JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – III Year / V Semester (2020-21)

Subject –Structural Analysis-I

Unit – III

Presented by – Akhil Maheshwari *(Asst. Prof., Department of Civil Engineering)*

VISSION AND MISSION OF INSTITUTE

Vision

To become a renowned center of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission

M-1: Focus on evaluation of learning outcomes and motivate students to inculcate research Aptitude by project based learning.
M-2: Identify, based on informed perception of Indian, Regional and global needs, areas of focus and provide platform to gain knowledge and solutions.
M-3: Offer opportunities for interaction between academia and industry.
M-4: Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

VISSION AND MISSION OF DEPARTMENT

VISION

To become a role model in the field of Civil Engineering for the sustainable development of the society.

MISSION

To provide outcome base education.

To create a learning environment conducive for achieving academic excellence.

To prepare civil engineers for the society with high ethical values.

PROGRAMME OUTCOMES (PO)

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering Fundamentals and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes (CO)

CO1. Students will be able to understand the Static and Kinematic Indeterminacy.

CO 2. Students will be able to understand the different types of Prop, Fixed and Continuous Beam.

CO 3. Students will be able to understand the Slope Deflection and Moment Distribution Method.

CO 4. Students will be able to understand Mechanical vibrations.

CO-PO MAPPING

CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	3	2	2	1	-	-	1	1	1	2
CO2	3	3	3	2	2	1	-	-	2	1	1	2
CO3	3	3	3	2	2	1	-	-	1	1	2	2
CO4	3	2	2	2	3	2	-	-	2	1	3	3

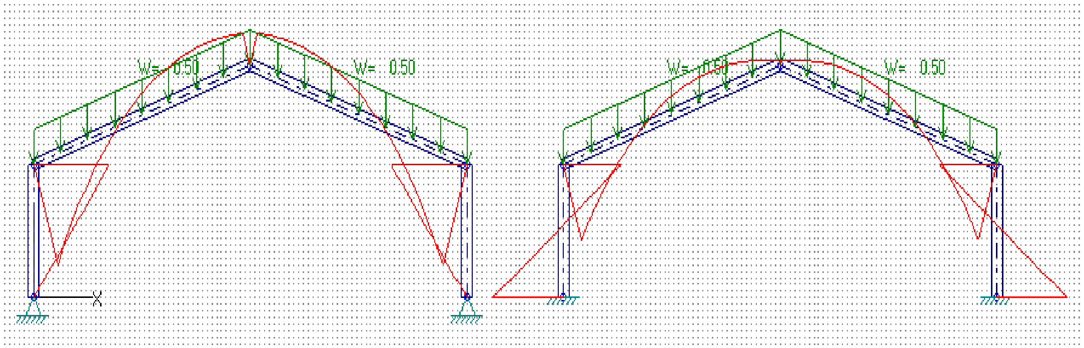
Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
1	1.1	Introduction,Scope, and Coutcome of subject	July	1	PPT
2	2.1	Introduction to Indeterminate structures	July	1	PPT
3	2.2	Degrees of freedom per node		1	PPT
4	2.3	Static and Kinematic indeterminacy (i.e. for beams, frames & portal with & without sway etc.)		1	PPT
5	2.4	Releases in structures		1	PPT
6	2.5	Maxwell's reciprocal theorem and Betti's theorem.	August	1	PPT
7	2.6	Analysis of prop cantilever structures		1	PPT
8	2.7	Analysis of Indeterminate Structure (fixed and continues beams) using Area moment method		1	PPT
9	2.8	Conjugate beam method		1	PPT
10	2.9	Three moments Theorem.		1	PPT

Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
11	3.1	Analysis of Statically Indeterminate Structures using Slope-deflection method	September	1	PPT
12	3.2	Moment-distribution method applied to continuous beams and portal frames with and without inclined members		1	PPT
13	4.1	Vibrations: Elementary concepts of structural vibration, Mathematical models, basic elements of vibratory system.		1	PPT
14	4.2	Degree of freedom. Equivalent Spring stiffness of springs in parallel and in series.		1	PPT
15	4.3	Simple Harmonic Motion: vector representation, characteristic, addition of harmonic motions, Angular oscillation.	October	1	PPT
16	4.4	Undamped free vibration of SDOF system: Newton's law of motion		1	PPT
17	4.5	D Almbert's principle, deriving equation of motions, solution of differential equation of motion, frequency & period of vibration, amplitude of motion; Introduction to damped and forced vibration.		1	PPT

SLOPE DEFLECTION METHOD



Presentation Overview:

1. INTRODUCTION.(Basic Idea of Slope Deflection Method)
2. ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD .
3. APLICATION OF SLOPE DEFLECTION METHOD.
4. SIGN CONVENTION.
5. PROCEDURE.
6. SLOPE DEFLECTION EQUATION.
7. EXAMPLE.

INTRODUCTION:

- This method was developed by axel bendexon in Germany in 1914. This method is applicable to all types of statically indeterminate beams & frames and in this method, we solve for unknown joint rotations, which are expressed in terms of the applied loads and the bending moments.Deflections due to shear and axial stresses are not considered as the effect are small.

- Indeterminate structure: the structure which can not be analyzed by the equations of static equilibriums alone are called indeterminate structures.

ASSUMPTIONS IN THE SLOPE DEFLECTION METHOD:

This method is based on the following simplified assumptions:

- All the joints of the frame are rigid,
- Distortion, due to axial and shear stresses, being very small, are neglected.

Applications of slope deflection method:

1. Continuous Beams
2. Frames with out side sway
3. Frames with side sway

SIGN CONVENTION:–

(1) ROTATIONS:– Clockwise joint rotations are considered as (-ve).

(2) END MOMENTS:– clockwise end moments are considered as (+ve).

PROCEDURE

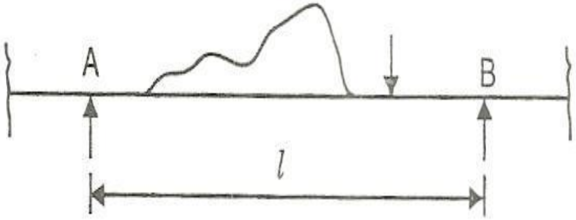
The procedure is as follows:

1. Determine the fixed end moments at the end of each span due to applied loads acting on span by considering each span as fixed ended. Assign ± Signs w.r.t. above sign convention.
2. Express all end moments in terms of fixed end moments and the joint rotations by using slope – deflection equations.
3. Establish simultaneous equations with the joint rotations as the unknowns by applying the condition that sum of the end moments acting on the ends of the two members meeting at a joint should be equal to zero.
4. Solve for unknown joint rotations.
5. Substitute back the end rotations in slope – deflection equations and compute the end moments.
6. Determine all reactions and draw S.F. and B.M. diagrams and also sketch the elastic curve

SLOPE DEFLECTION EQUATION:

General slope-deflection equations:

$$M_{AB}=M_{FAB}+\frac{2EI}{l}\left(2\theta_A+\theta_B+\frac{3\Delta}{l}\right)$$



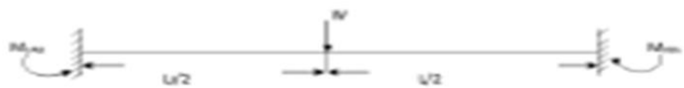
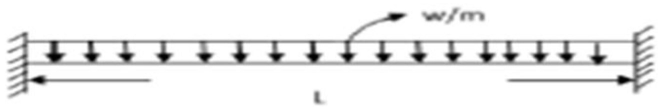
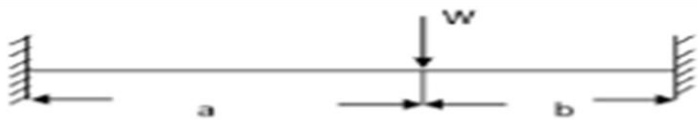
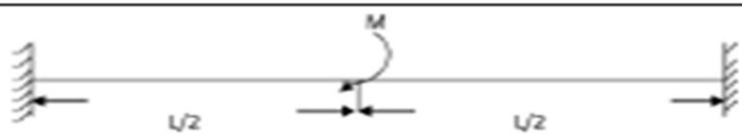
$$M_{BA}=M_{FBA}+\frac{2EI}{l}\left(2\theta_B+\theta_A+\frac{3\Delta}{l}\right)$$

M_{FAB}, M_{FBA} - Fixed end moments at A and B respectively due to the given loading

θ_A, θ_B - Slopes at A and B respectively

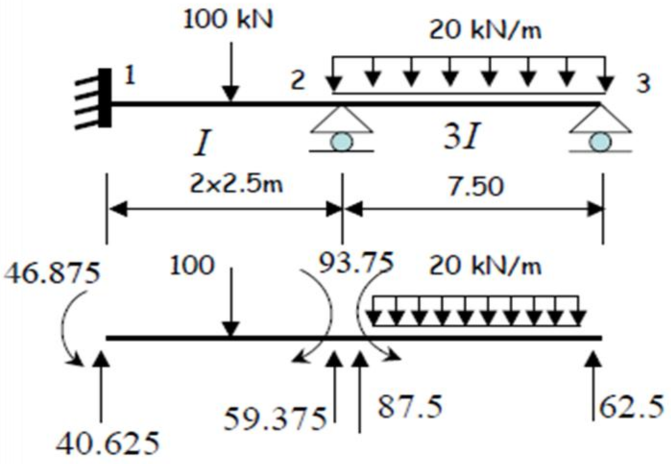
Δ - Sinking of support A with respect to B

EQUATION FOR FIXED END MOMENT:

S.No	CASES	BOTH ENDS FIXED	
		M_{FAB}	M_{FBA}
1		$-\frac{wl}{8}$	$+\frac{wl}{8}$
2		$-\frac{wl^2}{12}$	$+\frac{wl^2}{12}$
3		$-\frac{wab^2}{l^2}$	$+\frac{wab^2}{l^2}$
4		$-$ $\frac{wa}{12l^2}(6l^2-8al+3a^2)$	$+$ $\frac{wa^3}{12l^2}(4l^2-3a)$
5		$-\frac{wl^2}{30}$	$+\frac{wl^2}{20}$
6		$+\frac{m}{4}$	$+\frac{m}{4}$
7		$\frac{Mb}{l^2}(3a-l)$	$\frac{Ma}{l^2}(3b-l)$

EXAMPLE FOR BEAM:

Example: It is required to determine the support moments for the continuous beam.



$$-M_{12}^F = M_{21}^F = \frac{100*5}{8} = 62.5kNm$$

$$-M_{23}^F = M_{32}^F = \frac{20*7.5^2}{12} = 93.75kNm$$

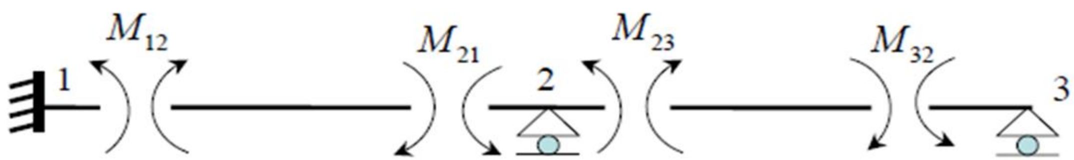
Slope – Deflection.Equations

$$M_{12} = \frac{2EI}{5}\theta_2 - 62.5 =$$

$$M_{21} = \frac{2EI}{5}2\theta_2 + 62.5 =$$

$$M_{23} = \frac{6EI}{7.5}(2\theta_2 + \theta_3) - 93.75 =$$

$$M_{32} = \frac{6EI}{7.5}(\theta_2 + 2\theta_3) + 93.75 =$$



Equilibrium.equations.of.joints

$$M_{21} + M_{23} = 0$$

$$M_{32} = 0$$

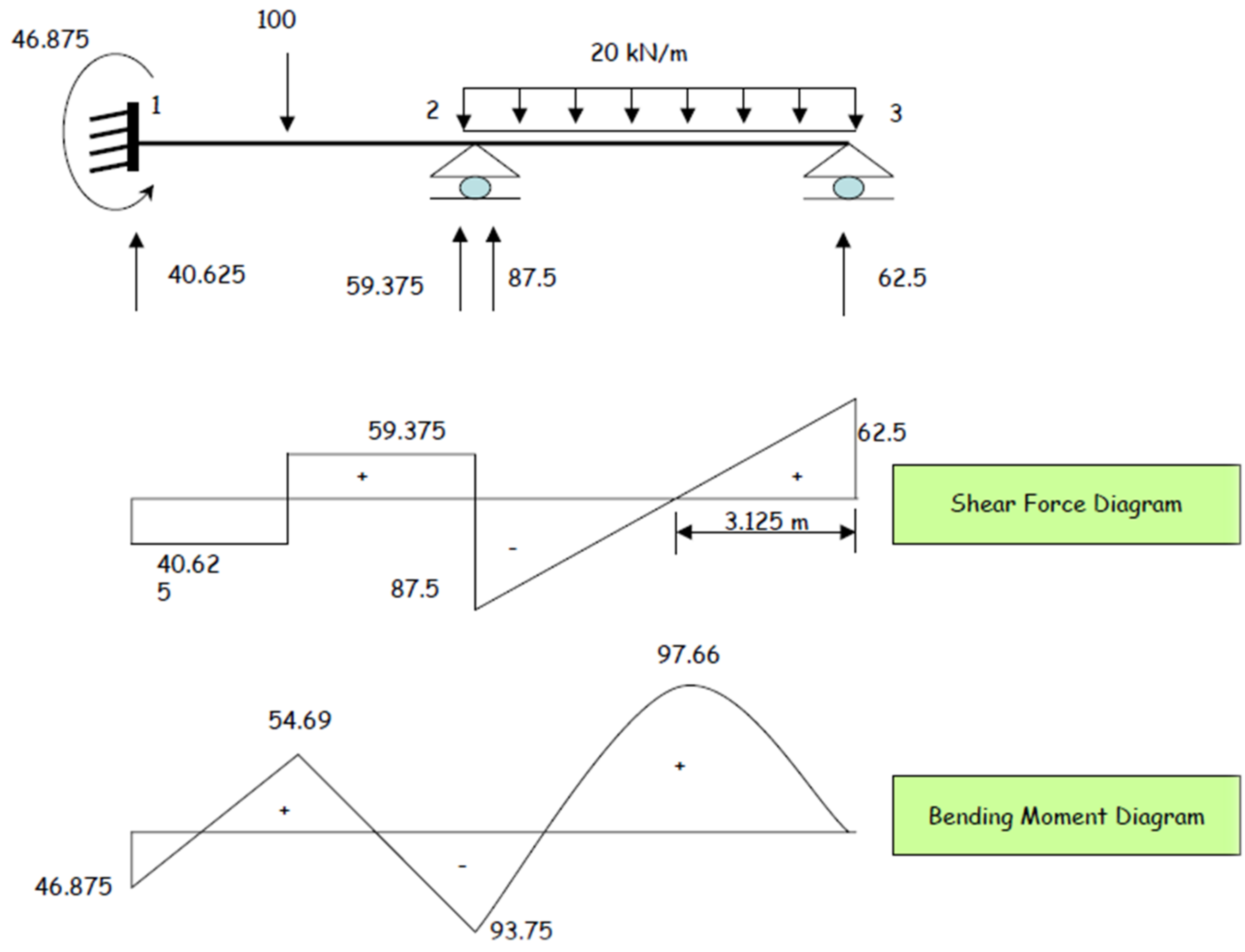
$$2.4EI\theta_2 + 0.8EI\theta_3 = 31.25$$

$$0.8EI\theta_2 + 1.6EI\theta_3 = -93.75 \rightarrow \theta_2 = \frac{39.0625}{EI} \rightarrow \theta_3 = \frac{-78.125}{EI}$$

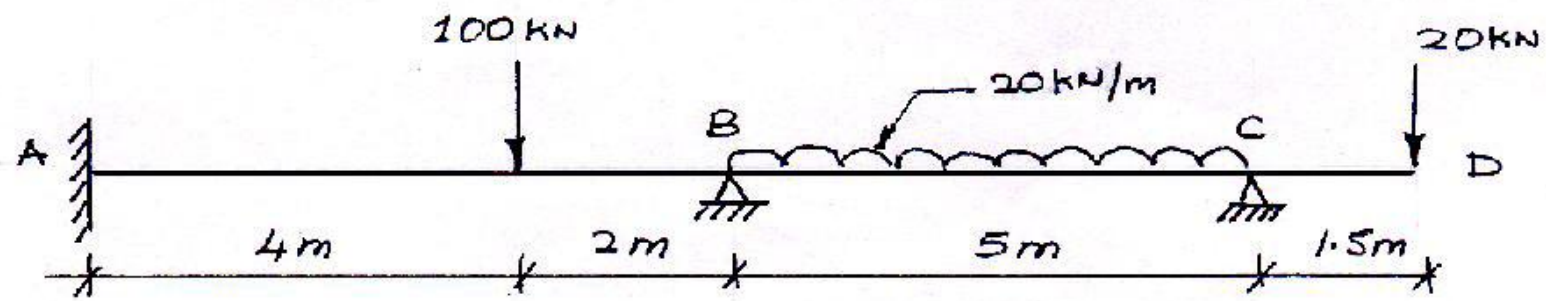
Substitude.these.results.in.slope.deflection.equations

$$M_{12} = -46.875kNm, \rightarrow M_{21} = 93.75kNm$$

$$M_{23} = -93.75kNm, \rightarrow M_{32} = 0kNm$$


46.875
100
20 kN/m
1
2
3
40.625
59.375
87.5
62.5
59.375
62.5
+
+
-
3.125 m
40.62
5
87.5
Shear Force Diagram
97.66
54.69
+
+
-
46.875
93.75
Bending Moment Diagram

Example: Analyze continuous beam ABCD by slope deflection method and then draw bending moment diagram. (Take EI constant)



Solution:

$$\theta_A = 0, \quad \theta_B \neq 0, \quad \theta_C \neq 0$$

FEMS

$$F_{AB} = -\frac{Wab^2}{L^2} = -\frac{100 \times 4 \times 2^2}{6^2} = -44.44 \quad KNM$$

$$F_{BA} = +\frac{Wa^2b}{L^2} = +\frac{100 \times 4^2 \times 2}{6^2} = +88.88 \quad KNM$$

$$F_{BC} = -\frac{wL^2}{12} = -\frac{20 \times 5^2}{12} = -41.67 \quad KNM$$

$$F_{CB} = +\frac{wL^2}{12} = +\frac{20 \times 5^2}{12} = +41.67 \quad KNM$$

$$F_{CD} = -20 \times 1.5 = -30 \quad KNM$$

Slope deflection equations:

$$M_{AB}=F_{AB}+\frac{2EI}{L}\left(2\theta_A+\theta_B\right)=-44.44+\frac{1}{3}EI\theta_B \qquad \text{--------->}(1)$$

$$M_{BA}=F_{BA}+\frac{2EI}{L}\left(2\theta_B+\theta_A\right)=+88.89+\frac{2}{3}EI\theta_B \qquad \text{--------->}(2)$$

$$M_{BC}=F_{BC}+\frac{2EI}{L}\left(2\theta_B+\theta_C\right)=-41.67+\frac{4}{5}EI\theta_B+\frac{2}{5}EI\theta_C \qquad \text{--------->}(3)$$

$$M_{CB}=F_{CB}+\frac{2EI}{L}\left(2\theta_C+\theta_B\right)=+41.67+\frac{4}{5}EI\theta_C+\frac{2}{5}EI\theta_B \qquad \text{--------->}(4)$$

$$M_{CD}=-30\ \text{KNM}$$

In the above equations we have two unknown rotations θ_B and θ_C, accordingly the boundary conditions are:

$$M_{BA} + M_{BC} = 0$$
$$M_{CB} + M_{CD} = 0$$

Now, $M_{BA} + M_{BC} = 88.89 + \frac{2}{3}EI\theta_B - 41.67 + \frac{4}{5}EI\theta_B + \frac{2}{5}EI\theta_C$

$$= 47.22 + \frac{22}{15}EI\theta_B + \frac{2}{5}EI\theta_C = 0 \qquad \text{---------} > (5)$$

And, $M_{CB} + M_{CD} = +41.67 + \frac{4}{5}EI\theta_C + \frac{2}{5}EI\theta_B - 30$

$$= 11.67 + \frac{2}{5}EI\theta_B + \frac{4}{5}EI\theta_C \qquad \text{-------} > (6)$$

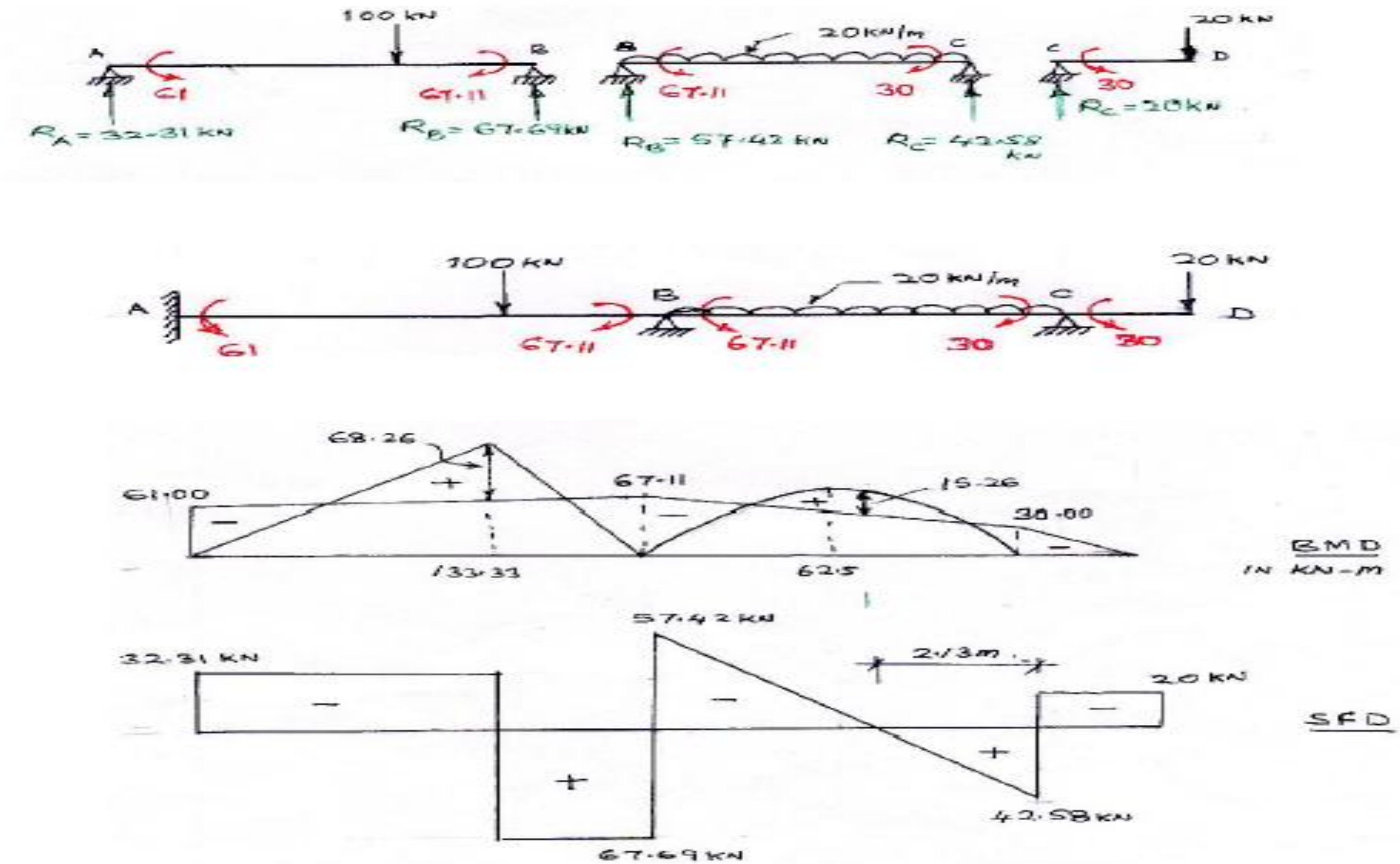
Solving (5) and (6) we get

$EI\theta_B = -32.67$ Rotation @ B anticlockwise
$EI\theta_C = +1.75$ Rotation @ B clockwise

Substituting value of $EI\theta_B$ and $EI\theta_C$ in slope deflection equations we have

$$M_{AB} = -44.44 + \frac{1}{2}(-32.67) = -61.00 \quad KNM$$
$$M_{BA} = +88.89 + \frac{2}{3}(-32.67) = +67.11 \quad KNM$$
$$M_{BC} = -41.67 + \frac{4}{5}(-32.67) = +\frac{2}{5}(1.75) = -67.11 \quad KNM$$
$$M_{CB} = +41.67 + \frac{4}{5}(1.75) + \frac{2}{5}(-32.67) = +30.00 \quad KNM$$
$$M_{CD} = -30 \quad KNM$$

Reactions: Consider free body diagram of beam AB, BC and CD as shown



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Span AB

$$R_B \times 6 = 100 \times 4 + 67.11 - 61$$
$$R_B = 67.69 \quad KN$$
$$R_A = 100 - R_B = 32.31 \quad KN$$

Span BC

$$R_C \times 5 = 20 \times \frac{5}{2} \times 5 + 30 - 67.11$$
$$R_C = 42.58 \quad KN$$
$$R_B = 20 \times 5 - R_B = 57.42 \quad KN$$

Maximum Bending Moments:

Span AB: Occurs under point load

$$Max = 133.33 - 61 - \left(\frac{67.11 - 61}{6} \times 4 = 68.26 \quad KNM \right)$$

Span BC: where SF=0, consider SF equation with C as reference

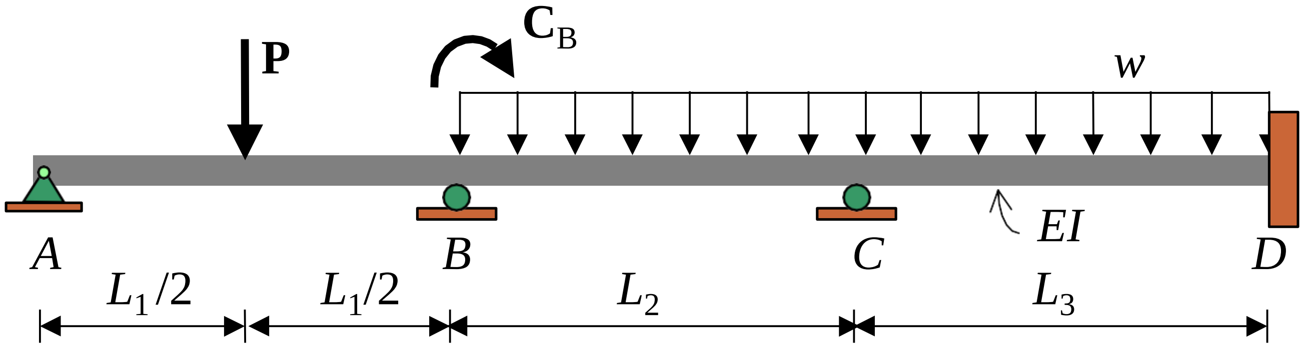
$$S_x = 42.58 - 20x = 0$$
$$x = \frac{42.58}{20} = 2.13 \quad m$$
$$\therefore \; M_{max} = 42.58 \times 2.13 - 20 \times \frac{2.13^2}{2} - 30 = 15.26 \quad KNM$$

MOMENT DISTRIBUTION METHOD

MOMENT DISTRIBUTION METHOD:

- Member Stiffness Factor (*K*)
- Distribution Factor (DF)
- Carry-Over Factor
- Distribution of Couple at Node
- Moment Distribution for Beams
  - General Beams
  - Symmetric Beams
- Moment Distribution for Frames: No Sidesway
- Moment Distribution for Frames: Sidesway

Member Stiffness Factor (*K*) & Carry-Over Factor (COF)

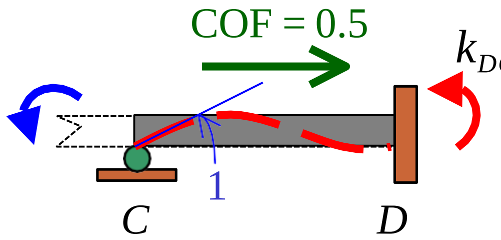


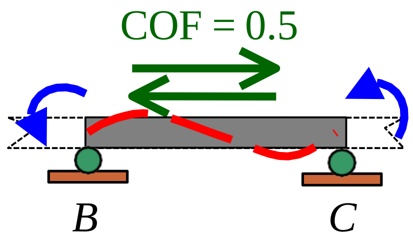
Internal members and **far-end member fixed** at **end support**:

$$K = \frac{4EI}{L}$$

$$k_{CC} = \frac{4EI}{L}$$

$$k_{DC} = \frac{2EI}{L}$$





$K_{(BC)} = 4EI/L_2$, $\quad K_{(CD)} = 4EI/L_3$

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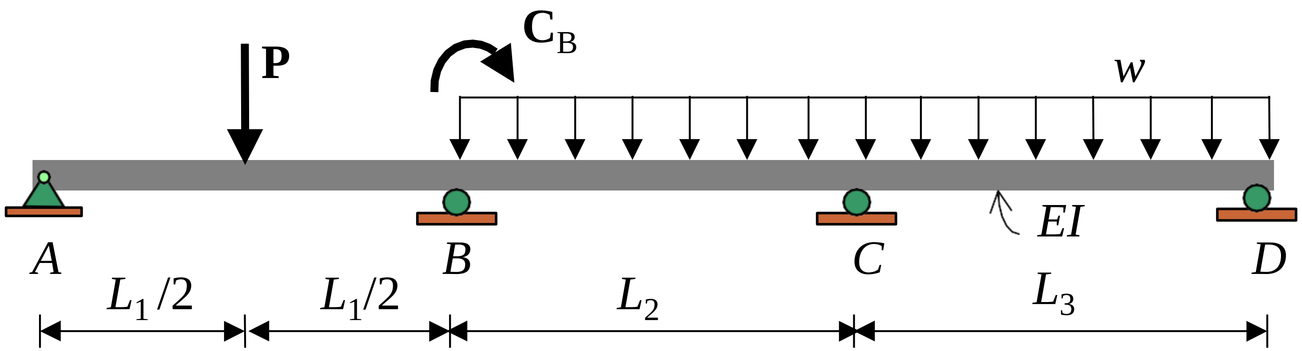
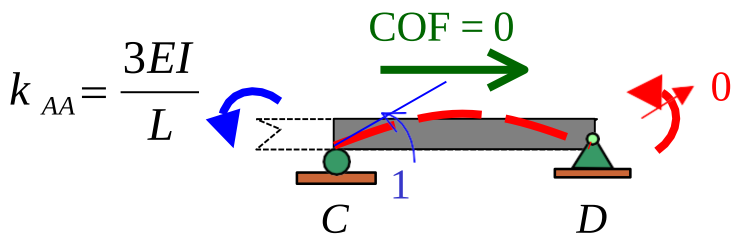


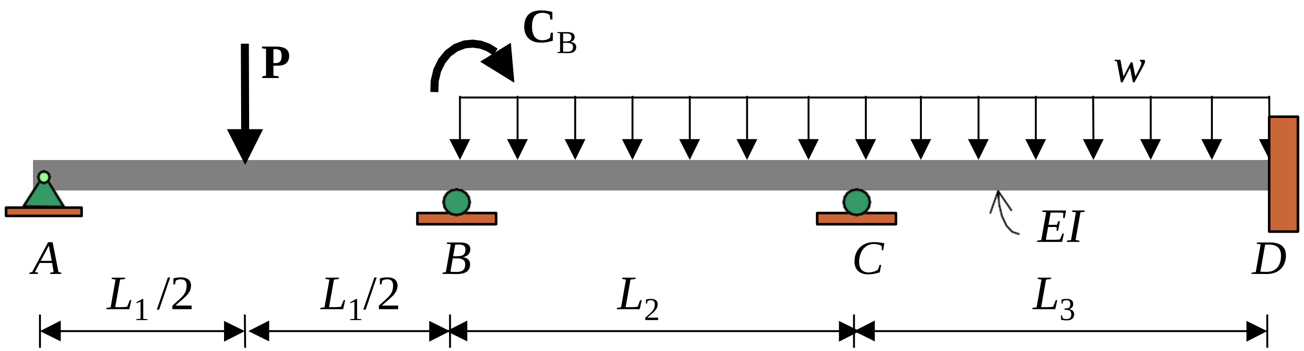
Far-end member pinned or roller end support:

$$K = \frac{3EI}{L}$$



$K_{(AB)} = 3EI/L_1$, $K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$

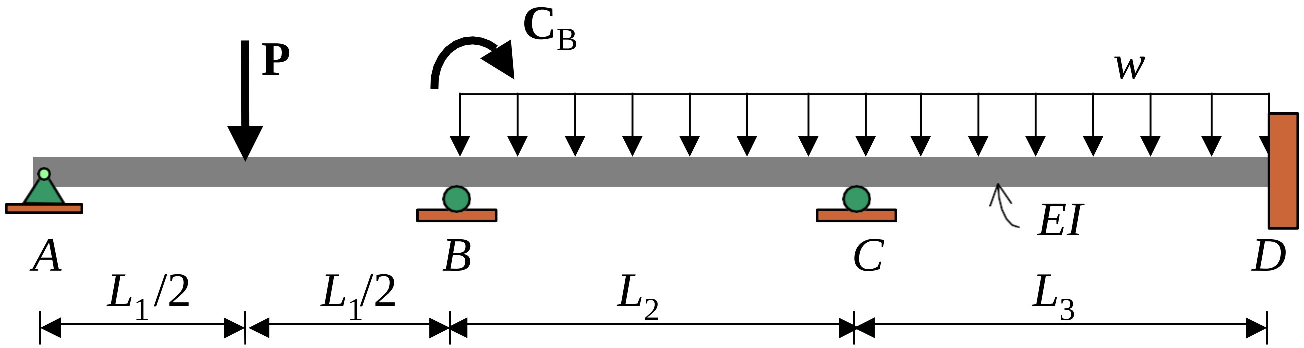
Joint Stiffness Factor (K)



