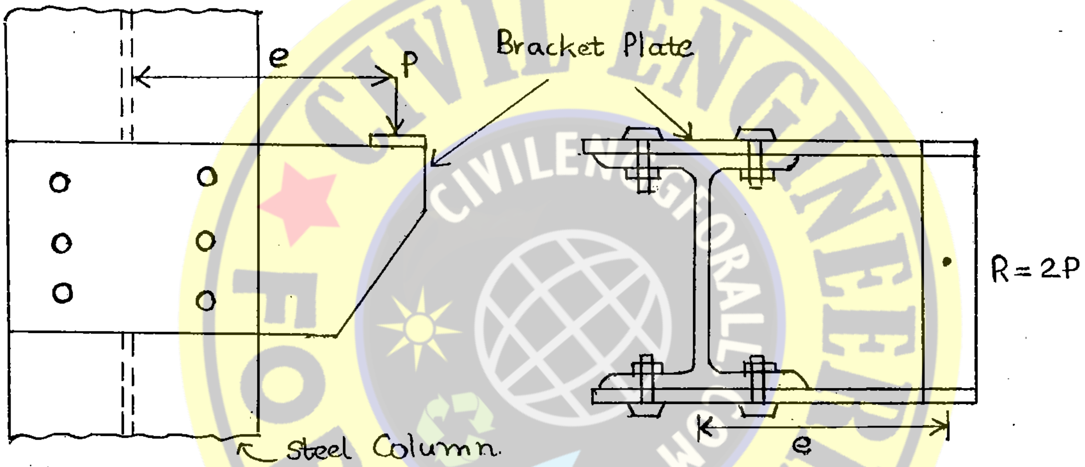
7th Sept,
SUNDAY

4. ECCENTRIC CONNECTIONS

→ ECCENTRIC BOLTED CONNECTIONS

1. Bracket Type Connection - I

- When load or moment is lying in the plane of bearing type bolt group.



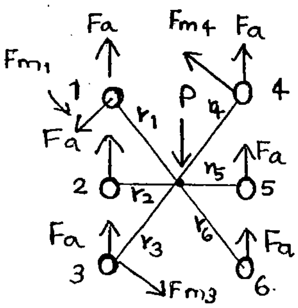
P → Factored load or Design Load.

e → eccentricity of the load. (Distance from CG of Bolt/weld group to applied load line)

- Bolt group is subjected to :
  (i) Direct concentric load (P)
  (ii) Twisting moment ($M = Pe$) [in-plane moment]

- Vertical SF in each bolt due to P is Fa

$$F_a = \frac{P}{n} \quad (n = \text{no. of bolts in bolt group}$$



$$\Rightarrow \frac{T}{J} = \frac{f_s}{r} \Rightarrow f_s = \frac{T}{I_p} \cdot r$$

$$f_s = \frac{M}{I_p} \cdot r$$

$$F_m = f_s A = \frac{M}{I_p} \cdot r \cdot A \Rightarrow F_m \propto r$$

Fm is force any bolt due to twisting moment ($M = Pe$).

