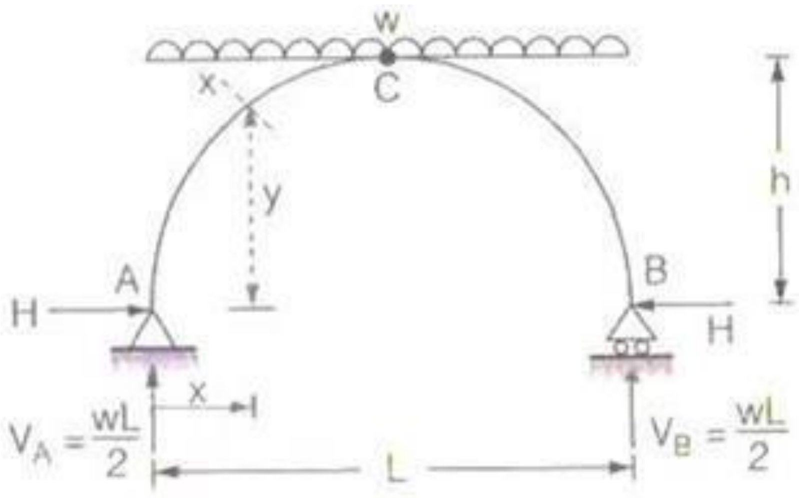
Unit III :- Arches

- A **two hinged arch** is statically indeterminate to single degree, since there are four reaction components to be determined while the number of equations available from static equilibrium is only three.
- **Arches**
- **Three Hinged Arches**
- (i) Three Hinged Parabolic Arch of Span L and rise 'h' carrying a UDL over the whole span



A two-hinged arch has hinges only at the supports (Fig. 4.2a). Such an arch is statically indeterminate. Determination of the horizontal and vertical components of each reaction requires four equations, whereas the laws of equilibrium supply only three (Art. 4.1).

Another equation can be written from knowledge of the elastic behavior of the arch. One procedure is to assume that one of the supports is on rollers. The arch then becomes statically determinate. Reactions VL and VR and horizontal movement of the support Sx can be computed for this condition with the laws of equilibrium (Fig. 4.2b). Next, with the support still on rollers, the horizontal force H required to return the movable support to its original position can be calculated (Fig. 4.2c). Finally, the reactions of the two-hinged arch of Fig. 4.2a are obtained by adding the first set of

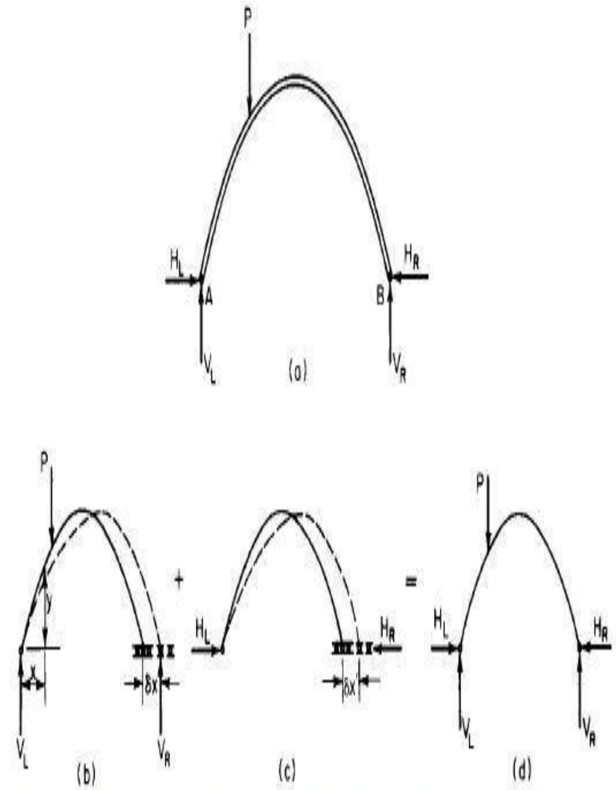


FIGURE 4.2 Two-hinged arch. Reactions of loaded arches (*a*) and (*d*) may be found as the sum of reactions in (*b*) and (*c*) with one support movable horizontally.

- I moment of inertia of arch cross section
  A cross-sectional area of arch at the section
  E modulus of elasticity
  ds differential length along arch axis
  dx differential length along the horizontal
  N normal thrust on the section due to loads
- **Circular Two-Hinged Arch** Example. A circular two-hinged arch of 175-ft radius with a rise of 29 ft must support a 10-kip load at the crown. The modulus of elasticity E is constant, as is I/A, which is taken as 40.0. The arch is divided into 12 equal segments, 6 on each symmetrical half. The elements of Eq. (4.8) are given in Table 4.1 for each arch half.
  Since the increment along the arch is as a constant, it will factor out of Eq. 4.8. In addition, the modulus of elasticity will cancel when factored. Thus, with A and I as constants, Eq. 4.8 may be simplified to

$$H = \frac{\sum_B^A My - \frac{I}{A}\sum_B^A N\cos\alpha}{\sum_B^A y^2 + \frac{I}{A}\sum_B^A \cos^2\alpha}$$

TABLE 4.1 Example of Two-Hinged Arch Analysis

α radians	My, kip-ft^2	y^2, ft^2	$N\cos\alpha$ kips	$\cos^2\alpha$
0.0487	12,665	829.0	0.24	1.00
0.1462	9,634	736.2	0.72	0.98
0.2436	6,469	568.0	1.17	0.94
0.3411	3,591	358.0	1.58	0.89
0.4385	1,381	154.8	1.92	0.82
0.5360	159	19.9	2.20	0.74
TOTAL	33,899	2,665.9	7.83	5.37

$$H = \frac{2.0(33899)}{2.0(2665.9)} = 12.71 \text{ kips}$$

Addition of the axial contribution yields

$$H = \frac{2.0[33899 - 40.0(7.83)]}{2.0[2665.9 + 40.0(5.37)]} = 11.66 \text{ kips}$$

It may be convenient to ignore the contribution of the thrust in the arch under actual loads. If this is the case, $H = 11.77$ kips.

(F. Arbabi, *Structural Analysis and Behavior,* McGraw-Hill Inc. New York.)