JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – III Year / V Semester (2020-21)
Subject –Structural Analysis-I
Unit – IV
Presented by – Akhil Maheshwari *(Asst. Prof., Department of Civil Engineering)*

VISSION AND MISSION OF INSTITUTE

<u>Vision</u>

To become a renowned center of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

<u>Mission</u>

M-1: Focus on evaluation of learning outcomes and motivate students to inculcate research Aptitude by project based learning.
M-2: Identify, based on informed perception of Indian, Regional and global needs, areas of focus and provide platform to gain knowledge and solutions.
M-3: Offer opportunities for interaction between academia and industry.
M-4: Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

VISSION AND MISSION OF DEPARTMENT

VISION

To become a role model in the field of Civil Engineering for the sustainable development of the society.

MISSION

To provide outcome base education.

To create a learning environment conducive for achieving academic excellence.

To prepare civil engineers for the society with high ethical values.

PROGRAMME OUTCOMES (PO)

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering Fundamentals and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes (CO)

CO1. Students will be able to understand the Static and Kinematic Indeterminacy.

CO 2. Students will be able to understand the different types of Prop, Fixed and Continuous Beam.

CO 3. Students will be able to understand the Slope Deflection and Moment Distribution Method.

CO 4. Students will be able to understand Mechanical vibrations.

CO-PO MAPPING

CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	3	2	2	1	-	-	1	1	1	2
CO2	3	3	3	2	2	1	-	-	2	1	1	2
CO3	3	3	3	2	2	1	-	-	1	1	2	2
CO4	3	2	2	2	3	2	-	-	2	1	3	3

Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
1	1.1	Introduction,Scope, and Coutcome of subject	July	1	PPT
2	2.1	Introduction to Indeterminate structures	July	1	PPT
3	2.2	Degrees of freedom per node		1	PPT
4	2.3	Static and Kinematic indeterminacy (i.e. for beams, frames & portal with & without sway etc.)		1	PPT
5	2.4	Releases in structures		1	PPT
6	2.5	Maxwell's reciprocal theorem and Betti's theorem.	August	1	PPT
7	2.6	Analysis of prop cantilever structures		1	PPT
8	2.7	Analysis of Indeterminate Structure (fixed and continues beams) using Area moment method		1	PPT
9	2.8	Conjugate beam method		1	PPT
10	2.9	Three moments Theorem.		1	PPT

Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
11	3.1	Analysis of Statically Indeterminate Structures using Slope-deflection method	September	1	PPT
12	3.2	Moment-distribution method applied to continuous beams and portal frames with and without inclined members		1	PPT
13	4.1	Vibrations: Elementary concepts of structural vibration, Mathematical models, basic elements of vibratory system.		1	PPT
14	4.2	Degree of freedom. Equivalent Spring stiffness of springs in parallel and in series.		1	PPT
15	4.3	Simple Harmonic Motion: vector representation, characteristic, addition of harmonic motions, Angular oscillation.	October	1	PPT
16	4.4	Undamped free vibration of SDOF system: Newton's law of motion		1	PPT
17	4.5	D Almbert's principle, deriving equation of motions, solution of differential equation of motion, frequency & period of vibration, amplitude of motion; Introduction to damped and forced vibration.		1	PPT

Fundamentals of Vibration

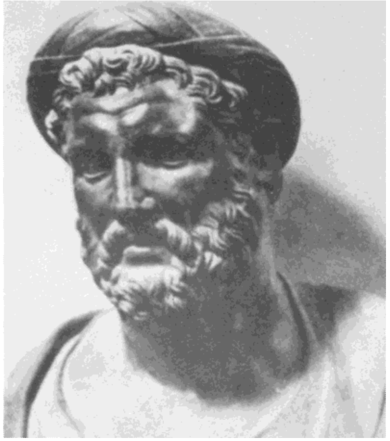
1.1 Preliminary Remarks

- Brief History of vibration
- Examination of vibration’s important role
- Vibration analysis of an engineering system
- Definitions and concepts of vibration
- Concept of harmonic analysis for general periodic motions

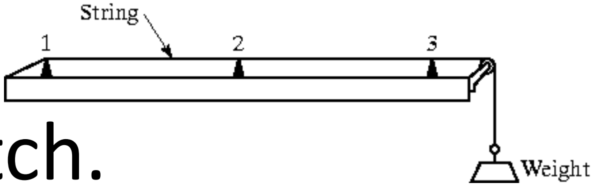
1.2 Brief History of Vibration

- Origins of vibration:

582-507 B.C. –



Pythagoras, the Greek philosopher and mathematician, is the first to investigate musical sounds on a scientific basis. He conducted experiments on a vibrating string by using a simple apparatus called a monochord. He further developed the concept of pitch.



1.2 Brief History of Vibration

Around 350 B.C. –

Aristotle wrote treatises on music and sound

In 320 B.C. –

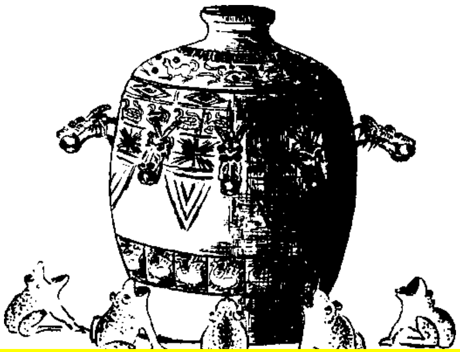
Aristoxenus wrote a three-volume work entitled *Elements of Harmony*

In 300 B.C. –

Euclid wrote a treatises *Introduction to Harmonics*

A.D. 132 –

Zhang Heng invented the world’s first seismograph to measure earthquakes



1.2 Brief History of Vibration

- **Galileo to Rayleigh:**

Galileo Galilei (1564 – 1642)

- founder of modern experimental science
- started experimenting on simple pendulum
- published a book, *Discourses Concerning Two New Sciences,* in 1638, describing resonance, frequency, length, tension and density of a vibrating stretched string

Robert Hooke (1635 – 1703)

- found relation between pitch and frequency of vibration of a string

1.2 Brief History of Vibration

Joseph Sauveur (1653 – 1716)

- coined the word "acoustics" for the science of sound
- founded *nodes, loops, harmonics* and *the fundamental frequency*
- calculated the frequency of a stretched string from the measured sag of its middle point

Sir Isaac Newton (1642-1727)

- published his monumental work, Philosophiae Naturalis Principia Mathematica, in 1686, discovering three laws of motion

1.2 Brief History of Vibration

Joseph Lagrange (1736 – 1813)

- found the analytical solution of the vibrating string and the wave equation

Simeon Poisson (1781 – 1840)

- solved the problem of vibration of a rectangular flexible membrane

R.F.A. Clebsch (1833 – 1872)

- studied the vibration of a circular membrane

Lord Baron Rayleigh

- founded Rayleigh-Ritz method, used to find frequency of vibration of a conservative system and multiple natural frequencies

1.2 Brief History of Vibration

- Recent contributions:

1902 – Frahm investigated the importance of torsional vibration study in the design of propeller shafts of steamships

Aurel Stodola (1859 – 1943)

- contributed to the study of vibration of beams, plates, and membranes.
- developed a method for analyzing vibrating beams which is applicable to turbine blades

C.G.P. De Laval (1845 – 1913)

- presented a practical solution to the problem of vibration of an unbalanced rotating disk

1.2 Brief History of Vibration

1892 – Lyapunov laid the foundations of modern stability theory which is applicable to all types of dynamical systems

1920 – Duffling and Van der Pol brought the first definite solutions into the theory of nonlinear vibrations and drew attention to its importance in engineering

– Introduction of the correlation function by Taylor

1950 – advent of high-speed digital computers

– generate approximate solutions

1.2 Brief History of Vibration

1950s – developed finite element method enabled engineers to conduct numerically detailed vibration analysis of complex mechanical, vehicular, and structural systems displaying thousands of degrees of freedom with the aid of computers

– Turner, Clough, Martin and Topp presented the finite element method as known today

1.3 Importance of the Study of Vibration

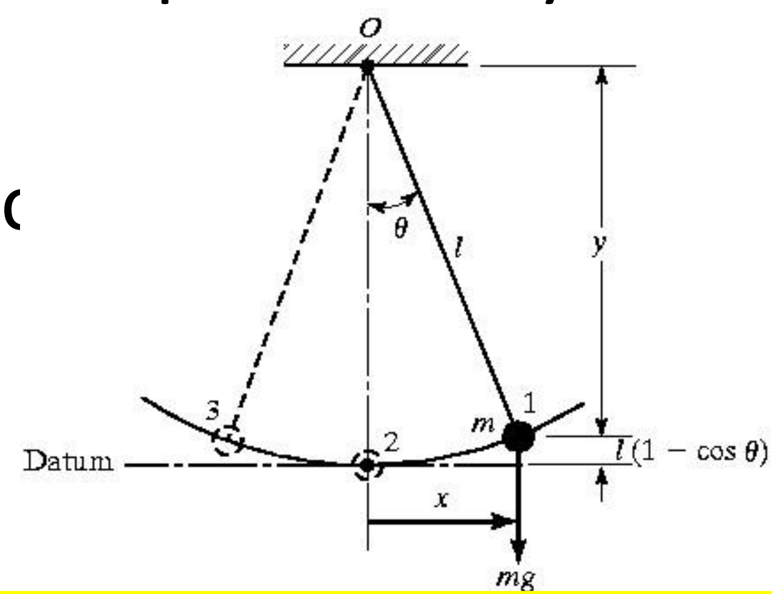
- Why study vibration?
  - ✓ Vibrations can lead to excessive deflections and failure on the machines and structures
  - ✓ To reduce vibration through proper design of machines and their mountings
  - ✓ To utilize profitably in several consumer and industrial applications
  - ✓ To improve the efficiency of certain machining, casting, forging & welding processes
  - ✓ To stimulate earthquakes for geological research and conduct studies in design of nuclear reactors

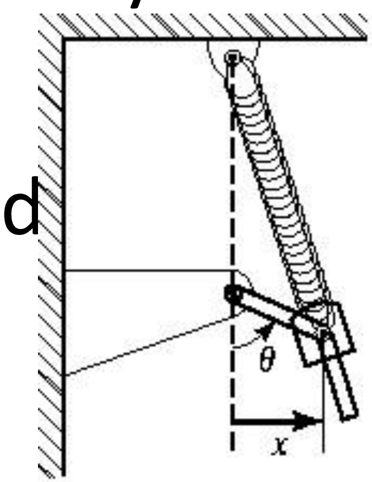
1.4 Basic Concepts of Vibration

- **Vibration** = any motion that repeats itself after an interval of time
- **Vibratory System** consists of:
  1) spring or elasticity
  2) mass or inertia
  3) damper
- Involves **transfer** of potential energy to kinetic energy and vice versa

1.4 Basic Concepts of Vibration

- Degree of Freedom *(d.o.f.)* = min. no. of independent coordinates required to determine completely the positions of all parts of a system at any instant of time
- Examples c ed systems:

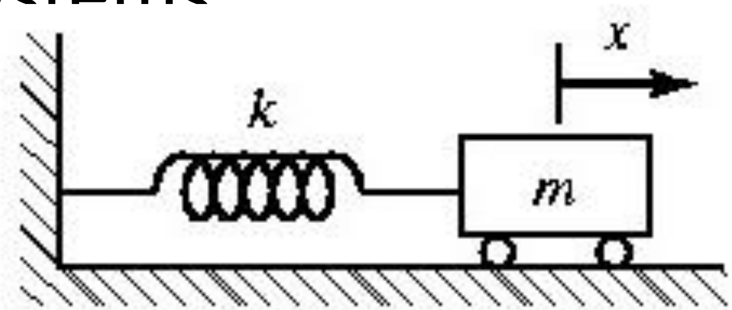




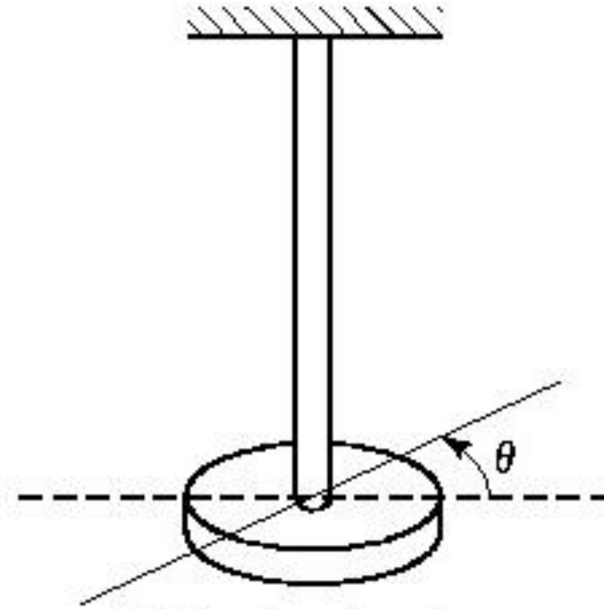
(a) Slider-crank-spring mechanism

1.4 Basic Concepts of Vibration

- Examples of single degree-of-freedom systems:



