 shows the time-scaled version of the network, for 10 weeks duration.

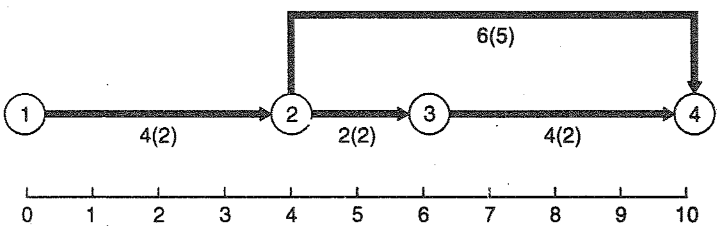


FIG. 9.19. NETWORK FOR 10 WEEKS DURATION.

Step 6. *Third stage crashing*

The remaining activities to be crashed (either fully or partly) are 1-2, 2-4 and 3-4. Out of these, activities 2-4 and 3-4 are to be crashed jointly, with a combined cost slope of 2500 + 1200 = 3700. The cost slope of activity 1-2 is 4000, which is higher. Hence activities 2-4 and 3-4 will be crashed *first*. Activity 2-4 has a crashing period of 6 – 5 = 1 week left. Hence only 1 week crashing will be done in this step, leading to a project duration of 9 weeks.

Cost of crashing 2-4 and 3-4 by 1 week.

$$= 3700 \times 1 = 3700.$$

∴ Direct cost of project for 9 weeks duration

$$= 21300 + 3700 = 25000.$$

The time scaled version of the network for 9 weeks duration is shown in Fig. 9.20. Upto this stage, we have crashed activity 2-4 also to its fullest extent.

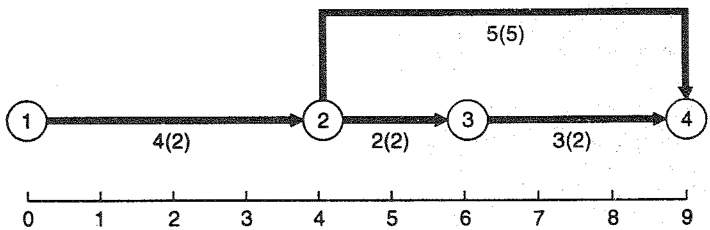


FIG. 9.20. NETWORK FOR 9 WEEKS DURATION.

Step 7. *Fourth stage crashing*

Out of remaining critical activities (*i.e.* 1-2 and 3-4), activity 3-4 cannot be further crashed to its fullest crash period (of 2 weeks), since it will affect activity 2-4 which has already been fully crashed.

Hence activity 1-2 is the only remaining activity to be crashed. The period by which it can be crashed is = 4 – 2 = 2 weeks, reducing the project duration to 9 – 2 = 7 weeks.

Extra cost of crashing 1-2 by 2 weeks = 2 × 4000 = 8000

∴ Direct cost of project of 7 weeks duration

$$= 25000 + 8000 = 33000.$$

The time-scale version of the network of the project of 7 weeks duration is shown in Fig. 9.21.

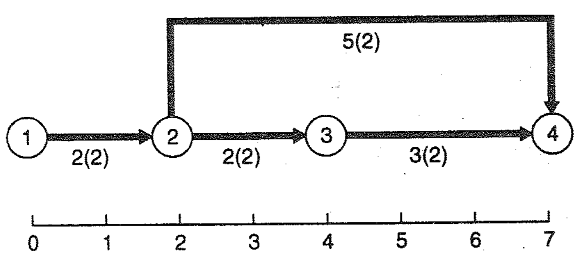


FIG. 9.21. NETWORK FOR 7 WEEKS DURATION.

It should be noted that the project duration has been crashed from its original duration of 13 weeks to the present duration of

7 weeks ; thus the period by which it has been crashed = Δt = 13 – 7 = 6 weeks only, as against the total available crashing period of 13 – 6 = 7 weeks along the original critical path.

Step 8. *Total cost of the project*

The total cost of the project is computed by adding the direct cost and the indirect cost, as illustrated in Table 9.8. The indirect cost is at the rate of Rs. 2000 per week.

Table 9.8. Total Costs

Project Duration (weeks)	*13 (normal)*	*11*	*10*	*9*	*7*
Direct cost (Rs.)	15600	18600	21200	25000	31000
Indirect cost (Rs.)	26000	22000	20000	18000	14000
Total cost (Rs.)	41600	40600	41300	43000	47000