$K_{(AB)} = 3EI/L_1$ $K_{(BC)} = 4EI/L_2$, $K_{(CD)} = 4EI/L_3$

$$K_{joint} = K_T = \Sigma K_{member}$$

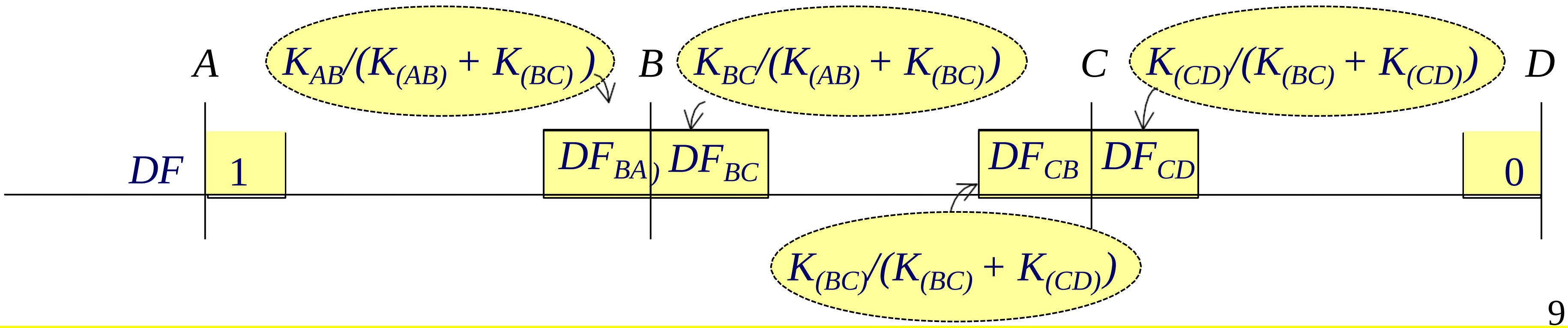
Distribution Factor (DF)



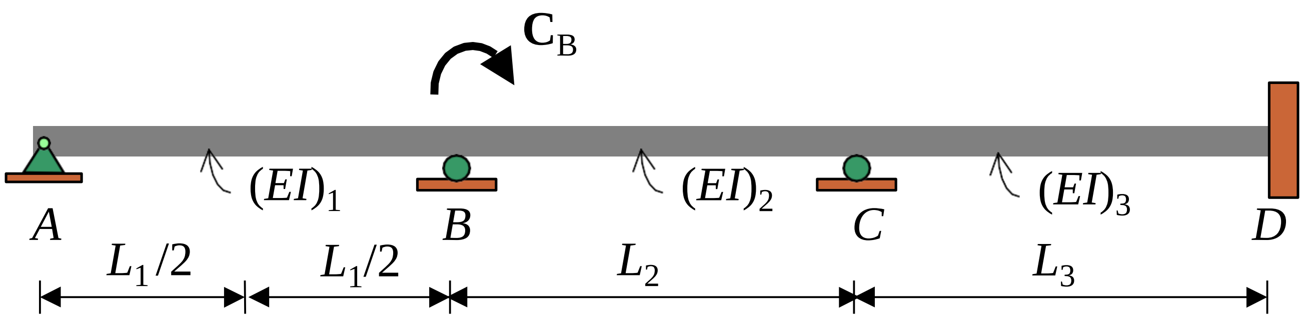
$$DF = \frac{K}{\Sigma K}$$

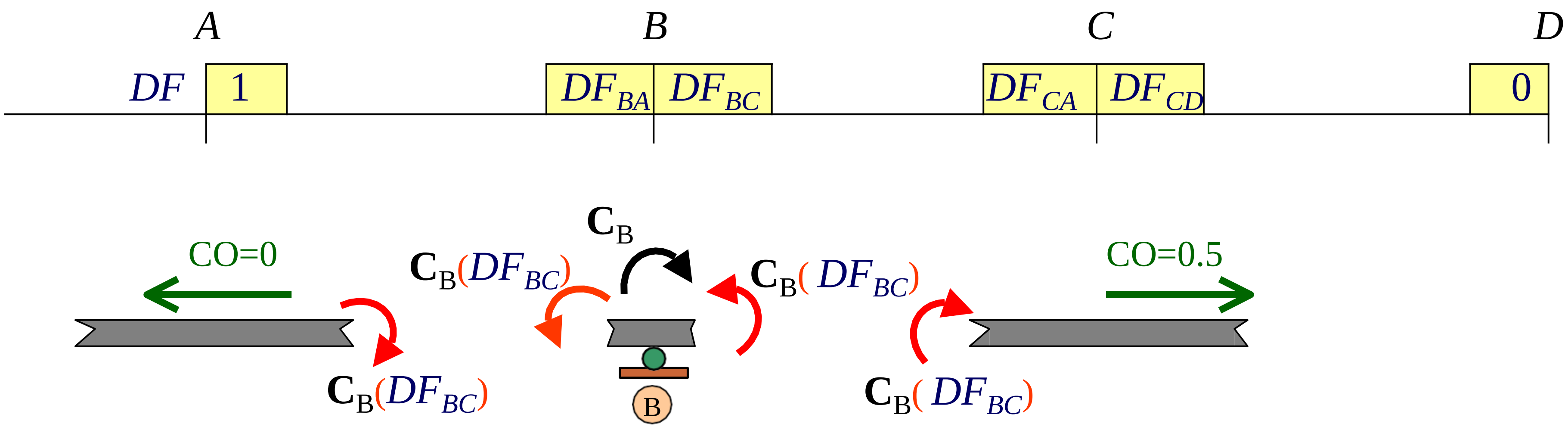
Notes:

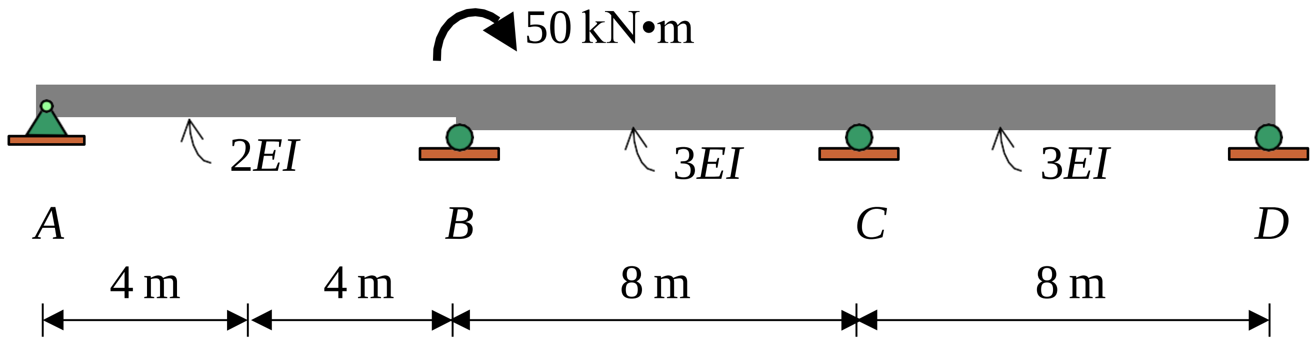
- far-end pined (DF = 1)
- far-end fixed (DF = 0)



Distribution of Couple at Node

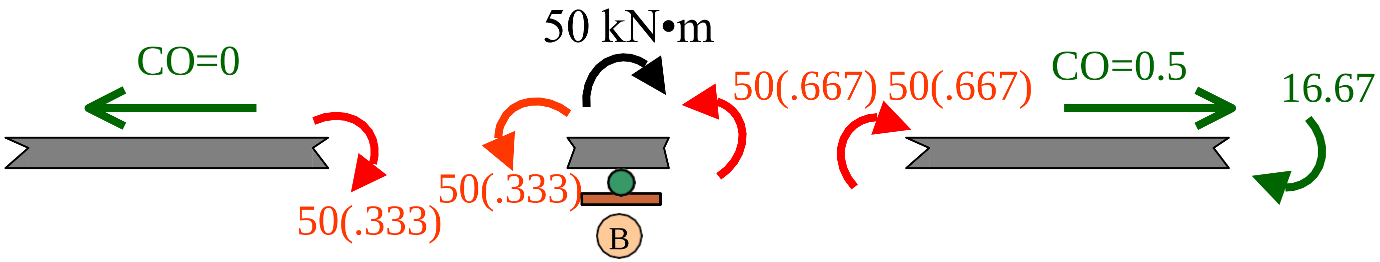




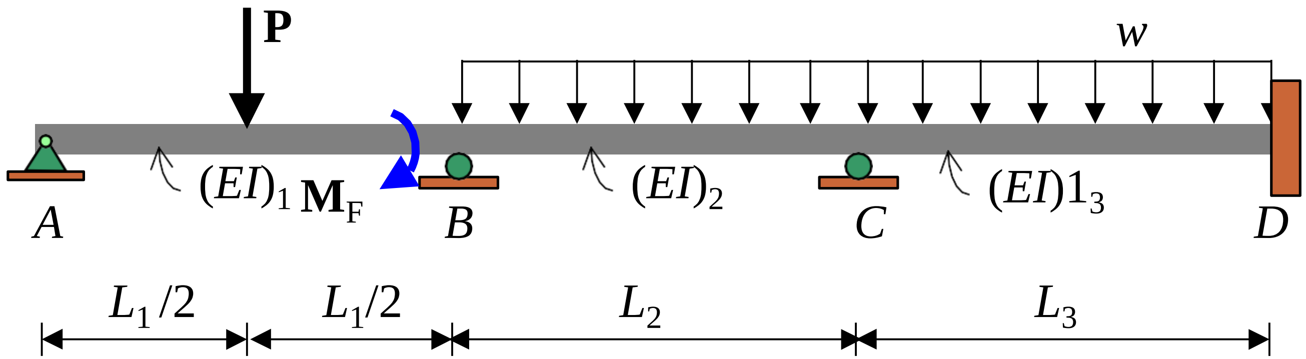


$L_1 = L_2 = L_3$

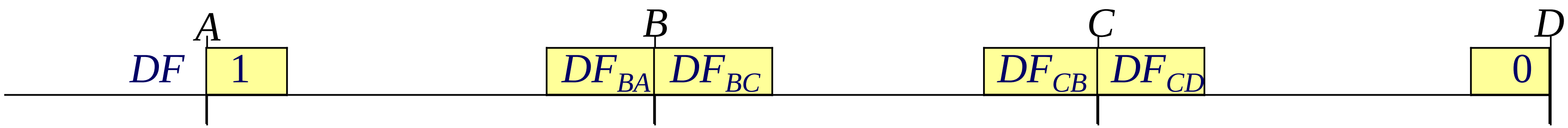
	A	B		C		D
DF	1	0.333	0.667	0.5	0.5	0

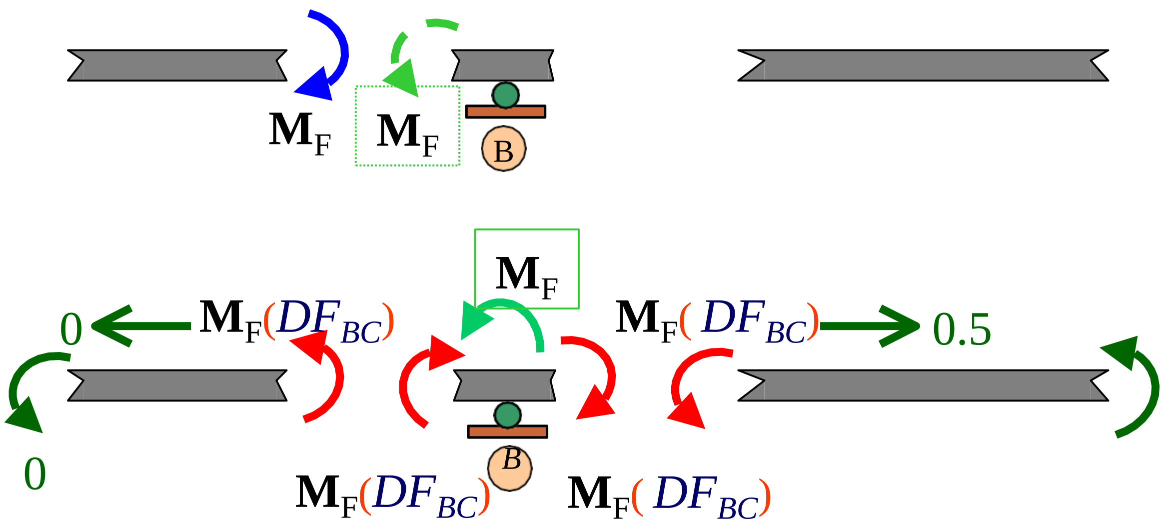


Distribution of Fixed-End Moments



$L_1 = L_2 = 8$ m, $L_3 = 10$ m





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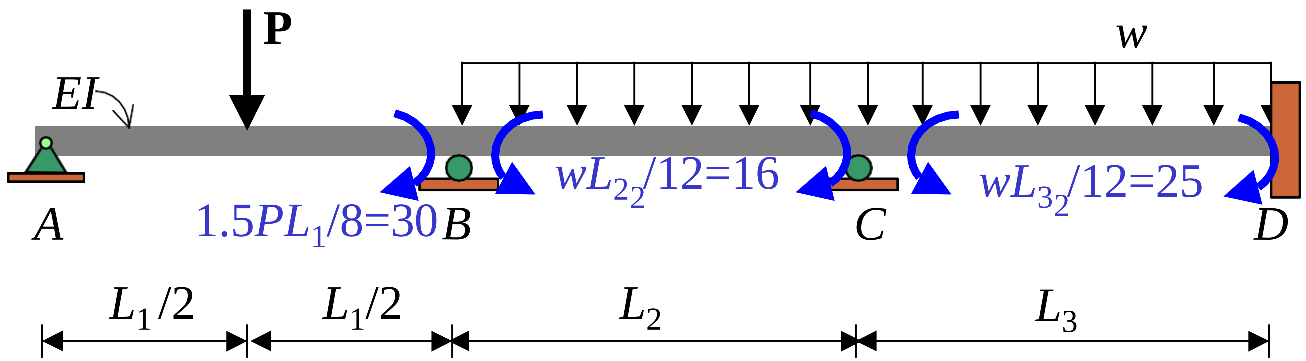
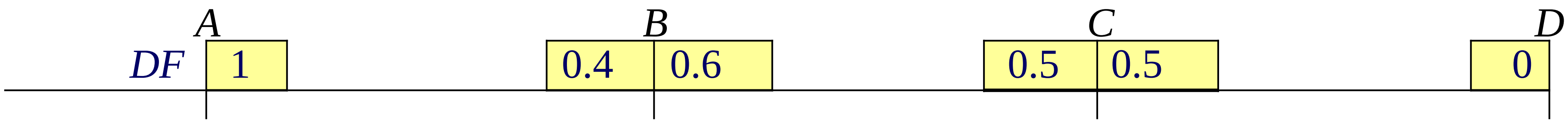
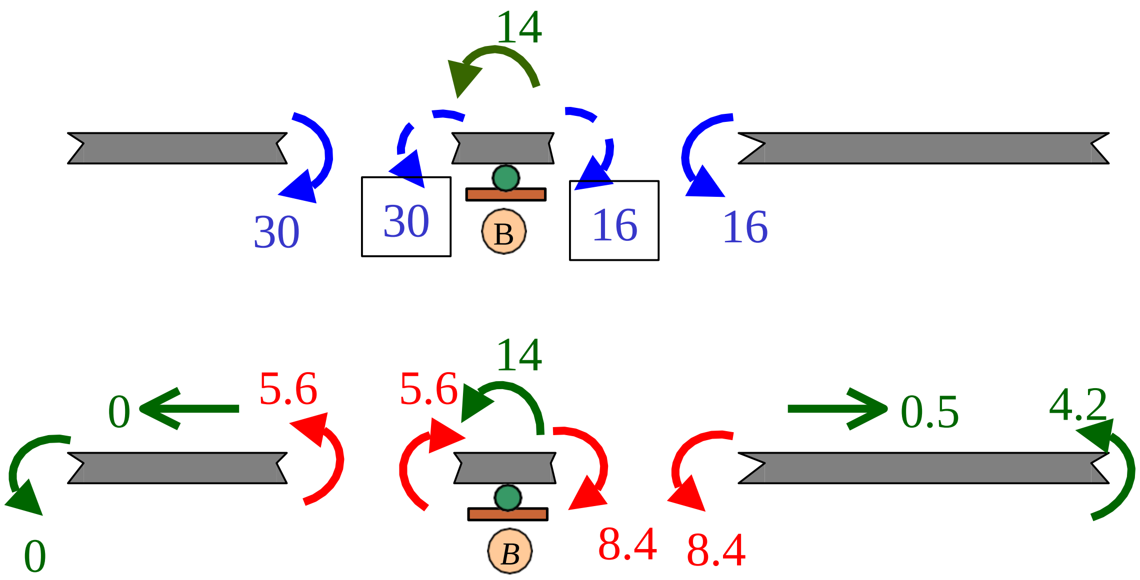


$L_1 = L_2 = 8$ m, $L_3 = 10$ m





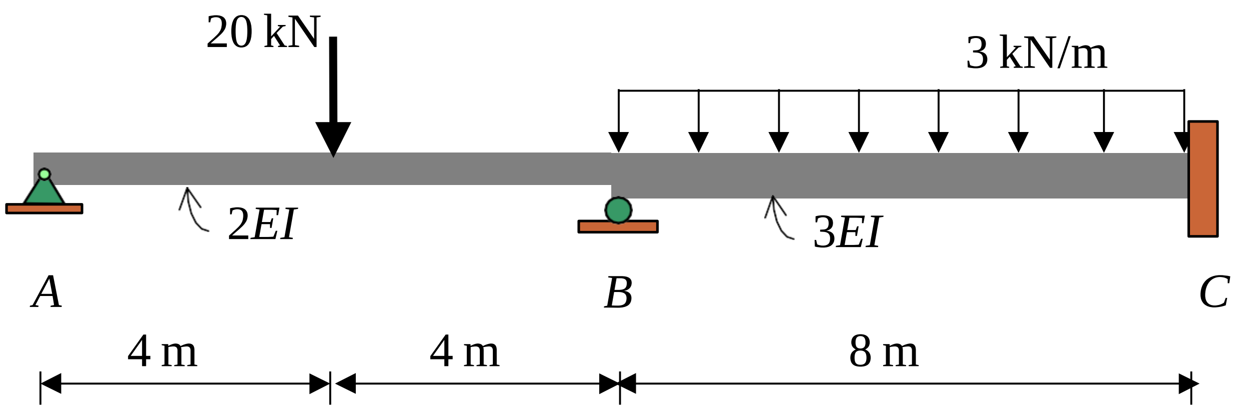
Moment Distribution for Beams

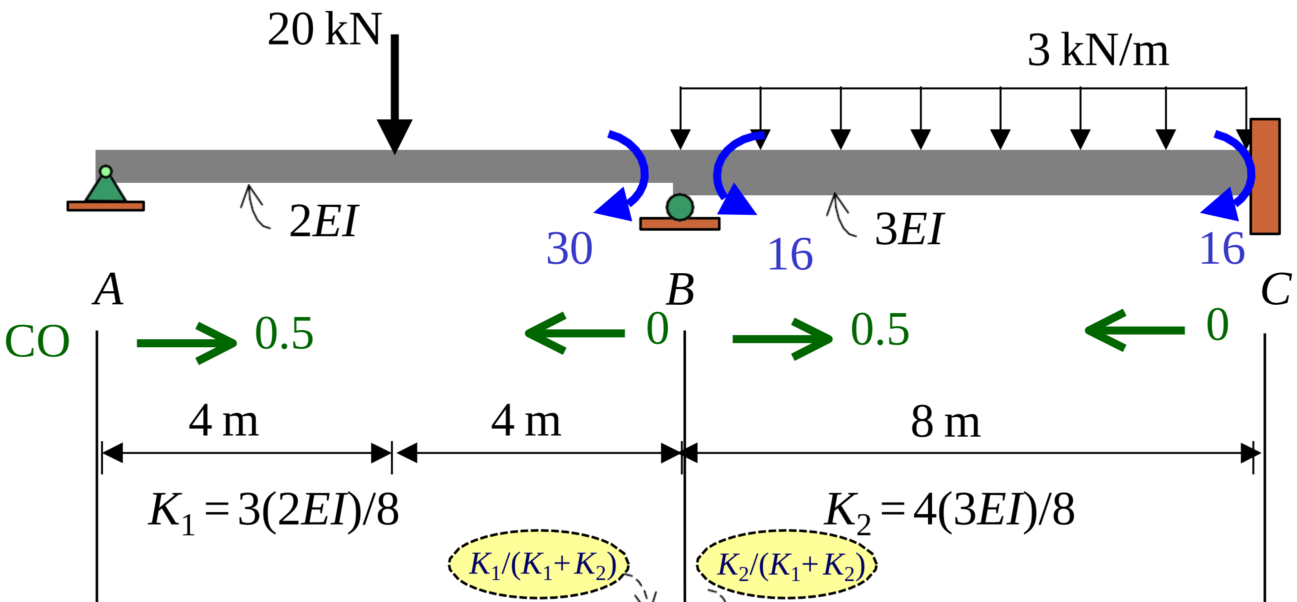
<u>**Example 1**</u>

The support *B* of the beam shown (E = 200 GPa, $I = 50\text{x}10^6\ \text{mm}^4$).
Use the moment distribution method to:

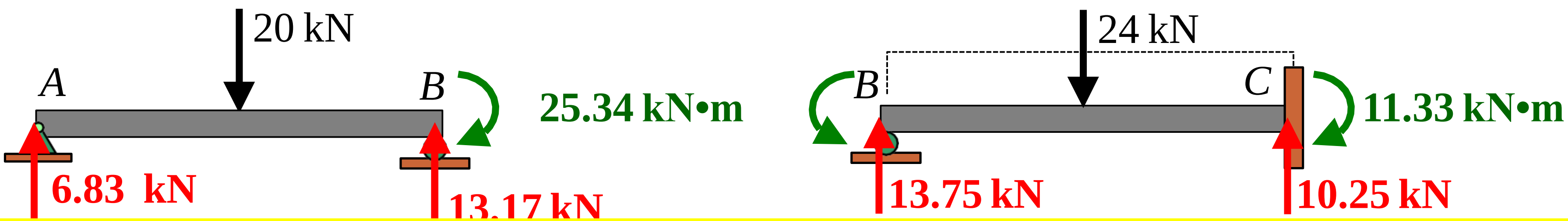
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**,and **qualitative deflected shape.**

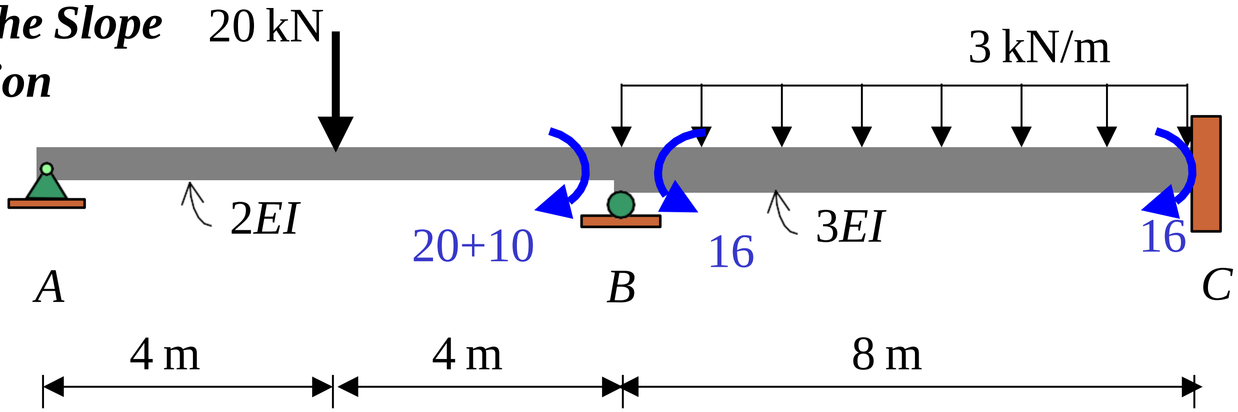




	A	B	B	C
DF	1	0.333	0.667	0
[FEM]$_{load}$		-30	16	-16
Dist.		4.662	9.338	
CO				4.669
Σ		**-25.34**	**25.34**	**-11.33**

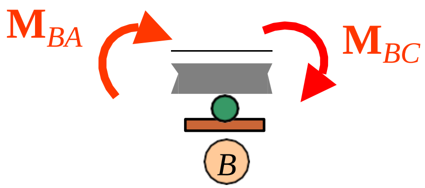


***Note :* Using the Slope Deflection**



$$M_{BA} = \frac{3(2EI)}{8}\theta_B - 30 \quad ---(1)$$

$$M_{BC} = \frac{4(3EI)}{8}\theta_B + 16 \quad ---(2)$$



$\Sigma \mathbf{M}_B = 0$: $-\mathbf{M}_{BA} - \mathbf{M}_{BC} = 0$

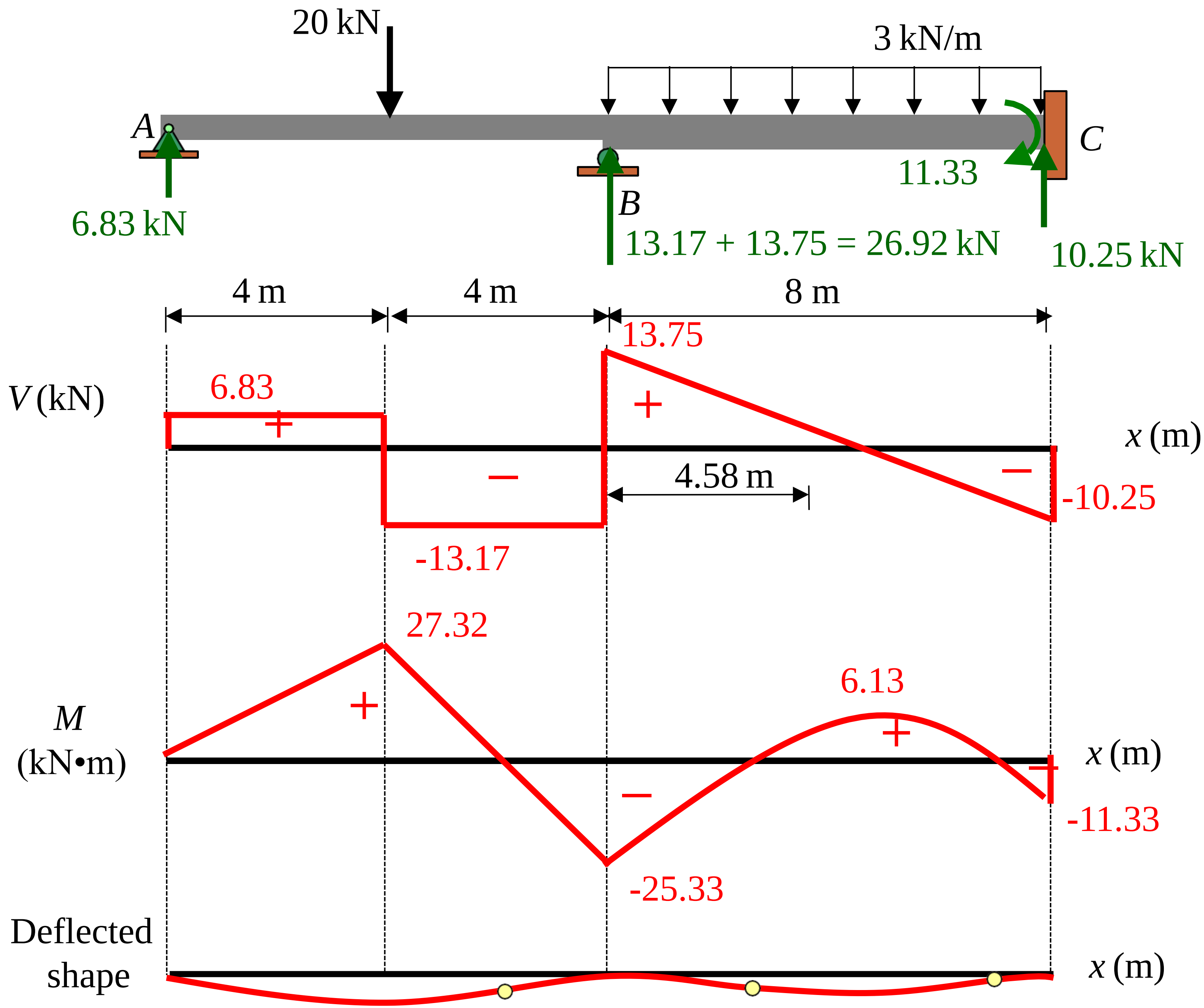
$(0.75 + 1.5)EI\theta_B - 30 + 16 = 0$

$\theta_B = 6.22/EI$

$M_{BA} = -25.33$ kN•m,

$M_{BC} = 25.33$ kN•m

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 = -11.33\, kN \bullet m$$

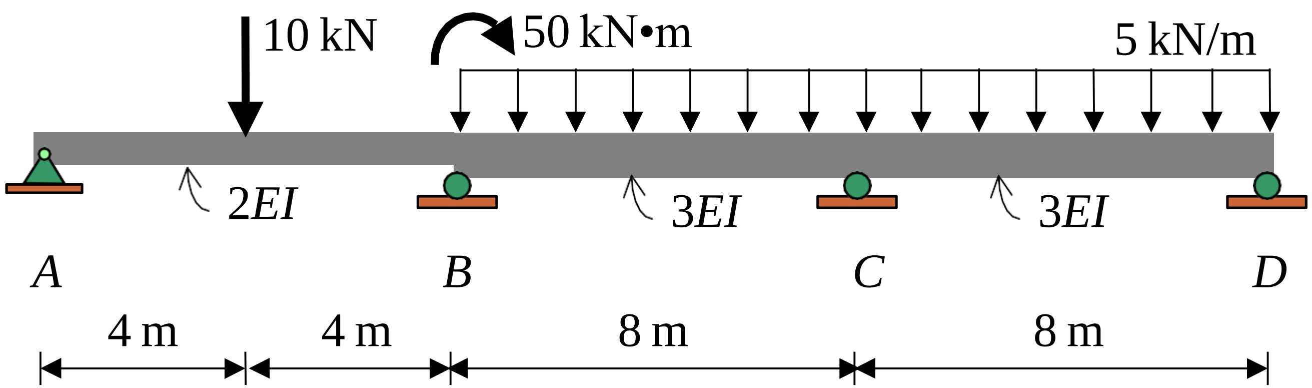

20 kN
3 kN/m
A
C
11.33
B
6.83 kN
13.17 + 13.75 = 26.92 kN
10.25 kN
4 m
4 m
8 m
13.75
6.83
V (kN)
x (m)
4.58 m
-10.25
-13.17
27.32
6.13
M
(kN•m)
x (m)
-11.33
-25.33
Deflected
shape
x (m)