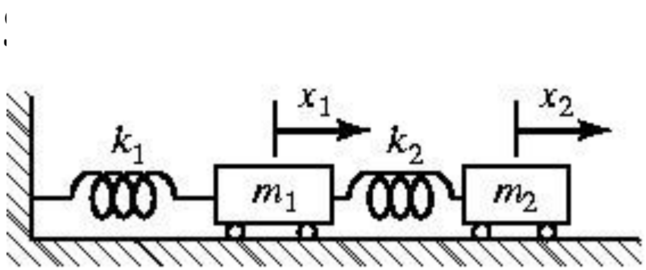
(b) Spring-mass system



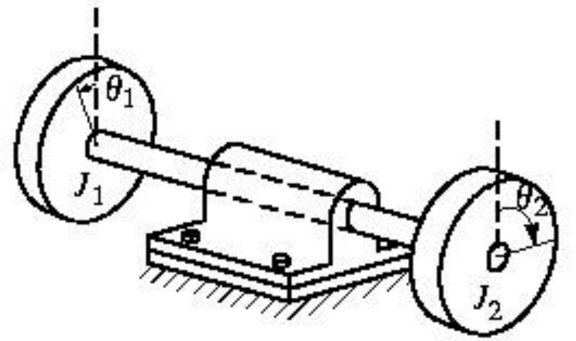
(c) Torsional system

1.4 Basic Concepts of Vibration

- Examples of Two degree-of-freedom



(a)



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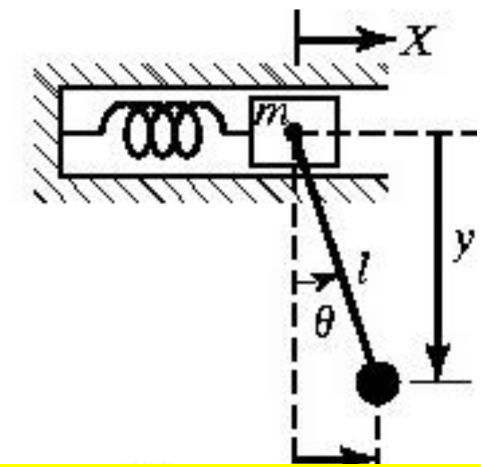

1.4 Basic Concepts of Vibration

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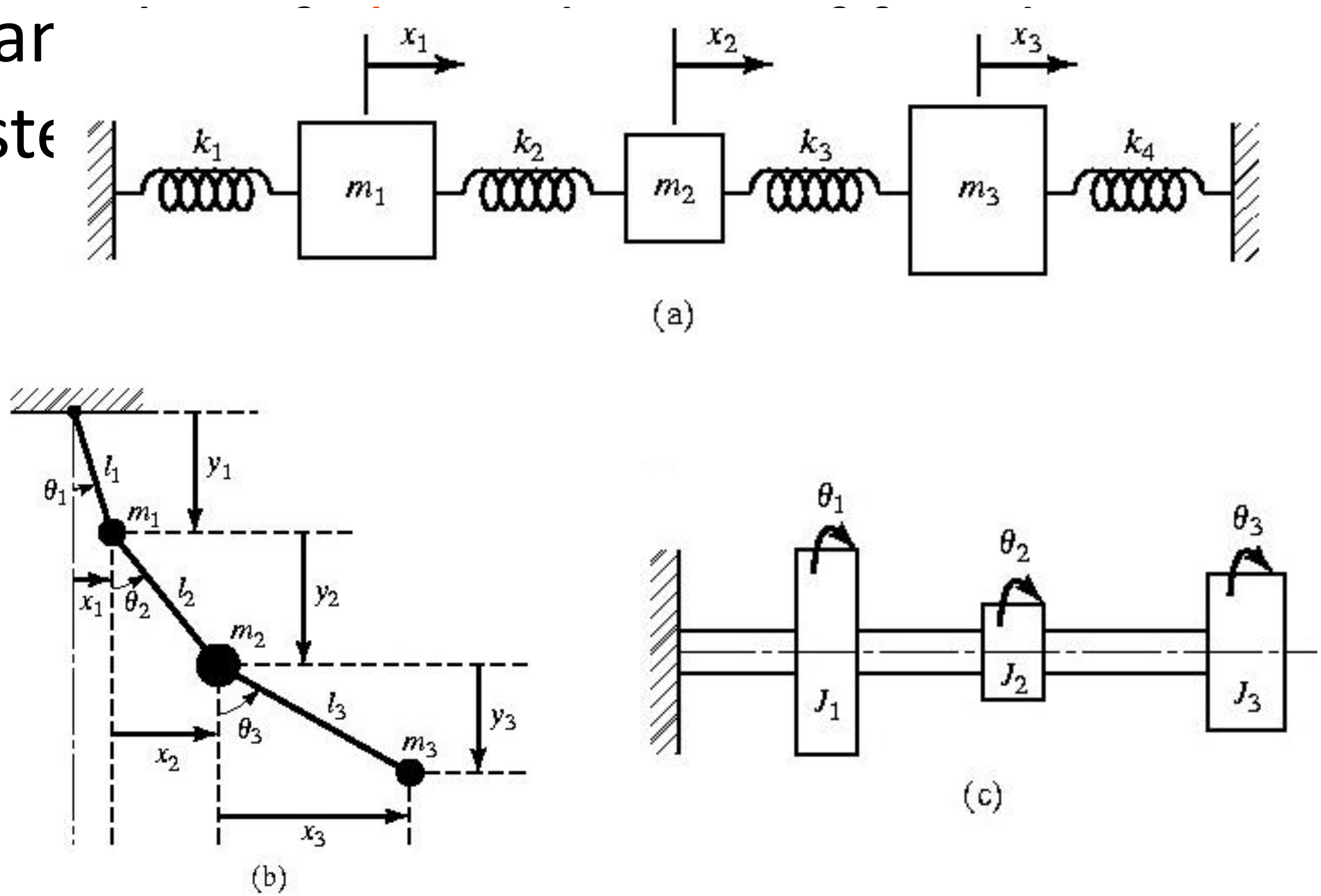
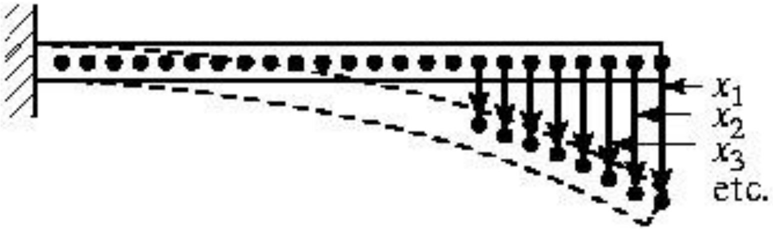


1.4 Basic Concepts of Vibration

- Example of Infinite-number-of-degrees-of-freedom system:



- *Infinite* number of degrees of freedom system are termed *continuous* or *distributed* systems
- *Finite* number of degrees of freedom are termed *discrete* or *lumped* parameter systems
- More accurate results obtained by increasing number of degrees of freedom

1.5 Classification of Vibration

- Free Vibration:
  A system is left to vibrate on its own after an initial disturbance and no external force acts on the system. E.g. simple pendulum
- Forced Vibration:
  A system that is subjected to a repeating external force. E.g. oscillation arises from diesel engines
  - *Resonance* occurs when the frequency of the external force coincides with one of the natural frequencies of the system

1.5 Classification of Vibration

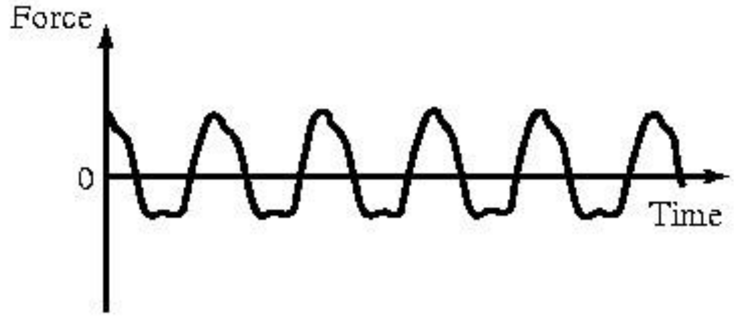
- **Undamped Vibration:**
  When ***no*** energy is lost or dissipated in friction or other resistance during oscillations
- **Damped Vibration:**
  When ***any*** energy is lost or dissipated in friction or other resistance during oscillations
- **Linear Vibration:**
  When ***all*** basic components of a vibratory system, i.e. the spring, the mass and the damper behave linearly

1.5 Classification of Vibration

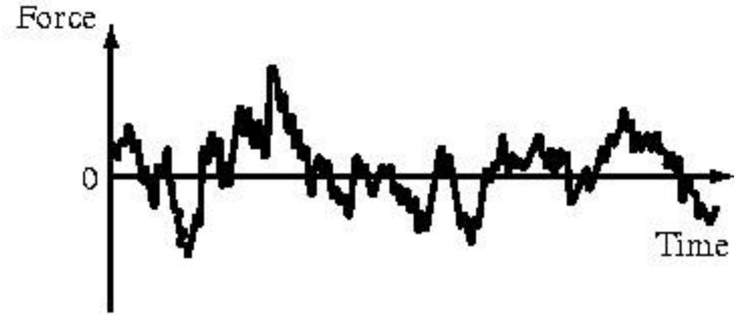
- Nonlinear Vibration:
  If ***any*** of the components behave nonlinearly
- Deterministic Vibration:
  If the value or magnitude of the excitation (force or motion) acting on a vibratory system is known at any given time
- **Nondeterministic or** random Vibration:
  When the value of the excitation at a given time cannot be predicted

1.5 Classification of Vibration

- Examples of deterministic and random excitation:



(a) A deterministic (periodic) excitation



(b) A random excitation

1.6 Vibration Analysis Procedure

Step 1: Mathematical Modeling

Step 2: Derivation of Governing Equations

Step 3: Solution of the Governing Equations

Step 4: Interpretation of the Results

1.6 Vibration Analysis Procedure

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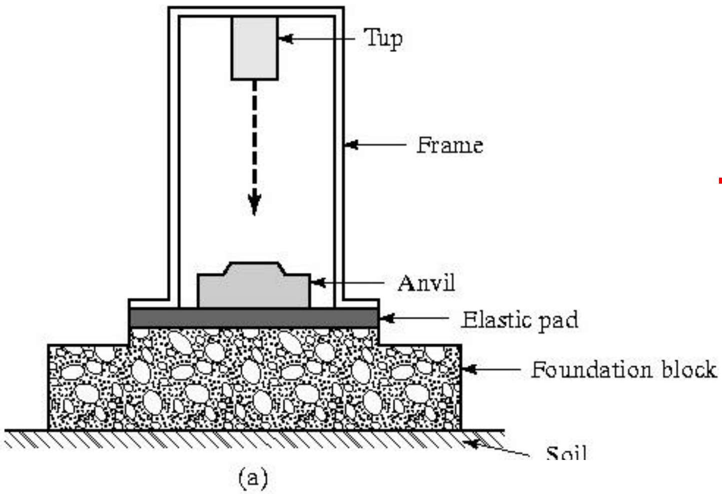
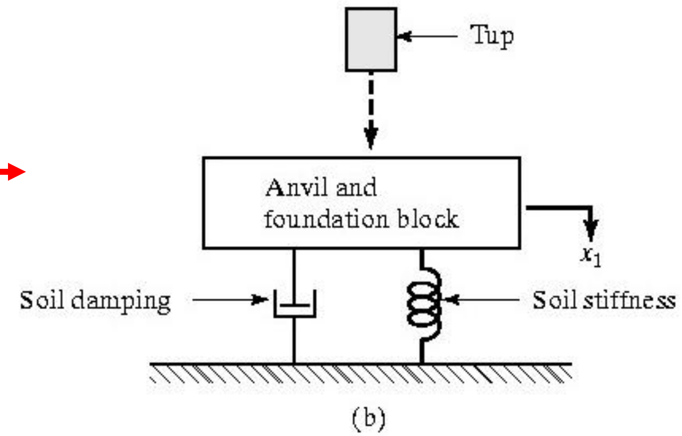
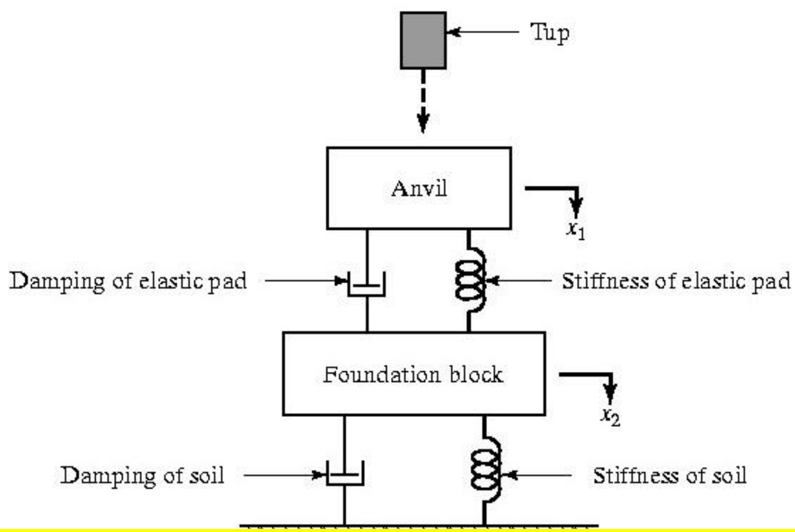
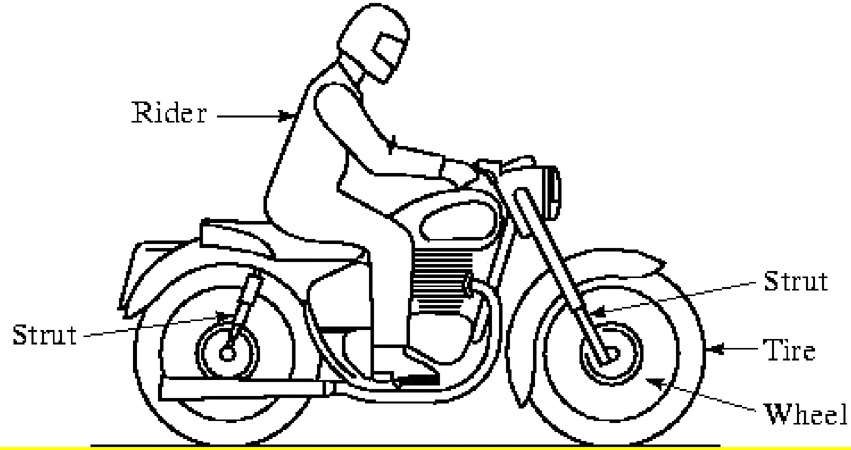






Example 1.1
Mathematical Model of a Motorcycle

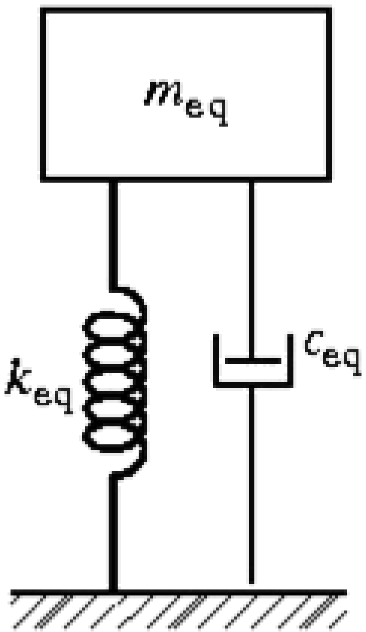
Figure 1.18(a) shows a motorcycle with a rider. Develop a sequence of three mathematical models of the system for investigating vibration in the vertical direction. Consider the elasticity of the tires, elasticity and damping of the struts (in the vertical direction), masses of the wheels, and elasticity, damping, and mass of the rider.



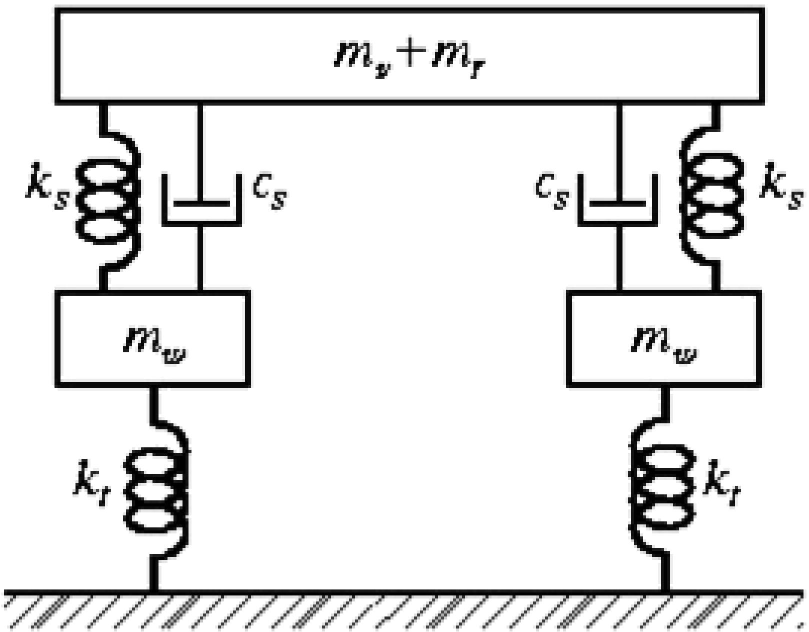
Example 1.1 Solution

We start with the simplest model and refine it gradually. When the equivalent values of the mass, stiffness, and damping of the system are used, we obtain a single-degree of freedom model of the motorcycle with a rider as indicated in Fig. 1.18(b). In this model, the equivalent stiffness (k_{eq}) includes the stiffness of the tires, struts, and rider. The equivalent damping constant (c_{eq}) includes the damping of the struts and the rider. The equivalent mass includes the mass of the wheels, vehicle body and the rider.

Example 1.1 Solution



(b)

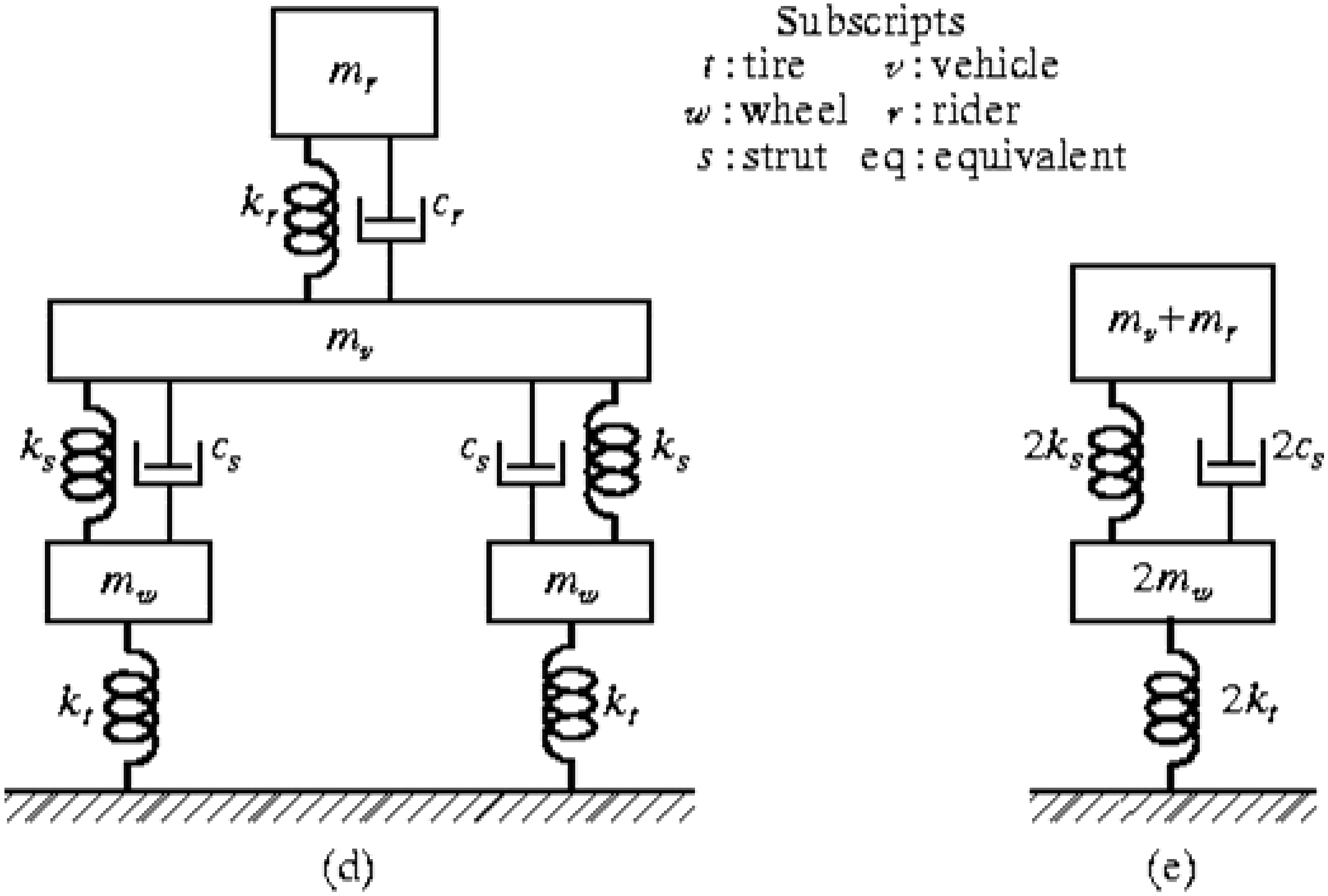


(c)

Example 1.1 Solution

This model can be refined by representing the masses of wheels, elasticity of tires, and elasticity and damping of the struts separately, as shown in Fig. 1.18(c). In this model, the mass of the vehicle body (m_v) and the mass of the rider (m_r) are shown as a single mass, $m_v + m_r$. When the elasticity (as spring constant k_r) and damping (as damping constant c_r) of the rider are considered, the refined model shown in Fig. 1.18(d) can be obtained.

Example 1.1 Solution



Example 1.1 Solution

Note that the models shown in Figs. 1.18(b) to (d) are not unique. For example, by combining the spring constants of both tires, the masses of both wheels, and the spring and damping constants of both struts as single quantities, the model shown in Fig. 1.18(e) can be obtained instead of Fig. 1.18(c).