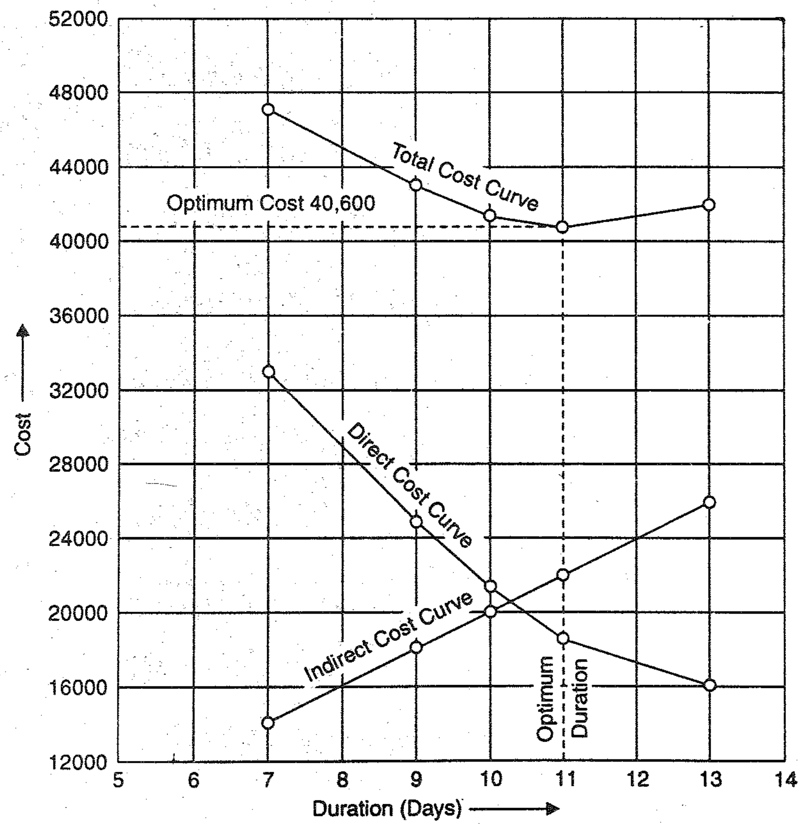


FIG. 9.22. DURATION-COST CURVES.

The direct cost curve, indirect cost curve and total cost curve are shown in Fig. 9.22. From Table 9.8 as well as Fig. 9.22, it is clear that the optimum project duration is **11 weeks** and the optimum cost corresponding to this period is Rs. **40600.** The time scaled network for this project duration is shown in Fig. 9.18.

PROBLEMS

1. Define the terms 'direct cost', 'indirect cost' and 'outage loss'.
2. What do you understand by 'cost-slope' ? How do you determine it ?
3. Define 'normal project time', 'normal cost', 'crash time' and 'crash cost'.
4. Draw a typical cost-duration curve and show on it optimum duration and minimum project cost.
5. Explain the method of times-cost optimization of project network.
6. Fig. 9.23 show the network for a project, the data for the duration and costs of each activity are given in Table 9.9.

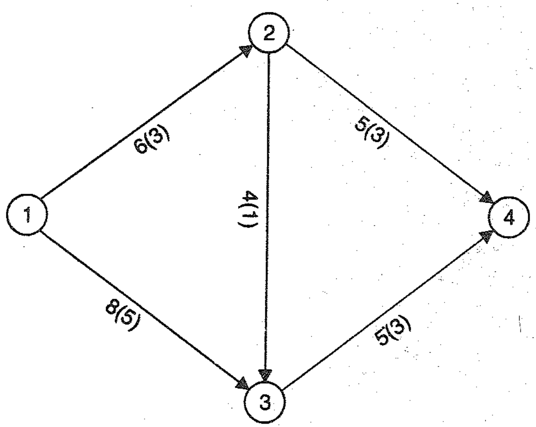


FIG. 9.23

Table 9.9

Activity	*Normal duration (weeks)*	*Normal cost (Rs.)*	*Crash duration (weeks)*	*Crash cost (Rs.)*
1—2	6	7000	3	14500
1—3	8	4000	5	8500
2—3	4	6000	1	9000
2—4	5	8000	3	15000
3—4	5	5000	3	11000

The direct cost of the project is Rs. 3000 per week. Determine the optimum duration of the project and the corresponding minimum cost. Draw the time scaled version of the network at each stage of crashing.

10

CPM : Updating

10.1. INTRODUCTION

Controlling is complementary to the planning. Once the scheduled plan has been prepared and execution commenced, control over the progress of work has to be exercised in order to complete the work by the stipulated date. Control involves comparing at regular intervals, the actual achievement with the original plans and then taking any necessary corrective action to bring things back on schedule. Therefore, *Controlling* requires an upward flow of information through a suitably designed reporting system. The information so fed is analysed and the project plan is brought upto date with necessary variations to keep performance as per the schedule.

10.2. UPDATING : PROCESS

During the process of implementing the plan according to the network, we may come across one or more of the following possibilities :

1. that some or all activities are progressing according to schedule ;
2. that some or all activities are ahead of schedule ; and
3. that some or all activities are behind schedule.

If all activities are progressing according to the schedule, there is no need for updating the network but this is seldom the case.