$$F_m \propto r$$

$$F_m = Kr \quad (K = \text{Elastic Constant})$$

Moment of resistance capacity of each bolt,

$$M_R = F_m \cdot r = K \cdot r \cdot r$$

$$M_R = Kr^2$$

Total moment of resistance capacity, $M_R = \Sigma K r^2 = K \Sigma r^2$

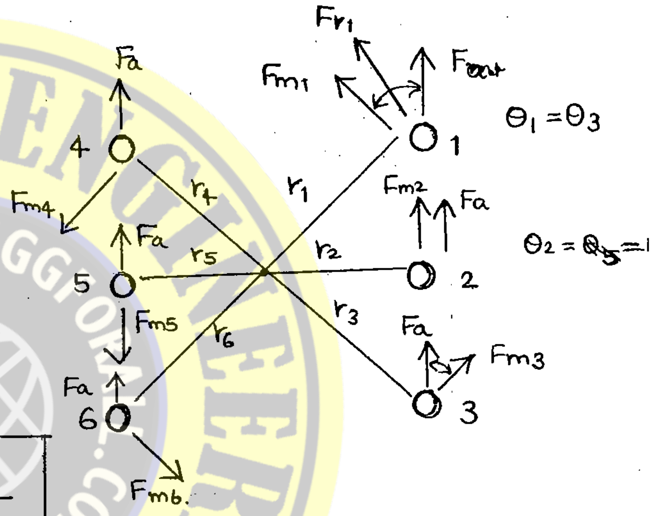
$$M_R = K \cdot \Sigma r^2$$

$$= \frac{F_m}{r} \Sigma r^2$$

$$M = M_R$$

$$Pe = \frac{F_m}{r} \Sigma r^2$$

$$\therefore \boxed{F_m = \frac{Mr}{\Sigma r^2} = \frac{Pe\, r}{\Sigma r^2}}$$



– Resultant force b/w Fa & Fm is F_R

$$F_R = \sqrt{F_a^2 + F_m^2 + 2 F_a \cdot F_m \cos\theta}$$

$$F_a \rightarrow \frac{P}{n}, \qquad F_m \rightarrow \frac{Mr}{\Sigma r^2}$$

* Conditions for Max. Resultant Force for Critical bo

$r \longrightarrow$ maximum (1, 3, 4, 6)

$\theta \longrightarrow$ minimum (1, 3)

Bolt which is closer to resultant loading – critical bolt (1 & 3)

* For safety of Bracket Connection –

$$(F_R)_{max} \leq V_{db}$$

V_{db} = design strength of one bolt = minimum of $\begin{cases} V_{dsb} \\ V_{dpb} \\ T_{db} \text{ (if exist)} \end{cases}$

F_R is max. for critical bolt.

* Critical Bolt

Critical bolt is one at which the resultant SF is maximum, which is farthest from CG of bolt group and which may be close to the applied load line.

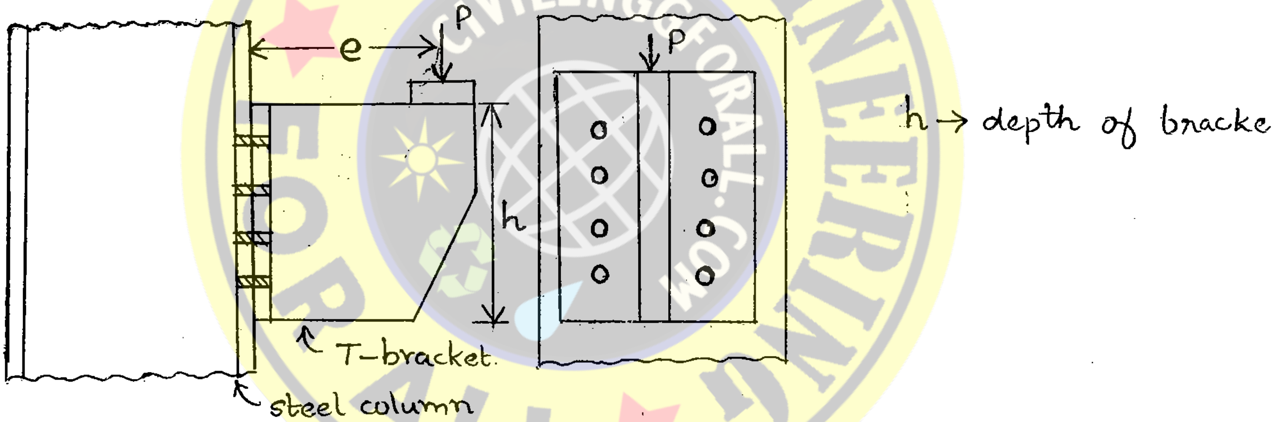
2. Bracket Type Connection - II

when load or moment is not lying in the plane of bearing type bolt group.