1.7 Spring Elements

- *Linear* spring is a type of mechanical link that is generally assumed to have negligible mass and damping
- *Spring force* is given by:

$$F = kx \qquad (1.1)$$

F = spring force,
k = spring stiffness or spring constant, and
x = deformation (displacement of one end with respect to the other)

1.7 Spring Elements

- *Work done (U)* in deforming a spring or the strain (potential) energy is given by:

$$U = \frac{1}{2}kx^2 \qquad (1.2)$$

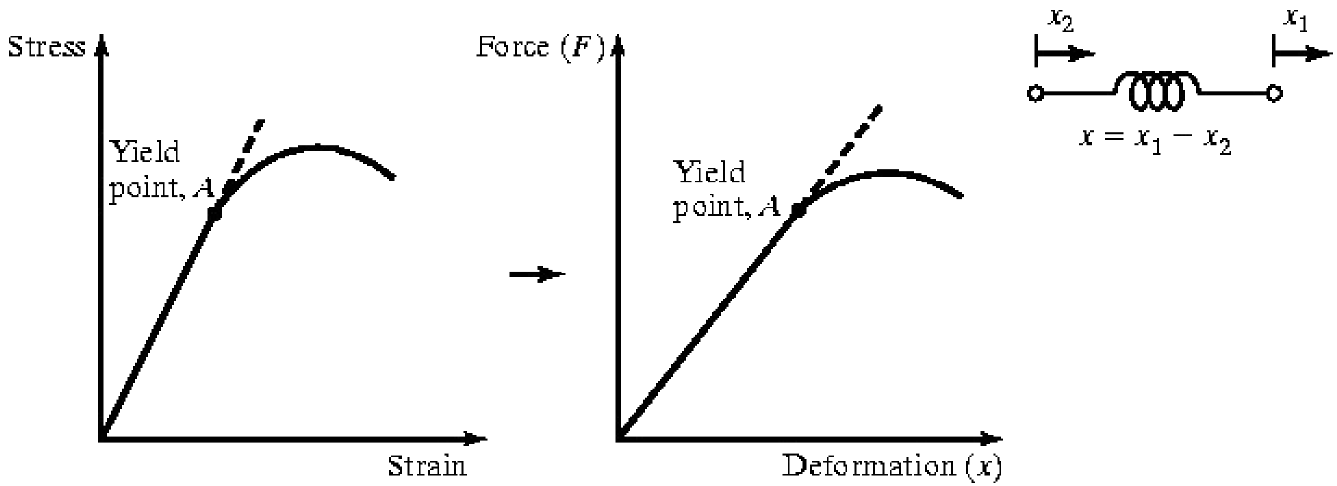
- When an incremental force *ΔF* is added to F:

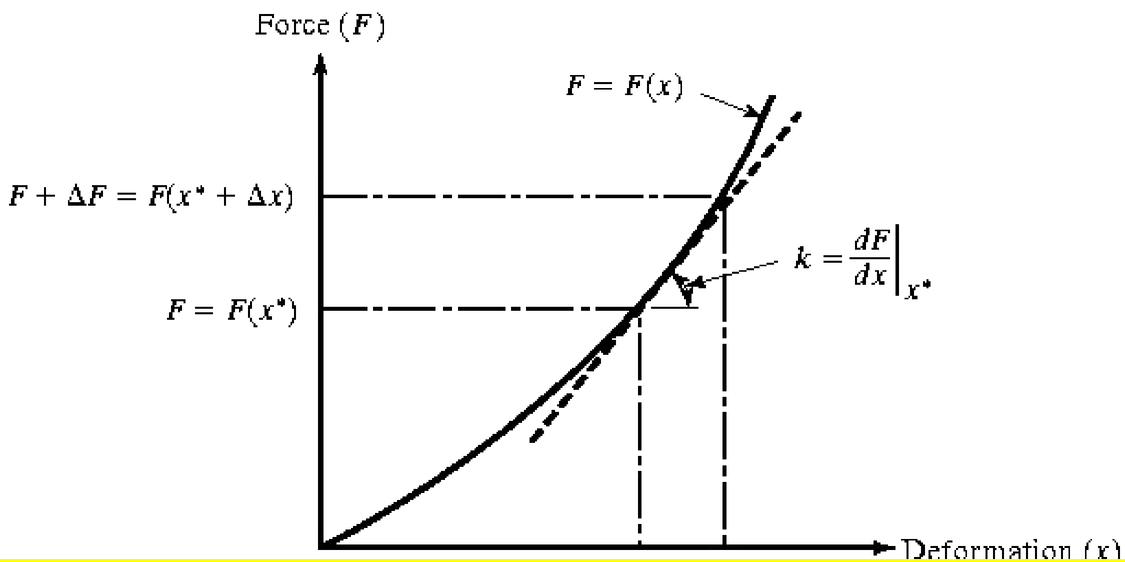
$$F + \Delta F = F(x^* + \Delta x)$$

$$= F(x^*) + \left.\frac{dF}{dx}\right|_{x^*} (\Delta x)$$

$$+ \frac{1}{2!}\left.\frac{d^2F}{dx^2}\right|_{x^*} (\Delta x)^2 + \ldots \qquad (1.3)$$

1.7 Spring Elements





1.7 Spring Elements

- *Static deflection* of a beam at the free end is given by:

$$\delta_{st} = \frac{Wl^3}{3EI} \qquad (1.6)$$

W = mg is the weight of the mass m,
E = Young's Modulus, and
I = moment of inertia of cross-section of beam

- *Spring Constant* is given by:

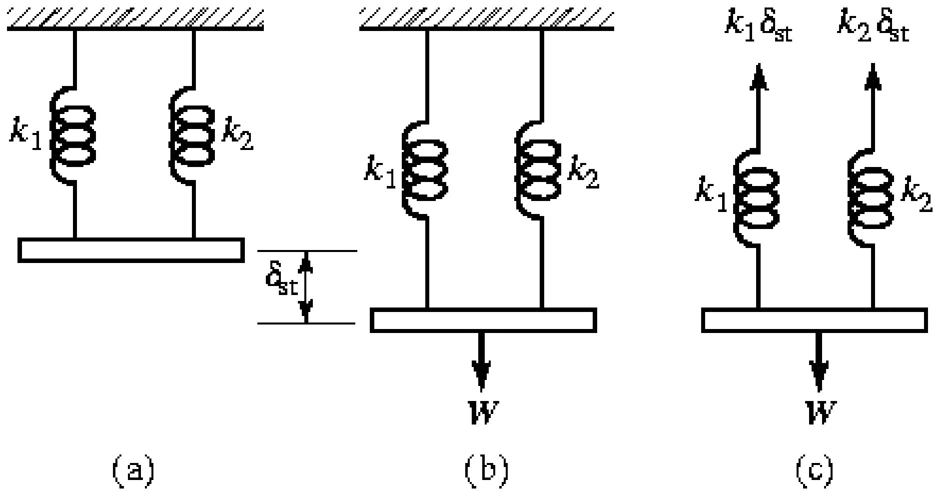
$$k = \frac{W}{\delta_{st}} = \frac{3EI}{l^3} \qquad (1.7)$$

1.7 Spring Elements

❑ Combination of Springs:

1) Springs in parallel – if we have *n* spring constants k_1, k_2, …, k_n in *parallel*, then the equivalent spring constant k_{eq} is:

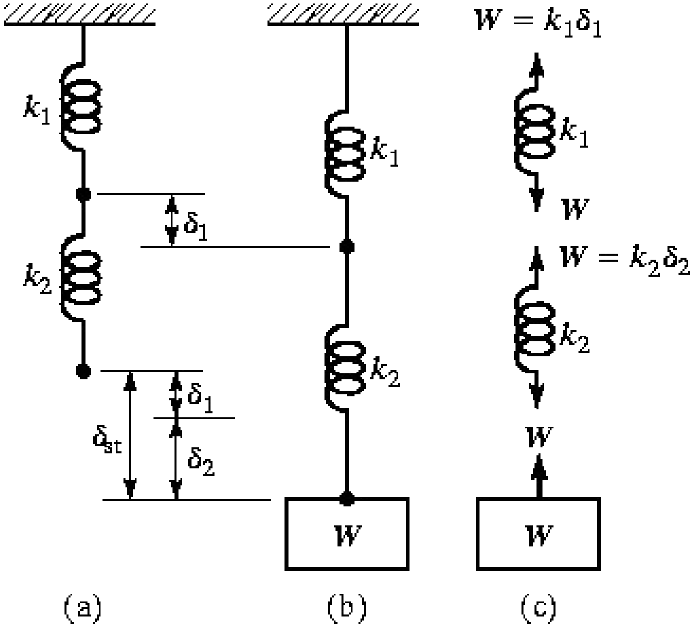
$$k_{eq} = k_1 + k_2 + \ldots + k_n \qquad (1.11)$$



1.7 Spring Elements

- Combination of Springs:

  2) Springs in series – if we have *n* spring constants $k_1, k_2, \ldots, k_n$ in *series*, then the equivalent spring constant k_{eq} is:

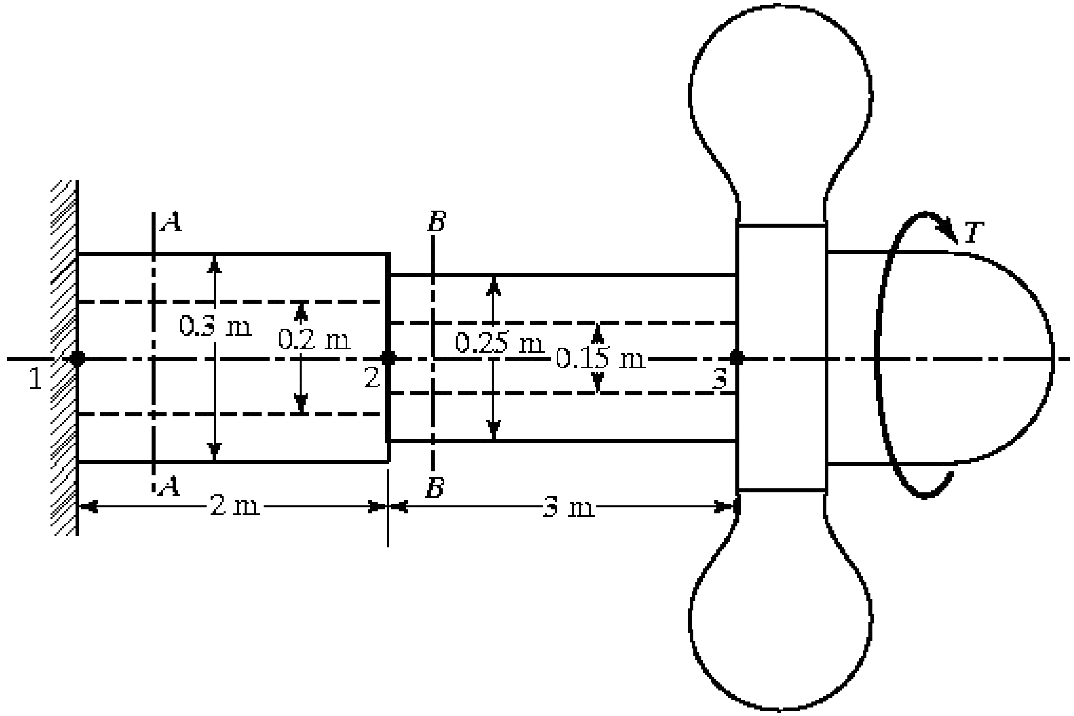


$$\frac{1}{k_{eq}} = \frac{1}{k_1} + \frac{1}{k_2} + \ldots + \frac{1}{k_n} \qquad (1.17)$$

Example 1.3

Torsional Spring Constant of a Propeller Shaft

Determine the torsional spring constant of the speed propeller shaft shown in Fig. 1.25.