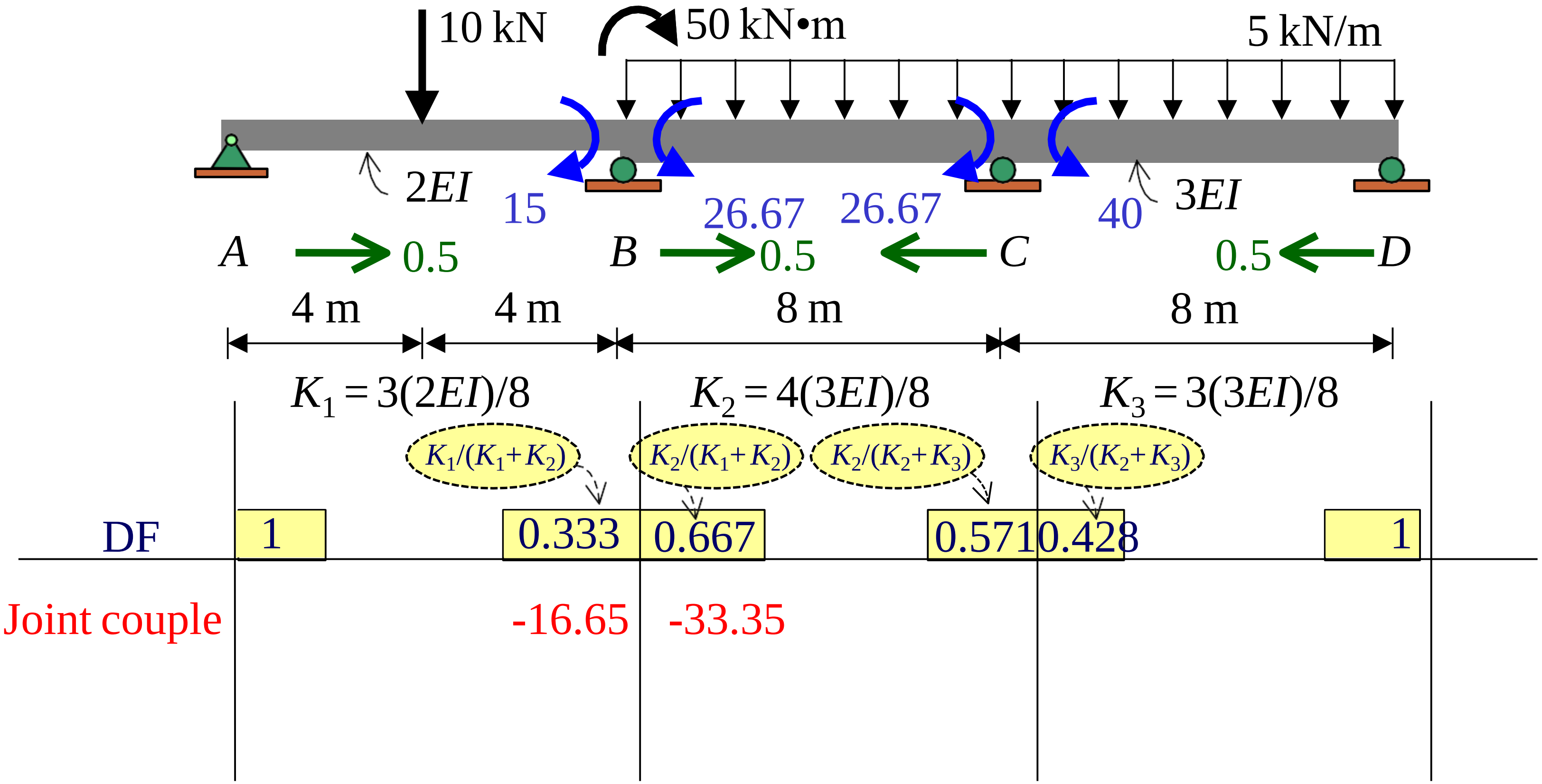
Example 2

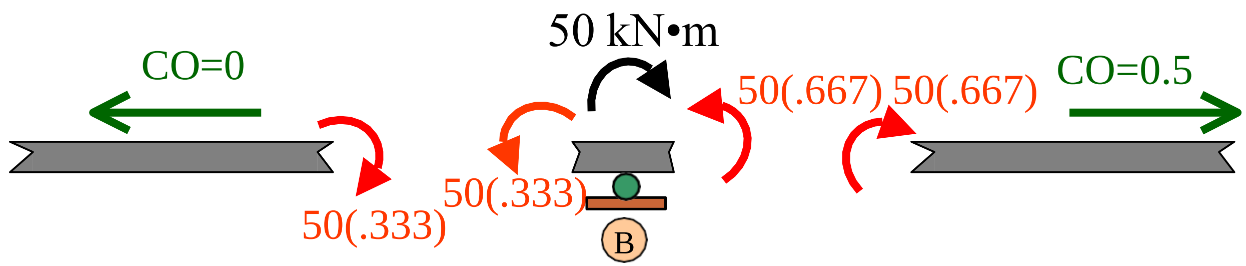
From the beam shown use the moment distribution method to:

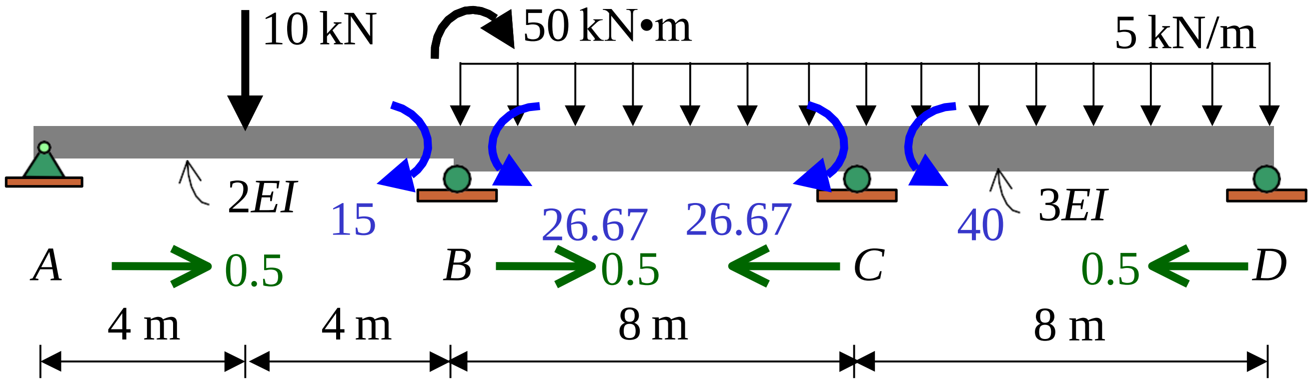
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

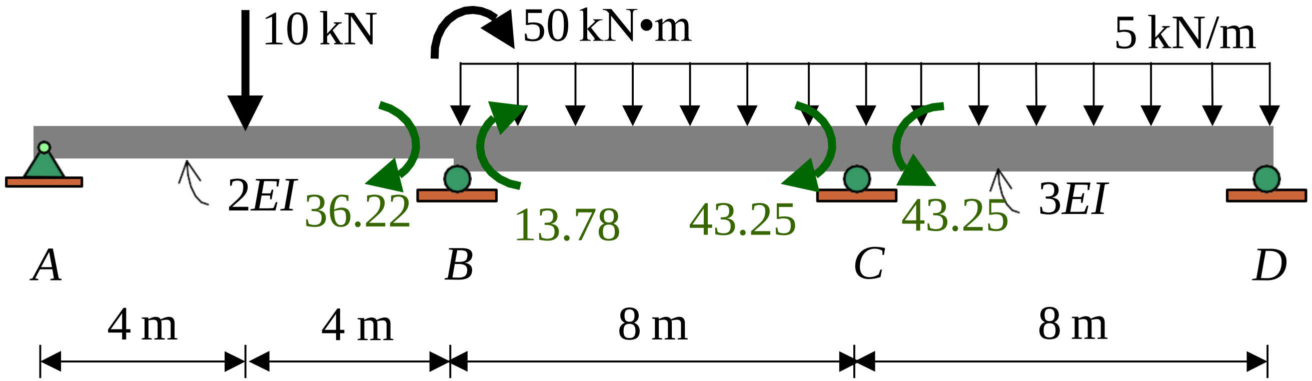


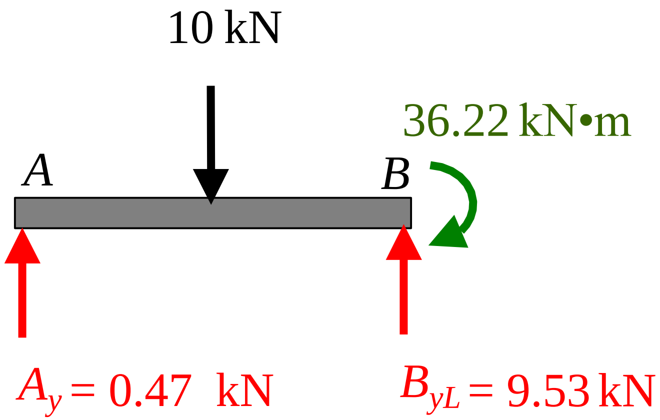

10 kN
50 kN•m
5 kN/m
2EI
3EI
15
26.67
26.67
40
A
B
C
D
0.5
0.5
0.5
4 m
4 m
8 m
8 m
K1 = 3(2EI)/8
K2 = 4(3EI)/8
K3 = 3(3EI)/8
K1/(K1+K2)
K2/(K1+K2)
K2/(K2+K3)
K3/(K2+K3)
DF
1
0.333
0.667
0.571
0.428
1
Joint couple
-16.65
-33.35

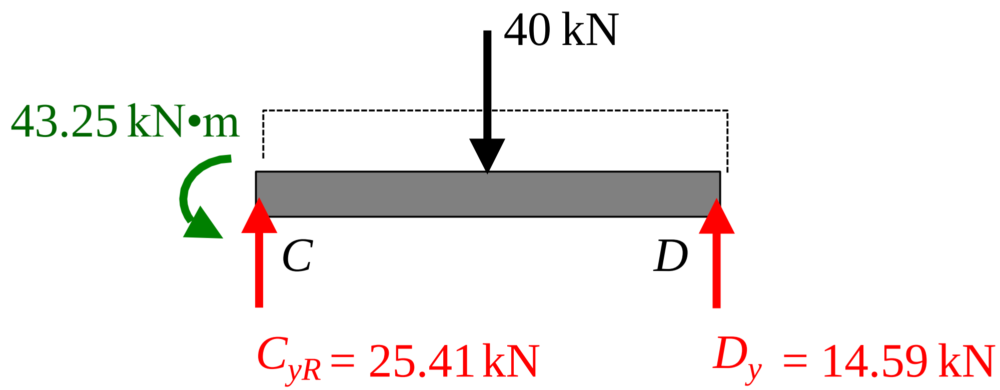

50 kN•m
CO=0
CO=0.5
50(.667)
50(.667)
50(.333)
50(.333)
B

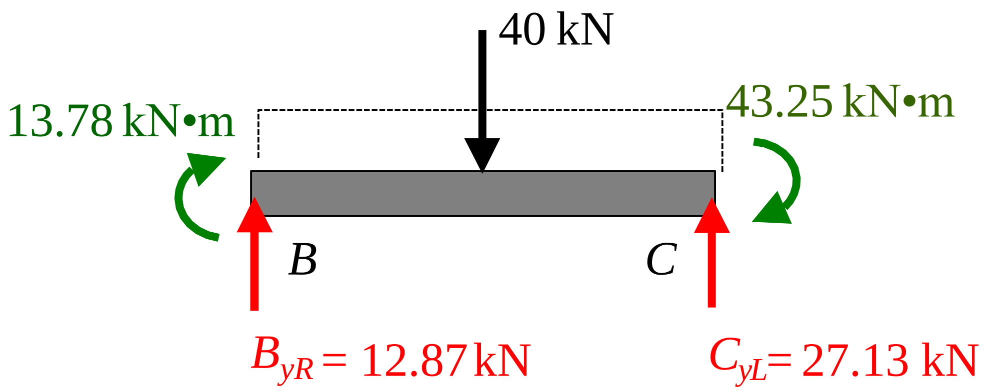


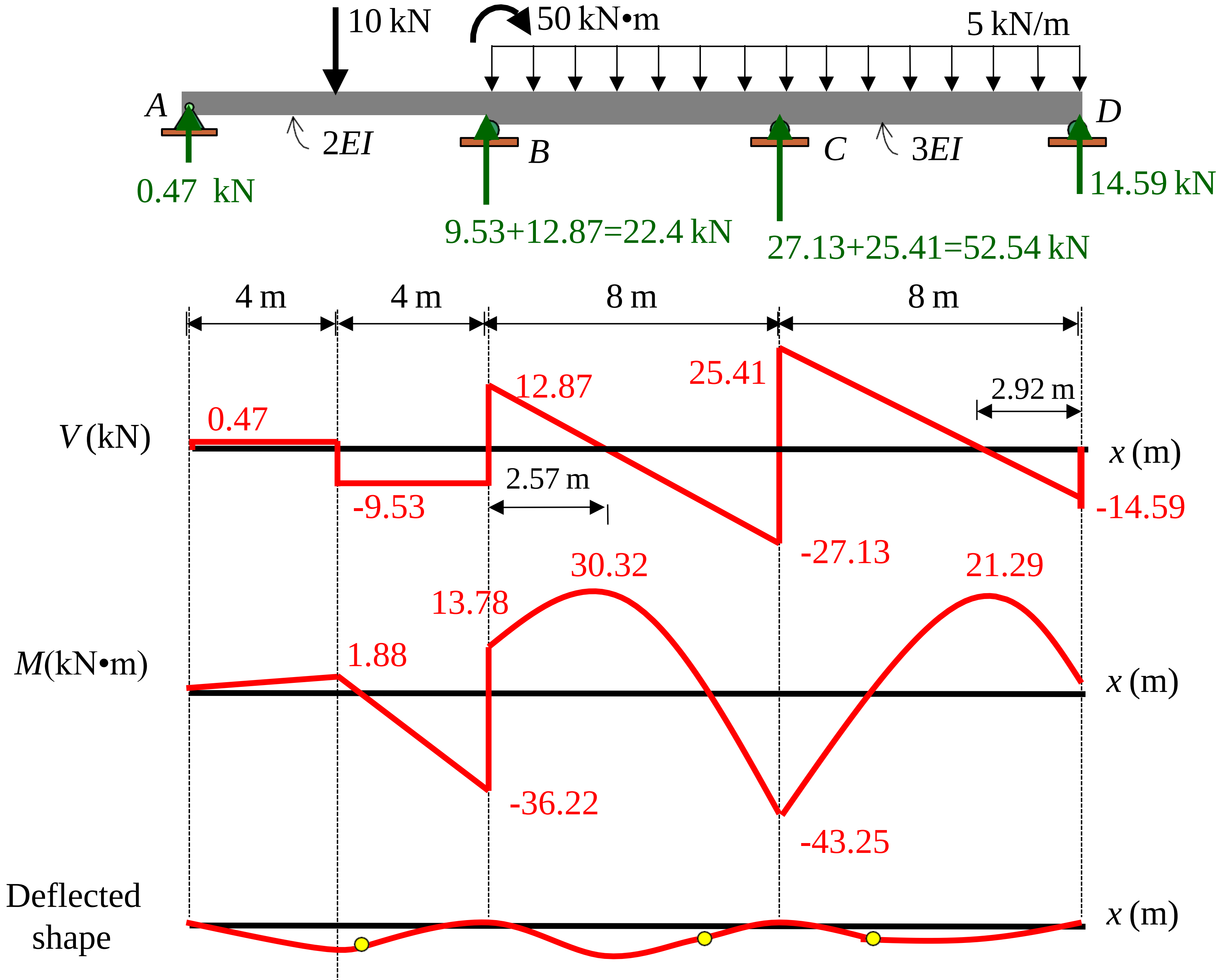
	$K_1 = 3(2EI)/8$		$K_2 = 4(3EI)/8$		$K_3 = 3(3EI)/8$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	$K_2/(K_2+K_3)$	$K_3/(K_2+K_3)$	
DF	1	0.333	0.667	0.571	0.429	1
Joint couple		-16.65	-33.35			
CO				-16.675		
FEM		-15	26.667	-26.667	40	
Dist.		-3.885	-7.782	1.905	1.437	
CO			0.953	-3.891		
Dist.		-0.317	-0.636	2.218	1.673	
CO			1.109	-0.318		
Dist.		-0.369	-0.740	0.181	0.137	
Σ		**-36.22**	**-13.78**	**-43.28**	**43.25**	


10 kN
50 kN•m
5 kN/m
2EI
36.22
13.78
43.25
43.25
3EI
A
B
C
D
4 m
4 m
8 m
8 m


10 kN
36.22 kN•m
A
B
Ay = 0.47 kN
ByL = 9.53 kN


40 kN
43.25 kN•m
C
D
CyR = 25.41 kN
Dy = 14.59 kN


40 kN
13.78 kN•m
43.25 kN•m
B
C
ByR = 12.87 kN
CyL = 27.13 kN

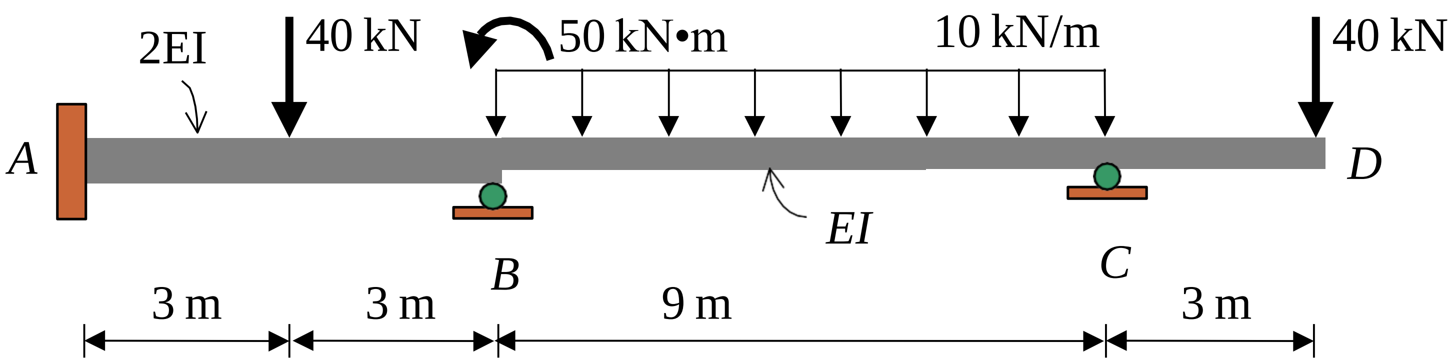

10 kN
50 kN•m
5 kN/m
A
D
2EI
B
C
3EI
0.47 kN
14.59 kN
9.53+12.87=22.4 kN
27.13+25.41=52.54 kN
4 m
4 m
8 m
8 m
V (kN)
0.47
12.87
25.41
2.92 m
x (m)
-9.53
2.57 m
-14.59
-27.13
30.32
21.29
13.78
1.88
M(kN•m)
x (m)
-36.22
-43.25
Deflected shape
x (m)

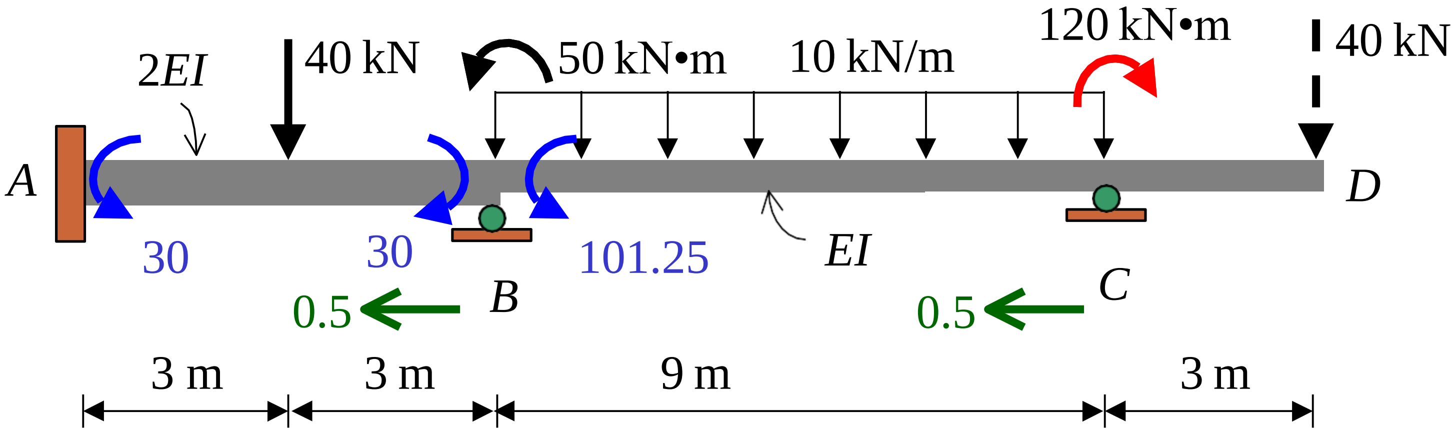
<u>Example 3</u>

From the beam shown use the moment distribution method to:

(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

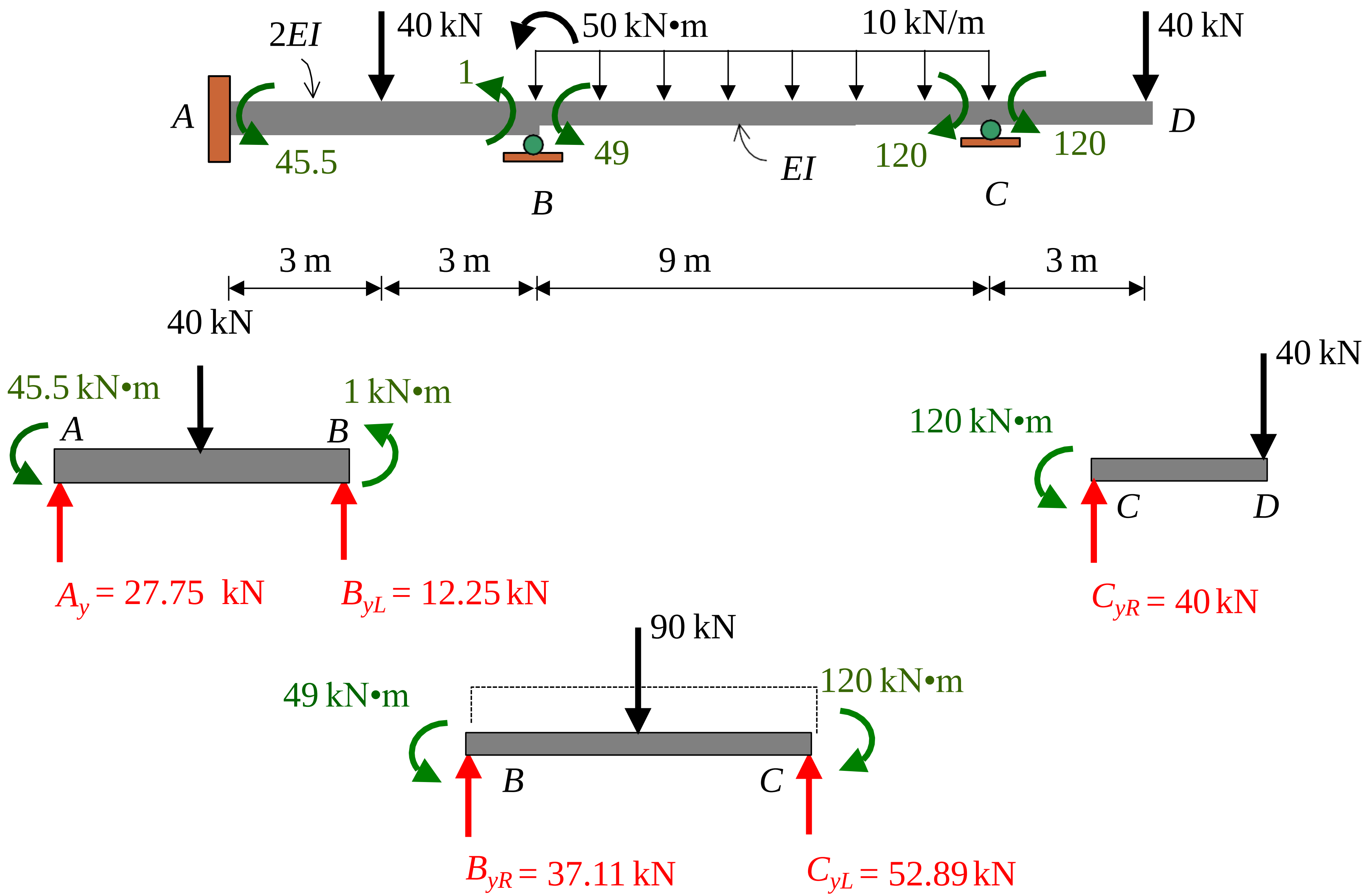


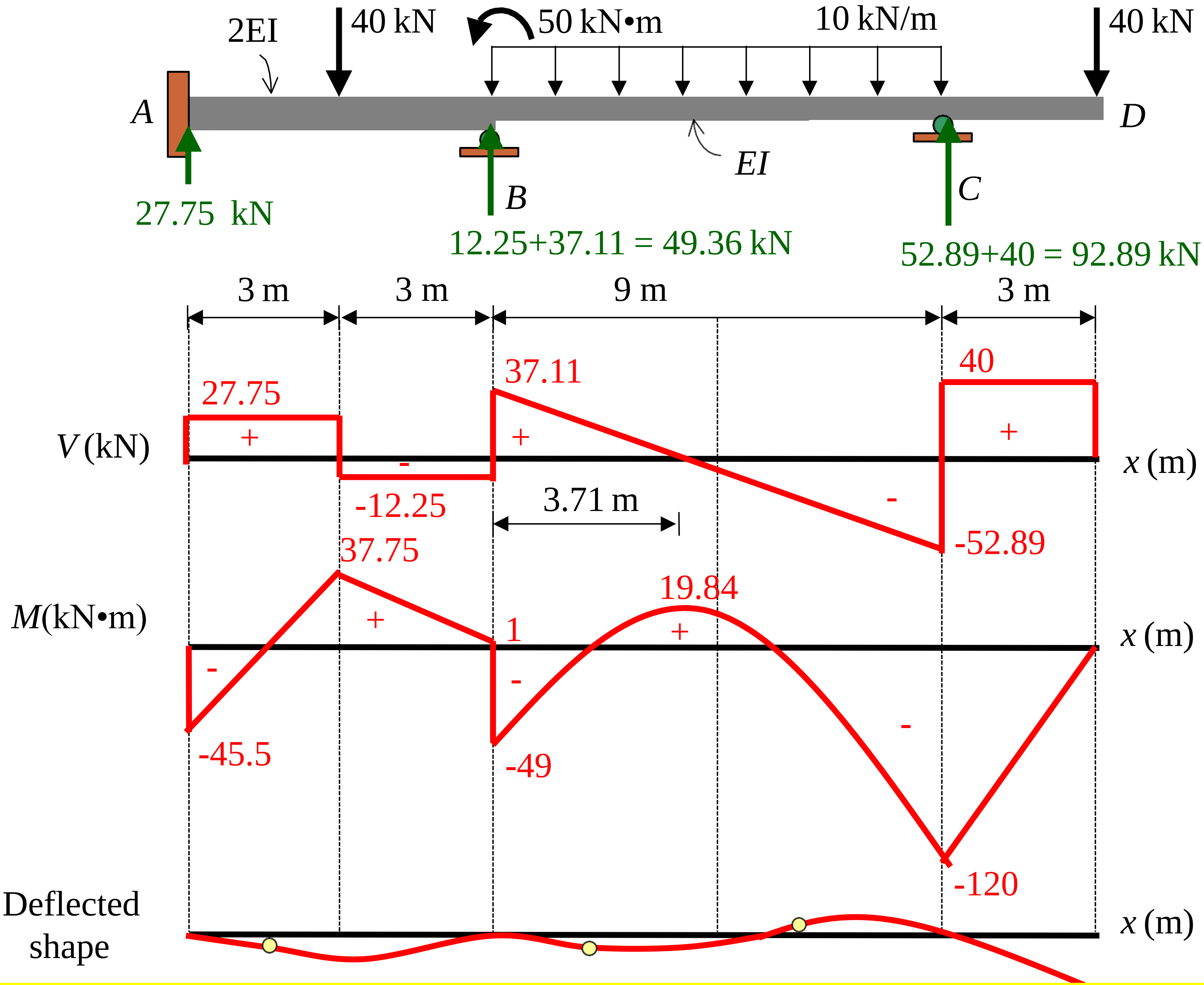


$K_1 = 4(2EI)/6$ $K_2 = 3(EI)/9$

$K_1/(K_1+K_2)$ $K_2/(K_1+K_2)$

DF	0	0.80	0.20	1
Joint couple		40	10	-120
CO	20		-60	
FEM	30	-30	101.25	
Dist.				
Dist.		-9	-2.25	
CO	-4.5			
Σ	**45.5**	**1**	**49**	**-120**


2EI
40 kN
50 kN•m
10 kN/m
40 kN
1
A
45.5
49
EI
120
120
D
B
C
3 m
3 m
9 m
3 m
40 kN
45.5 kN•m
1 kN•m
A
B
A_y = 27.75 kN
B_{yL} = 12.25 kN
40 kN
120 kN•m
C
D
C_{yR} = 40 kN
90 kN
49 kN•m
120 kN•m
B
C
B_{yR} = 37.11 kN
C_{yL} = 52.89 kN

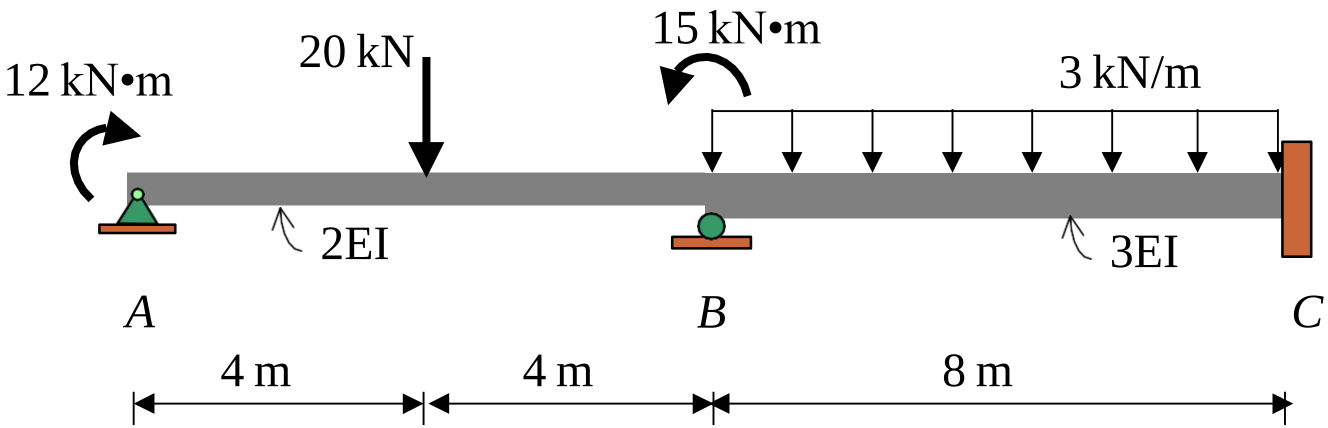

2EI
40 kN
50 kN•m
10 kN/m
40 kN
A
D
EI
B
C
27.75 kN
12.25+37.11 = 49.36 kN
52.89+40 = 92.89 kN
3 m
3 m
9 m
3 m
V (kN)
27.75
37.11
40
-12.25
3.71 m
-52.89
x (m)
M(kN•m)
37.75
19.84
1
-45.5
-49
-120
x (m)
Deflected shape
x (m)