Therefore, based on the progress of the work and the revised durations of unfinished activities due to delays, the network diagram has to be redrawn and this process is known as *Updating*.

As stated previously, critical path method is a numerical technique which project management can use as an aid in planning, scheduling and controlling any type of project. The calculations made previously were based on the assumption that the planner has taken an entirely new project in hand and tail event of the project has been taken as the base for the calculations. The networks previously developed can also be used to aid planners or managers in decision-making, after the commencement of the project *i.e.* when project is already in progress.

When the project is partially completed and is at an intermediate stage, it may be possible that :

1. the time durations originally assigned for some activities were erroneous and

2. the planner may himself feel it desirable as a result of experience or he may be enriched with additional information, to reconsider and re-estimate duration times of activities not yet being performed. Now, new information and considerations can be placed on the original network and fresh calculations are made for controlling the project.

The process of replanning and rescheduling based on the results which serve a guidance for decision by performing calculations made by taking into consideration the new knowledge and latest information at an intermediate stage of the project thus modifying the original network, is known as the process of Updating.

10.3. DATA REQUIRED FOR UPDATING

The following information is necessary to update the plan at an intermediate stage of execution of a project :

1. original network ;
2. original network calculation chart ;
3. stage at which updating is being done *i.e.*, a point in time of updating ;
4. execution position of the project at that stage and
5. new information and knowledge which will affect the duration time of the activities to be performed.

10.4. STEPS IN THE PROCESS OF UPDATING

1. DESCRIBE	:	the point in time at which updating is to be done according to the original plan.
2. RECORD	:	what has happened actually till the updating point.

3. SUMMARISE : the knowledge attained in the tabulated form as given below :

Table 10.1. Updating

Activity	*Whether completed or not*		*If in progress, additional time required for the completion*	*Completion required for activities yet to begin*
	Yes/No	*If yes, time taken for completion*		

4. PLACE : the information contained in the updating table on to the original network. This is done by :

1. assigning the time of update as the earliest occurrence time for the tail event of the project ;
2. alloting a zero time duration for all activities which have been completed ;
3. entering the remaining estimated durations of those activities which are in progress ; and
4. entering the estimated durations based on new knowledge of activities which are still to be commenced.

5. PERFORM calculations of earliest occurrence time and latest occurrence time and mark these on the network known as updated network.

The updating cycle is shown in Fig. 10.1.

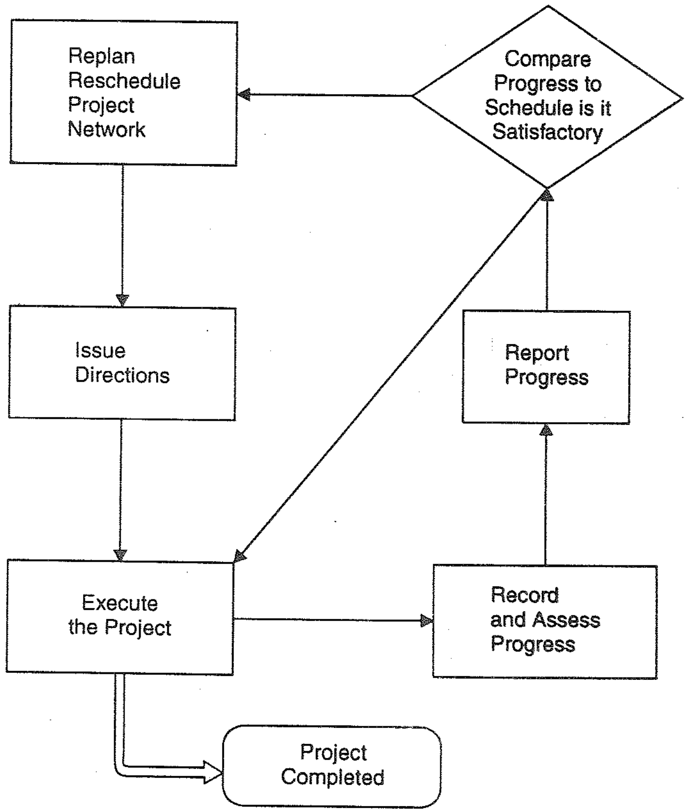


FIG. 10.1. UPDATING CYCLE.

10.5. WHEN TO UPDATE

The following points must be kept in view while deciding the time of Updating :

1. for shorter duration projects, the updating must be done frequently by taking into account the latest position of the execution of the project.

2. for large duration projects, the process of updating must be increased as the project is progressing toward completion. Duration of project goes on decreasing as project progresses, and behaving more or less like a small duration project.

3. whenever there is major change in the duration of any of the activity the updating is to be done.

4. updating is essential if there is change in the estimated duration of any activity falling on the critical path. If the duration of a critical activity increases, remedial measures are necessary and if the activity duration decreases, this may allow changes in the project plan which were not possible previously.