Example 1.3 Solution

We need to consider the segments 12 and 23 of the shaft as springs in combination. From Fig. 1.25, the torque induced at any cross section of the shaft (such as *AA* or *BB*) can be seen to be equal to the torque applied at the propeller, *T*. Hence, the elasticities (springs) corresponding to the two segments 12 and 23 are to be considered as series springs. The spring constants of segments 12 and 23 of the shaft (k_{t12} and k_{t23}) are given by

Example 1.3 Solution

$$k_{t_{12}} = \frac{GJ_{12}}{l_{12}} = \frac{G\pi(D_{12}^4 - d_{12}^4)}{32 l_{12}} = \frac{(80\times 10^9)\pi(0.3^4 - 0.2^4)}{32(2)}$$

$$= 25.5255\times 10^6 \text{ N - m/rad}$$

$$k_{t_{23}} = \frac{GJ_{23}}{l_{23}} = \frac{G\pi(D_{23}^4 - d_{23}^4)}{32 l_{23}} = \frac{(80\times 10^9)\pi(0.25^4 - 0.15^4)}{32(3)}$$

$$= 8.9012\times 10^6 \text{ N - m/rad}$$

Example 1.3 Solution

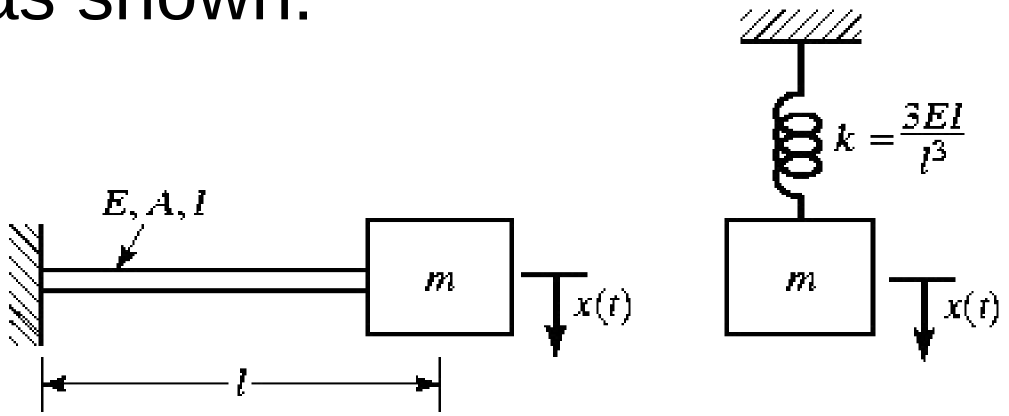
Since the springs are in series, Eq. (1.16) gives

$$k_{t_{eq}} = \frac{k_{t_{12}} k_{t_{23}}}{k_{t_{12}} + k_{t_{23}}} = \frac{(25.5255 \times 10^6)(8.9012 \times 10^6)}{(25.5255 \times 10^6 + 8.9012 \times 10^6)}$$

$$= 6.5997 \times 10^6 \text{ N - m/rad}$$

1.8 Mass or Inertia Elements

- Using mathematical model to represent the actual vibrating system
  - E.g. In figure below, the mass and damping of the beam can be disregarded; the system can thus be modeled as a spring-mass system as shown.



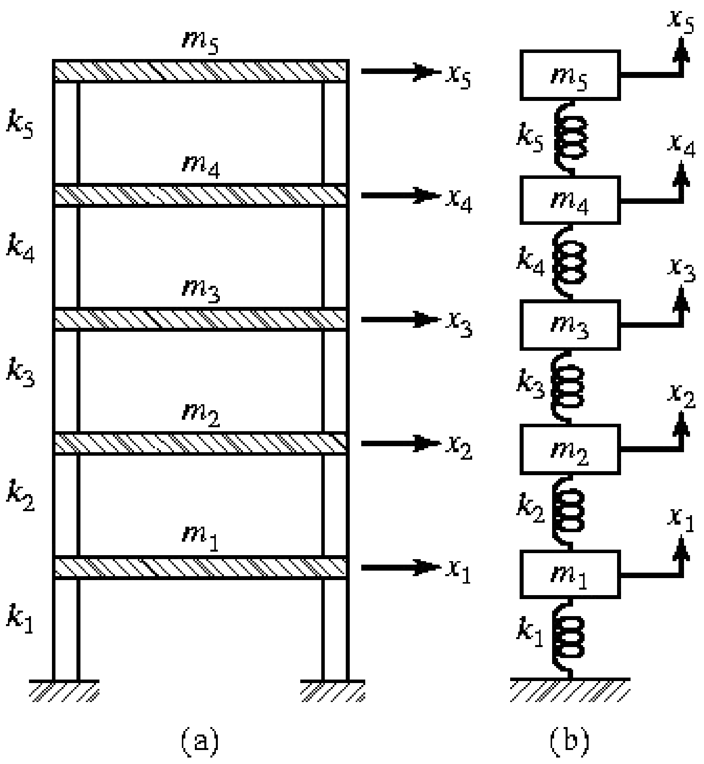
(a) Actual system

(b) Single degree of freedom model

1.8 Mass or Inertia Elements

Combination of Masses

- E.g. Assume that the mass of the frame is negligible compared to the masses of the floors. The masses of various floor levels represent the mass elements, and the elasticities of the vertical members denote the spring elements.



1.9 Damping Elements

- **Viscous Damping:**
  Damping force is proportional to the velocity of the vibrating body in a fluid medium such as air, water, gas, and oil.
- **Coulomb or Dry Friction Damping:**
  Damping force is constant in magnitude but opposite in direction to that of the motion of the vibrating body between dry surfaces
- **Material or Solid or Hysteretic Damping:**
  Energy is absorbed or dissipated by material during deformation due to friction between internal planes

1.9 Damping Elements

- *Shear Stress (τ)* developed in the fluid layer at a distance y from the fixed plate is:

$$\tau = \mu \frac{du}{dy} \qquad (1.26)$$

where $du/dy = v/h$ is the velocity gradient.

- *Shear or Resisting Force (F)* developed at the bottom surface of the moving plate is:

$$F = \tau A = \mu \frac{Av}{h} = cv \qquad (1.27)$$

where A is the surface area of the moving plate.

Where A is the surface area of the moving

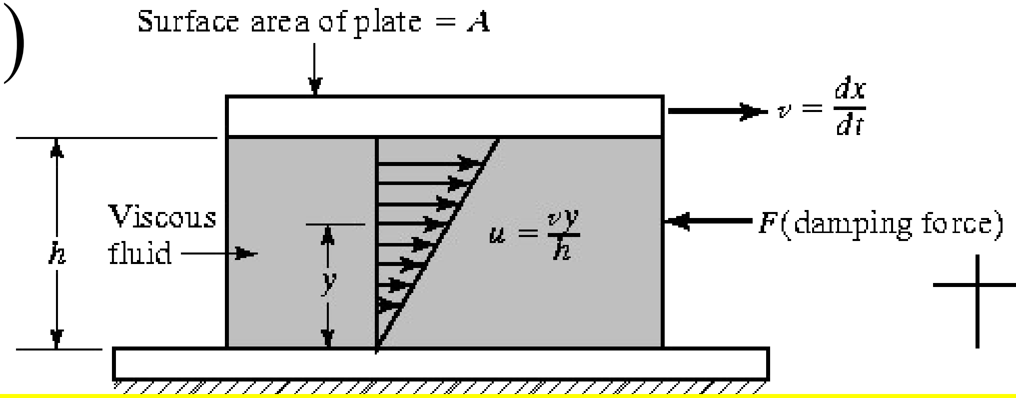
1.9 Damping Elements

and $$c = \frac{\mu A}{h} \qquad (1.28)$$

is called the damping constant.

- If a damper is nonlinear, a linearization process is used about the operating velocity (v^*) and the equivalent damping constant is:

$$c = \left.\frac{dF}{dv}\right|_{v^*} \qquad (1.29)$$



1.10 Harmonic Motion

- Periodic Motion: motion repeated after equal intervals of time
- Harmonic Motion: simplest type of periodic motion

$$x = A\sin\theta = A\sin\omega t \qquad (1.30)$$

- Displacement (x): *(on horizontal axis)*
- Velocity:

$$\frac{dx}{dt} = \omega A\cos\omega t \qquad (1.31)$$

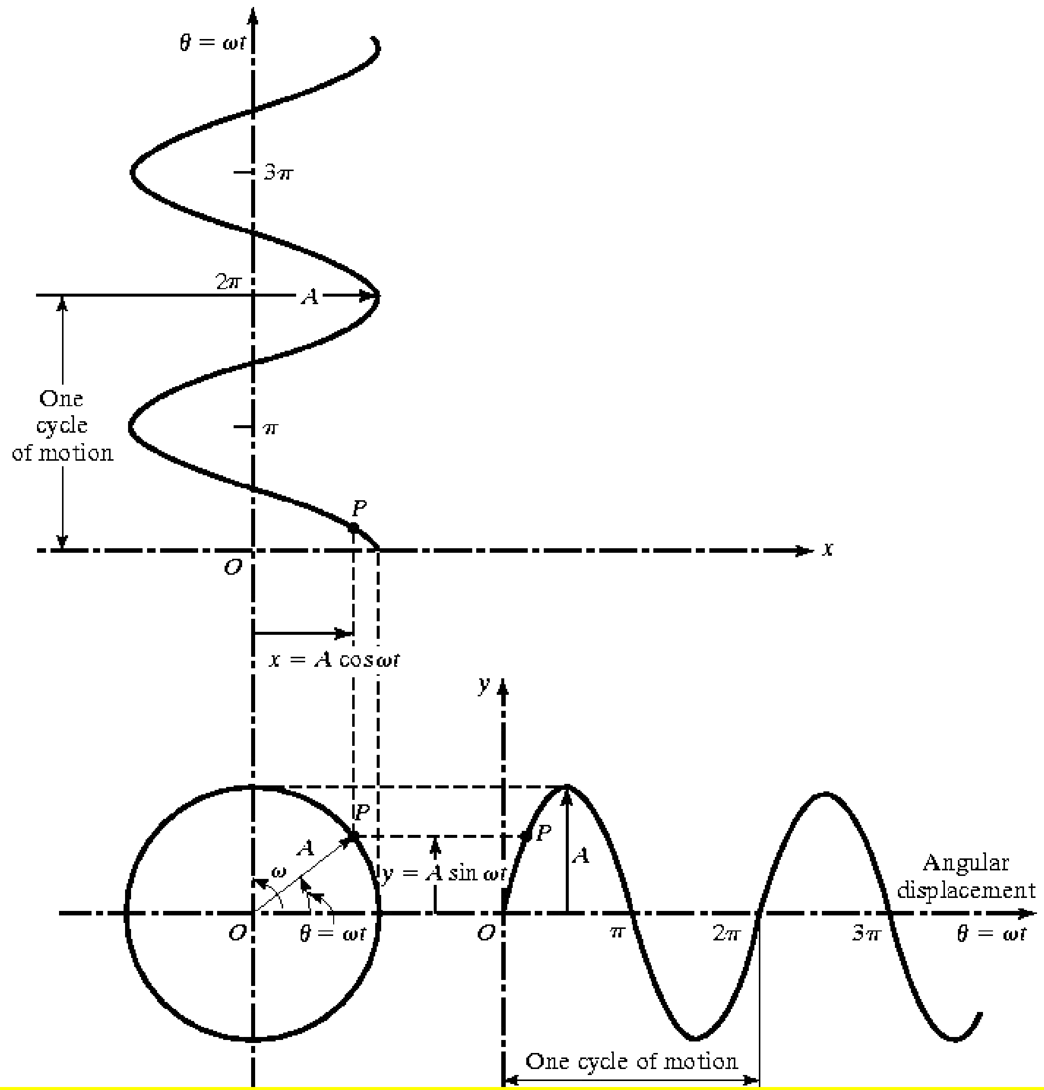
- Acceleration:

$$\frac{d^2x}{dt^2} = -\omega^2 A\sin\omega t = -\omega^2 x \qquad (1.32)$$

1.10 Harmonic Motion

- Scotch yoke mechanism:

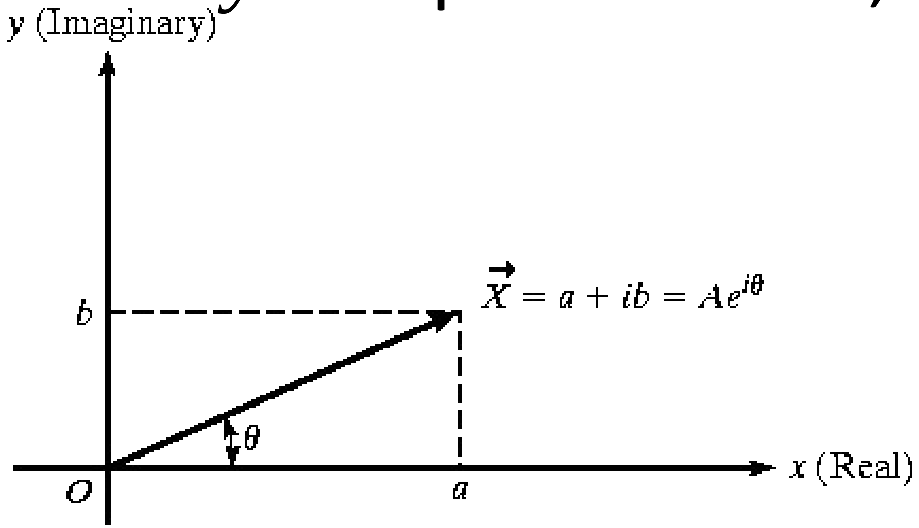
  The similarity between cyclic (harmonic) and sinusoidal motion.