Bolt group is subjected to:

(i) Direct Concentric Load (P) (shear)

(ii) Bending moment ($M = Pe$) (tensile)



-Vertical SF in each bolt due to P is V_b

$$V_b = \frac{P}{n} \quad (n = \text{no. of bolts present})$$

$$\Rightarrow \frac{M}{I} = \frac{f}{y}$$

$$f = \frac{M}{I} y$$

$$T_b = f \cdot A = \frac{M}{I} y \cdot A.$$

$$\boxed{T_b \propto y_i}$$

T_b is tensile force in each bolt due to BM

$$T_{bi} = k y_i \quad ; (k = \text{elastic constant})$$

M_R of i^{th} bolt $= T_{bi} \times y_i$

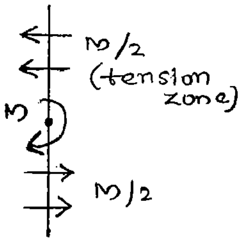
$$= K \cdot y_i \cdot y_i = K y_i^2$$

Total M_R of all bolts in tension zone, $M_R = \Sigma K y_i^2 = K \Sigma y_i^2$

$$M_R = \frac{T_{bi}}{y_i} \Sigma y_i^2$$

Assume axis of rotation is $\frac{h}{2}$ from top edge of T bracket

$$M' = M'' = M/2$$



$$M' = M_R$$

$$= \frac{T_{bi}}{y_i} \Sigma y_i^2$$

$$T_{bi} = \frac{M' y_i}{\Sigma y_i^2}$$

Tensile force in extreme bolt due to M is T_b,

$$\boxed{T_b = \frac{M' y_n}{\Sigma y_i^2}} \quad \text{(TENSION)}$$

Shear force in each bolt due to P is V_b,

$$\boxed{V_b = \frac{P}{n}} \quad \text{(SHEAR)}$$

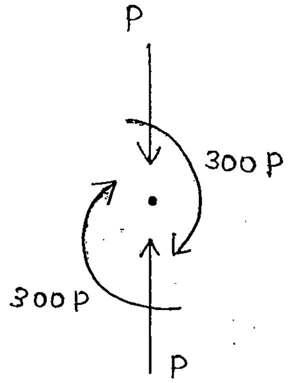
– For safety of connection, interaction equation must be satisfied as per IS 800 : 2007.

$$\boxed{\left(\frac{V_b}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1.0}$$

$V_{db} \rightarrow$ design shear capacity of bolt

$T_{db} \rightarrow$ design tensile capacity of bolt

1.



$$F_a = \frac{P}{n} = \frac{0}{4} = \underline{\underline{0}}$$

$$F_m = \frac{Mr}{\Sigma r^2}$$

r_{max} (1, 2, 3, 4)

θ_{min} (1, 2, 3, 4).

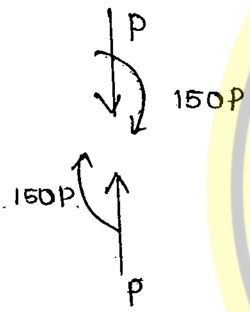
$$= \frac{600P \times 150}{4 \times 150^2}$$

$$= P.$$

$$(F_R)_{max} = \left.\begin{array}{l} F_{R1} \\ F_{R2} \\ F_{R3} \\ F_{R4} \end{array}\right\} = \sqrt{\cancelto{0}{F_a^2} + F_m^2 + 2\cancelto{0}{F_a} F_m \cos\theta} = F_m$$

$$= \underline{\underline{P}}$$

2.



$$F_a = \frac{P}{n} = \frac{0}{4} = \underline{\underline{0}}$$

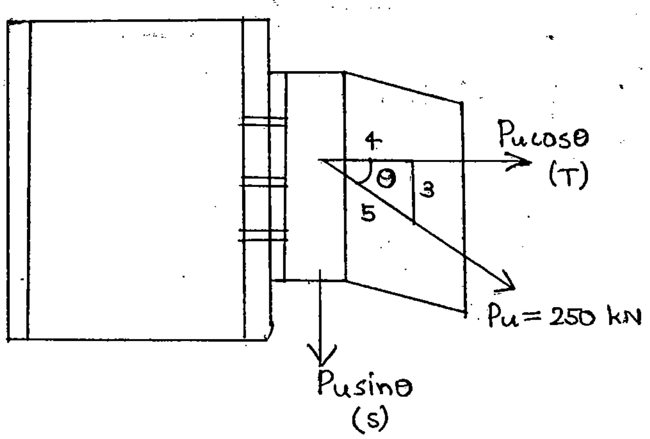
$$F_m = \frac{Mr}{\Sigma r^2}$$

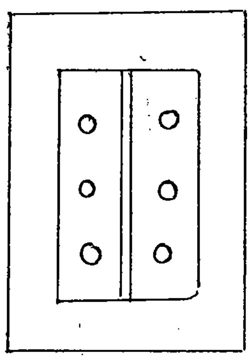
$$r = \sqrt{80^2 + 60^2} = 100 \text{ mm}.$$

$$\therefore F_m = \frac{300P \times 100}{4 \times 100^2} = \frac{3}{4} P.$$

$$(F_R)_{max} = F_m = \underline{\underline{\frac{3}{4} P}}$$

GATE 2014: Calculate SF and Tensile force (Both in kN) at bolted connection shown in fig below.