After updating there may be some changes in the completion time of the project. If updating time comes out to be more, the planner has two options :

1. He may ask the executing authorities to perform the operations on the critical path faster than previously estimated. Such execution will require the arrangement of more resources *i.e.*, man-power and material etc., and

2. He may redraw some portion of the network containing those activities which have still not commenced. Such alteration means change in the company's policy of execution which leads to revised inter-dependence of operations.

From the above it is clear that CPM is not only useful in the planning stage of a project but also aids the decision-maker during execution and to some extent in controlling the completion of project on schedule.

10.6. ILLUSTRATIVE EXAMPLES

Example 10.1. *Fig. 10.2 shows the network of a project which is to be updated at the end of 12 days. The following conditions exist at the time of updating :*

1. Activity 1-4 was completed as originally planned.

2. Activity 1-3 was executed more rapidly then originally scheduled, and it took 8 days for its completion.

3. Activity 3-4 commenced following the completion of activity 1-3 and was finished at the end of 11th day.

4. Activity 4-5 was commenced following the completion of activity 3-4 (i.e., at the end of 11th day), and still requires 6 more days for its completion.

5. Completion of activity 1-2 was delayed drastically, and it still requires 10 more days for its completion.

6. Activity 2-7 will commence following the completion of activity 1-2 and will require 9 days for its completion instead of 6 days originally estimated.

7. The time required to perform activity 5-8 has been revised, based on the experience on the project, gained to this point. It now requires 10 days in the place of 6 days originally estimated.

8. No other activities have been started, and the original time estimates for these activities still appear to be accurate.

Update the network, and determine the revised critical path.

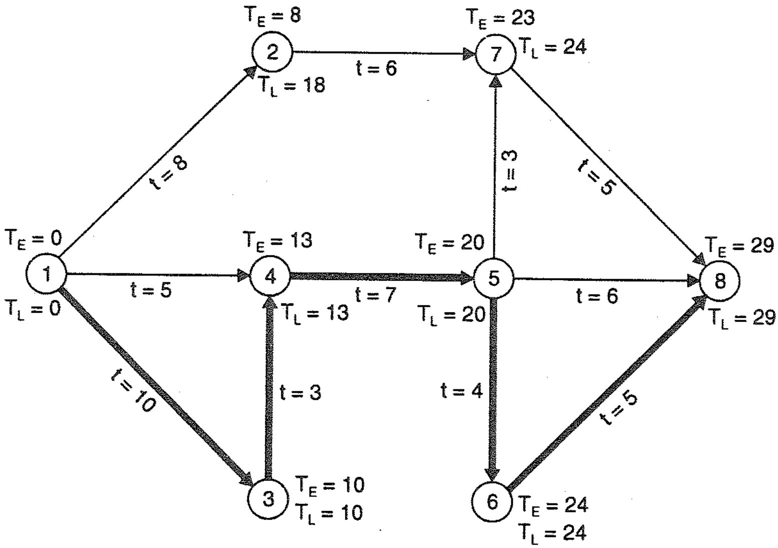


FIG. 10.2

Solution. Fig. 10.2 shows the original network, with T_E and T_L marked. The critical path, shown by dark lines is along activities 1-3, 3-4, 4-5, 4-6 and 6-8.

Table 10.2 gives the details of execution of the various activities at the end of 12 days.

The updated network can now be drawn on the basis of data of columns (1), (2), (4) and (5) of the above table. For those activities, which have already been completed, completion time t is taken to be zero, since they require zero time after the 12th day. Also the earliest event time (T_E) and leasted occurrence time (T_L) of each event is computed *with reference* to the original starting date of the project. This can be best achieved by taking T_E for event 1 as equal to 12.

Table 10.2
Review after 12 days

Activity	Whether completed or not		Additional time required for activities in progress (days)	Completion time required for activities yet to begin (days)
	Yes / No	If yes, time taken (days)		
(1)	(2)	(3)	(4)	(5)
1—2	No	—	10	—
1—3	Yes	8	—	—
1—4	Yes	5	—	—
2—7	No	—	—	9
3—4	Yes	3	—	—
4—5	No	—	6	—
5—6	No	—	—	4
5—7	No	—	—	3
5—8	No	—	—	10
6—8	No	—	—	5
7—8	No	—	—	5

After having determined the updated T_E for each event, corresponding T_L can be computed by the *backward pass*. The updated network is shown in Fig. 10.3. The critical path of the