1.10 Harmonic Motion

- Complex number representation of harmonic motion: $\breve{X} = a + ib \qquad (1.35)$

where $i = \sqrt{(-1)}$ and a and b denote the real and imaginary x and y components of X, respectively.



1.10 Harmonic Motion

- Also, Eqn. (1.36) can be expressed as

$$\vec{X} = A\cos\theta + iA\sin\theta \qquad (1.36)$$

$$\vec{X} = A(\cos\theta + i\sin\theta) = Ae^{i\theta} \qquad (1.43)$$

- Thus,

$$A_j = \sqrt{(a_j^2 + b_j^2)};\quad j = 1, 2 \qquad (1.47)$$

$$\theta_j = \tan^{-1}\left(\frac{b_j}{a_j}\right);\quad j = 1, 2 \qquad (1.48)$$

1.10 Harmonic Motion

- Operations on Harmonic Functions:
  - Rotating Vector, $\overset{\varpi}{X} = Ae^{i\omega t}$ (1.51)

$$\text{Displacement} = \text{Re}[Ae^{i\omega t}] = A\cos\omega t \qquad (1.54)$$

$$\text{Velocity} = \text{Re}[i\omega Ae^{i\omega t}] = -\omega A\sin\omega t$$

$$= -\omega A\cos(\omega t + 90°) \qquad (1.55)$$

$$\text{Acceleration} = \text{Re}[-\omega^2 Ae^{i\omega t}]$$

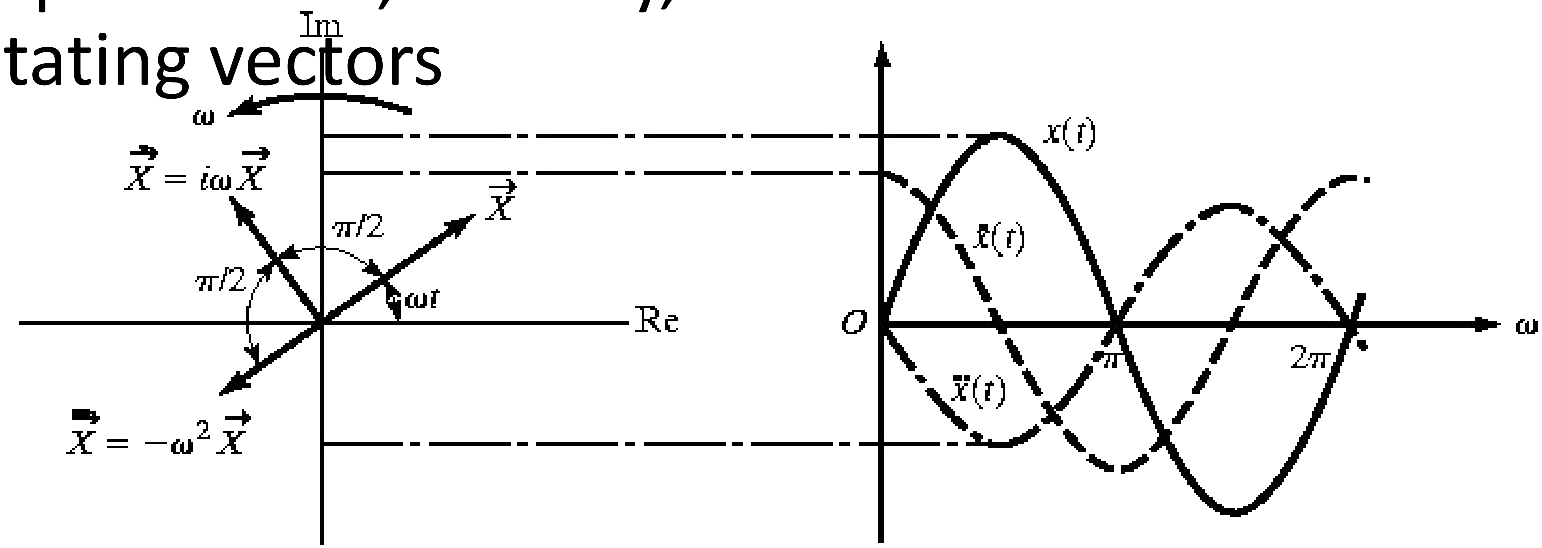
$$= -\omega^2 A\cos\omega t$$

$$= \omega^2 A\cos(\omega t + 180°) \qquad (1.56)$$

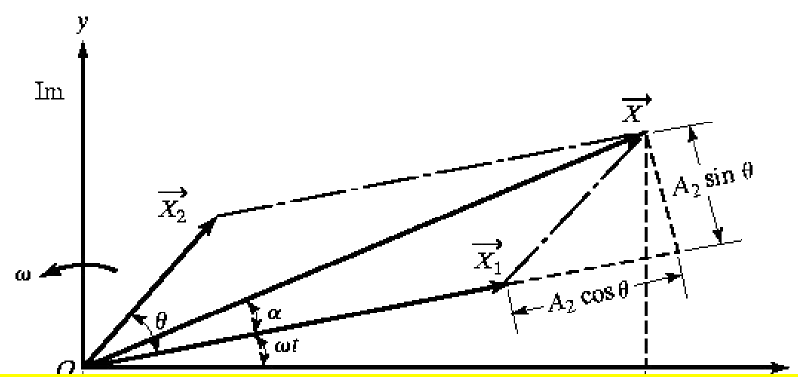
where Re denotes the real part

1.10 Harmonic Motion

- Displacement, velocity, and accelerations as rotating vectors



- Vectorial addition of harmonic functions



Example 1.11

Addition of Harmonic Motions

Find the sum of the two harmonic motions

$$x_1(t) = 10\cos\omega t \ \text{ and } \ x_2(t) = 15\cos(\omega t + 2).$$

Solution:

Method 1: By using trigonometric relations: Since the circular frequency is the same for both $x_1(t)$ and $x_2(t)$, we express the sum as

$$x(t) = A\cos(\omega t + \alpha) = x_1(t) + x_2(t) \qquad \text{(E.1)}$$

Example 1.11 Solution

That is,

$$A\left(\cos\omega t\cos\alpha-\sin\omega t\sin\alpha\right)=10\cos\omega t+15\cos(\omega t+2)$$

$$=10\cos\omega t+15(\cos\omega t\cos 2-\sin\omega t\sin 2) \qquad \text{(E.2)}$$

That is,

$$\cos\omega t(A\cos\alpha)-\sin\omega t(A\sin\alpha)=\cos\omega t(10+15\cos 2)$$

$$-\sin\omega t(15\sin 2) \qquad \text{(E.3)}$$

By equating the corresponding coefficients of cosωt and sinωt on both sides, we obtain

$$A\cos\alpha=10+15\cos 2$$

$$A\sin\alpha=15\sin 2$$

$$A=\sqrt{\left(10+15\cos 2\right)^2+(15\sin 2)^2}$$

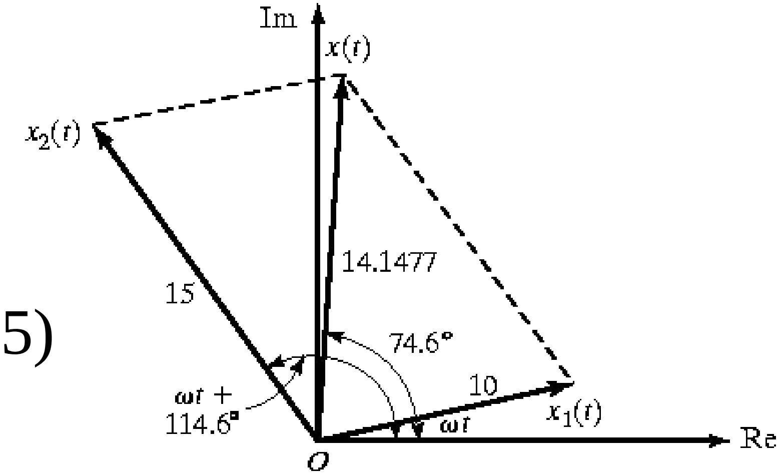
$$=14.1477 \qquad \text{(E.4)}$$

Example 1.11 Solution

and

$$\alpha = \tan^{-1}\left(\frac{15\sin 2}{10+15\cos 2}\right)$$

$$= 74.5963° \qquad \text{(E.5)}$$



Method 2: By using vectors: For an arbitrary value of *ωt*, the harmonic motions $x_1(t)$ and $x_2(t)$ can be denoted graphically as shown in Fig. 1.43. By adding them vectorially, the resultant vector *x(t)* can be found to be

$$x(t) = 14.1477\cos(\omega t + 74.5963°) \qquad \text{(E.6)}$$

Example 1.11 Solution

Method 3: By using complex number representation: the two harmonic motions can be denoted in terms of complex numbers:

$$x_1(t) = \mathrm{Re}\left[A_1 e^{i\omega t}\right] \equiv \mathrm{Re}\left[10 e^{i\omega t}\right]$$

$$x_2(t) = \mathrm{Re}\left[A_2 e^{i(\omega t+2)}\right] \equiv \mathrm{Re}\left[15 e^{i(\omega t+2)}\right] \qquad \text{(E.7)}$$

The sum of $x_1(t)$ and $x_2(t)$ can be expressed as

$$x(t) = \mathrm{Re}\left[A e^{i(\omega t+\alpha)}\right] \qquad \text{(E.8)}$$

where A and α can be determined using Eqs. (1.47) and (1.48) as A = 14.1477 and α = 74.5963º

1.10 Harmonic Motion

❑ Definitions of Terminology:

- Amplitude (A) is the maximum displacement of a vibrating body from its equilibrium position
- Period of oscillation (T) is time taken to complete one cycle of motion

$$T = \frac{2\prod}{\omega} \qquad (1.59)$$

- Frequency of oscillation (f) is the no. of cycles per unit time

$$f = \frac{1}{T} = \frac{\omega}{2\pi} \qquad (1.60)$$

1.10 Harmonic Motion

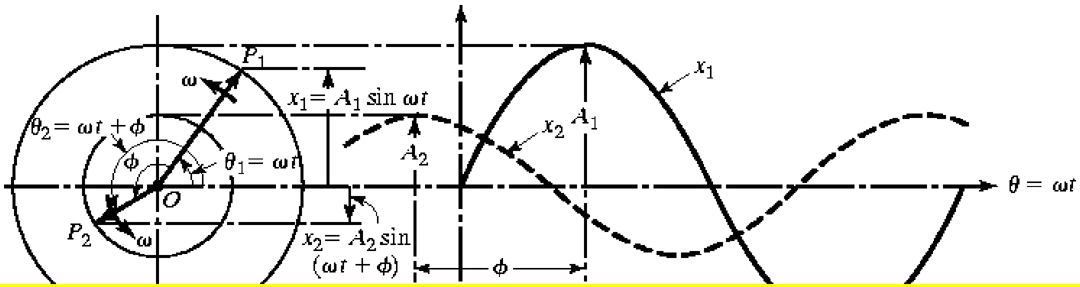
Definitions of Terminology:

- Natural frequency is the frequency which a system oscillates without external forces

- Phase angle (ϕ) is the angular difference between two synchronous harmonic motions

$$x_1 = A_1 \sin \omega t \qquad (1.61)$$

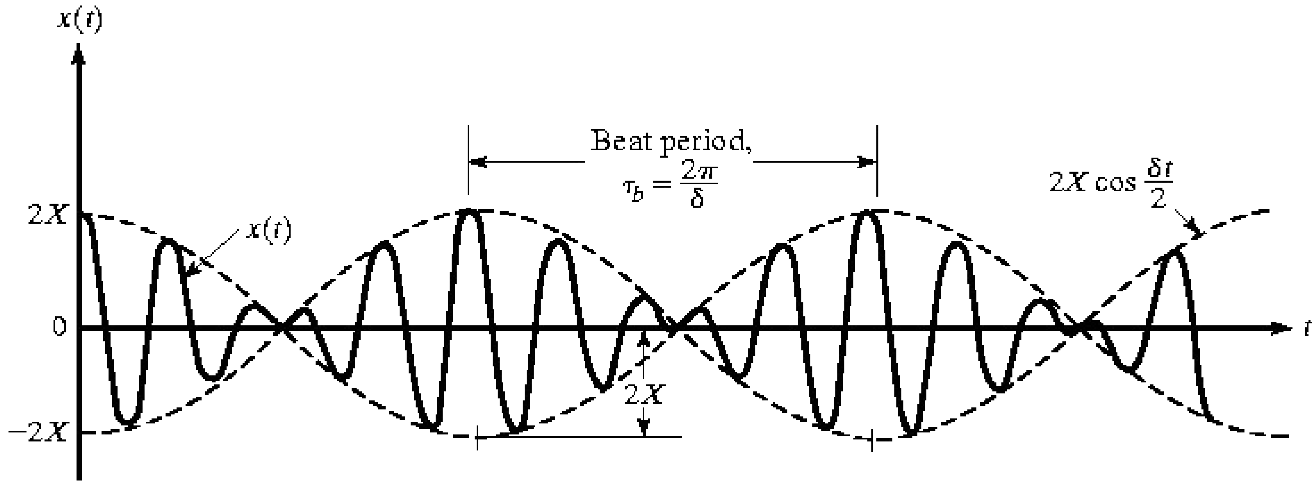
$$x_2 = A_2 \sin(\omega t + \phi) \qquad (1.62)$$



1.10 Harmonic Motion

❑ Definitions of Terminology:

- Beats are formed when two harmonic motions, with frequencies close to one another, are added



1.10 Harmonic Motion

Definitions of Terminology:

- Decibel is originally defined as a ratio of electric powers. It is now often used as a notation of various quantities such as displacement, velocity, acceleration, pressure, and power

$$\mathrm{dB} = 10 \log\left(\frac{P}{P_0}\right) \tag{1.68}$$

$$\mathrm{dB} = 20 \log\left(\frac{X}{X_0}\right) \tag{1.69}$$

where P_0 is some reference value of power and X_0 is specified reference voltage.

1.11 Harmonic Analysis

- Fourier Series Expansion:
  If $x(t)$ is a periodic function with period τ, its Fourier Series representation is given by

$$x(t) = \frac{a_0}{2} + a_1 \cos \omega t + a_2 \cos 2\omega t + \ldots$$

$$+ b_1 \sin \omega t + b_2 \sin 2\omega t + \ldots$$

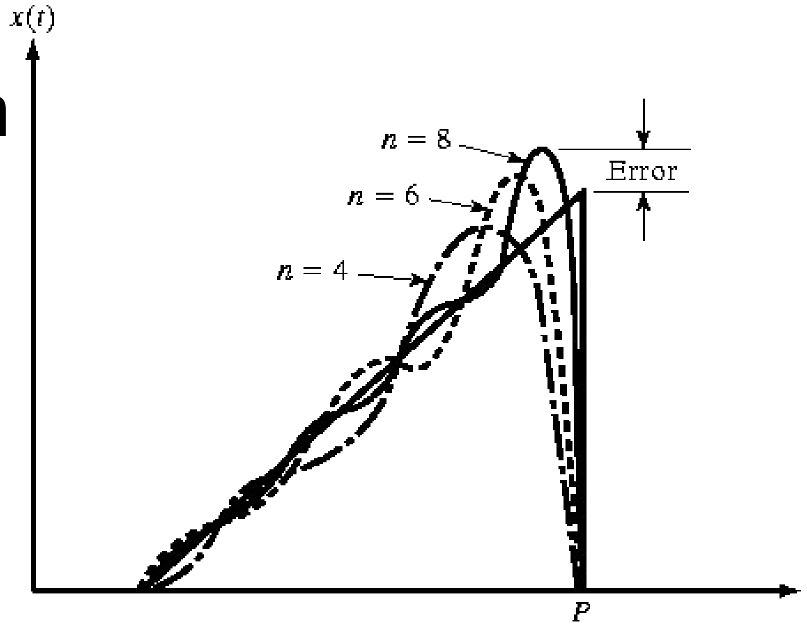
$$= \frac{a_0}{2} + \sum_{n=1}^{\infty} (a_n \cos n\omega t + b_n \sin n\omega t) \qquad (1.70)$$

1.11 Harmonic Analysis

• Gibbs Phenomenon:

An anomalous behavior observed from a periodic function that is being represented by Fourier series.

As n increases, the approximation can be seen to improve everywhere except in the vicinity of the discontinuity, P. The error in amplitude remains at approximately 9 percent, even when $k \rightarrow \infty$.



1.11 Harmonic Analysis

- Complex Fourier Series:

The Fourier series can also be represented in terms of complex numbers.

$$e^{i\omega t} = \cos\omega t + i\sin\omega t \qquad (1.78)$$

and

$$e^{-i\omega t} = \cos\omega t - i\sin\omega t \qquad (1.79)$$

Also,

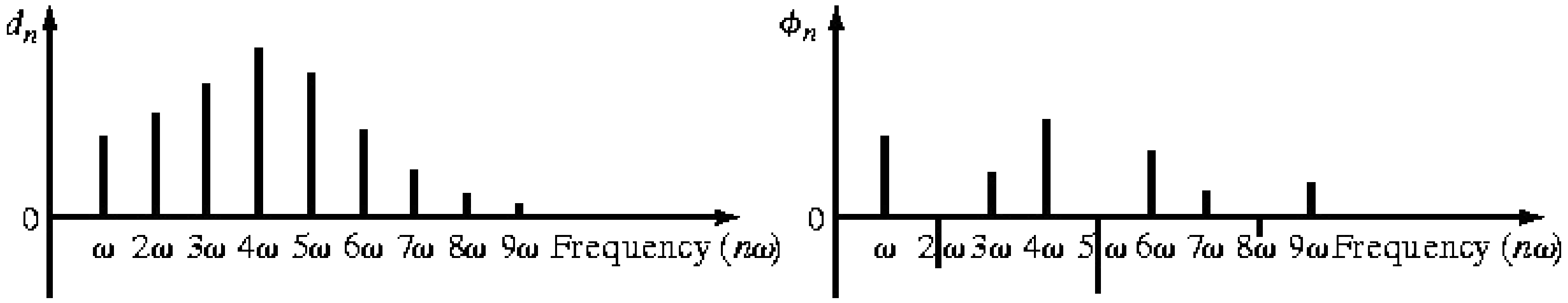
$$\cos\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2} \qquad (1.80)$$

$$\sin\omega t = \frac{e^{i\omega t} + e^{-i\omega t}}{2i} \qquad (1.81)$$

1.11 Harmonic Analysis

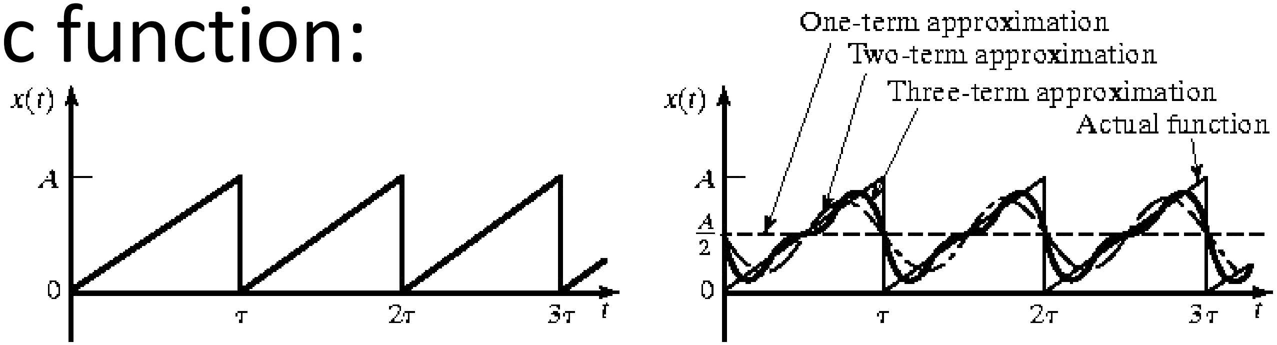
- Frequency Spectrum:

  Harmonics plotted as vertical lines on a diagram of amplitude (a_n and b_n or d_n and Φ_n) versus frequency ($n\omega$)



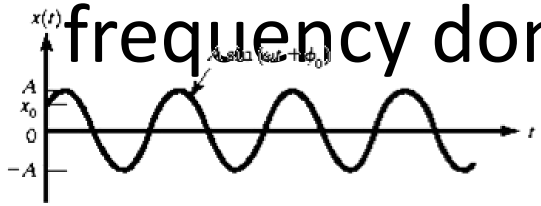
1.11 Harmonic Analysis

- A periodic function:

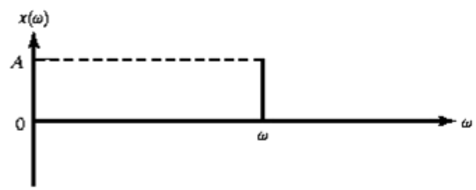


(a) (b)

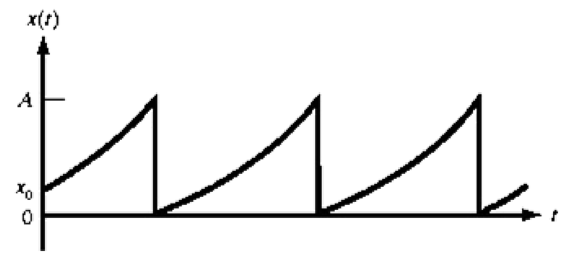
- Representation of a function in time and frequency domain:



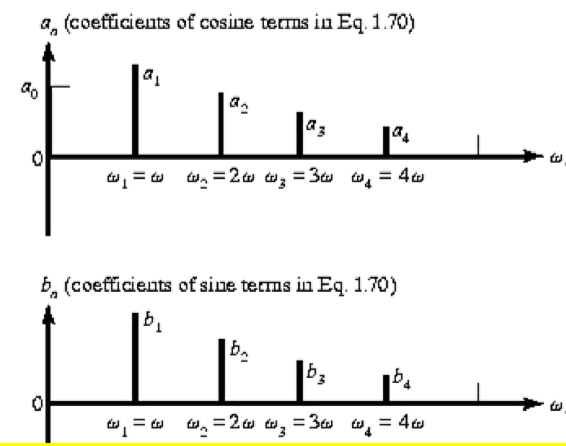
(a)



(b)



(c)



1.11 Harmonic Analysis

- Even and odd functions:

Even function & its Fourier series expansion

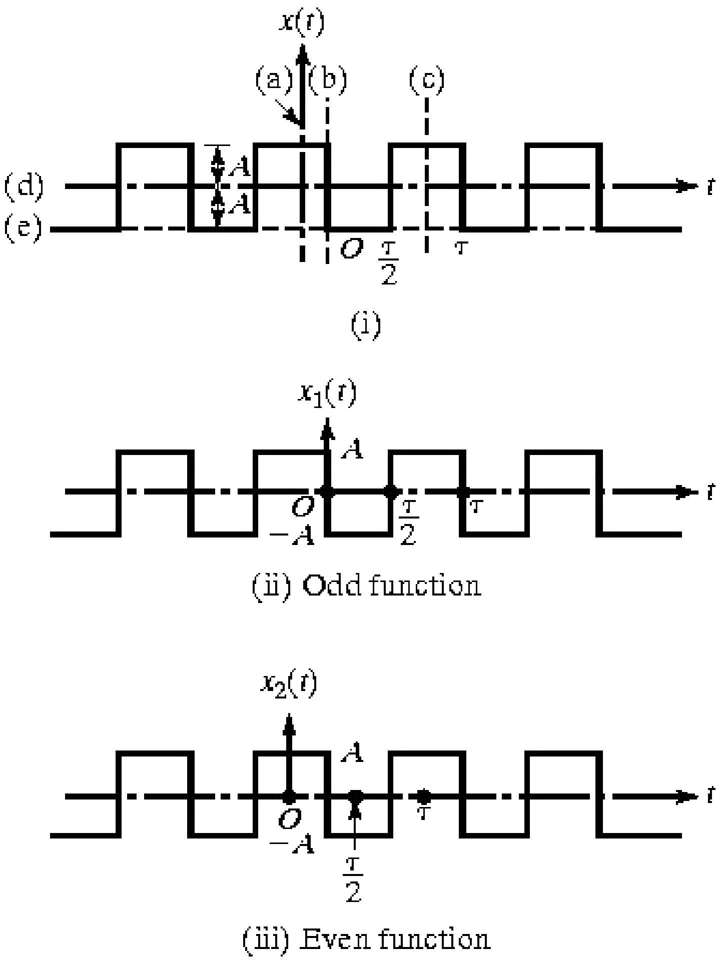
$$x(-t) = x(t) \qquad (1.87)$$

$$x(t) = \frac{a_0}{2} + \sum_{n=1}^{\infty} a_n \cos n\omega t \qquad (1.88)$$

Odd function & its Fourier series expansion

$$x(-t) = -x(t) \qquad (1.89)$$

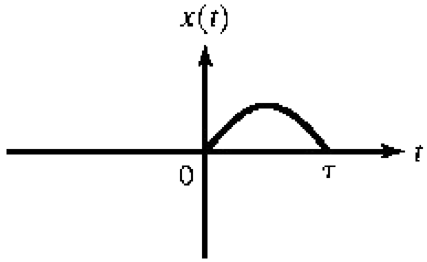
$$x(t) = \sum_{n=1}^{\infty} b_n \sin n\omega t \qquad (1.90)$$



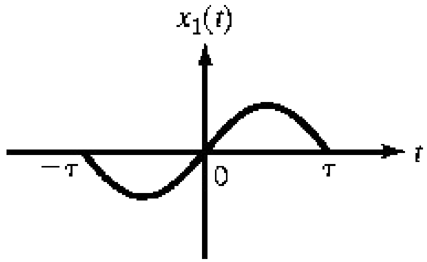
1.11 Harmonic Analysis

- Half-Range Expansions:

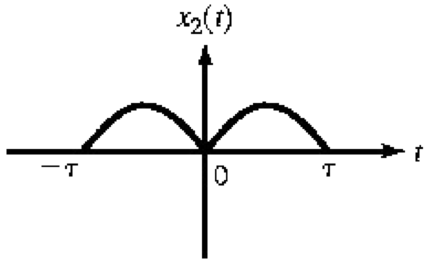
The function is extended to include the interval $-\tau$ to 0 as shown in the figure. The Fourier series expansions of $x_1(t)$ and $x_2(t)$ are known as half-range expansions.



(a) Original function



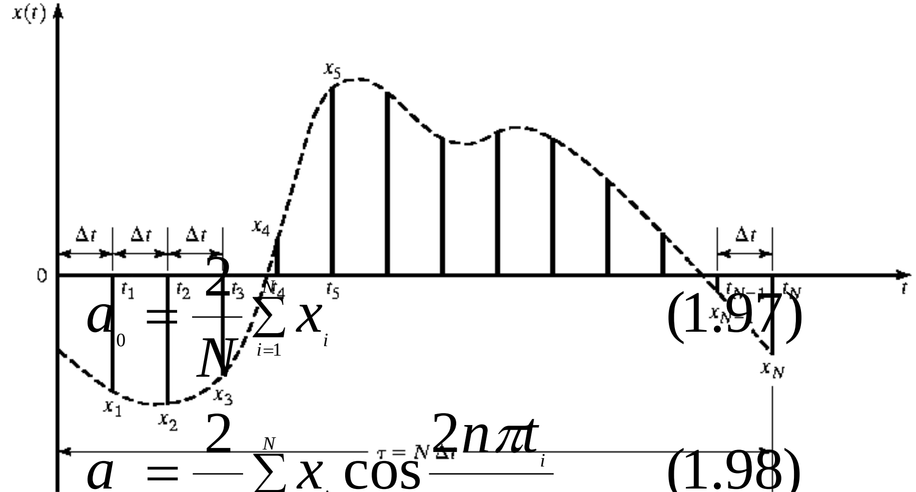
(b) Extension as an odd function



(c) Extension as an even function

1.11 Harmonic Analysis

- Numerical Computation of Coefficients
  If $x(t)$ is not in a simple form, experimental determination of the amplitude of vibration and numerical integration procedure like the trapezoidal or Simpson's rule is used to determine the coefficients a_n and b_n.



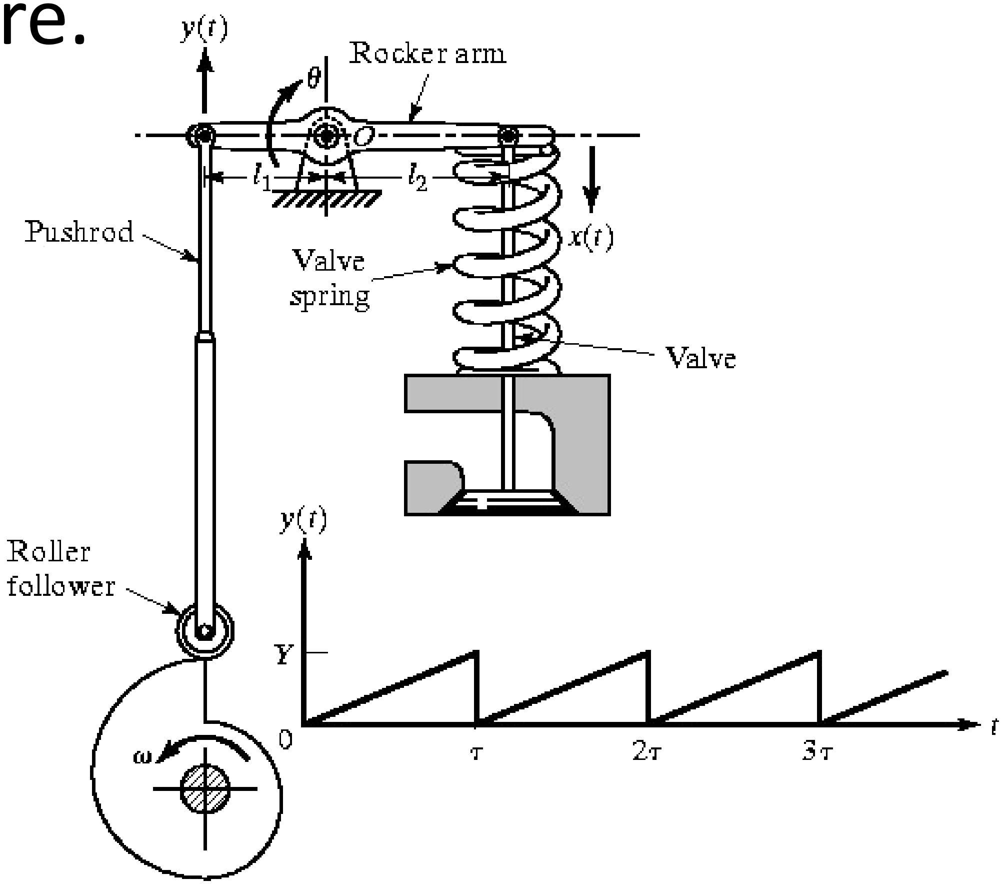
$$a_0 = \frac{2}{N}\sum_{i=1}^{N} x_i \qquad (1.97)$$

$$a_n = \frac{2}{N}\sum_{i=1}^{N} x_i \cos\frac{2n\pi t_i}{\tau} \qquad (1.98)$$

$$b_n = \frac{2}{N}\sum_{i=1}^{N} x_i \sin\frac{2n\pi t_i}{\tau} \qquad (1.99)$$

Example 1.12
Fourier Series Expansion

Determine the Fourier series expansion of the motion of the valve in the cam-follower system shown in the Figure.



Example 1.12 Solution

If $y(t)$ denotes the vertical motion of the pushrod, the motion of the valve, $x(t)$, can be determined from the relation:

$$\tan\theta = \frac{y(t)}{l_1} = \frac{x(t)}{l_2}$$

or

$$x(t) = \frac{l_2}{l_1} y(t) \qquad \text{(E.1)}$$

where

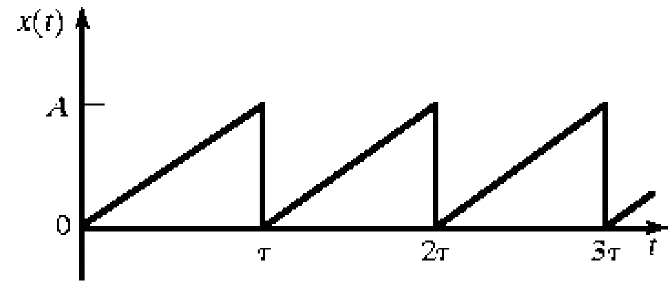
$$y(t) = Y\frac{t}{\tau}; \quad 0 \le t \le \tau \qquad \text{(E.2)}$$

and the period is given by $\tau = \dfrac{2\pi}{\omega}$.

Example 1.12 Solution

By defining

$$A = \frac{Y l_2}{l_1}$$



(a)

$x(t)$ can be expressed as

$$x(t) = A\frac{t}{\tau}; \quad 0 \le t \le \tau \qquad \text{(E.3)}$$

Equation (E.3) is shown in the Figure.
To compute the Fourier coefficients a_n and b_n, we use Eqs. (1.71) to (1.73):

$$a_0 = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)dt = \frac{\omega}{\pi}\int_0^{2\pi/\omega} A\frac{t}{\tau}dt = \frac{\omega}{\pi}\frac{A}{\tau}\left(\frac{t^2}{2}\right)_0^{2\pi/\omega} = A \qquad \text{(E.4)}$$

Example 1.12 Solution

$$a_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\cos n\omega t\cdot dt = \frac{\omega}{\pi}\int_0^{2\pi/\omega} A\frac{t}{\tau}\cos n\omega t\cdot dt$$

$$= \frac{A\omega}{\pi\tau}\int_0^{2\pi/\omega} t\cos n\omega t\cdot dt = \frac{A}{2\pi^2}\left[\frac{\cos n\omega t}{n^2} + \frac{\omega t\sin n\omega t}{n}\right]_0^{2\pi/\omega}$$

$$= 0, \qquad n = 1, 2, .. \qquad \text{(E.5)}$$

$$b_n = \frac{\omega}{\pi}\int_0^{2\pi/\omega} x(t)\sin n\omega t\cdot dt = \frac{\omega}{\pi}\int_0^{2\pi/\omega} A\frac{t}{\tau}\sin n\omega t\cdot dt$$

$$= \frac{A\omega}{\pi\tau}\int_0^{2\pi/\omega} t\sin n\omega t\cdot dt = \frac{A}{2\pi^2}\left[\frac{\sin n\omega t}{n^2} + \frac{\omega t\cos n\omega t}{n}\right]_0^{2\pi/\omega}$$

$$= \frac{A}{n\pi}, \quad n = 1, 2, .. \qquad \text{(E.6)}$$

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