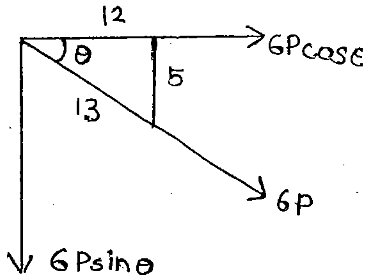


SF in each bolt due to $P_u \sin\theta = \dfrac{P_u \sin\theta}{n}$ $\qquad \sin\theta = \dfrac{3}{5}$

$$= \frac{250}{6} \times \frac{3}{5} = \underline{\underline{25 \text{ kN}}}$$

TF in each bolt due to $P_u \cos\theta = \dfrac{250}{6} \times \dfrac{4}{5} = 33.33$.

03. $V_{db} = 45$ kN ; $T_{db} = 36$ kN



Tensile force in each bolt due to

$$6P\cos\theta = T_b$$

$$= \frac{6P\cos\theta}{n} = \frac{6P}{6} \times \frac{12}{13} = \underline{\underline{\frac{12P}{13}}}$$

SF in each bolt due to $6P\sin\theta$,

$$V_b = \frac{6P\sin\theta}{n} = P\sin\theta = \underline{\underline{\frac{5P}{13}}}$$

Interaction equation as per IS 800 : 2007,

$$\left(\frac{V_b}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1$$

$$\left(\frac{5P}{13} \cdot \frac{1}{45}\right)^2 + \left(\frac{12P}{13} \cdot \frac{1}{36}\right)^2 \leq 1$$

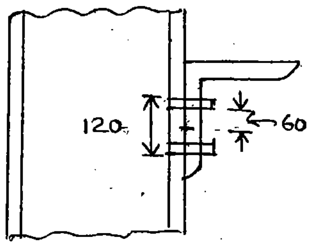
$$\left(\frac{P}{117}\right)^2 + \underline{\underline{\left(\frac{P}{39}\right)^2}} \leq 1.0$$

04. SF in each bolt due to P is $V_b = \dfrac{P}{n} = \dfrac{P}{4}$.

TF in any bolt due to M is $T_b = \dfrac{M' y_n}{\Sigma y_i^2}$.

$$M = P \times e = 150P$$

$$M' = M'' = \frac{M}{2} = 75\,P \text{ kN mm}$$



$$\therefore\ T_b = \frac{75P \times 60}{2 \times 60^2} = \frac{5P}{8}$$

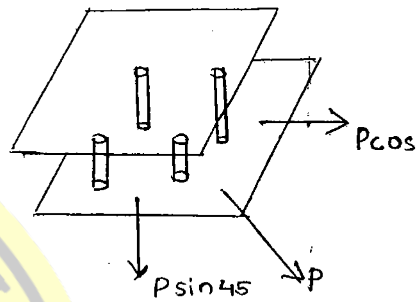
$$\left(\frac{V_b}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1.0$$

$$\left(\frac{P}{4}\cdot\frac{1}{20}\right)^2 + \left(\frac{5P}{8}\cdot\frac{1}{15}\right)^2 \leq 1.0$$

$$\left(\frac{P}{80}\right)^2 + \left(\frac{P}{24}\right)^2 \leq 1.0$$

5. SF in each bolt due $P\cos45$,

$$V_b = \frac{P\cos45}{n} = \frac{P}{4\sqrt{2}}$$



TF in each bolt due to $P\sin45$,

$$T_b = \frac{P\sin45}{n} = \frac{P}{4\sqrt{2}}.$$

For safety of connection, interaction eqn as per IS 800:2007

$$\left(\frac{V_b}{V_{db}}\right)^2 + \left(\frac{T_b}{T_{db}}\right)^2 \leq 1.$$