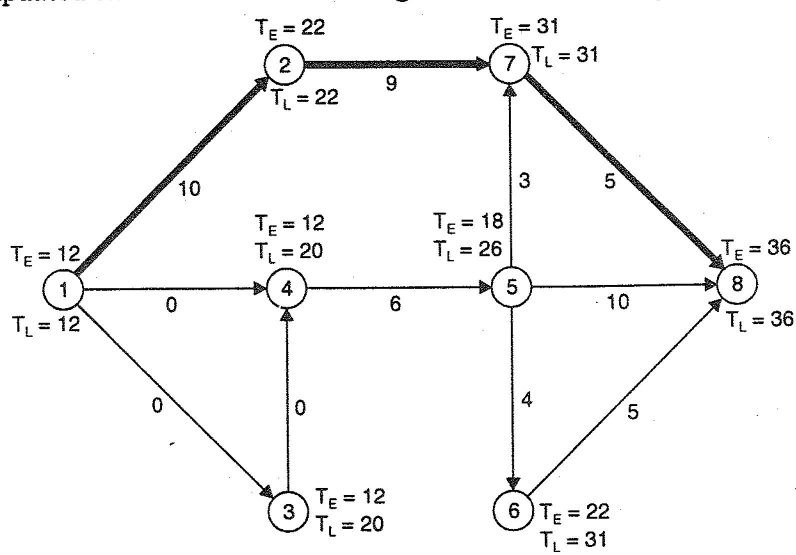


FIG. 10.3. UPDATED NETWORK.

updated network has now changed ; it is now along activities 1-2, 2-7, 7-8, shown by dark lines. According to the updated network, the project will take a total time of 36 days, instead to 29 days originally planned. On the day of updating, the remaining duration of project = 36 – 13 = 24 days.

PROBLEMS

1. What do you understand by updating ? Why is it essential ?
2. Illustrate the method of updating a network during its execution period.
3. A network for a project is shown in Fig. 10.4. The network is to be updated after 10 days of its execution. The following conditions exist at the end of 10 days :

   (*i*) Activity 1-2, 1-3 and 1-4 have been completed as originally scheduled.

   (*ii*) Activity 4-5 is in progress and will require 6 more days for its completion.

   (*iii*) Activity 4-6 is in progress and will require 6 more days for its completion.

   (*iv*) Activity 3-6 is in progress and will be completed in one day.

   (*v*) Other activities have not been commenced and their original predicted durations will hold good, except for activity 5-7 which will require only three days instead of 5 days originally planned.

   Update the network and determine the critical path of the updated network. What is the total increase in the project duration ?

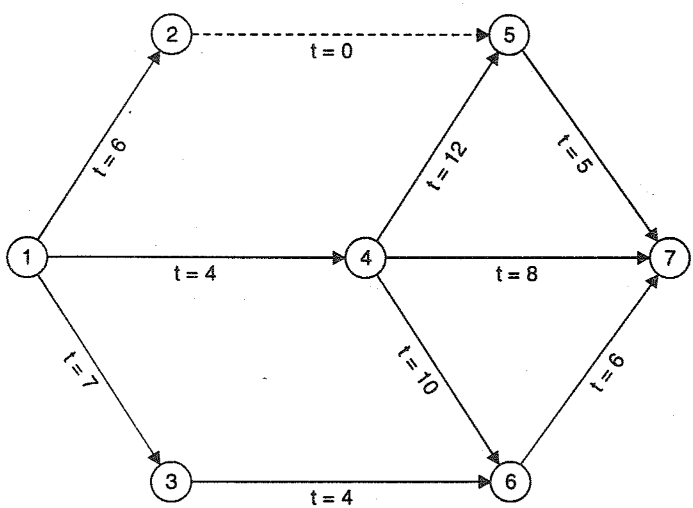


FIG. 10.4

11

Resources Allocation

11.1. INTRODUCTION

A resource is a physical variable, such as men, materials, machines, space, money that is required for completing various activities/jobs of a project. The network analysis for PERT/CPM, carried out so far is valid only if the availability of resources is liberal or unlimited. But all the necessary resources are not available in unlimited quantities. Availability of some of the resources may be restricted. Availability of manpower (supervisory staff, technical and specialist personnel, skilled and unskilled labour etc.) and materials etc. may be restricted. Availability of funds, credits, capital investment and heavy equipment may be restricted. In certain cases, there may be space limitations, which prevent more than one or two technicians working simultaneously. Supervisory, technical and skilled manpower, space and equipment are usually the most important resources that need be allocated carefully. The various activities of the project are to be scheduled in such a way that the demand of various resources is more or less uniform all along the project duration. Large fluctuations in their demand may cause problems in the project execution.