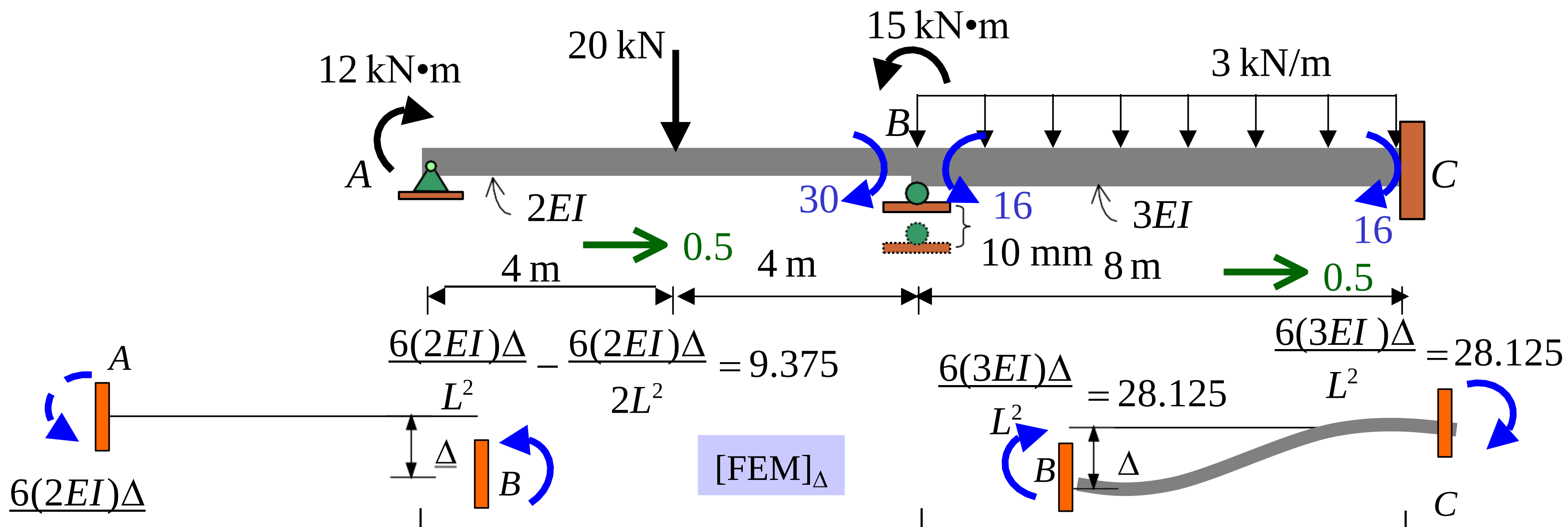
Example 4

The support *B* of the beam shown (E = 200 GPa, I = 50x10^6 mm^4) settles 10 mm.
Use the moment distribution method to:

(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**



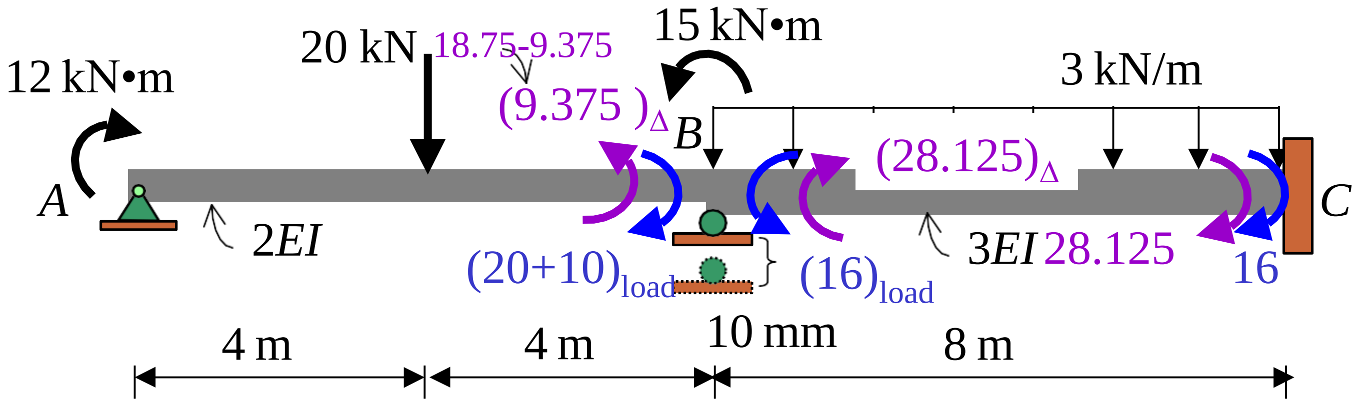


$K_1 = 3(2EI)/8$ $K_2 = 4(3EI)/8$

$K_1/(K_1+K_2)$ $K_2/(K_1+K_2)$

DF	1	0.333	0.667	0
Joint couple	-12	5	10	
CO		-6		5
$[FEM]_{load}$		-30	16	-16
$[FEM]_\Delta$		9.375	-28.125	-28.125
Dist.		12.90	25.85	
CO				12.92
Σ	**-12**	**-8.72**	**23.72**	**-26.20**

Note : ***Using the slope deflection***



$$M_{AB} = \frac{4(2EI)}{8}\theta_A + \frac{2(2EI)}{8}\theta_B + 20 - 18.75 \quad ---(1)$$

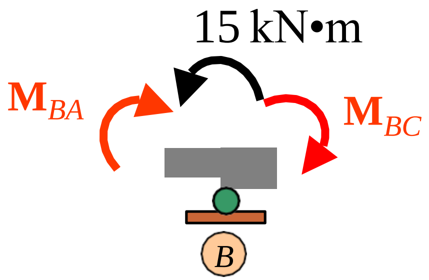
-12

$$M_{BA} = \frac{2(2EI)}{8}\theta_A + \frac{4(2EI)}{8}\theta_B - 20 + 18.75 \quad ---(2)$$

$$\frac{(2)-(1)}{2} : M_{BA} = \frac{3(2EI)}{8}\theta_B - 30 + 9.375 - 12/2 \quad ---(2a)$$

$$M_{BC} = \frac{4(3EI)}{8}\theta_B + 16 - 28.125 \quad ---(3)$$

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 - 28.125 \quad ---(4)$$



$+\circlearrowleft$ $\Sigma M_B = 0$: $- \mathbf{M}_{BA} - \mathbf{M}_{BC} + 15 = 0$

$$(0.75 + 1.5)EI\theta_B - 38.75 - 15 = 0$$

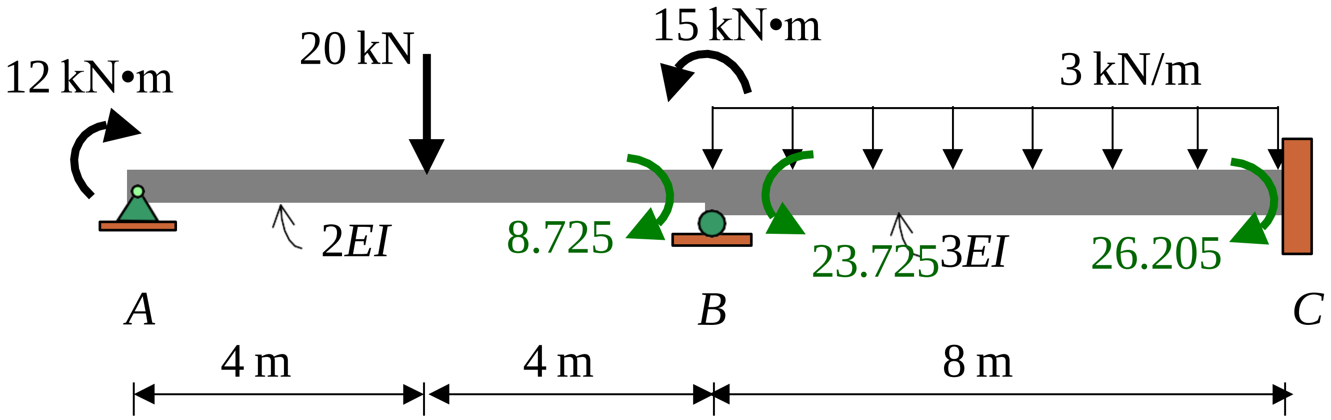
$$\theta_B = 23.9/EI$$

$$M_{BA} = -8.7 \text{ kN•m},$$

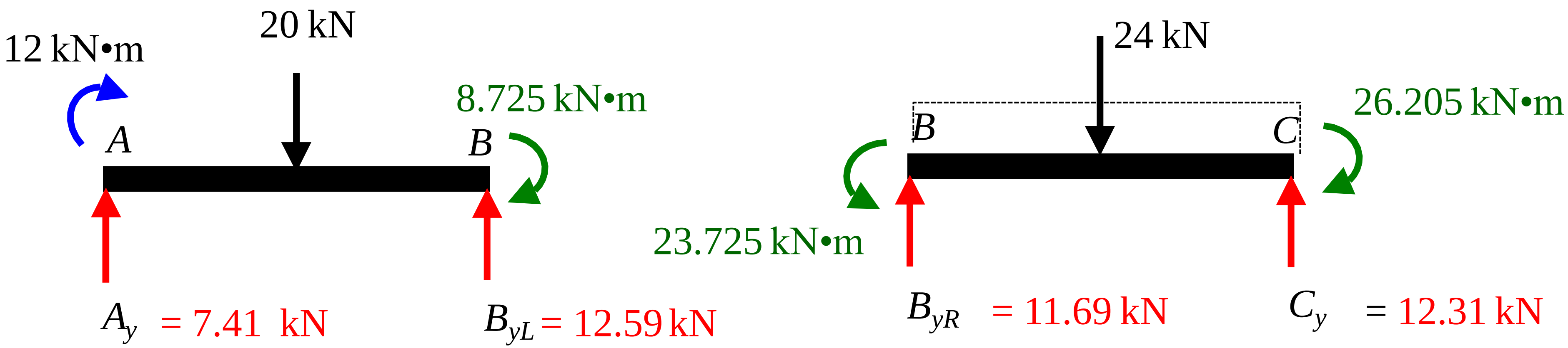
$$M_{BC} = 23.72 \text{ kN•m}$$

$$M_{CB} = \frac{2(3EI)}{8}\theta_B - 16 - 28.125$$

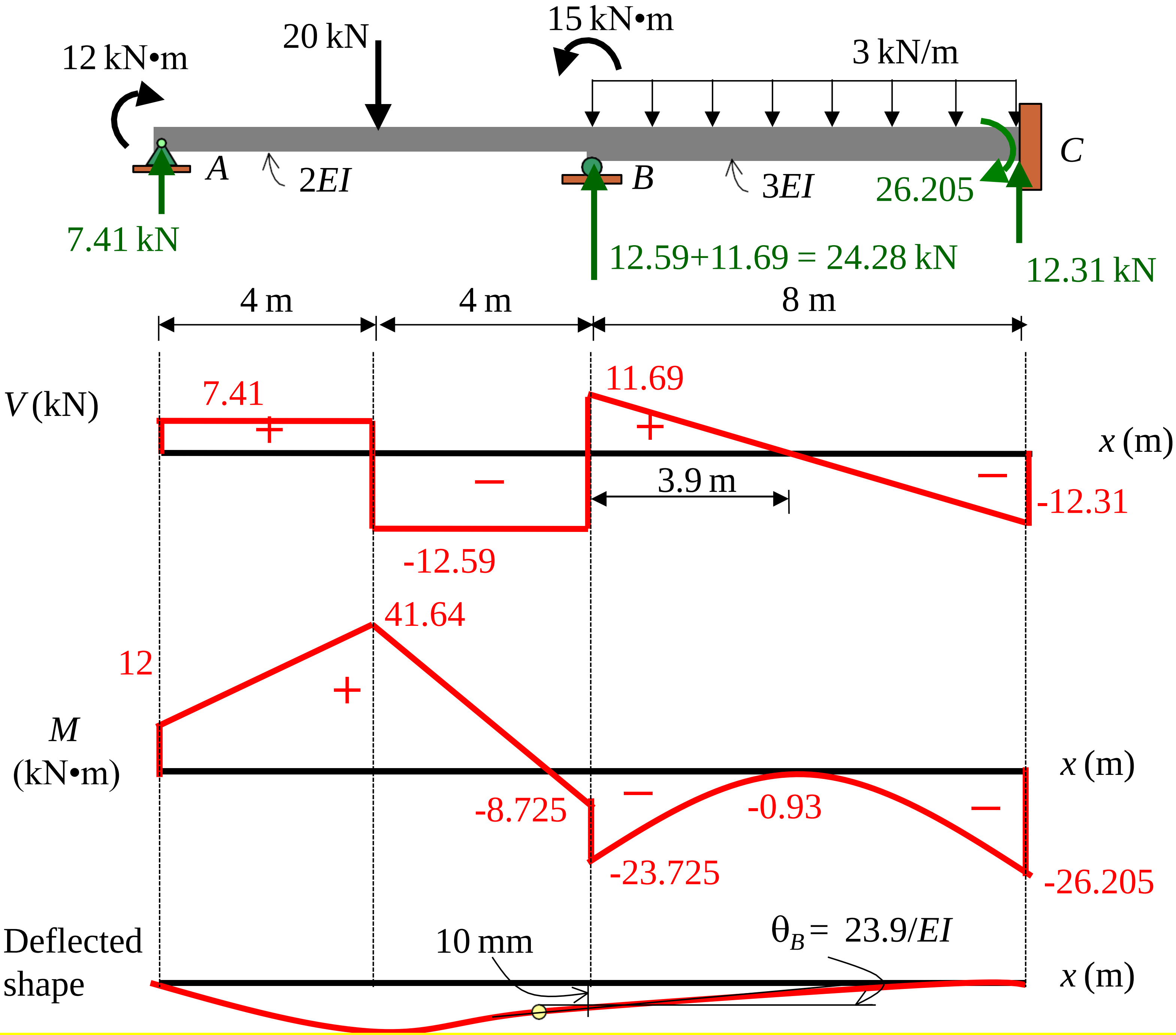
$$= -26.2\ kN \bullet m$$



12 kN•m
20 kN
15 kN•m
3 kN/m
2EI
8.725
23.725
3EI
26.205
A
B
C
4 m
4 m
8 m



12 kN•m
20 kN
A
B
8.725 kN•m
A_y = 7.41 kN
B_{yL} = 12.59 kN
24 kN
B
C
26.205 kN•m
23.725 kN•m
B_{yR} = 11.69 kN
C_y = 12.31 kN


12 kN•m
20 kN
15 kN•m
3 kN/m
A
2EI
B
3EI
C
26.205
7.41 kN
12.59+11.69 = 24.28 kN
12.31 kN
4 m
4 m
8 m
V (kN)
7.41
11.69
x (m)
3.9 m
-12.31
-12.59
41.64
12
M
(kN•m)
x (m)
-8.725
-0.93
-23.725
-26.205
Deflected
shape
10 mm
θB = 23.9/EI
x (m)

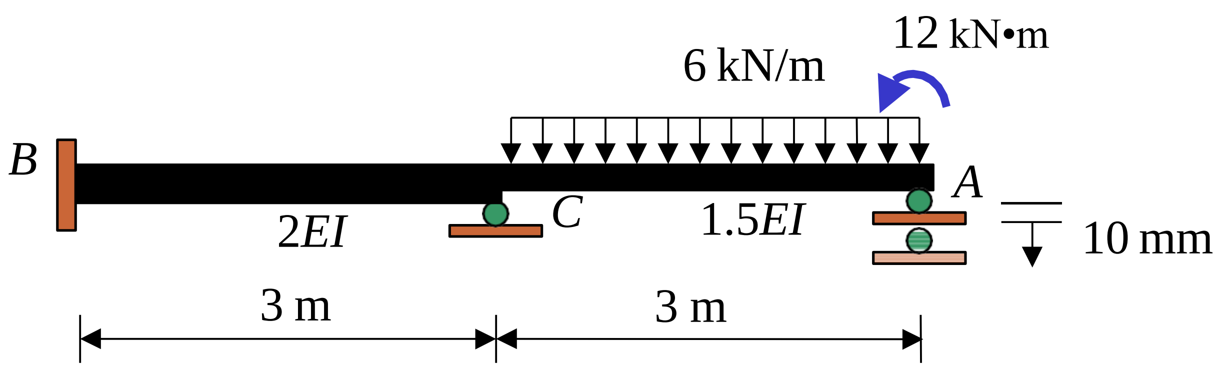
Example 5

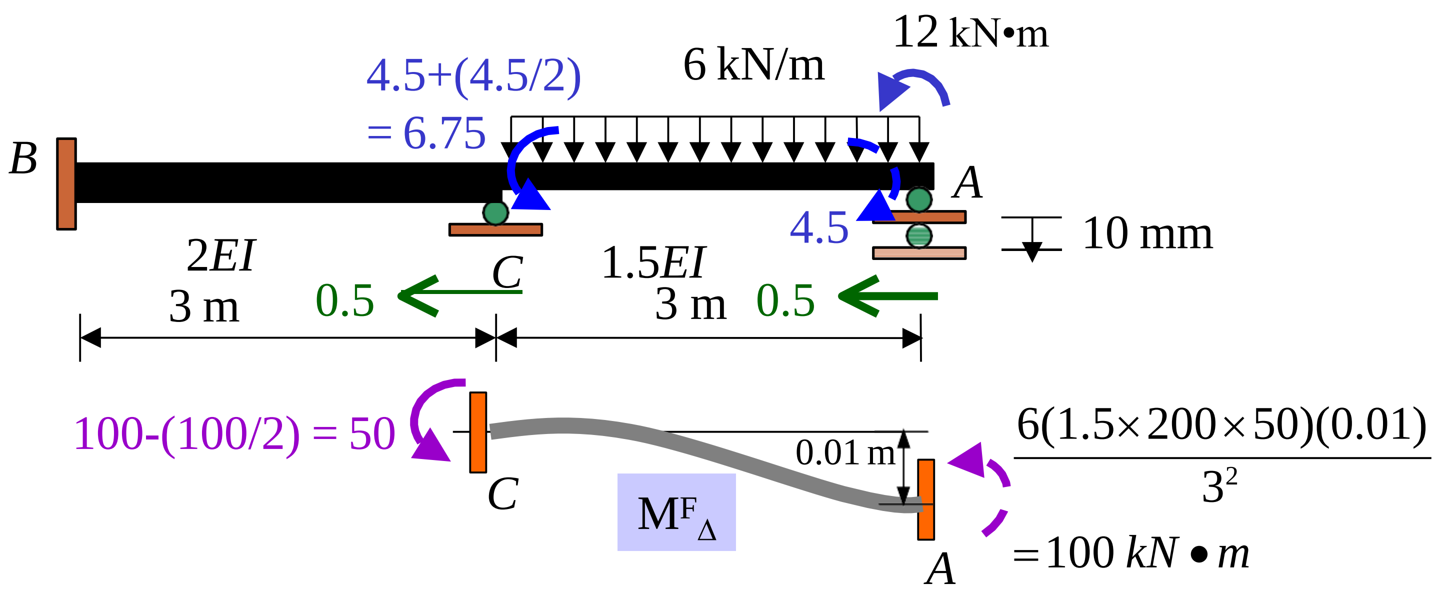
For the beam shown, support A settles 10 mm downward, use the moment distribution method to

(a) Determine all the **reactions** at supports

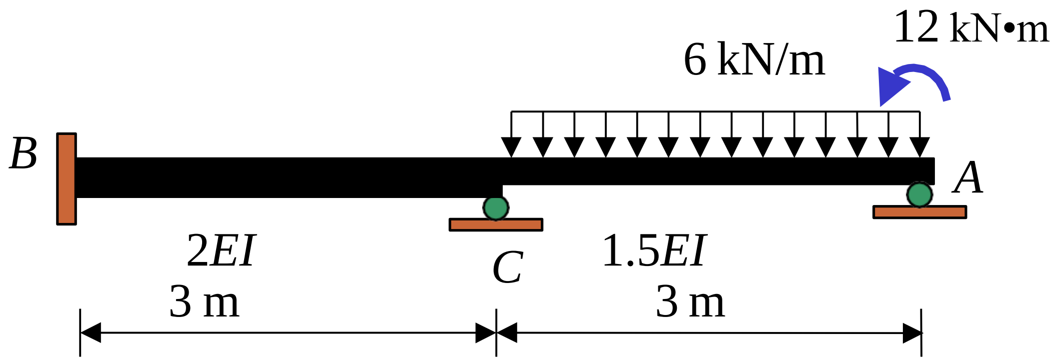
(b) Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**.

Take E= 200 GPa, I = 50(10^6) mm^4.

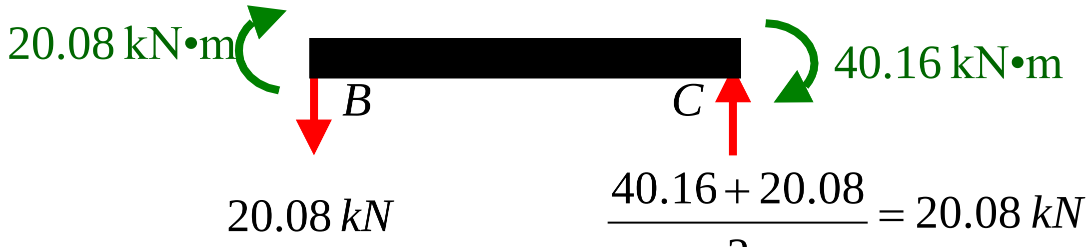




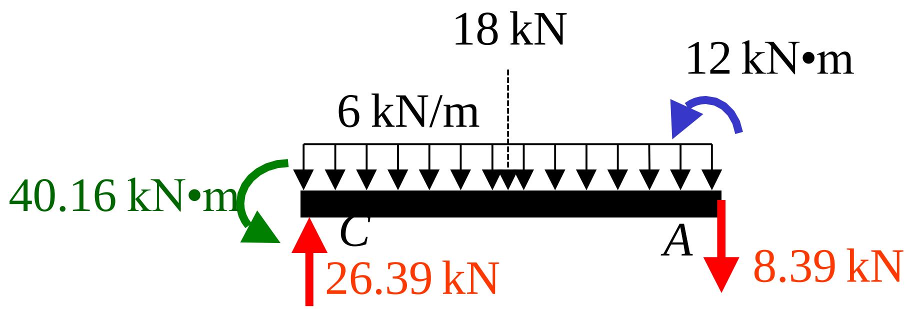
	$K_1 = 4(2EI)/3$		$K_2 = 3(1.5EI)/3$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	
DF	0	0.64	0.36	1
Joint couple				12
CO			6	
$[FEM]_{load}$			6.75	
$[FEM]_\Delta$			50	
Dist.		-40.16	-22.59	
CO	-20.08			
Σ	**-20.08**	**-40.16**	**40.16**	**12**

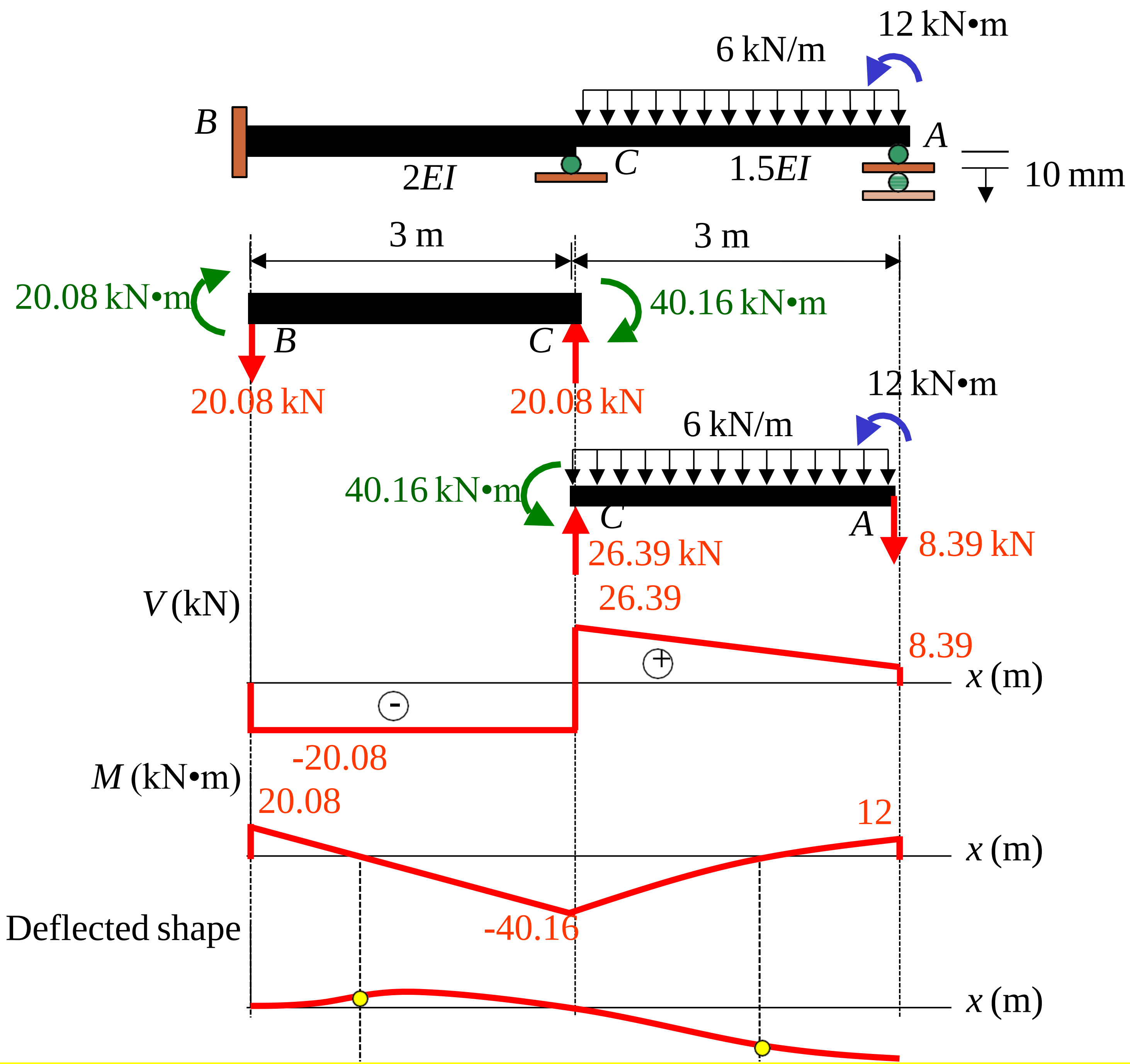


ΣM	**-20.08**	**-10.16**	**40.16**	**12**



$$\frac{40.16+20.08}{3}=20.08\ kN$$




12 kN•m
6 kN/m
B
A
C
2EI
1.5EI
10 mm
3 m
3 m
20.08 kN•m
40.16 kN•m
B
C
20.08 kN
20.08 kN
12 kN•m
6 kN/m
40.16 kN•m
C
A
26.39 kN
8.39 kN
V (kN)
26.39
8.39
x (m)
-20.08
M (kN•m)
20.08
12
x (m)
-40.16
Deflected shape
x (m)

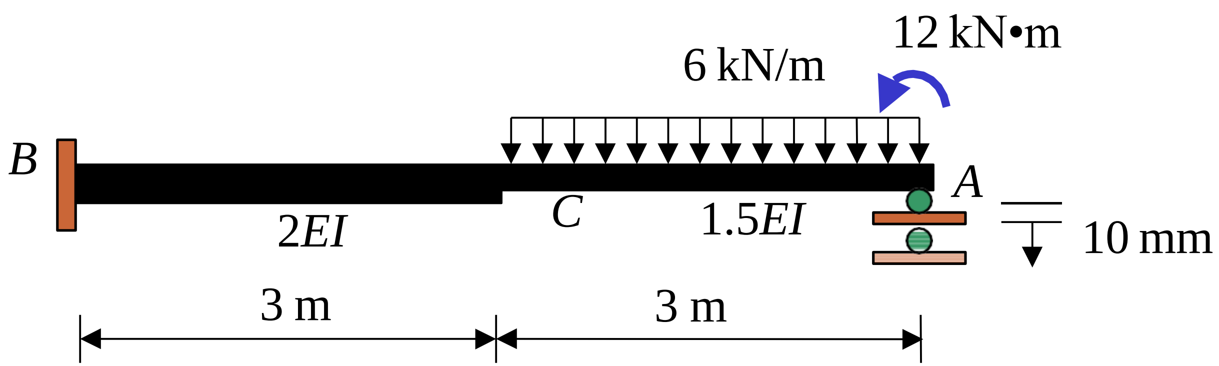
Example 6

For the beam shown, support A settles 10 mm downward, use the moment distribution method to

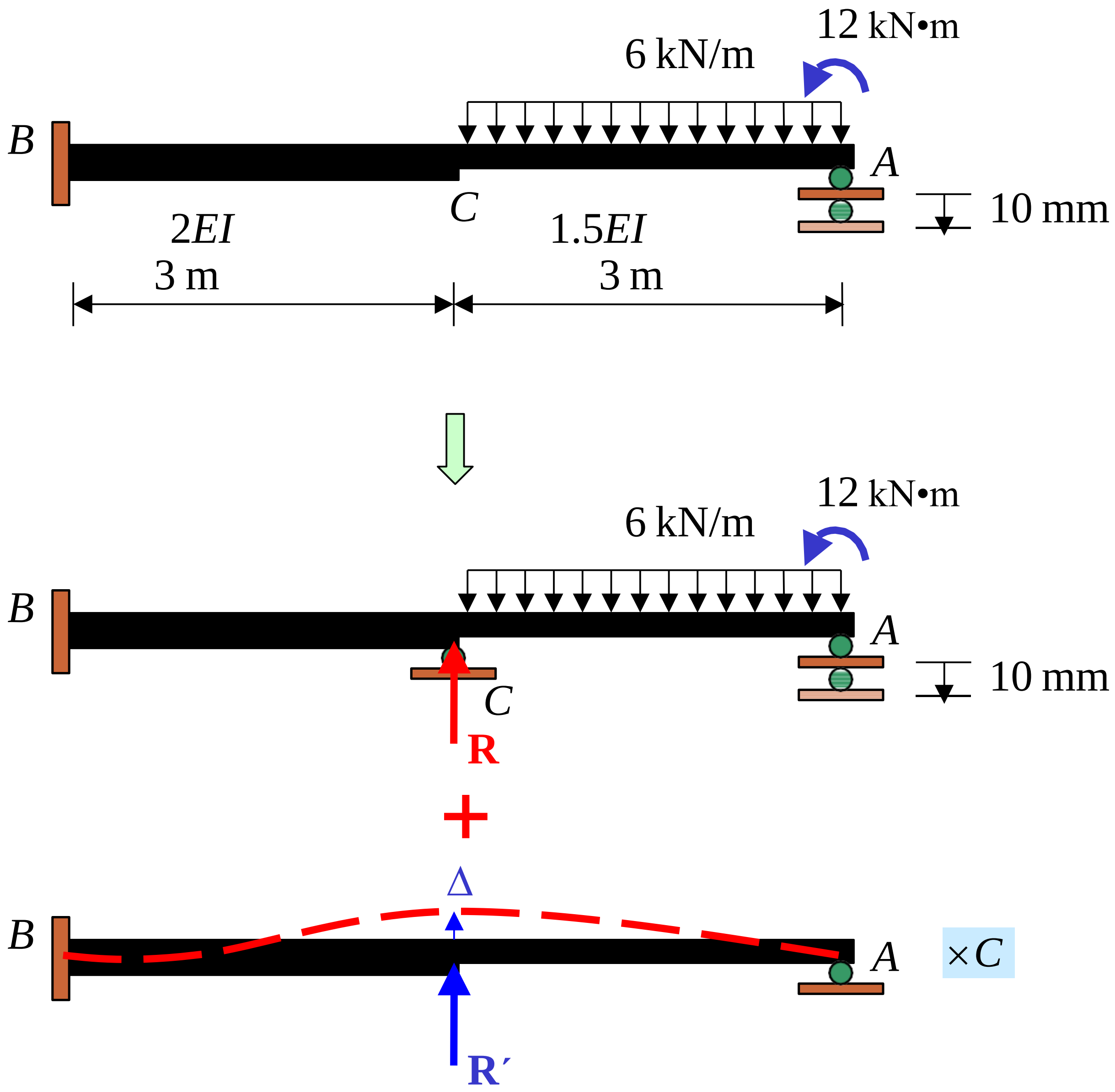
(a) Determine all the **reactions** at supports

(b) Draw its **quantitative shear, bending moment diagrams**, and **qualitative deflected shape**.

Take E= 200 GPa, $I = 50(10^6)$ mm^4.