$$\left(\frac{P}{4\sqrt{2}}\times\frac{1}{30}\right)^2 + \left(\frac{P}{4\sqrt{2}}\cdot\frac{1}{40}\right)^2 = 1$$ (for max. value of

$$P = 135.76 \text{ kN}$$

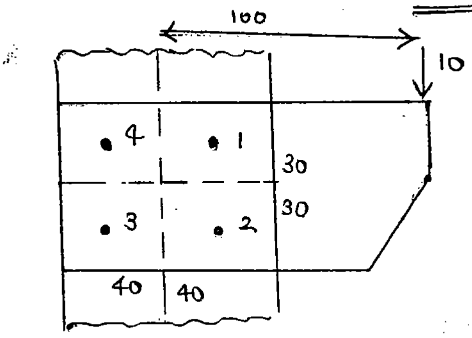
6. SF in any bolt due to M is Fm.

$$Fm_A = \frac{M r_A}{\Sigma r^2} = \frac{80\times10^3\times100}{4\times100^2}$$

$$r_A = \sqrt{\left(\frac{160}{2}\right)^2 + \left(\frac{120}{2}\right)^2} = 100 \text{ mm}$$

$$= 200 \text{ kN}$$

07.



r_{max} = 1, 2, 3, 4

θ_{min} = 1 & 2

Critical bolts : 1 & 2

$$P(e) \rightarrow \begin{cases} P = 10 \text{ kN} \rightarrow F_a \\ M = Pe = 1000 \rightarrow F_m \end{cases} \rightarrow F_R$$

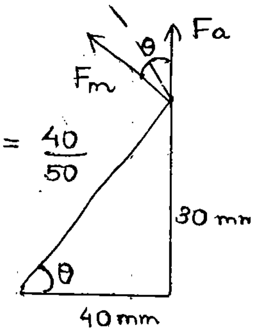
Vertical SF in each bolt due to P = 10 kN is Fa

$$F_a = \frac{P}{n} = \frac{10}{4} = 2.5 \text{ kN}$$

SF in any bolt due to M (1000 kNmm) is Fm,

$$F_m = \frac{M r_1}{\Sigma r^2} = \frac{1000 \times 50}{4 \times 50^2} = 5 \text{ kN} \qquad r_1 = \sqrt{30^2 + 40^2} = 50$$

$$(F_R)_{max} = F_{R1} = F_{R2}.$$



$$= \sqrt{F_a^2 + F_m^2 + 2 F_a \cdot F_m \cdot \cos\theta}$$

$$= \sqrt{2.5^2 + 5^2 + 2 \times 2.5 \times 0.8} = 7.16 \text{ kN}$$

8. For grade Fe 410 steel,

$f_u = 410$ MPa.

For bolt, 4.6 grade, $f_{ub} = 400$ MPa.

$d = 24$ mm

$P = 40$ kN

Critical bolt is bolt no: 4.

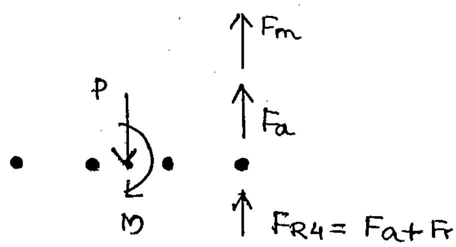
$P = 40$ kN

$M = Pe$

$$e = 200 + \frac{220}{2} = 310 \text{ mm}.$$

$M = 40 \times 310 = 12400$ kNm

$$F_a = \frac{P}{n} = \frac{40}{4} = 10 \text{ kN}$$



$$F_m = \frac{M r_4}{\Sigma r^2} = \frac{12400 \times \frac{220}{2}}{2 \times \left(\frac{220}{2}\right)^2 + 2 \times \left(\frac{140}{2}\right)^2} = 40.11 \text{ kN}$$

Max. resultant force $= (F_R)_{max} = F_{R4}$

$$= F_a + F_m = 10 + 40.11 = 50.11 \text{ kN}$$

For safety of connection,

$$(F_R)_{max} \leq V_{db}.$$

V_{db} = lesser of V_{dsb} or V_{dpb}.