11.2. RESOURCES USAGE PROFILES : HISTOGRAMS

For a given network, the requirements of various resources are determined, using the early start schedule of each activity. In a network, various activities are involved, and each activity requires some resources to perform it. There may be activities which are to be performed simultaneously and may require common resources. The requirements of resources to execute these simultaneous activities may exceed the available resources. However, at some other

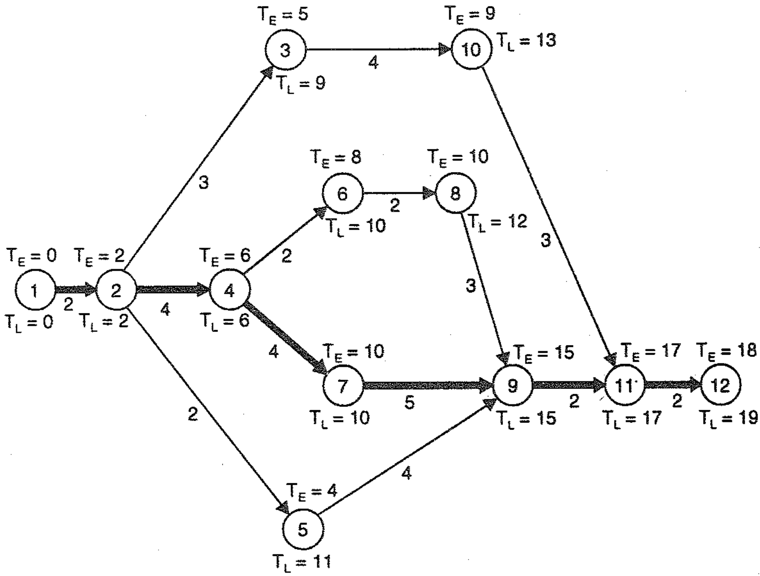


FIG. 11.1

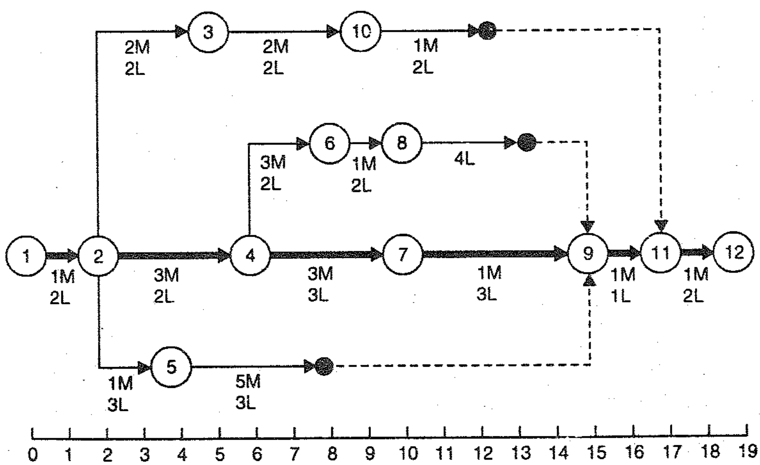
period of the execution of the same project, there may be very few activities which may require these resources. Hence the requirement of a particular type of resource may not be uniform during the project duration. This can be best known by plotting the *resources usage profiles* or *histograms*.

Consider a network shown in Fig. 11.1, having 14 activities. The duration of each activity is marked under its activity arrow. The early event times and late event times are marked near each event circle. The critical path is along activities 1-2, 2-4, 4-7, 7-9, 9-11 and 11-12, shown by thick lines. Table 11.1 shows the requirements of masons (marked by *M*) and labourers (marked by *L*) for each activity. Let us analyse the project from resources requirements point of view.

Table 11.1
Resources Requirements

Activity	*Duration*	*Masons (M)*	*Labourers (L)*
1—2	2	1	2
2—3	3	2	2
2—4	4	3	2
2—5	2	1	3
3—10	4	2	2
4—6	2	3	2
4—7	4	3	3
5—9	4	5	3
6—8	2	1	2
7—9	5	1	3
8—9	3	—	4
9—11	2	1	1
10—11	3	1	2
11—11	2	1	2

Fig. 11.2 shows the time scaled version of the network, assuming early start times for each activity. The activities along the