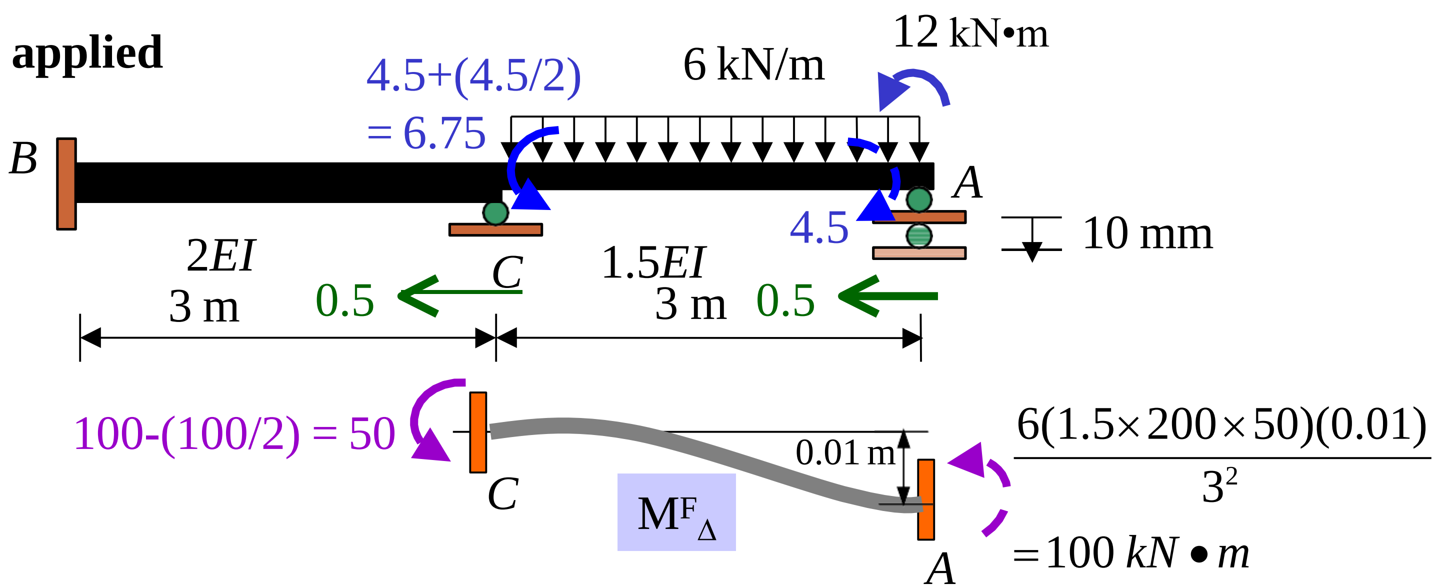


• **Overview**

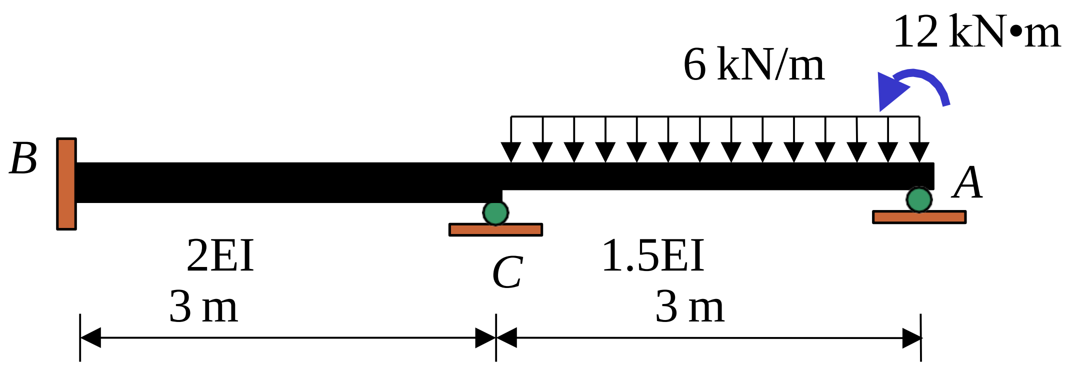


$$R + R'C = 0 \quad ---(1^*)$$

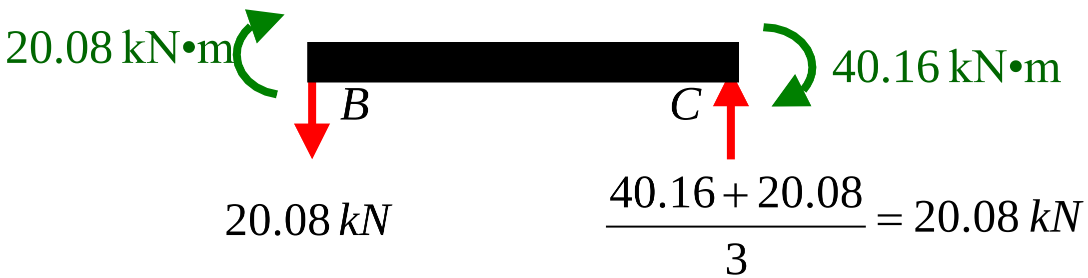
- **Artificial joint applied**

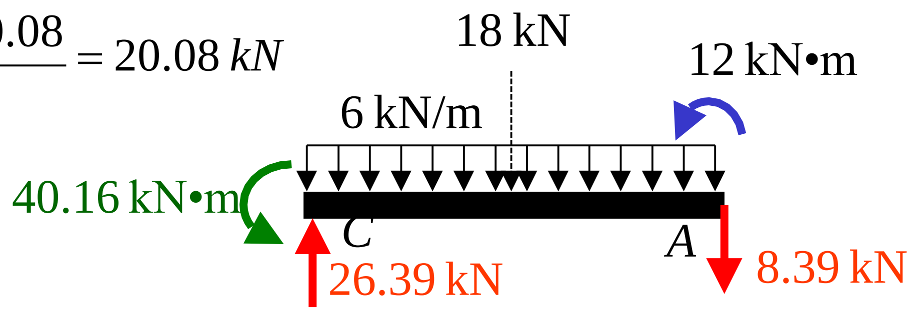


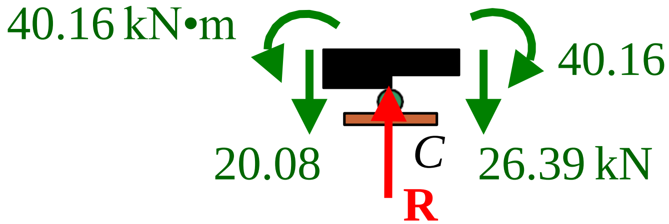
	$K_1 = 4(2EI)/3$		$K_2 = 3(1.5EI)/3$	
		$K_1/(K_1+K_2)$	$K_2/(K_1+K_2)$	
DF	0	0.64	0.36	1
Joint couple				12
CO			6	
[FEM]$_{load}$			6.75	
[FEM]$_\Delta$			50	
Dist.		-40.16	-22.59	
CO	-20.08			
Σ	**-20.08**	**-40.16**	**40.16**	**12**



ΣM	**-20.08**	**-40.16**	**40.16**	**12**







$$+\uparrow \quad \Sigma F_y = 0: \quad -20.08 - 26.39 + R = 0$$

$$R = 46.47\ kN$$

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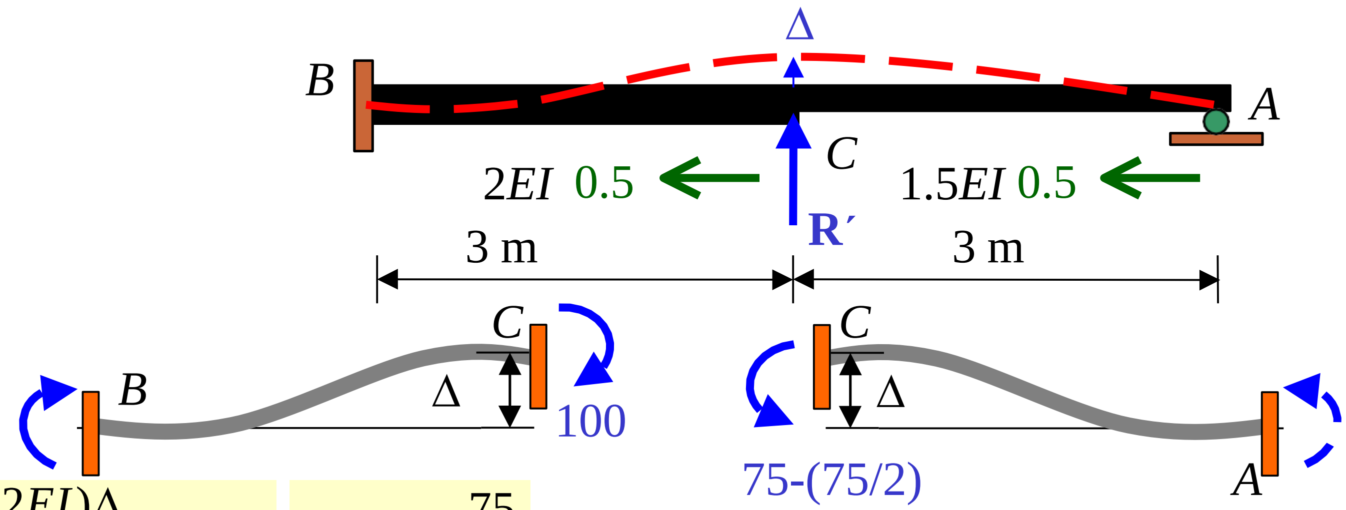
JECRC, JAIPUR

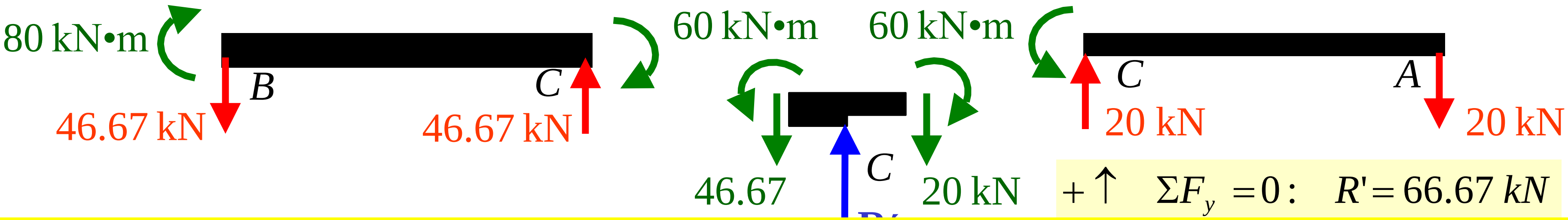
- **Artificial joint removed**



$$\frac{6(2EI)\Delta_C}{3^2} = 100 \rightarrow \Delta = \frac{75}{EI}$$

$$\frac{6(1.5EI)(\frac{75}{EI})}{3^2} = 75$$

	B	C	C	A
DF	0	0.64	0.36	1
$[FEM]_\Delta$	-100	-100	+37.5	
Dist.		40	22.5	
CO	20			
Σ	**-80**	**-60**	**60**	

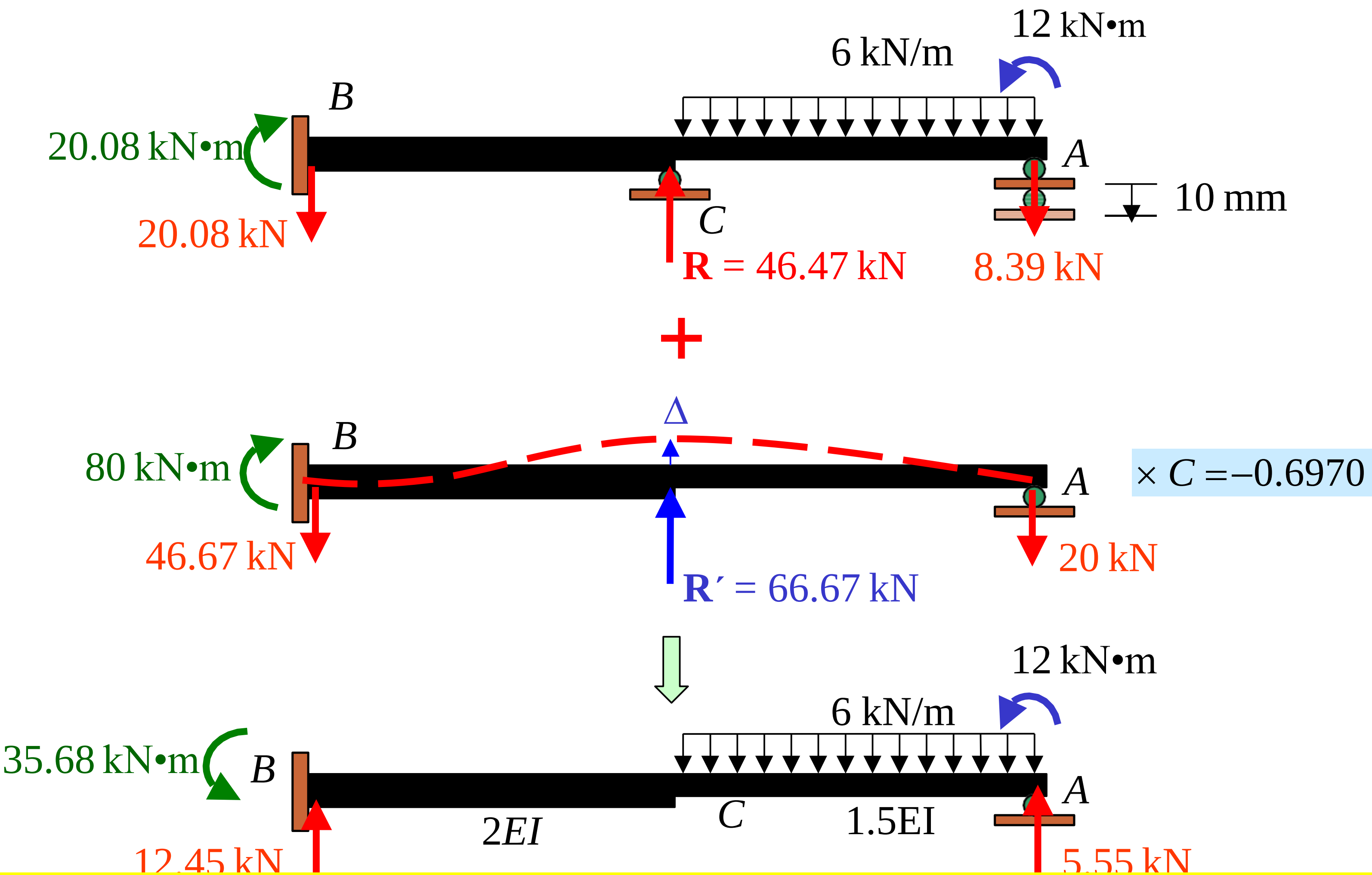


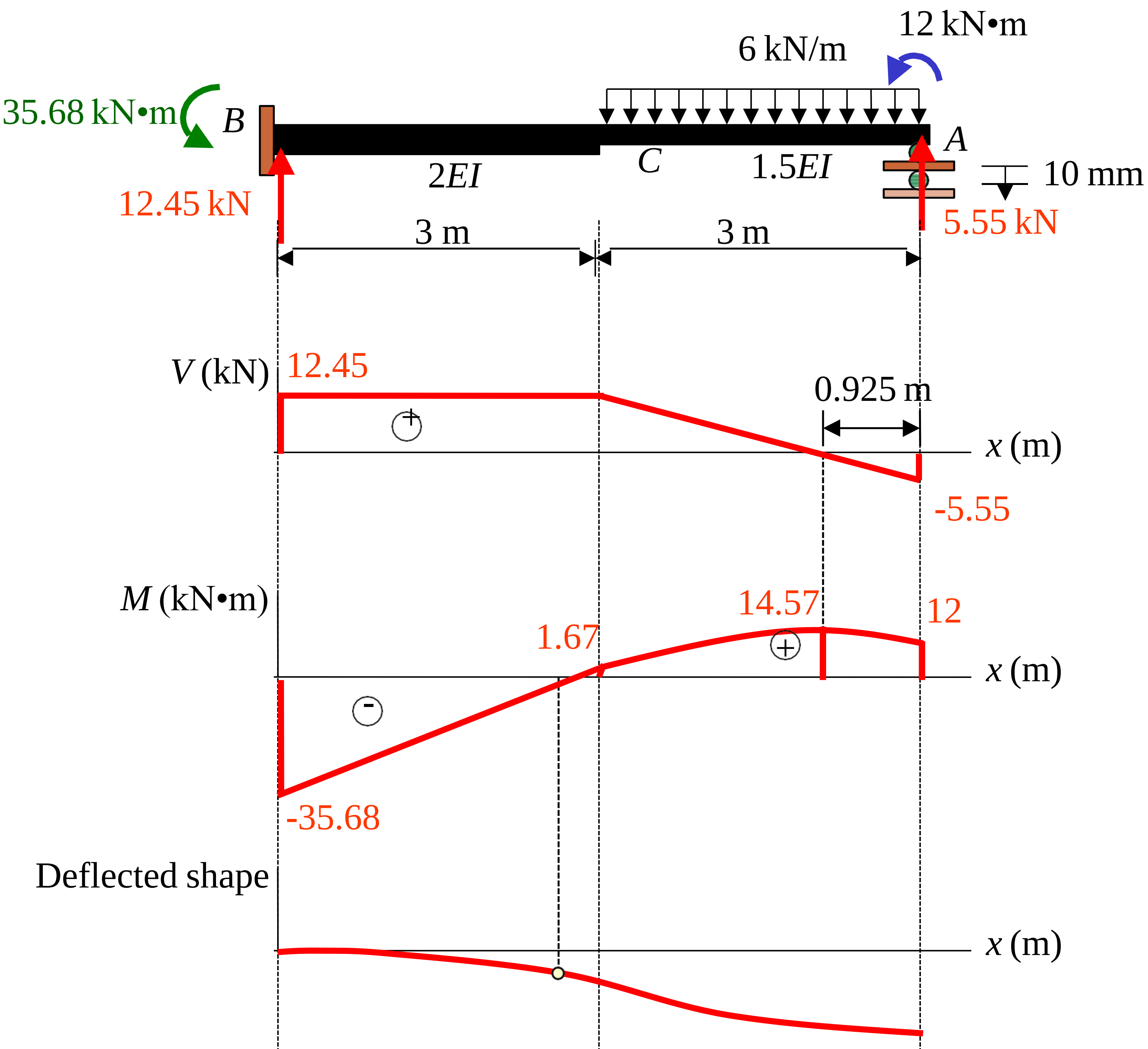
$$+\uparrow \quad \Sigma F_y = 0 : \quad R' = 66.67\ kN$$

- **Solve equation**

$$Substitute\ R = 46.47\ kN\ and\ R' = 66.67\ kN\ in\ (1^*)$$

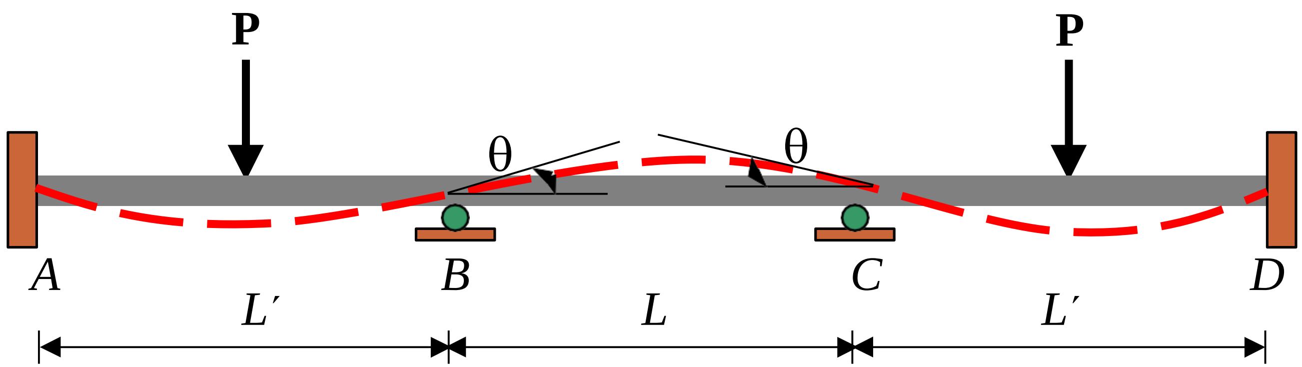
$$46.47 + 66.67C = 0$$
$$C = -0.6970$$



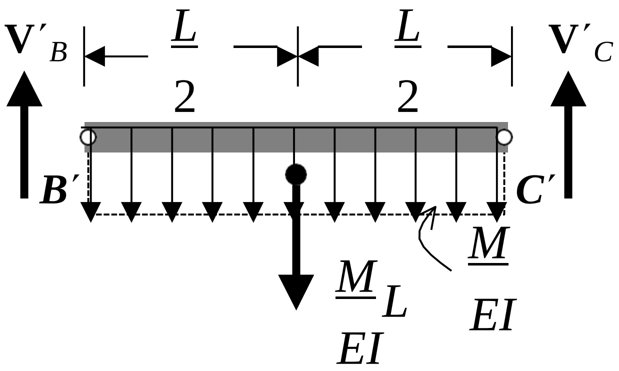

12 kN•m
6 kN/m
35.68 kN•m
B
A
C
2EI
1.5EI
10 mm
12.45 kN
5.55 kN
3 m
3 m
V (kN)
12.45
0.925 m
x (m)
-5.55
M (kN•m)
14.57
12
1.67
-35.68
x (m)
Deflected shape
x (m)

Symmetric Beam

- **Symmetric Beam and Loading**



real beam



conjugate beam

$$+\circlearrowright \Sigma M_{C'} = 0: \quad -V_{B'}(L) + \frac{M}{EI}(L)\left(\frac{L}{2}\right) = 0$$

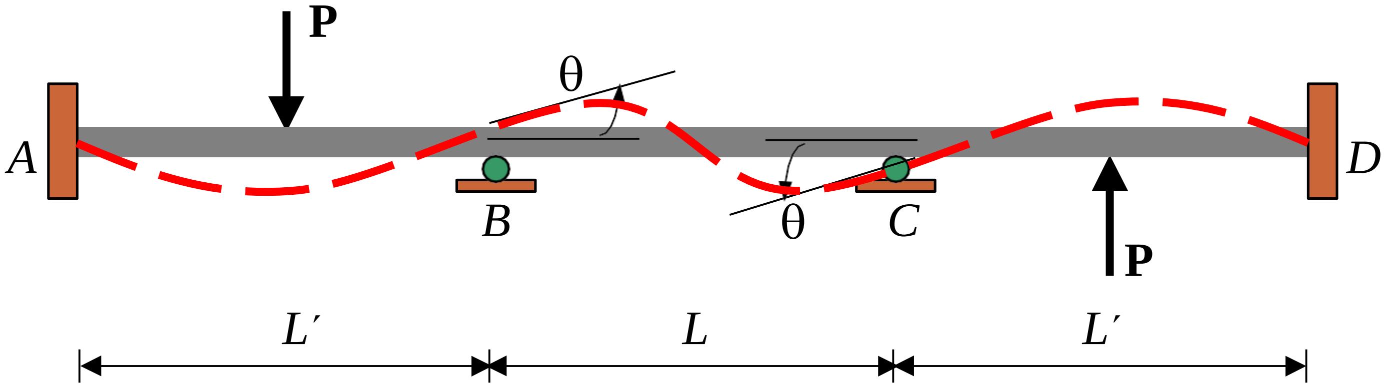
$$V_{B'} = \theta = \frac{ML}{2EI}$$

$$M = \frac{2EI}{L}\theta$$

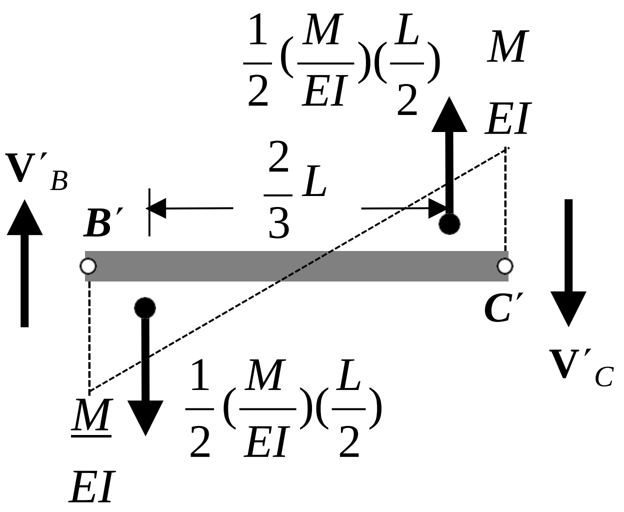
The stiffness factor for the center span is, therefore,

$$K = \frac{2EI}{L}$$

- **Symmetric Beam with Antisymmetric Loading**



real beam



conjugate beam

$$+\circlearrowright \Sigma M_{C'} = 0: \quad -V_{B'}(L) + \frac{1}{2}(\frac{M}{EI})(\frac{L}{2})(\frac{2L}{3}) = 0$$

$$V_{B'} = \theta = \frac{ML}{6EI}$$

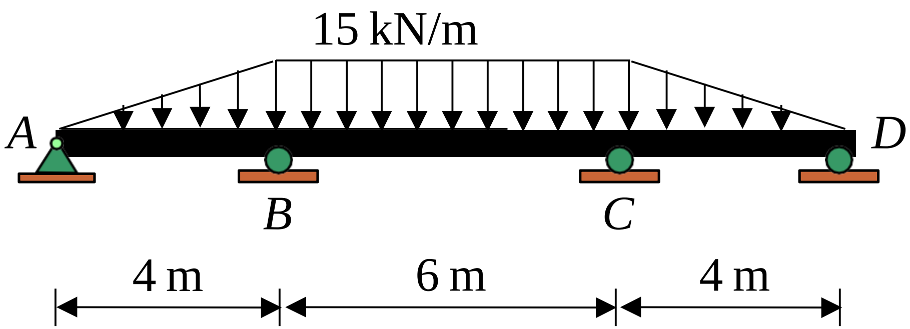
$$M = \frac{6EI}{L}\theta$$

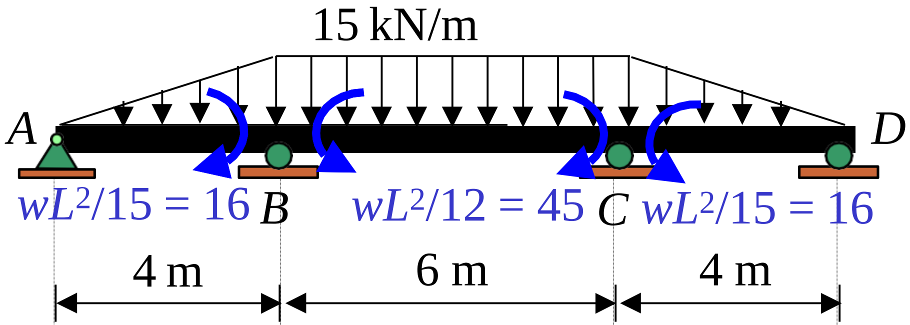
The stiffness factor for the center span is, therefore,

$$K = \frac{6EI}{L}$$

Example 5a

Determine all the reactions at supports for the beam below. *EI* is constant.



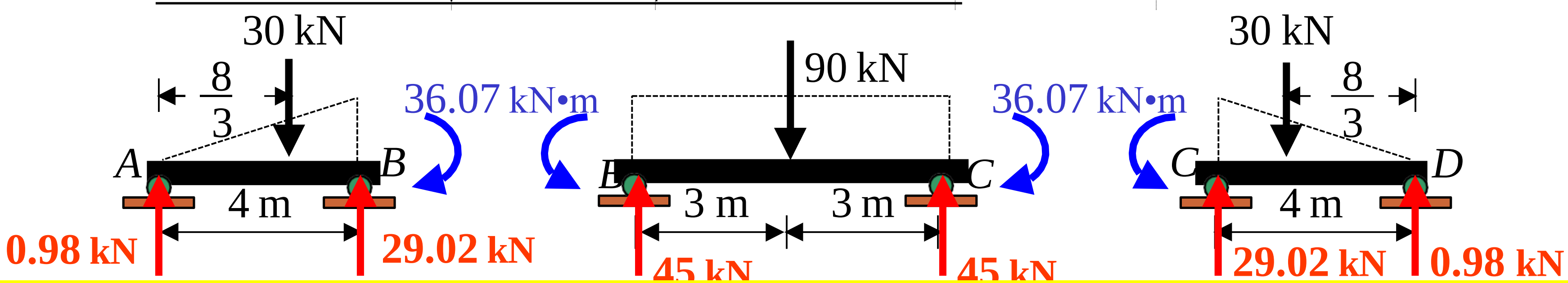


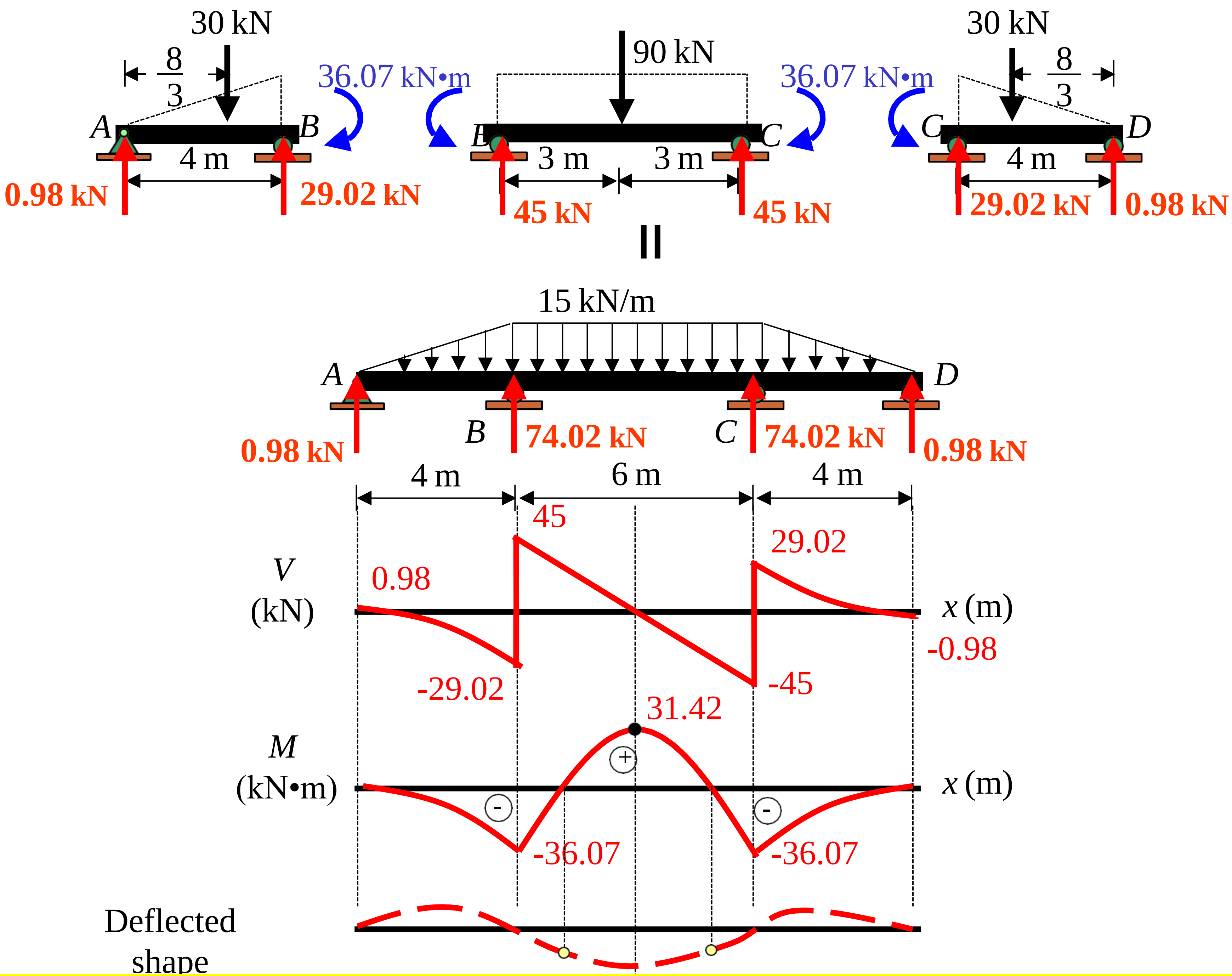
$$K_{(AB)} = \frac{3EI}{L} = \frac{3EI}{4}, \quad K_{(BC)} = \frac{2EI}{L} = \frac{2EI}{6}$$

$$(DF)_{AB} = \frac{K_{(AB)}}{K_{(AB)}} = 1, (DF)_{BA} = \frac{K_{(AB)}}{K_{(AB)} + K_{(BC)}} = \frac{(3EI/4)}{(3EI/4) + (2EI/6)} = 0.692,$$

$$(DF)_{BC} = \frac{K_{(BC)}}{K_{(AB)} + K_{(BC)}} = \frac{(2EI/6)}{(3EI/4) + (2EI/6)} = 0.308$$