$$V_{dsb} = \frac{f_{ub}}{\sqrt{3}\,\gamma_{mb}}\left(n_n A_{nb} + n_s A_{sb}\right)$$

$$= \frac{400}{\sqrt{3}\times 1.25}\left(1\times 1\times 0.78\times \frac{\pi}{4}\times 24^2 + 0\right) = \underline{\underline{65.18\ kN}}$$

$$V_{dpb} = 2.5\,d t f_{ub}.\frac{K_b}{\gamma_{mb}}.$$

K_b is min of :

$$\frac{e}{3d_0} = \frac{40}{3\times 26} = 0.512$$

$$\frac{p}{3d_0} - 0.25 = 0.51 - 0.25 = 0.26$$

$$\frac{f_{ub}}{f_u} = \frac{400}{410} = 0.97$$

$$= 2.5\times 24\times 8\times \frac{400\times 0.26}{1.25}$$

$$= \underline{\underline{38}}$$

$$\underline{\underline{V_{db} = 38\ kN}}$$

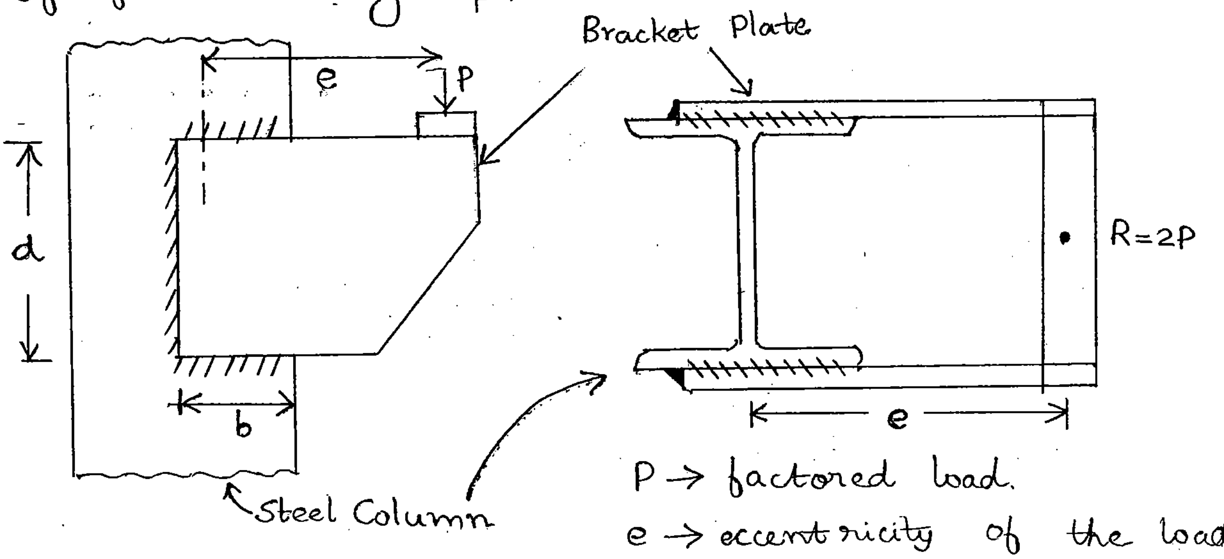
$(F_R)_{max} > V_{db} \Rightarrow$ hence connection is unsafe.

→ ECCENTRIC WELDED CONNECTIONS

1. Fillet Welds

(i) Bracket Type Connection – I

When load or moment is lying in the plane of fillet weld group.



P → factored load.

e → eccentricity of the load

d → depth of bracket plate

- Vertical shear stress in weld due to P is q_1.

$$q_1 = \frac{P}{\text{Effective sectional area.}}$$

$$q_1 = \frac{P}{L_w \cdot t_t}$$

L_w = effective length of fillet weld $= (d+2b)$

t_t = Effective throat thickness $= K.S.$

S = size of fillet weld.