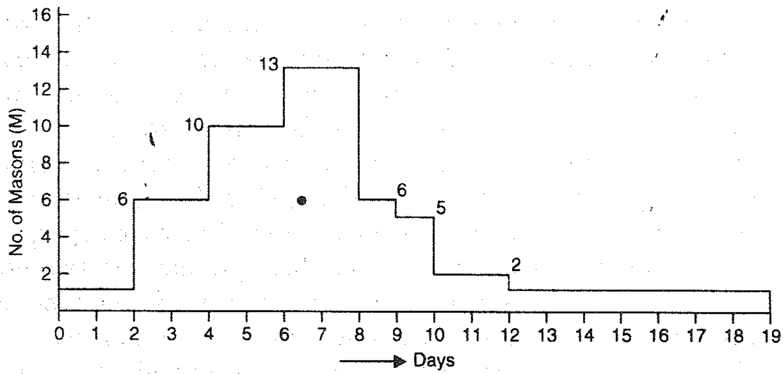


M	1	1	6	6	10	10	13	13	6	5	2	2	1	1	1	1	1	1	1
L	2	2	7	7	7	7	10	10	7	7	9	9	7	3	3	1	1	2	2

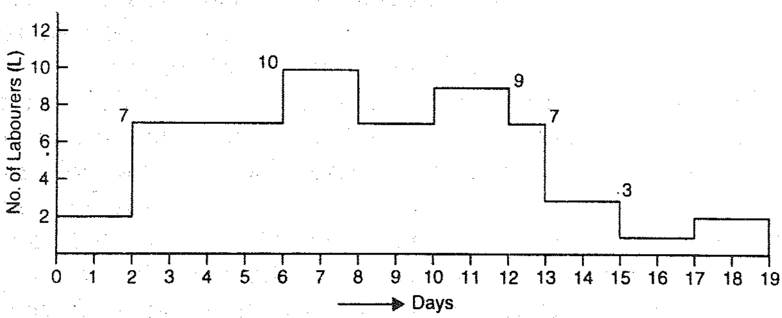
FIG. 11.2

critical path have been arranged along horizontal line. The dotted lines show the total float of each activity. The requirements of masons (M) and labourers (L) for each activity is marked under the activity arrow. The table below the time scale shows total requirements of masons and labourers each day.

Fig. 11.3 (a) and (b) show the variation in the requirements of masons and labourers respectively, with time. These diagrams are known as *resources usage profiles* or *histograms*.



(a) Requirements of Masons



(b) Requirements of Labourers

FIG. 11.3. RESOURCES USAGE PROFILES.

From Fig. 11.2 as well as 11.3, it is clear that the requirements of masons and labourers is not uniform along the project duration.

The demand of the masons is as high as 13 on 7th and 8th days, while it is as low as 1 in the beginning and end of the project. Similarly, the demand of the labourers is as high as 10 on 7th and 8th days, while it is as low as 2 in the beginning and the end of the project. This shows the great variation in the resources (*i.e.* manpower) requirements. If 13 masons are employed to meet the peak demand, they will sit idle during the non-peak periods. This will be highly uneconomical unless we employ them on temporary basis only as per actual requirements each day. However, skilled persons such as masons, foreman etc. are required to be employed on the permanent basis. Therefore, the planning should be done in such a manner that resources are utilized in a more or less uniform manner. This can be achieved by the following two approaches :

(*a*) Resources smoothing

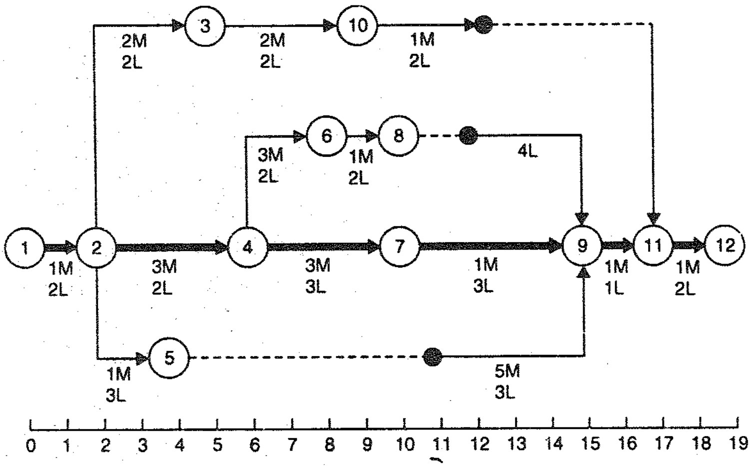
(*b*) Resources levelling.

The above nomenclature for the two approaches to solve the resources allocation problem has not been standardized so far with the result that some people use them interchangely. In the *first approach*, known as *resources smoothing method*, the total project duration is not changed, but some of the activities start times are shifted by their available floats so that a uniform demand for the resources is generated. However, the resources are considered to be unlimited. In the *second approach*, known as *resources levelling*, the activity start times are so re-scheduled that the peak demand for a particular resource does not cross the available limit of the resources. Thus, the resources are considered to be *limited*. In rescheduling the activities, the floats are first used, but if it does not give the desirable results, the total project duration may be changed.

11.3. RESOURCES SMOOTHING

This is the first approach of solving the resources allocation problem, in which the resources are considered to be unlimited. The original project duration (*i.e.* duration along the critical path) is however maintained. The start times of some of the activities are so shifted within their available floats that uniform demand is created for the resources.