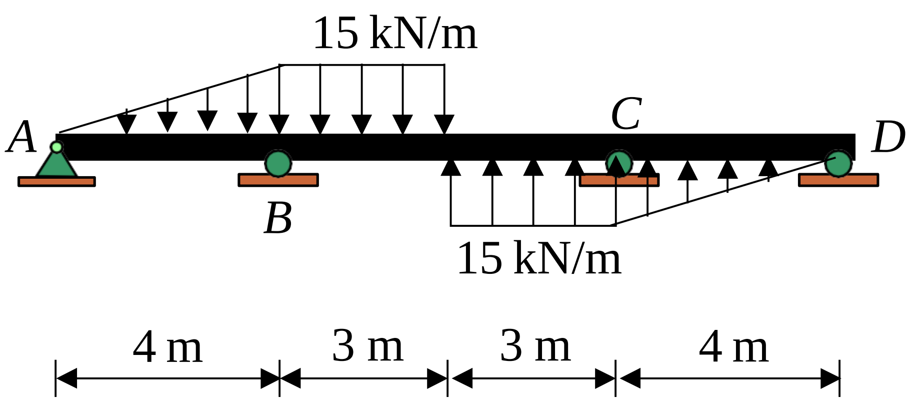
DF	1.0	0.692	0.308
[FEM]$_{load}$	0	-16	+45
Dist.		-20.07	-8.93
ΣM		-36.07	+36.07



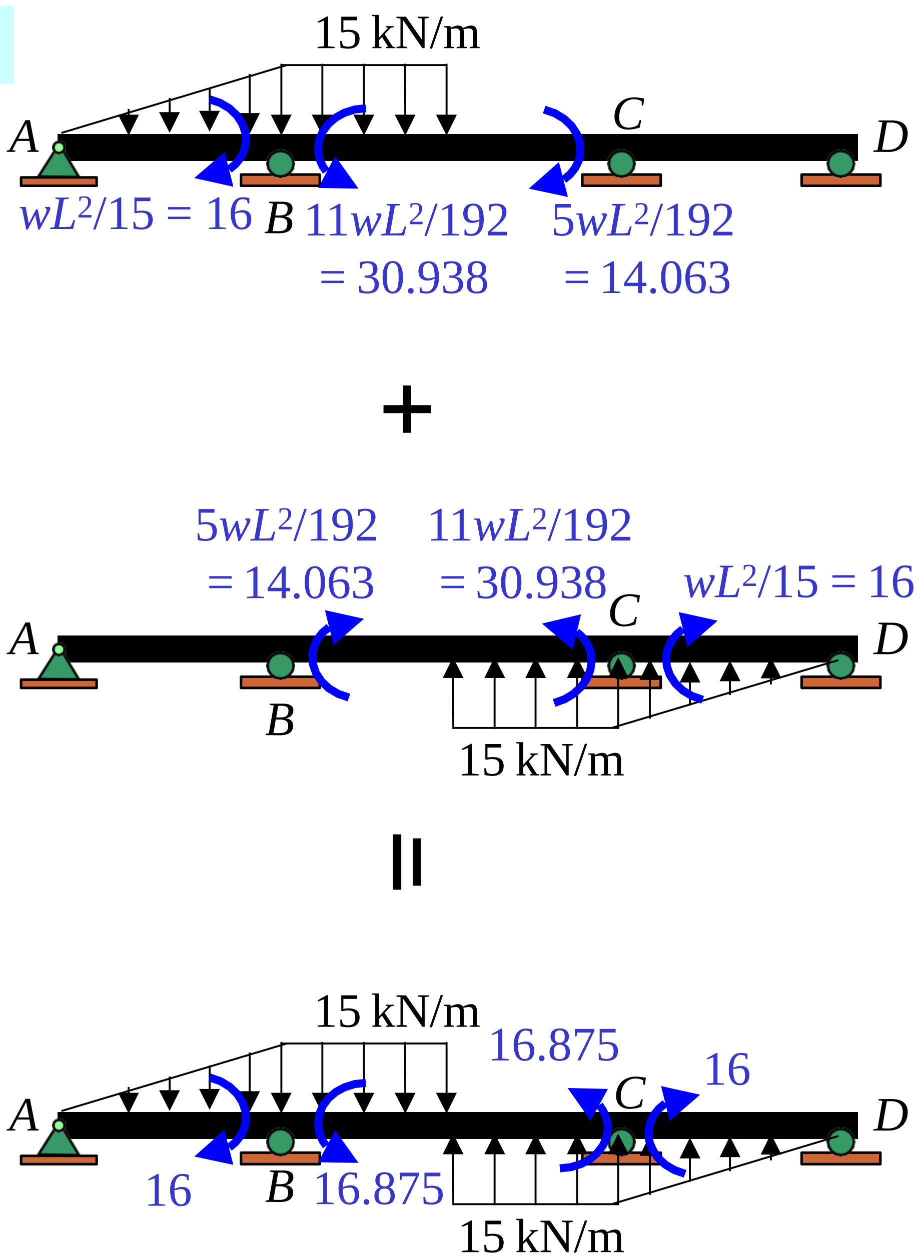

30 kN
8/3
A
B
4 m
0.98 kN
29.02 kN
36.07 kN•m
90 kN
B
C
3 m
3 m
45 kN
45 kN
36.07 kN•m
30 kN
8/3
C
D
4 m
29.02 kN
0.98 kN
=
15 kN/m
A
D
B
C
0.98 kN
74.02 kN
74.02 kN
0.98 kN
4 m
6 m
4 m
V
(kN)
0.98
-29.02
45
-45
29.02
-0.98
x (m)
M
(kN•m)
31.42
-36.07
-36.07
x (m)
Deflected
shape

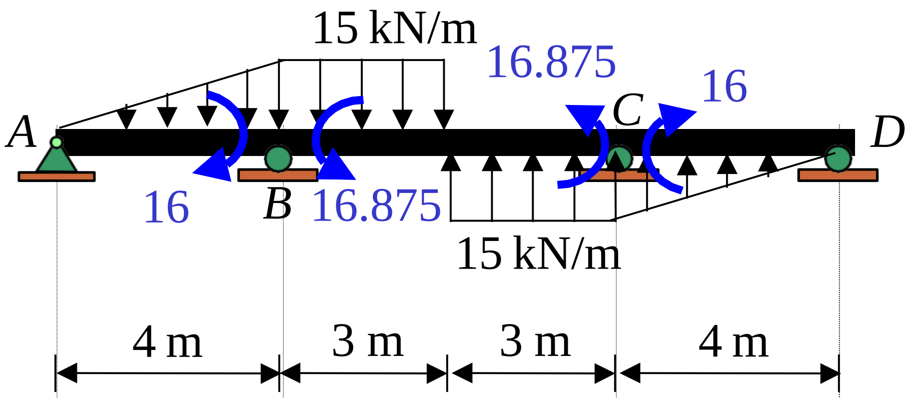
Example 5b

Determine all the reactions at supports for the beam below. *EI* is constant.



Fixed End Moment

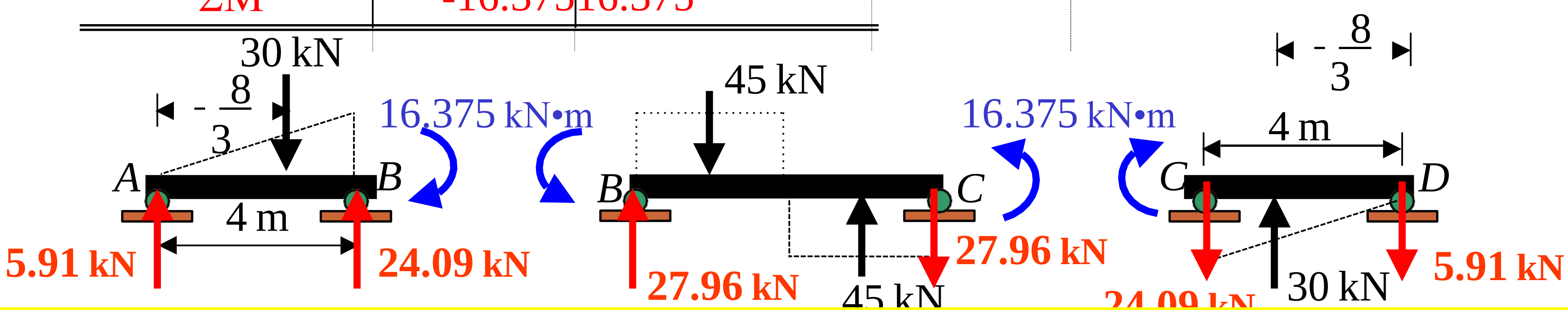


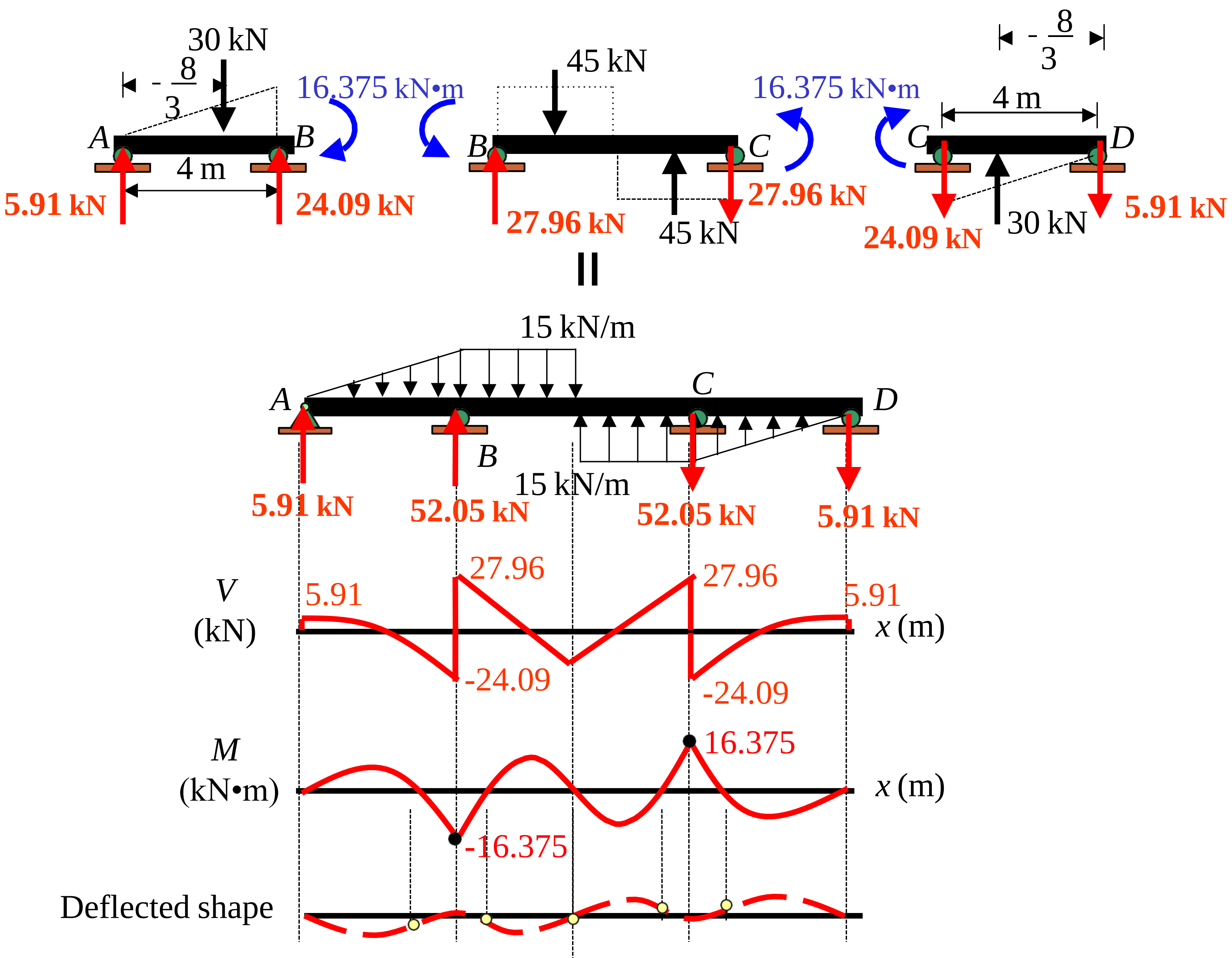


$$K_{(AB)} = \frac{3EI}{L} = \frac{3EI}{4} = 0.75EI, \quad K_{(BC)} = \frac{6EI}{L} = \frac{6EI}{6} = EI$$

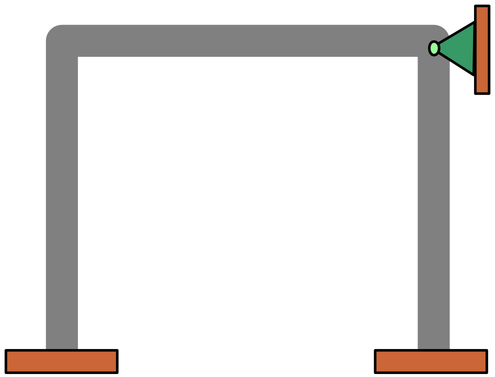
$$(DF)_{AB} = 1, \qquad (DF)_{BA} = \frac{0.75}{0.75+1} = 0.429, \qquad (DF)_{BC} = \frac{1}{0.75+1} = 0.571$$

	AB	BA	BC
DF	1.0	0.429	0.571
$[FEM]_{load}$	0	-16	16.875
Dist.		-0.375	-0.50
ΣM		-16.375	16.375



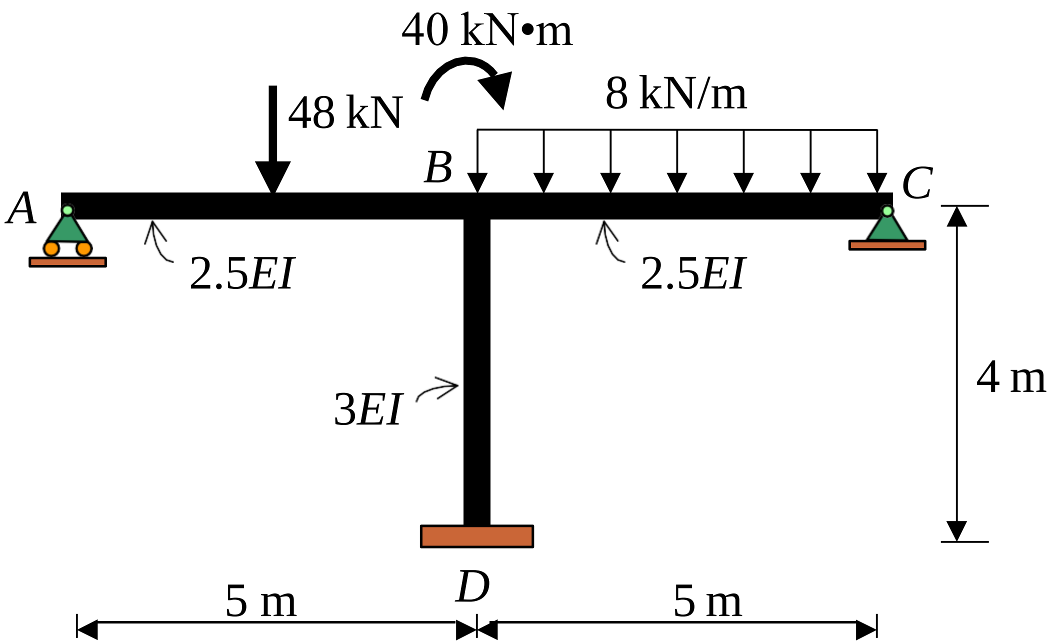

30 kN
8/3
16.375 kN•m
45 kN
16.375 kN•m
8/3
4 m
A
B
B
C
C
D
4 m
5.91 kN
24.09 kN
27.96 kN
45 kN
27.96 kN
24.09 kN
30 kN
5.91 kN
15 kN/m
A
C
D
B
15 kN/m
5.91 kN
52.05 kN
52.05 kN
5.91 kN
V
(kN)
5.91
27.96
-24.09
27.96
-24.09
5.91
x (m)
M
(kN•m)
16.375
-16.375
x (m)
Deflected shape

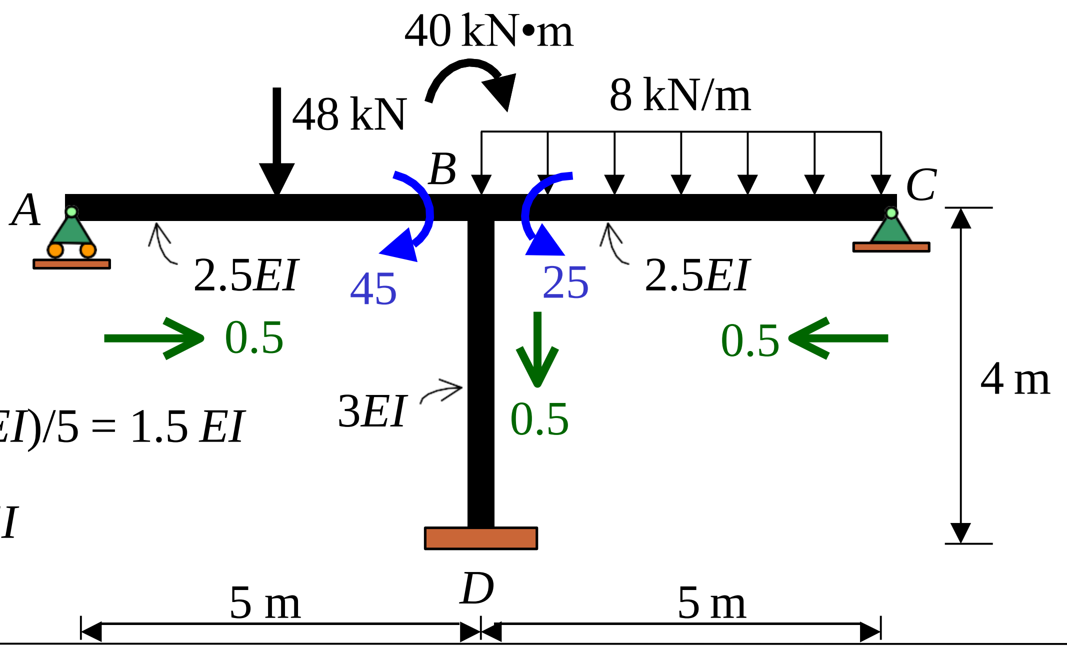
Moment Distribution Frames: No Sidesway



Example 6

From the frame shown use the moment distribution method to:
(a) Determine all the **reactions** at supports
(b)Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

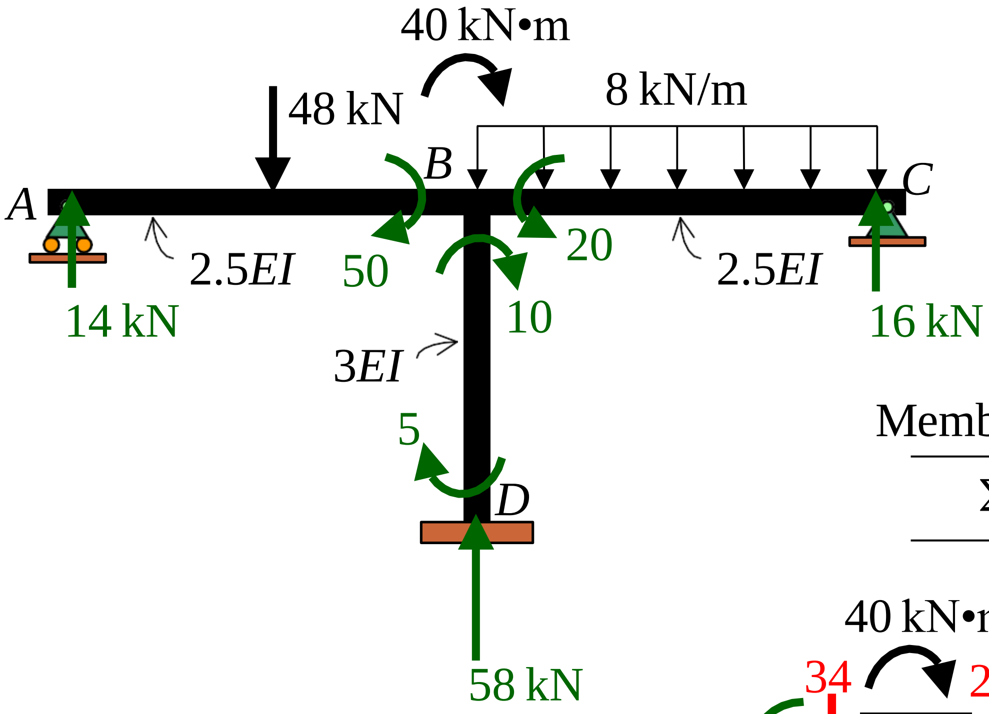




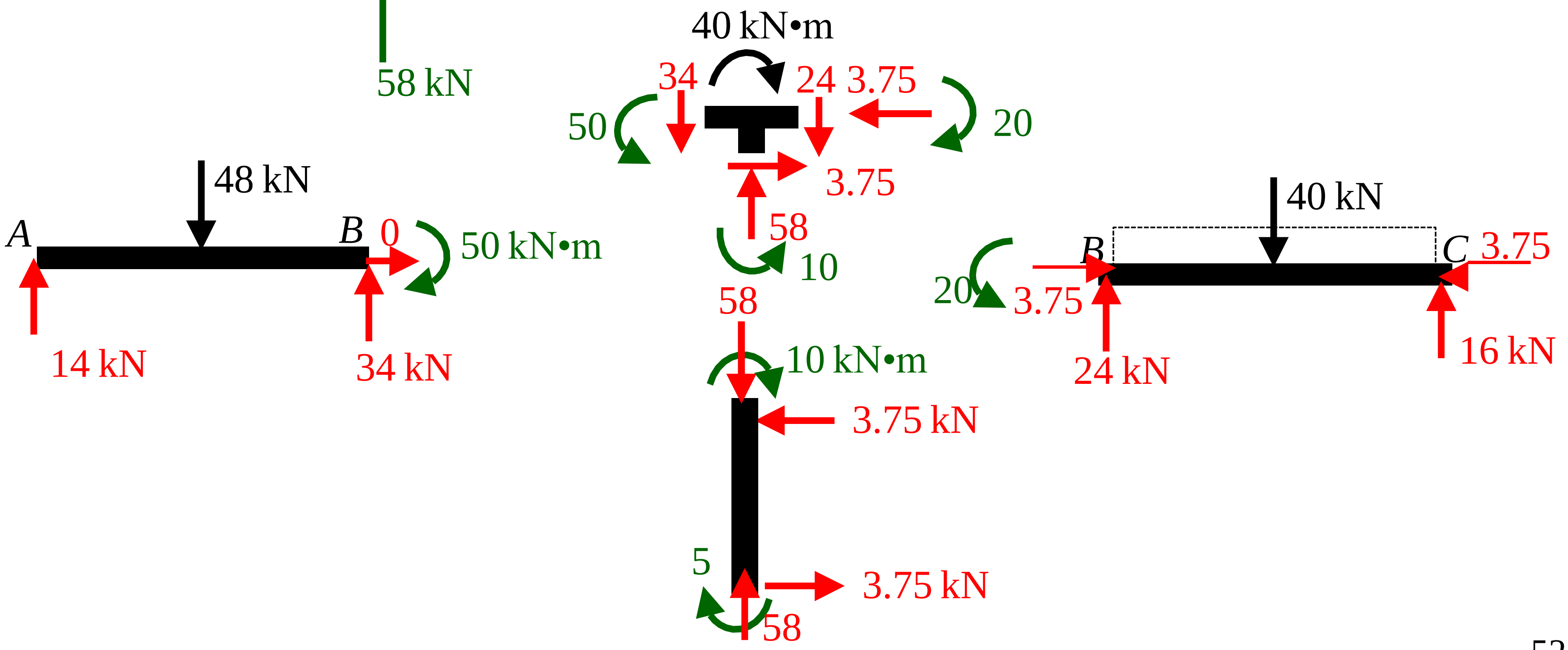
$K_{AB} = K_{BC} = 3(2.5EI)/5 = 1.5\ EI$

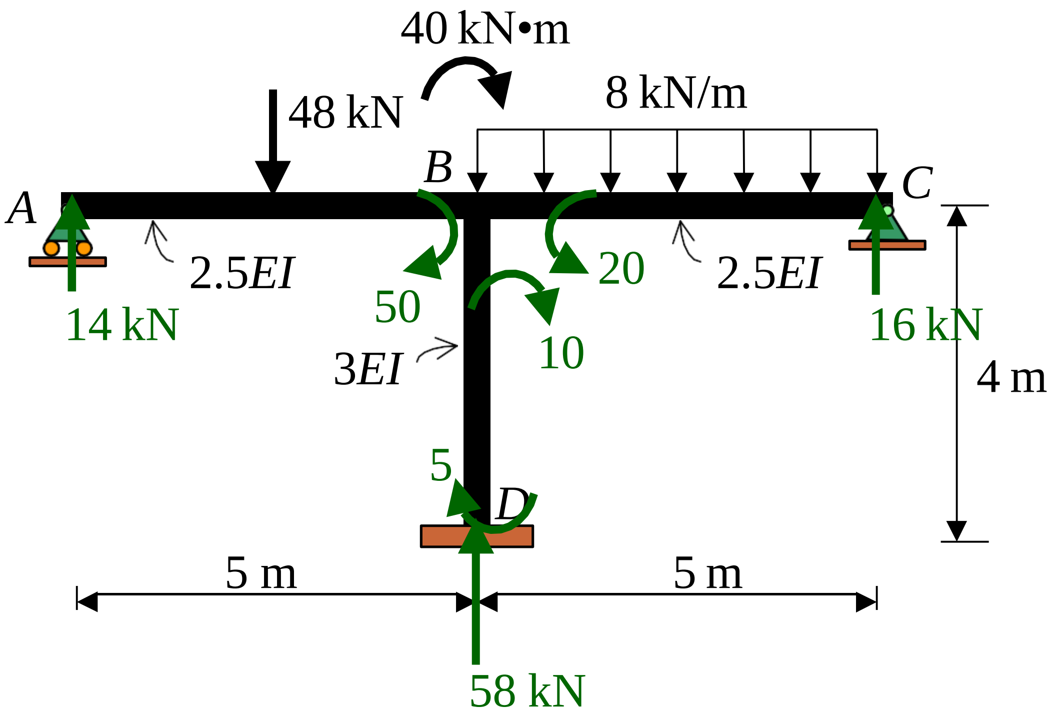
$K_{BD} = 4(3EI)/4 = 3EI$

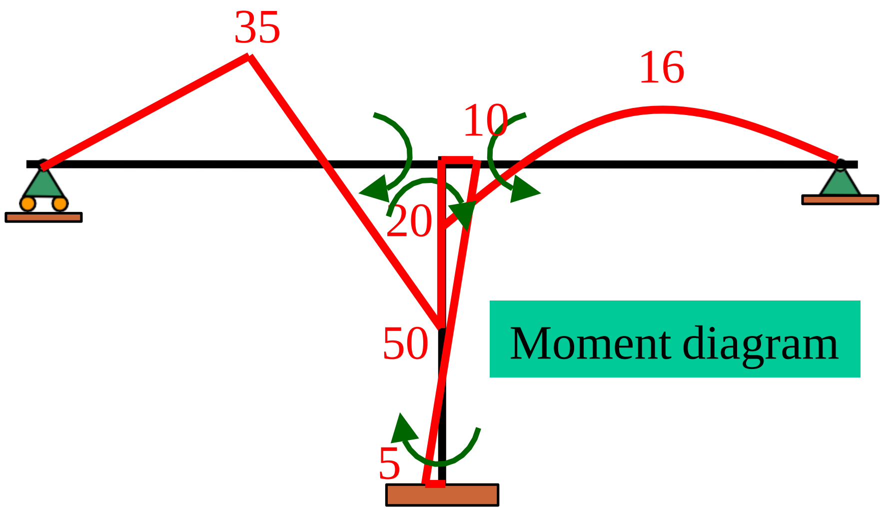
	A	B			D	C
Member	AB	BA	BC	BD	DB	CB
DF	1	0.25	0.25	0.5	0	1
Joint load		-10	-10	-20		
CO					-10	
FEM		-45	25			
Dist.		5	5	10		
CO					5	
Σ	**0**	**-50**	**20**	**-10**	**-5**	**0**



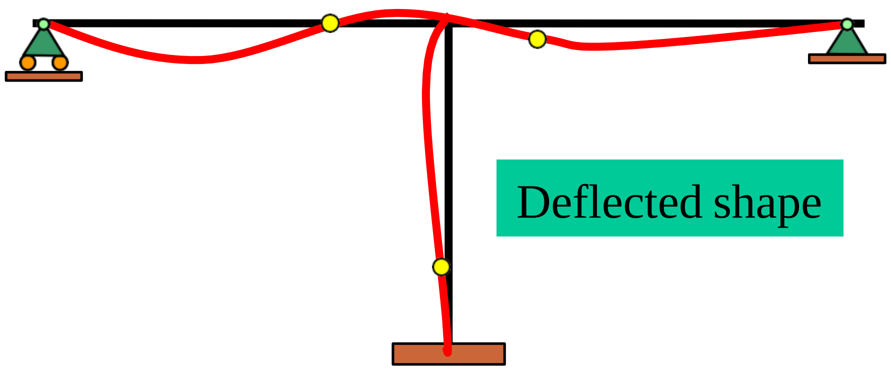
Member	*AB*	*BA*	*BC*	*BD*	*DB*	*CB*
Σ	**0**	**-50**	**20**	**-10**	**-5**	**0**