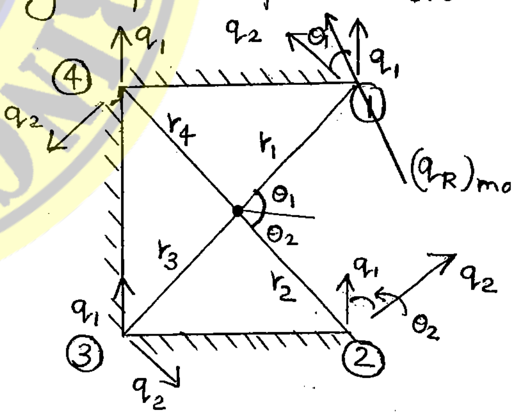
- Shear stress in weld due to twisting moment (M) is q_2

$$q_2 = \frac{M}{I_p} \times r$$

$M = Pe$

I_p = polar MI of fillet weld $= I_{zz} + I_{yy}$ of fillet weld.

r = radial distance from CG weld group to point on weld length.



* Resultant shear stress b/w q_1 & q_2 is q_R.

$$q_R = \sqrt{q_1^2 + q_2^2 + 2q_1 q_2 \cos\theta}$$

- condition for $(Q_R)_{max}$:

$r \rightarrow$ maximum

$\theta \rightarrow$ minimum.

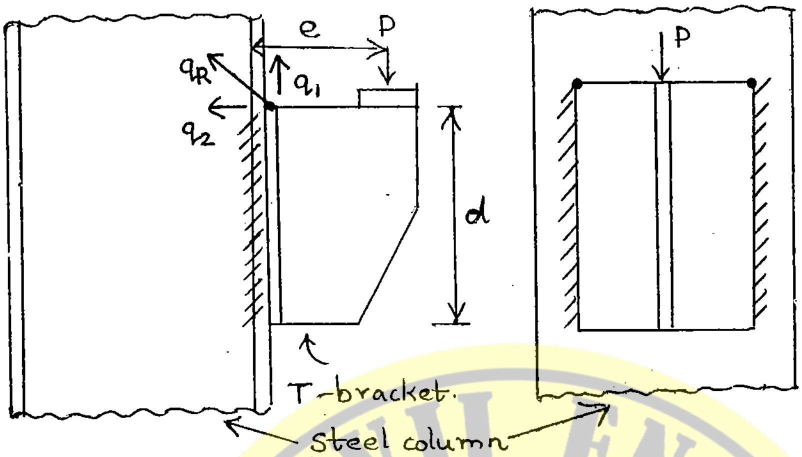
- for safety of fillet weld:

$(q_R)_{max} \leq f_{wd}$ = design shear capacity of fillet weld

$$f_{wd} = \frac{f_u}{\sqrt{3}}$$

(ii) Bracket Type Connection – II

When load or moment is not lying in the plane of fillet weld group.



Vertical shear stress in weld due to P is q_1,

$$q_1 = \frac{P}{\text{Eff. sectional area}} = \frac{P}{L_w \cdot t_t}$$

$L_w \rightarrow$ Effective length of fillet weld $= 2d$

$t_t \rightarrow$ Effective throat thickness $= K \cdot S$

$S \rightarrow$ size of fillet weld.

Stress in weld due to M is q_2,

$$q_2 = \frac{M}{I} y$$

$M = Pe$

I = MI of fillet weld about bending axis.

$$= \frac{t_t d^3}{12} \times 2 = \frac{t_t d^3}{6}$$

$y = \frac{d}{2}$ (for max q_2)

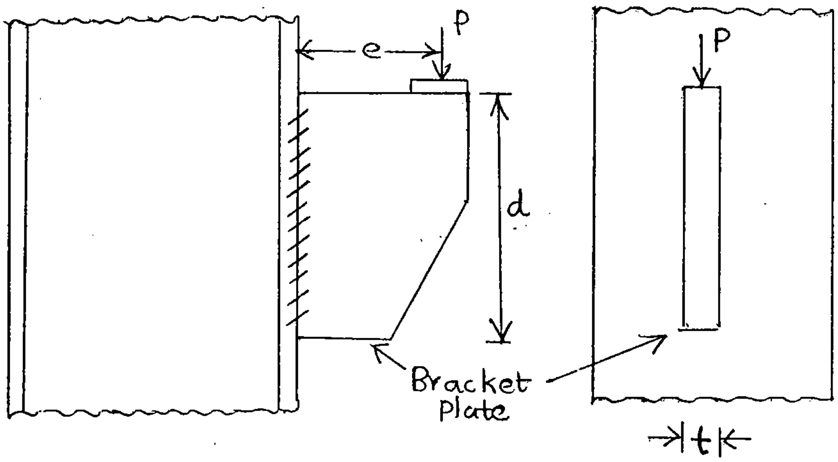
– Combined stress b/w q_1 & q_2 is q_R.