To illustrate the procedure, let us consider the network of Fig. 11.2. We find that the peak requirements of masons are there on 7th and 8th day. Also, the requirements of masons on 5th and 6th day is high. Also, the requirements of mason on 11th day and



M	1	1	6	6	5	5	8	8	6	5	2	7	6	6	6	1	1	1	1
L	2	2	7	7	4	4	7	7	7	7	5	8	10	10	10	1	1	2	2

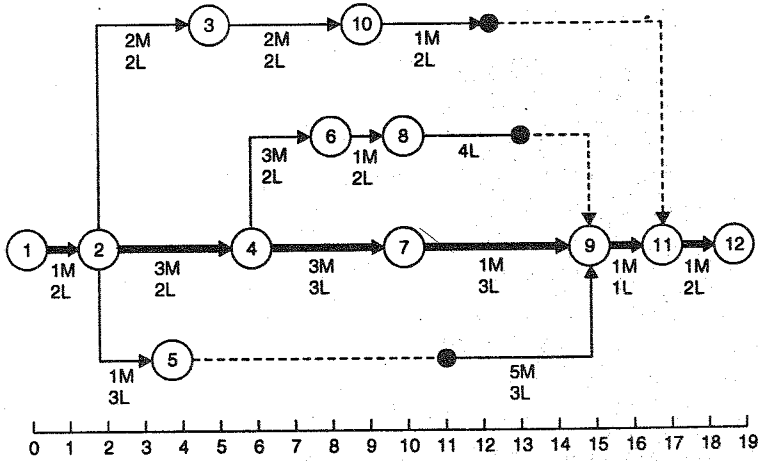
FIG. 11.4

onwards is very low. By inspection, we find the activities 1-5 and 5-9, have a total float of 7 days. Hence the start time of activity 5-9 can be shifted by 7 days. This will give encouraging results since this activity requires 5 masons. As a first trial therefore, let us shift activity 5-9 by 7 days, so that it starts on 12th day instead of 5th day. Fig. 11.4 shows revised network, along with the modified resources accumulation table.

From Fig. 11.4, we find that the peak demand for masons has decreased from 13 (for 7th and 8th day) to 8 (for 7th and 8th day). Also, the demand of masons has decreased from 10 to 12.

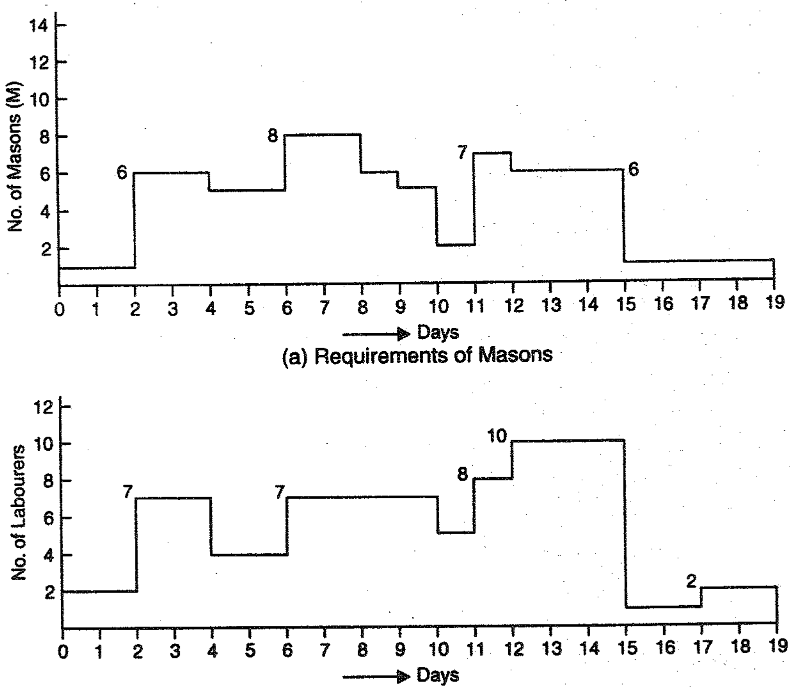
In the second trial, we can shift activity 8-9 by its total float period of 2 days. This will result in smoothing the labour requirements.

From Fig. 11.5, we observe that the demand of labourers has been decreased from 12 to 10. Fig. 11.6 shows the corresponding histograms for the masons and the labourers. Thus, following this procedure, it is always possible to smoothen the resources requirements, without affecting the project duration.



M	1	1	6	6	5	5	8	8	6	5	2	7	6	6	6	1	1	1	1
L	2	2	7	7	4	4	7	7	7	7	9	12	10	6	6	1	1	2	2

FIG. 11.5



(a) Requirements of Masons

(b) Requirements of Labourers

FIG. 11.6

11.4. RESOURCES LEVELLING

In the resources levelling process, the activities are so rescheduled that the maximum or peak resources requirement does not cross the limit of available resources. The available resources should, however, not be less than the maximum number or quantity required for any activity of the project. In rescheduling, the available floats are first used. If by doing so, the resources demand is more than the available resources, the duration of some of the activities is *increased* so that the resources requirements for these activities is decreased. Thus in the sources levelling process, the project duration, initially planned, might be changed.

PROBLEMS

1. Discuss in brief the resources allocation problem. What are the methods of solving the problem ?
2. With the help of an illustrative example, explain the resources smoothing method.