$$q_R = \sqrt{q_1^2 + q_2^2}$$

– Condition for Max. q_R : $y \rightarrow \max\left(\frac{d}{2}\right)$

– For safety of fillet weld : $(q_R)_{max} \leq f_{wd} = \frac{f_u}{\sqrt{3} \cdot \gamma_{mw}}$

2. Butt Weld is loaded eccentrically.



Butt weld is subjected:

(i) Direct concentric load (P)

(ii) Bending moment ($M = Pe$).

– Vertical shear stress in butt weld

$$q_{cal} = \frac{P}{\text{Eff. sectional area}} = \frac{P}{L_w \times t_e} = \frac{P}{d \times t}$$

L_w = eff. length of butt weld $= d$.

t_e = eff. throat thickness $= t$.

– Bending stress in weld due to M is f_{cal}.

$$f_{cal} = \frac{M}{I} \cdot y$$

$$M = P.e$$

$$y = \frac{d}{2}$$

$$I = MI \text{ of butt weld} = \frac{t d^3}{12}$$

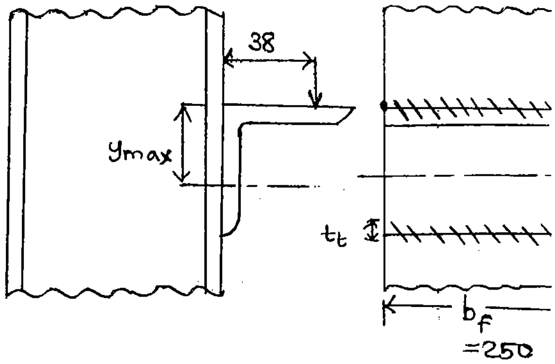
– Equivalent stress in weld, $f_e = \sqrt{3 q_{cal}^2 + f_{cal}^2}$

– For safety of butt weld:

$$(f_e)_{max} \leq \frac{f_y}{\gamma_{mo}} \quad (\text{design equivalent stress})$$

10. Let size of fillet weld is S.

For Fe 410 grade steel, $f_u = 410$ MPa



Vertical shear stress in weld due to P is q_1

$$q_1 = \frac{P}{L_w t_t} = \frac{75\times10^3}{2\times250\, t_t} = \frac{150}{t_t}\ \text{N/mm}^2.$$

Stress in weld due to M is q_2,

$$q_2 = \frac{M}{I}\cdot y$$

$$M = 75\times10^3\times38$$

$$y = \frac{d}{2} = \frac{125}{2} = 62.5 \text{ mm}$$

$$I_{zz} = (I_{CG} + Ay^2)\,2.$$

MI of fillet weld about bending axis,

$$I = 2\left(\frac{250\, t_t^3}{12} + 250\, t_t \times \left(\frac{125}{2}\right)^2\right)$$

Neglecting t_t^3 and t_t^2 terms,

$$I = 500\, t_t \times 62.5^2 \text{ mm}^4.$$

$$q_2 = \frac{75\times10^3\times38}{500\, t_t\times62.5^2}\times62.5 = \frac{91.2}{t_t}\ \text{N/mm}^2.$$

Max. combined stress b/w q_1 & q_2 is $(q_R)_{max}$,

$$(q_R)_{max} = \sqrt{q_1^2+q_2^2}$$

$$= \sqrt{\left(\frac{150}{t_t}\right)^2 + \left(\frac{91.2}{t_t}\right)^2} = \frac{175.54}{t}\ \text{N/mm}^2$$

For safety of fillet weld,

$$(q_R)_{max} \leq f_{wd} = \frac{f_u}{\sqrt{3}\,\gamma_{mw}}$$

$$\frac{175.54}{t_t} = \frac{410}{\sqrt{3}\times1.52} \Rightarrow t_t = 1.18 \text{ mm}; \quad S = \frac{t_t}{0.7}$$