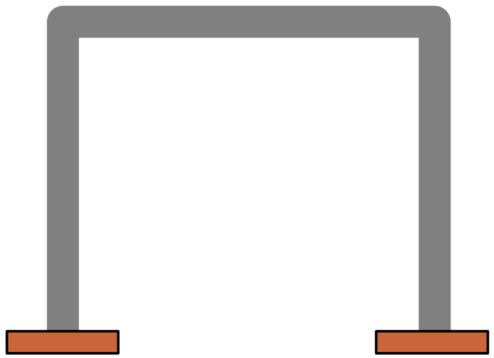


Moment diagram

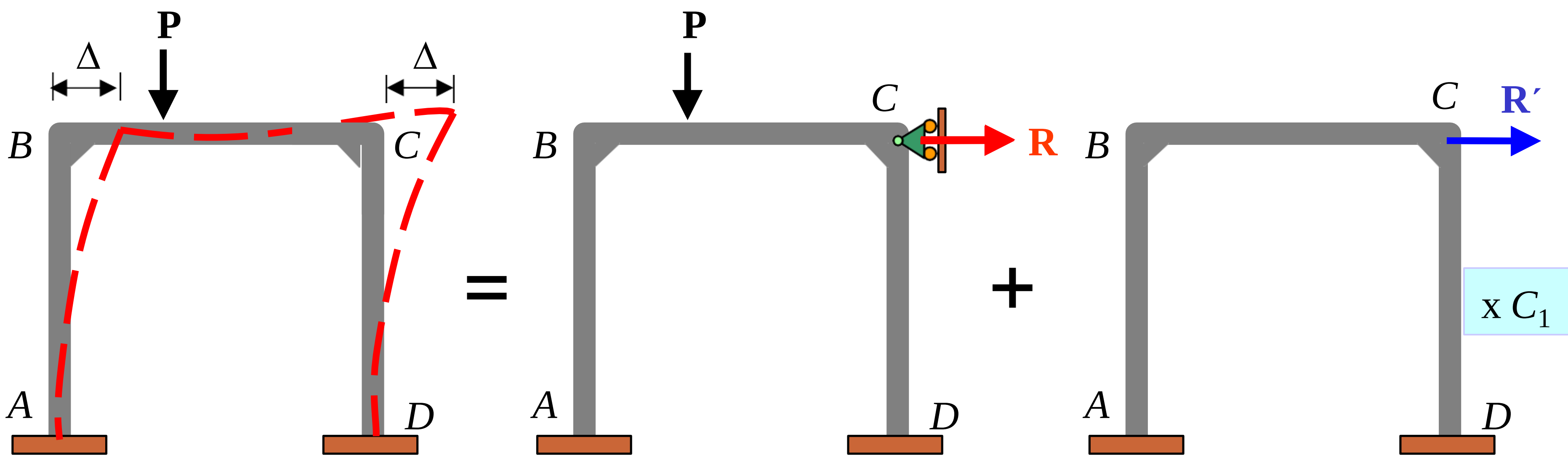


Deflected shape

Moment Distribution for Frames: Sidesway



Single Frames

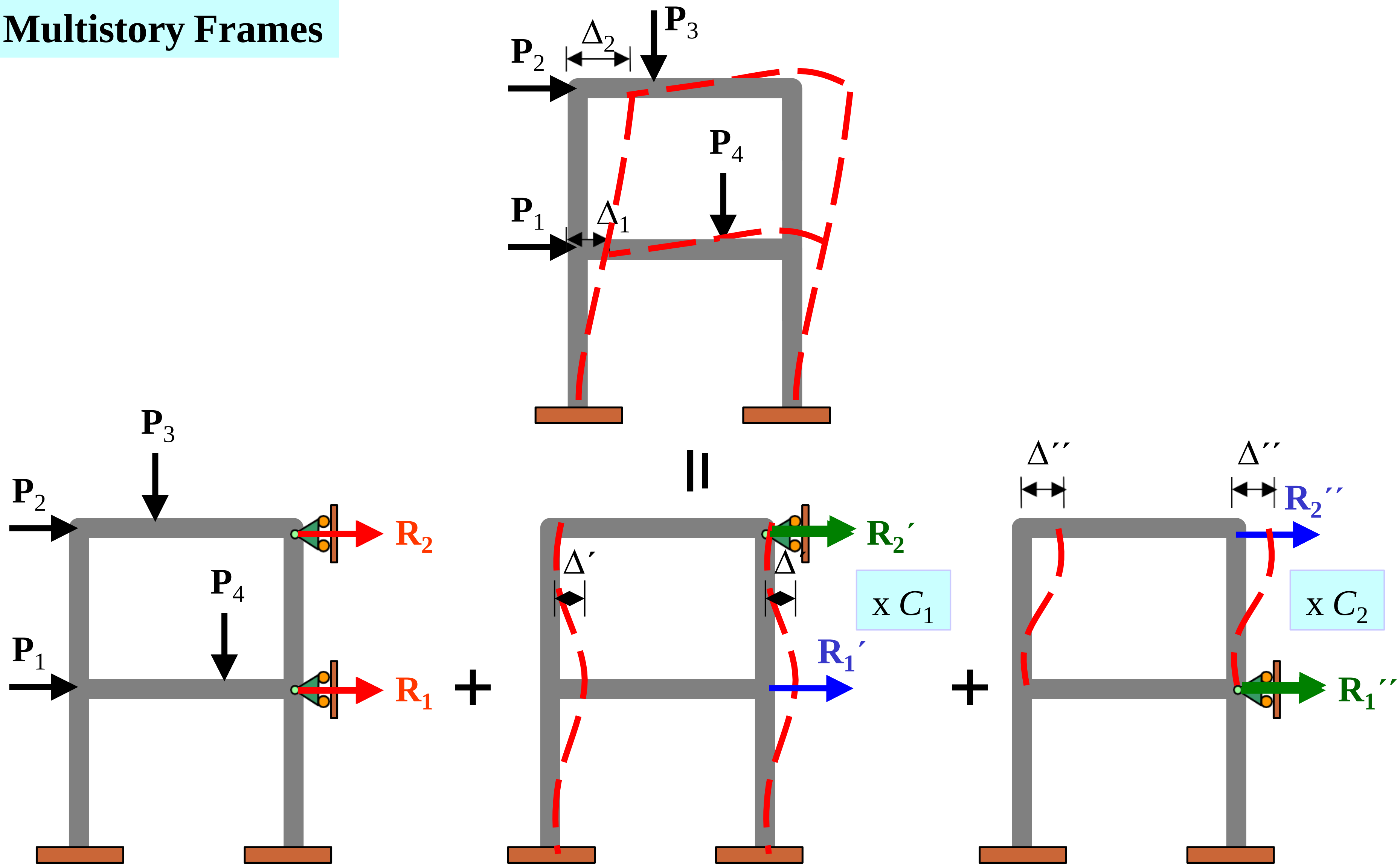


Artificial joint applied
(no sidesway)

Artificial joint removed
(sidesway)

$$0 = R + C_1 R'$$

Multistory Frames



$$0 = R_2 + C_1 R_2' + C_2 R_2''$$

$$0 = R_1 + C_1 R_1' + C_2 R_2''$$

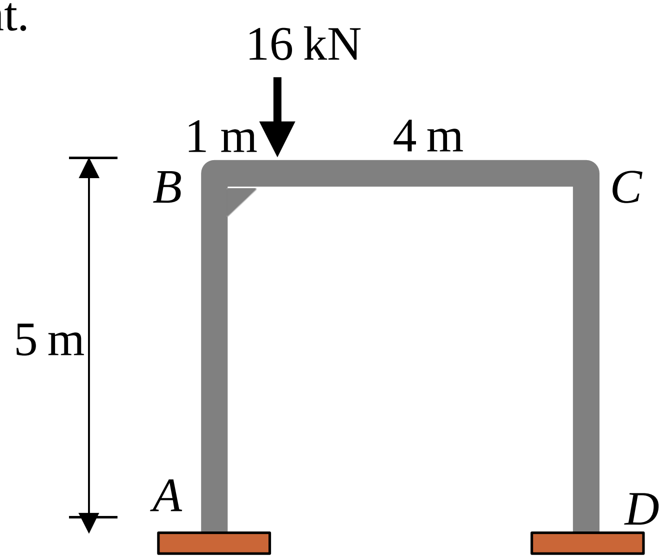
Example 7

From the frame shown use the moment distribution method to:

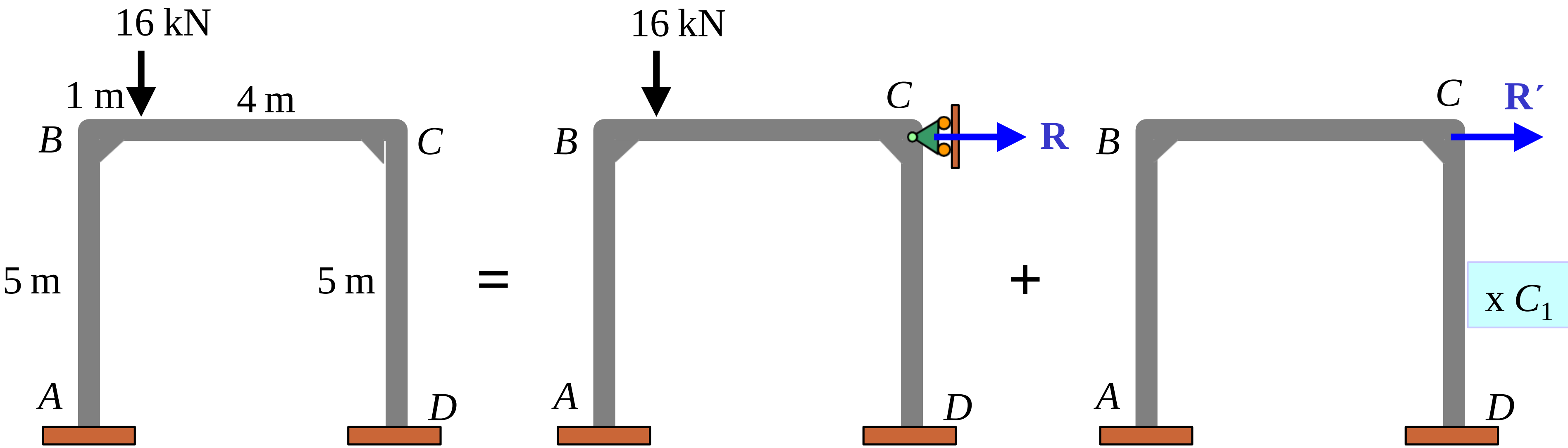
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

EI is constant.



- **Overview**



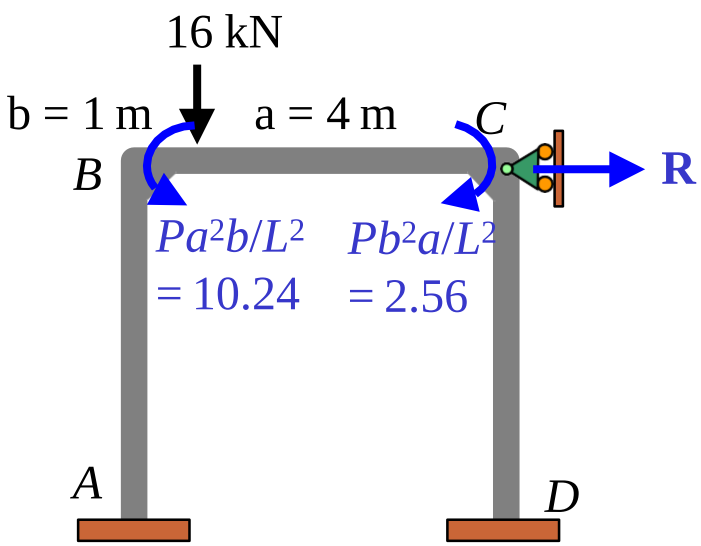
artificial joint applied
(no sidesway)

artificial joint removed
(sidesway)

$$R + C_1 R' = 0 \quad \text{---------(1)}$$

- **Artificial joint applied (no sidesway)**

Fixed end moment:



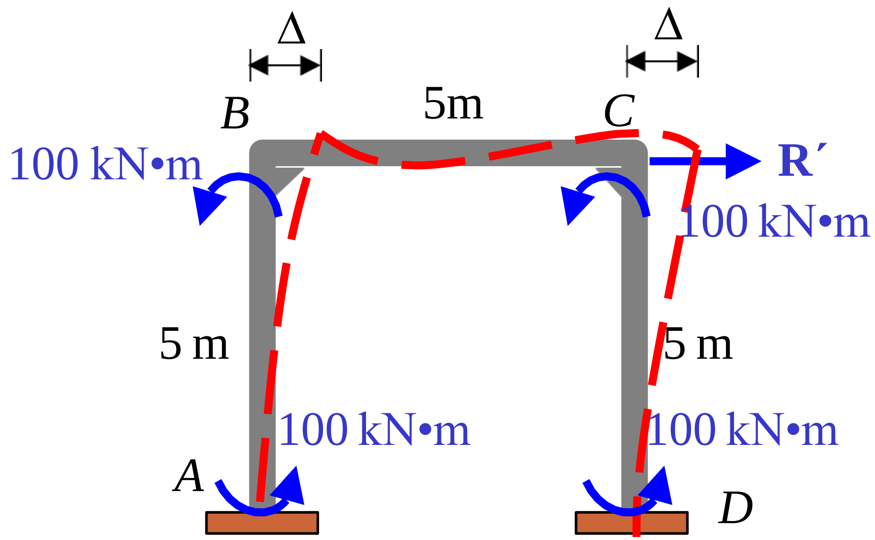
Equilibrium condition :

$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + D_x + R = 0$

- **Artificial joint removed (sidesway)**

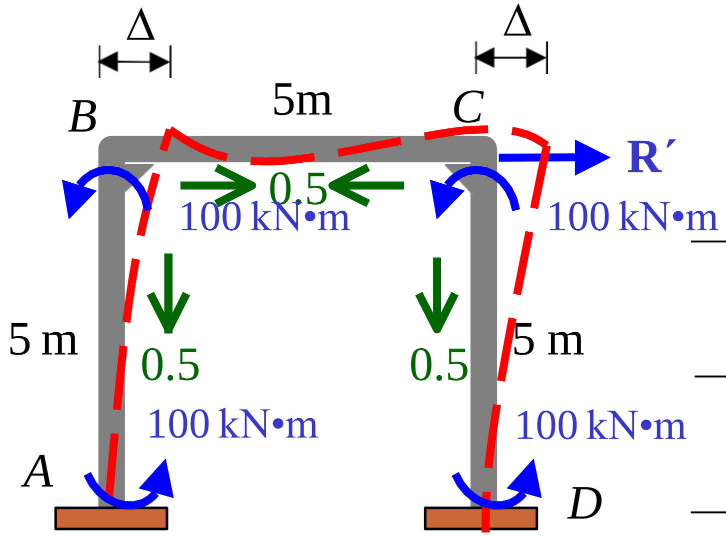
Fixed end moment:

Since both *B* and *C* happen to be displaced the same amount Δ, and *AB* and *DC* have the same *E*, *I*, and *L* so we will assume fixed-end moment to be 100 kN•m.

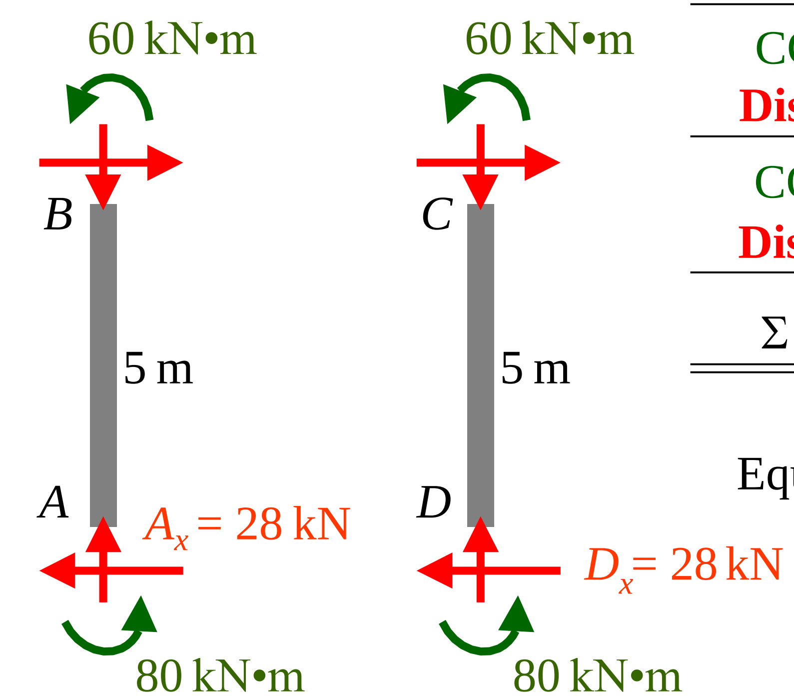


Equilibrium condition :

$$\xrightarrow{+} \Sigma F_x = 0: \quad A_x + D_x + R' = 0$$



	A	B		C		D
DF	0	0.50	0.50	0.50	0.50	0
FEM	100	100			100	100
Dist.		-50	-50	-50	-50	
CO	-25.0		-25.0	-25.0		-25.0
Dist.		12.5	12.5	12.5	12.5	
CO	6.5		6.5	6.5		6.5
Dist.		-3.125	-3.125	-3.125	-3.125	
CO	-1.56		-1.56	-1.56		-1.56
Dist.		0.78	0.78	0.78	0.78	
CO	0.39		0.39	0.39		0.39
Dist.		-0.195	-0.195	-0.195	-0.195	
Σ	**80**	**60**	**-60**	**-60**	**60**	**80**



Equilibrium condition: $\xrightarrow{+} \Sigma F_x = 0$:

$$-28 - 28 + R' = 0$$

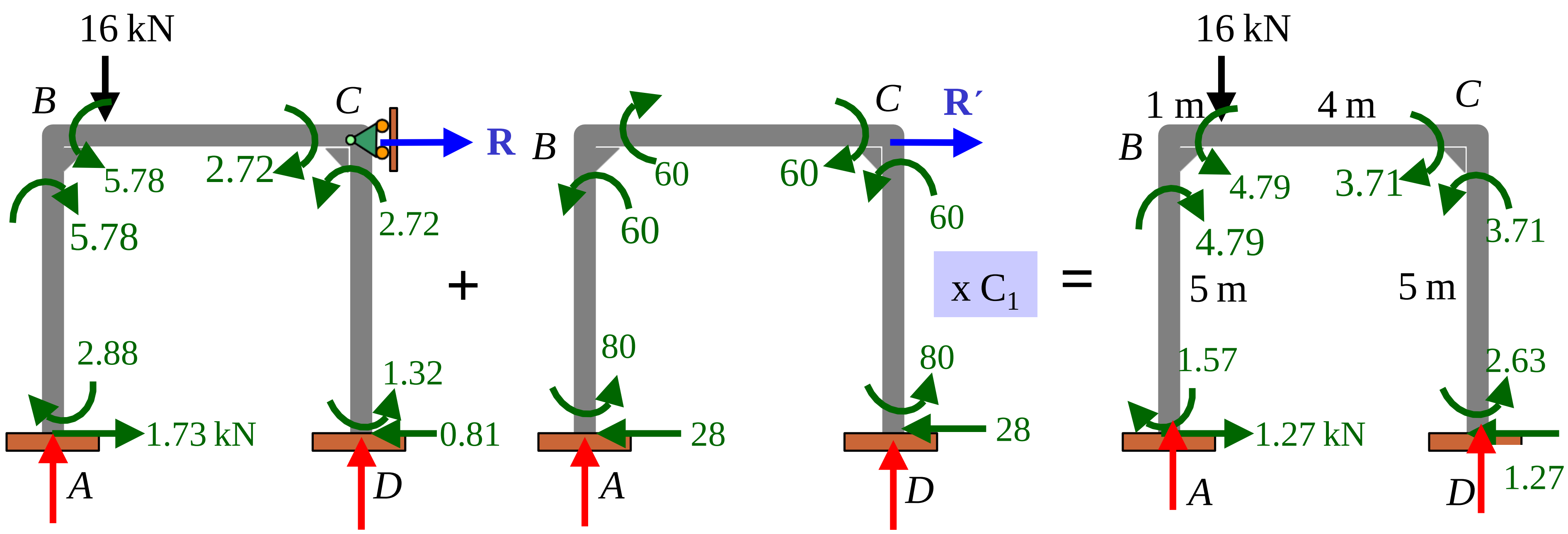
$$\boldsymbol{R'} = 56 \text{ kN}$$

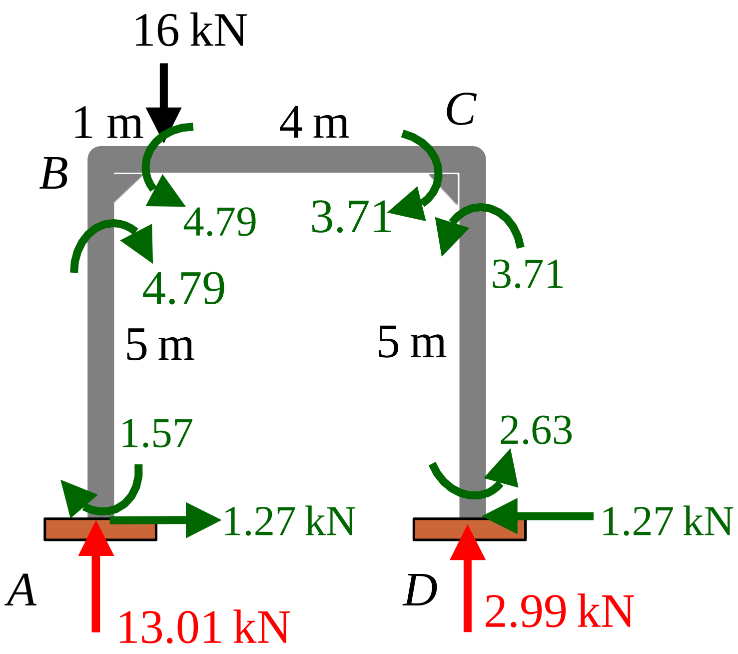
Substitute R = -0.92 and R' = 56 in (1) :

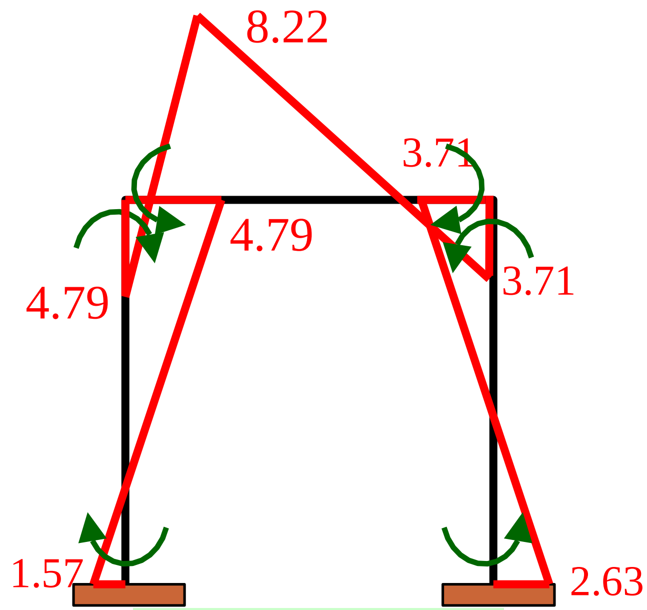
$$R + C_1 R' = 0$$

$$-0.92 + C_1(56) = 0$$

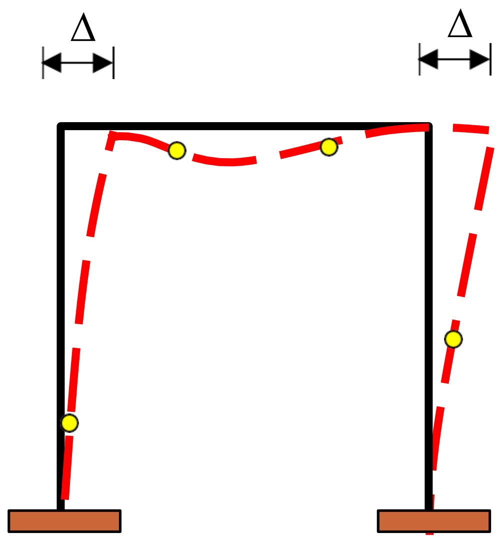
$$C_1 = \frac{0.92}{56}$$







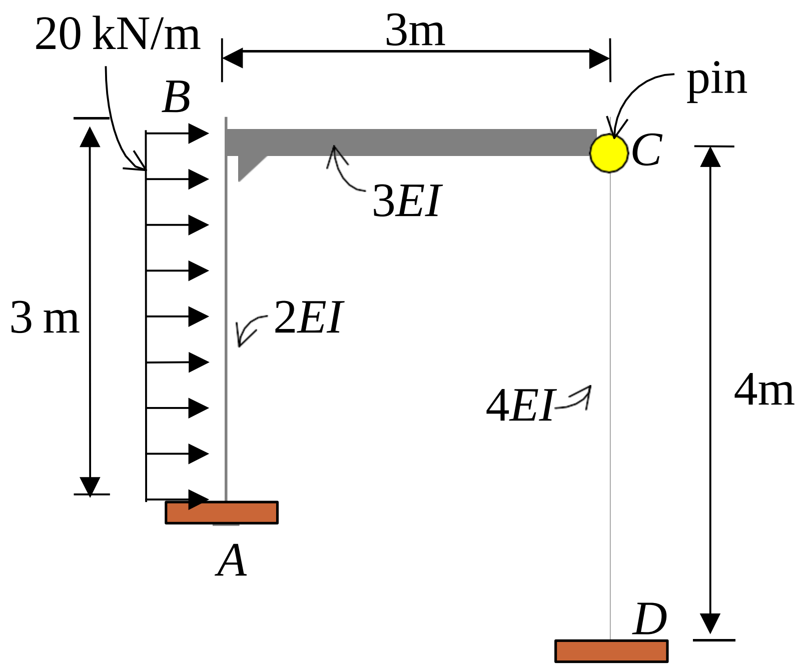
Bending moment diagram (kN•m)



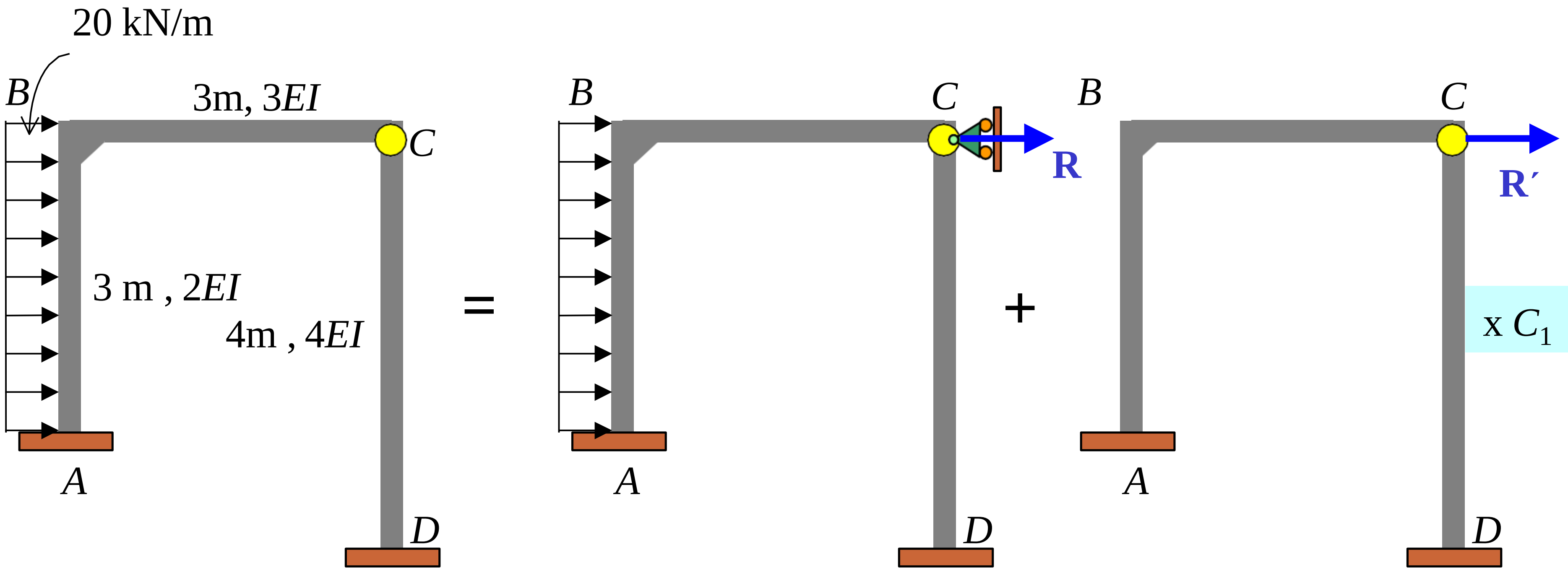
Deflected shape

Example 8

From the frame shown use the moment distribution method to:
(a) Determine all the reactions at supports, and also
(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**



• **Overview**

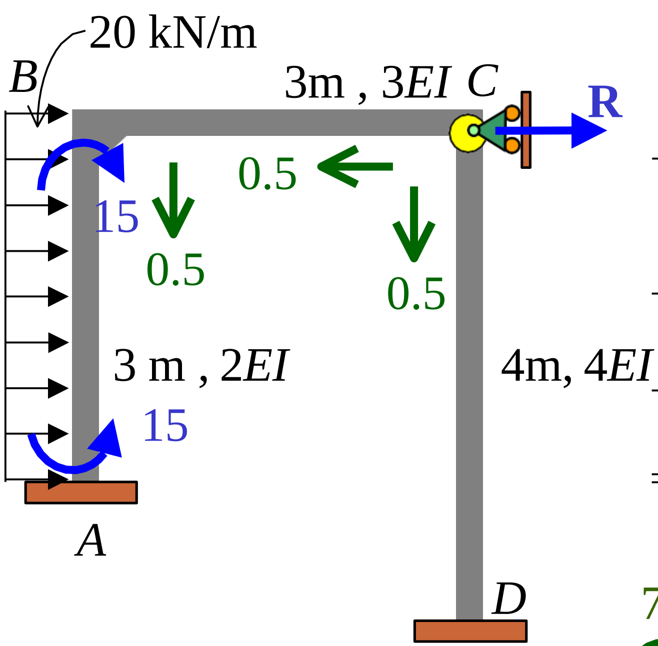


artificial joint applied
(no sidesway)

artificial joint removed
(sidesway)

$$R + C_1 R' = 0 \qquad \text{----------(1)}$$

- **Artificial joint applied (no sidesway)**

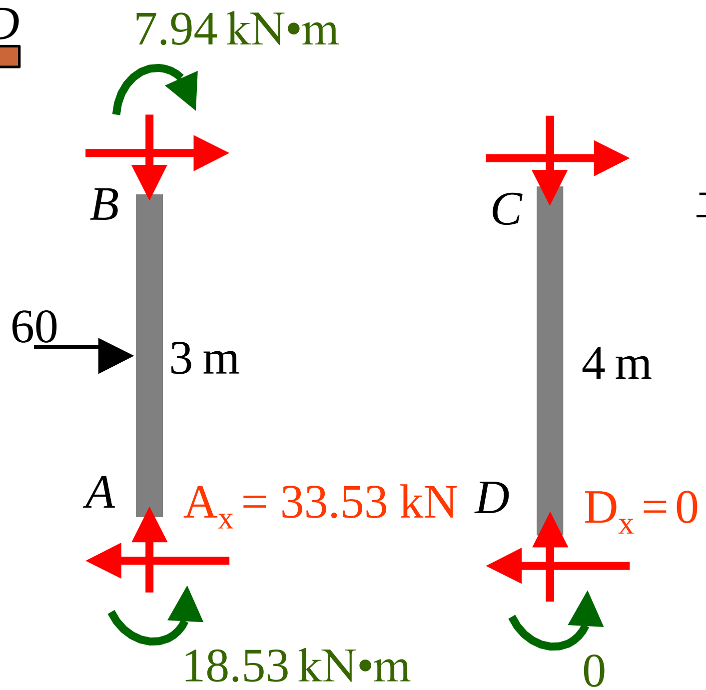


	A	B		C		D
DF	0	0.471	0.529	1.00	1.00	0
FEM	15.00	-15.00				
Dist.		7.065	7.935			
CO	3.533					
Σ	**18.53**	**-7.94**	**7.94**			

$K_{BA} = 4(2EI)/3 = 2.667EI$

$K_{BC} = 3(3EI)/3 = 3EI$

$K_{CD} = 3(4EI)/4 = 3EI$

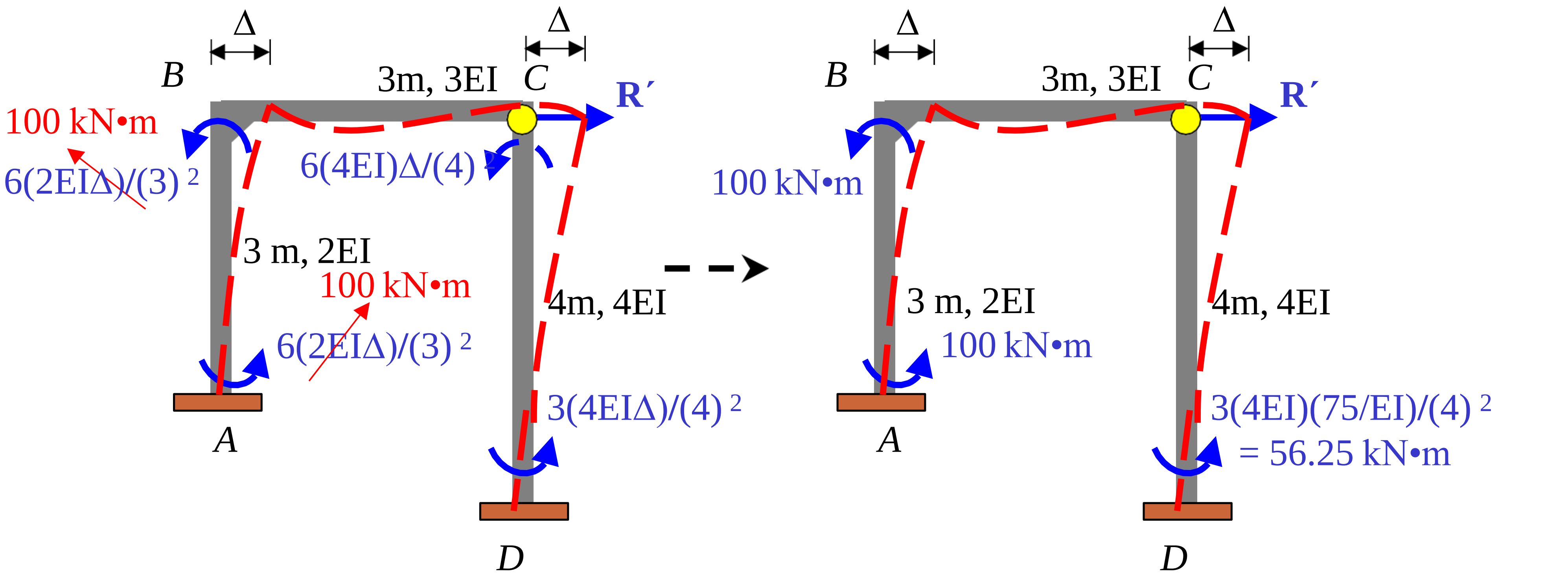


$\overset{+}{\rightarrow} \Sigma F_x = 0$:

$60 - 33.53 - 0 + R = 0$

$R = -26.47$ kN ←

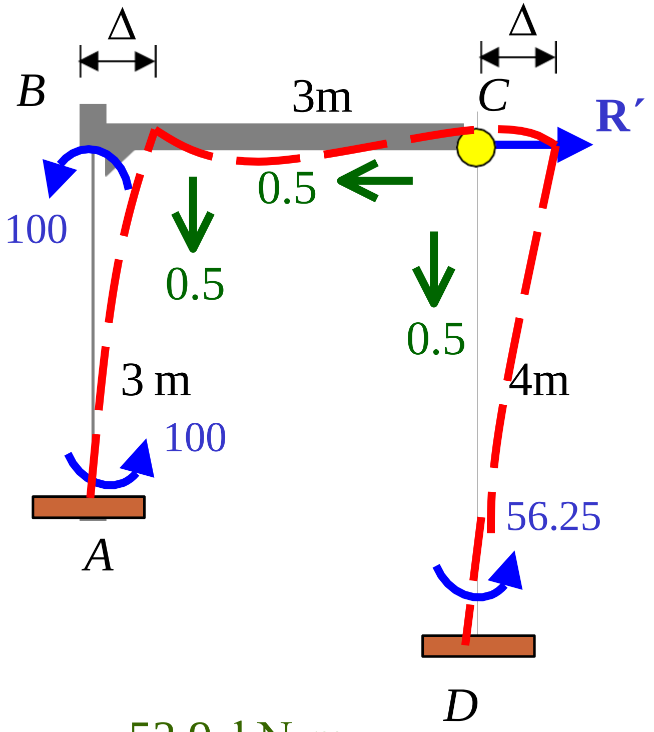
- **Artificial joint removed (sidesway)**
  - **Fixed end moment**



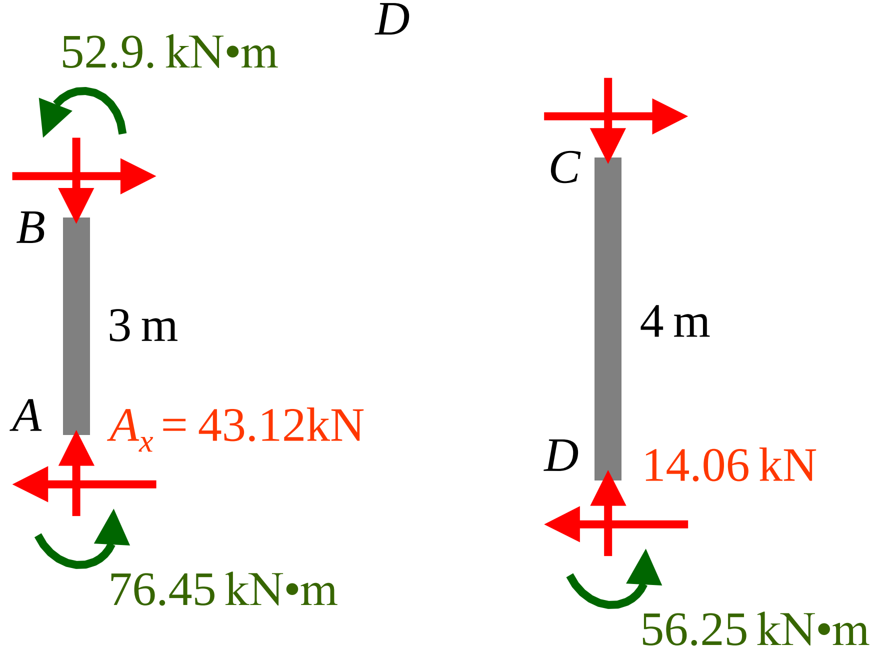
Assign a value of $(FEM)_{AB} = (FEM)_{BA} = 100$ kN•m

$$\frac{6(2EI)\Delta}{3^2} = 100$$

$$\Delta_{AB} = 75/EI$$



	A	B		C		D
DF	0	0.47	1 0.529	1.00	1.00	0
FEM	100	100				56.25
Dist.		-47.1	-52.9			0
CO	-28.55					
Σ	**76.45**	**52.9**	**-52.9**			**56.25**



$\xrightarrow{+} \Sigma F_x = 0:$

$-43.12 - 14.06 + R' = 0$

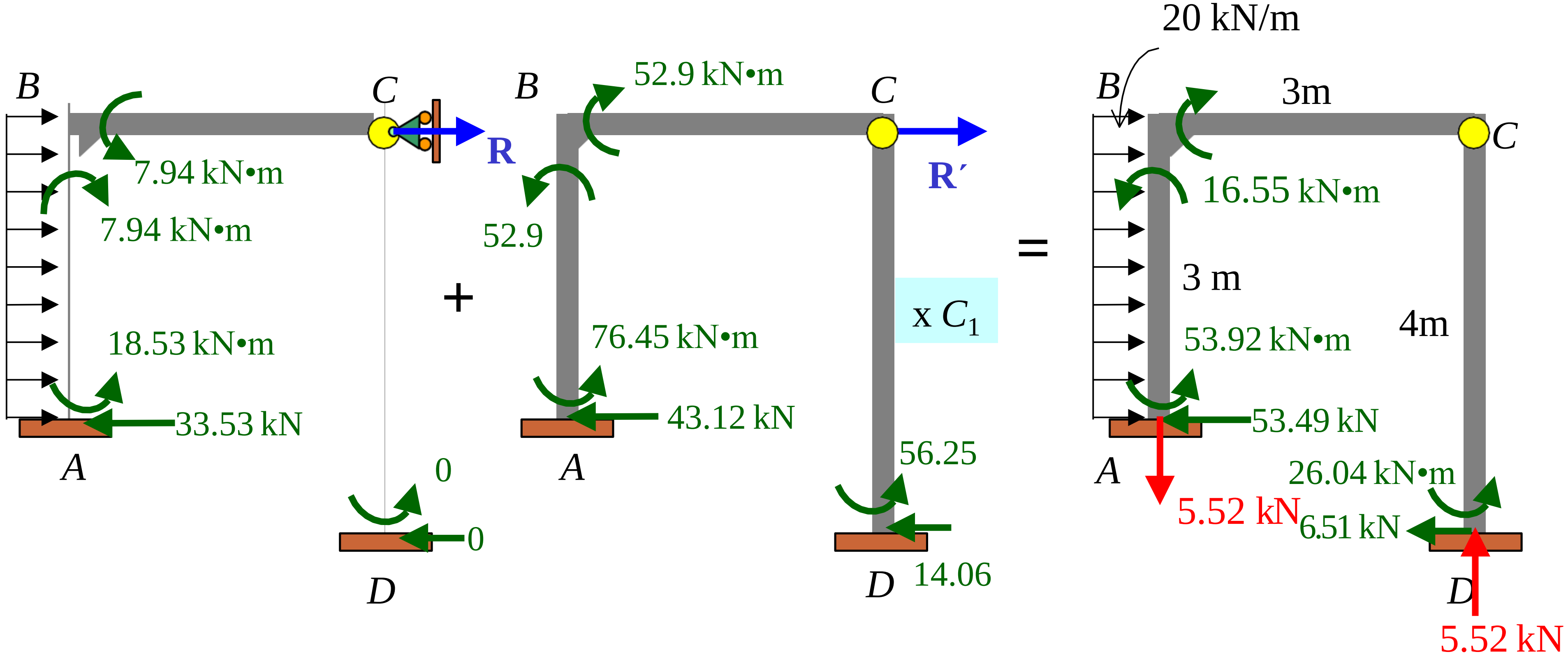
$R' = 57.18$ kN

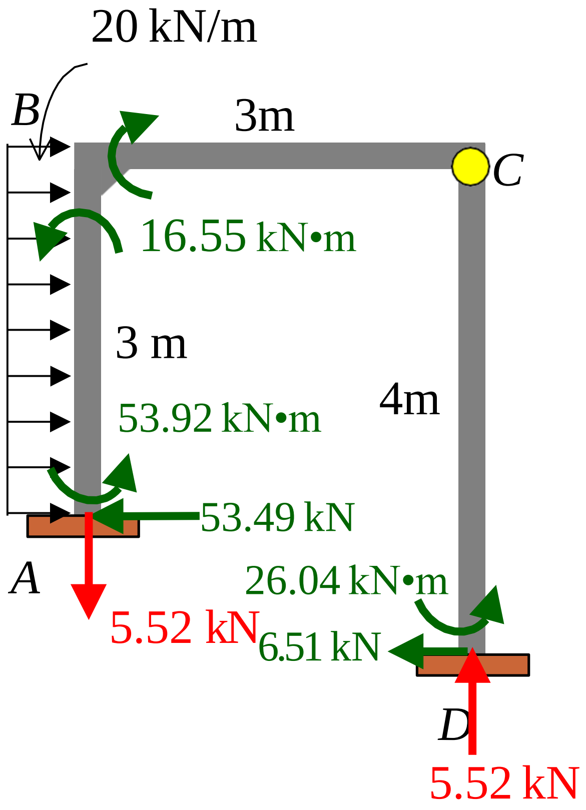
Substitute R = -26.37 and R′ = 57.18 in (1) :

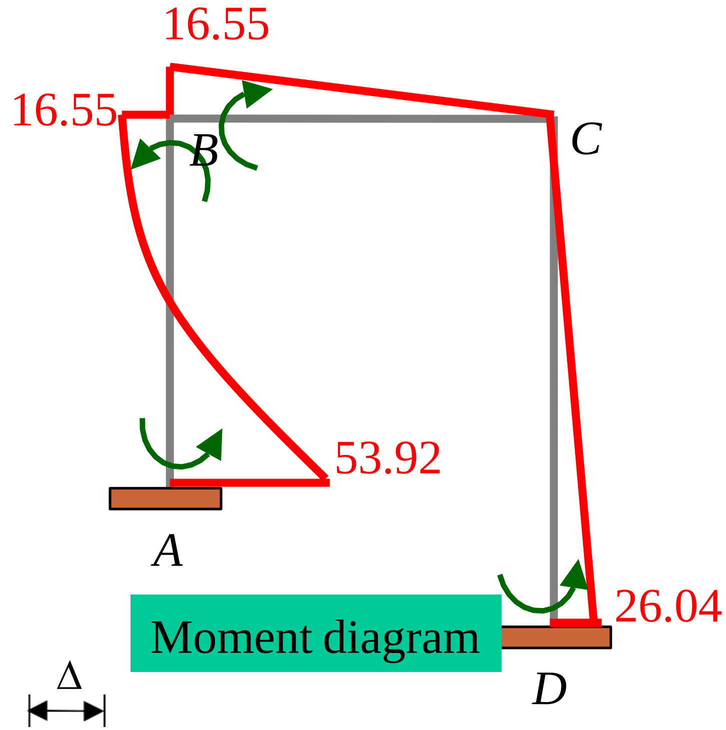
$$R + C_1 R' = 0$$

$$-26.47 + C_1(57.18) = 0$$

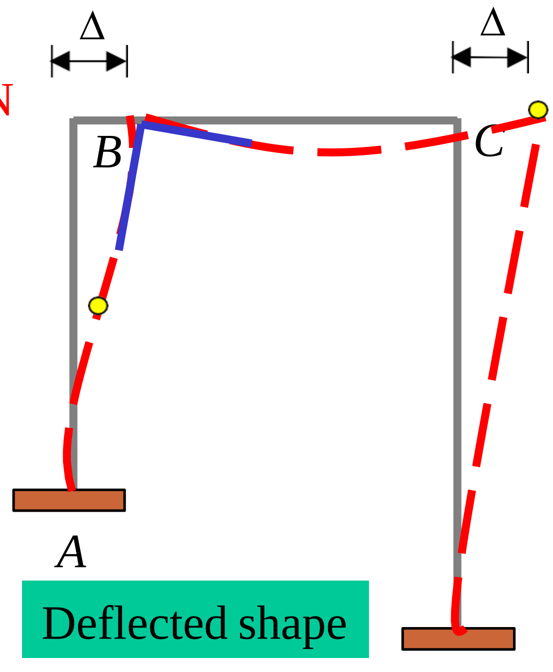
$$C_1 = \frac{26.47}{57.18}$$







Moment diagram



Deflected shape

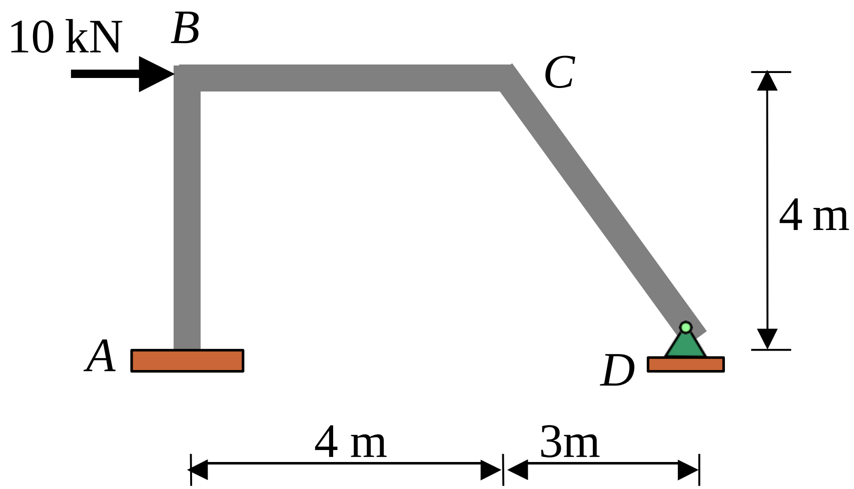
Example 8

From the frame shown use the moment distribution method to:

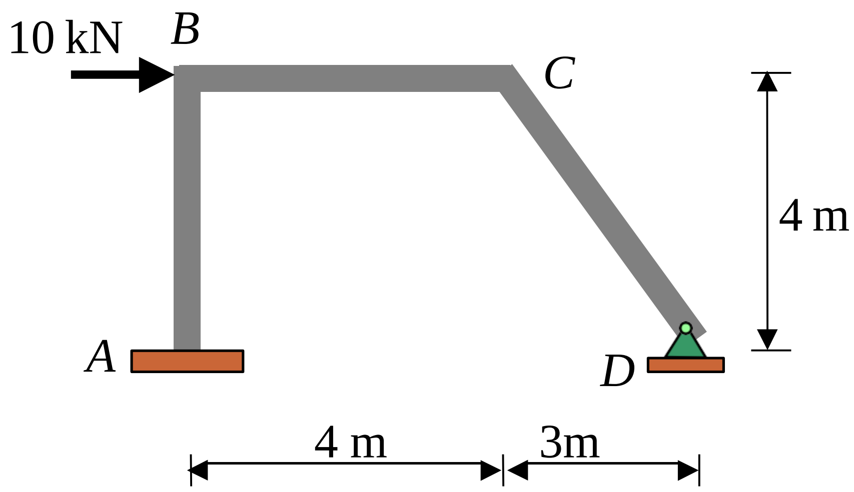
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**, and **qualitative deflected shape.**

EI is constant.

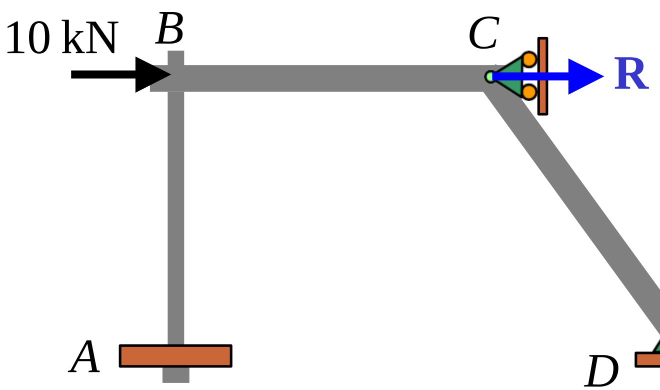


- **Overview**



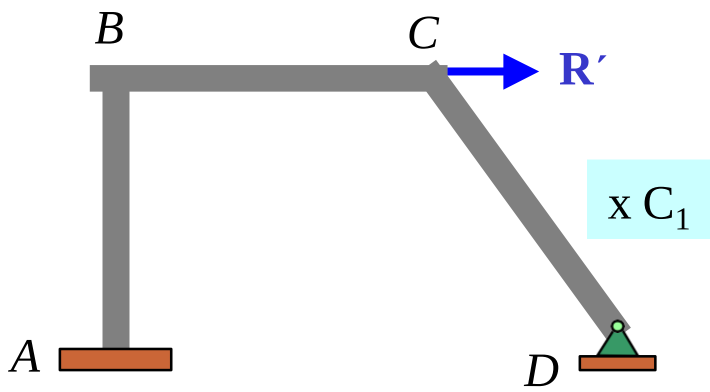
$$R + C_1R' = 0 \quad \text{---------(1)}$$

=



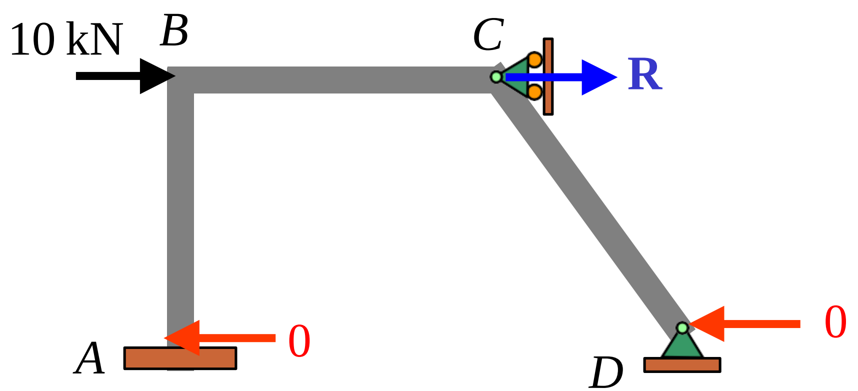
artificial joint applied
(no sidesway)

+



artificial joint removed
(sidesway)

- **Artificial joint applied (no sidesway)**



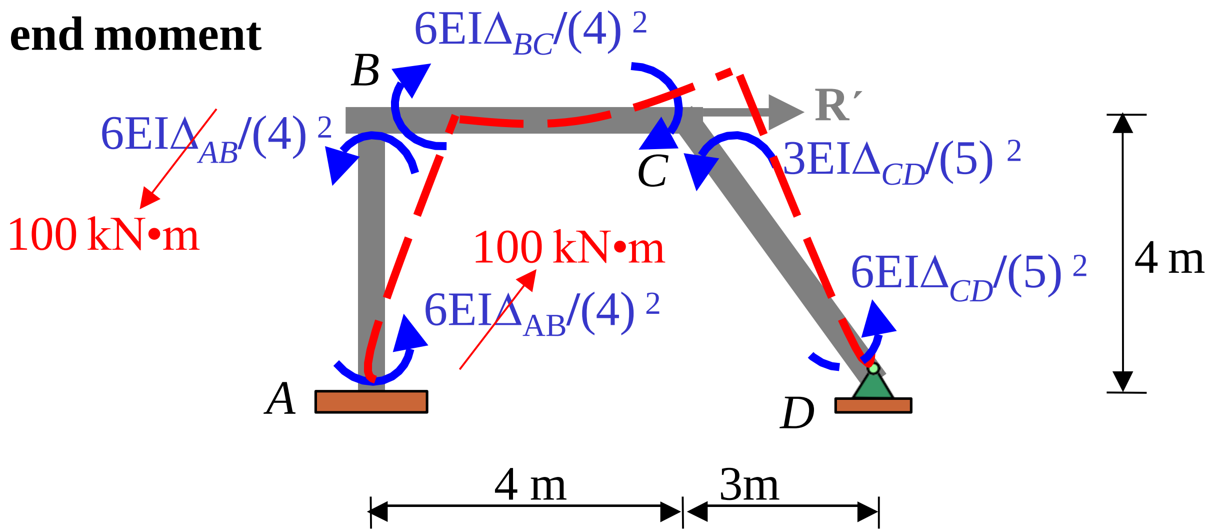
Equilibrium condition : $\overset{+}{\rightarrow} \Sigma F_x = 0:$

$$10 + R = 0$$

$$R = -10 \text{ kN} \quad \leftarrow$$

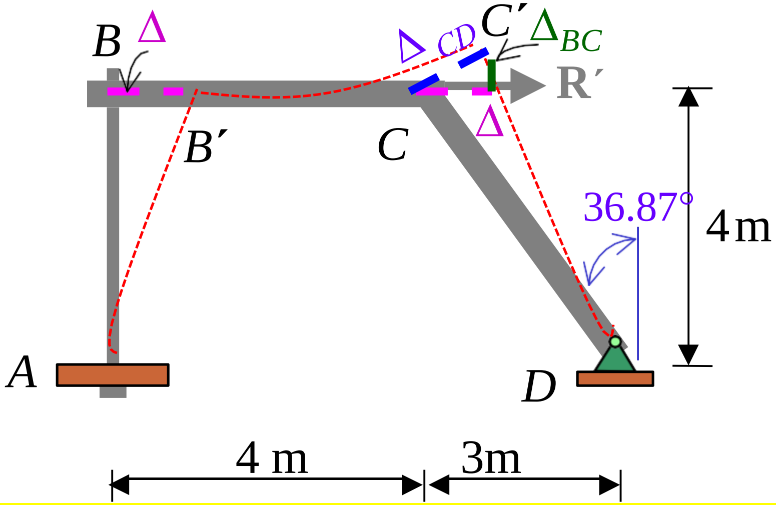
- **Artificial joint removed (sidesway)**

  - **Fixed end moment**



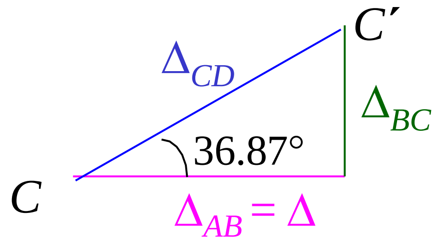
Assign a value of $(FEM)_{AB} = (FEM)_{BA} = 100$ kN•m : $\dfrac{6EI\Delta_{AB}}{4^2} = 100$

$$\Delta_{AB} = 266.667/EI$$



$\Delta_{CD} = \Delta / \cos 36.87° = 1.25\,\Delta = 1.25(266.667/EI)$

$= 333.334/EI$

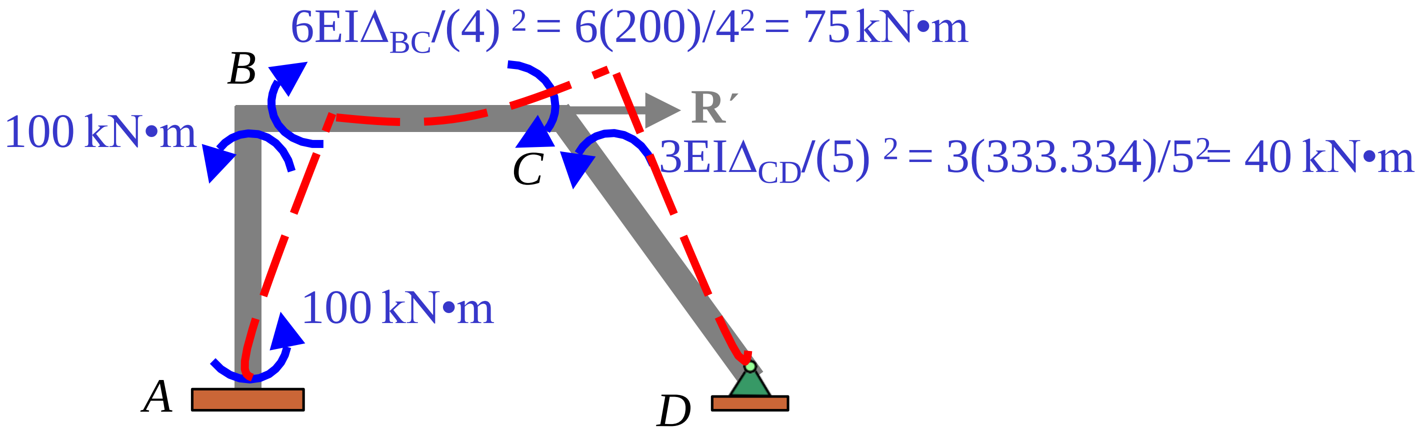


$\Delta_{BC} = \Delta \tan 36.87° = 0.75\,\Delta$

$= 0.75(266.667/EI)$

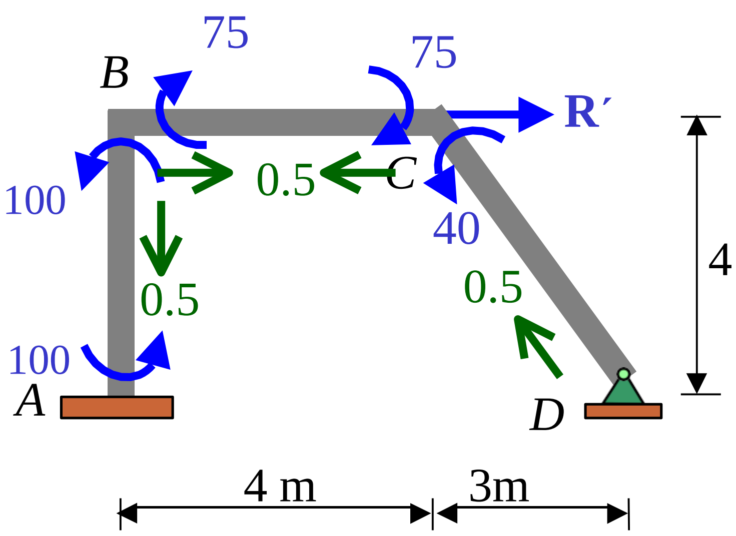
$= 200/EI$

$\Delta_{BC} = 200/EI,\ \Delta_{CD} = 333.334/EI$



Equilibrium condition :

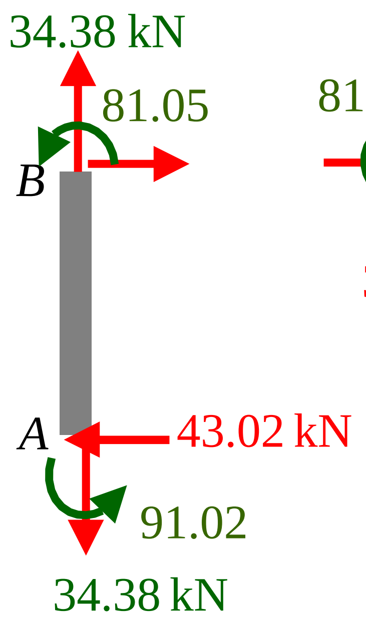
$$\overset{+}{\rightarrow}\ \Sigma F_x = 0:\quad A_x + D_x + R' = 0$$

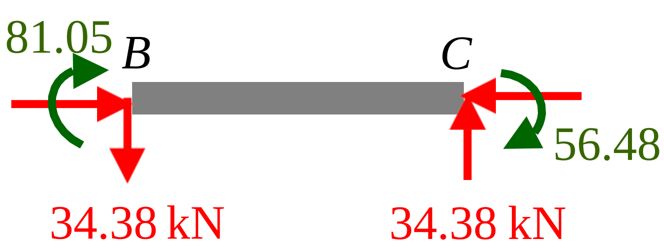


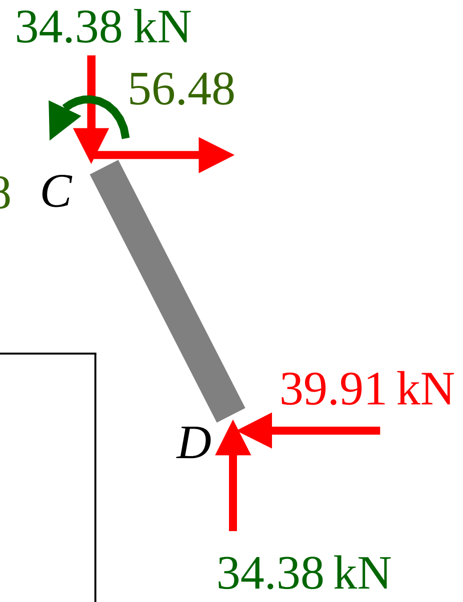
	A	B		C		D
DF	0	0.50	0.50	0.625	0.375	1
FEM	100	100	-75	-75	40	
Dist.		-12.5	-12.5	21.875	13.125	
CO	-6.25		10.938	-6.25		
Dist.		-5.469	-5.469	3.906	2.344	
CO	-2.735		1.953	-2.735		
Dist.		-0.977	-0.977	1.709	1.026	
Σ	**91.02**	**81.05**	**-81.05**	**-56.48**	**56.48**	

$K_{BA} = 4EI/4 = EI$, $K_{BC} = 4EI/4 = EI$,

$K_{CD} = 3EI/5 = 0.6EI$







$\xrightarrow{+}$ $\Sigma F_x = 0$:

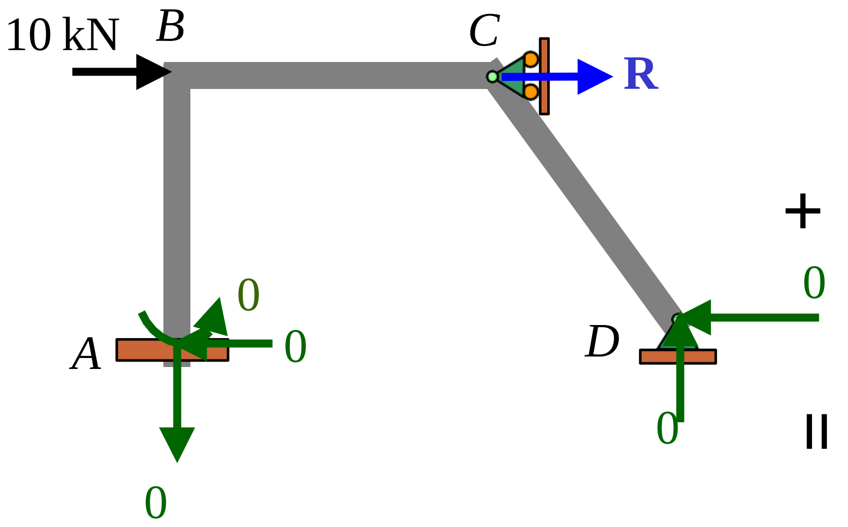
$-43.02 - 39.91 + R' = 0$

$R' = 82.93$ kN

Substitute R = -10 kN and R′= 82.93 kN in (1) : $-10 + C_1(82.93) = 0$

$R + C_1R' = 0$ ---------(1)

$C_1 = 10/82.93$



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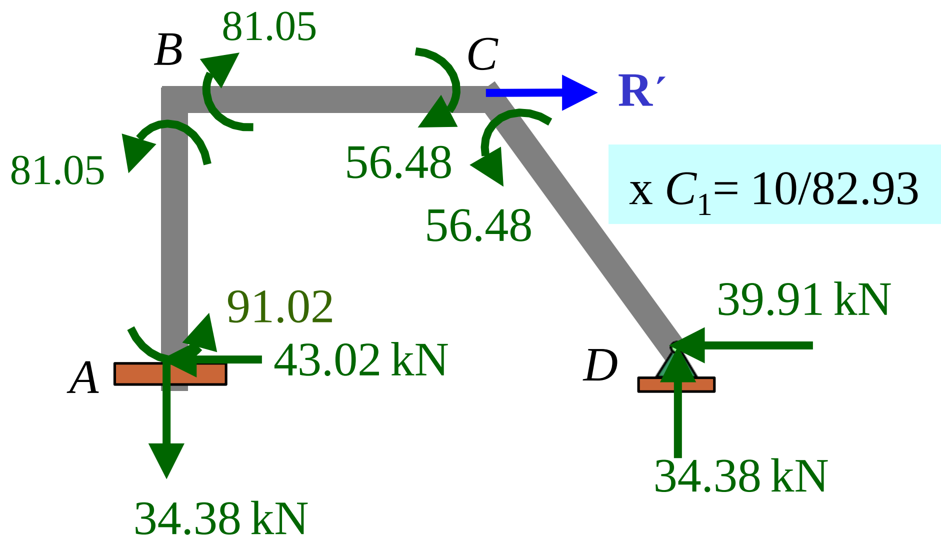

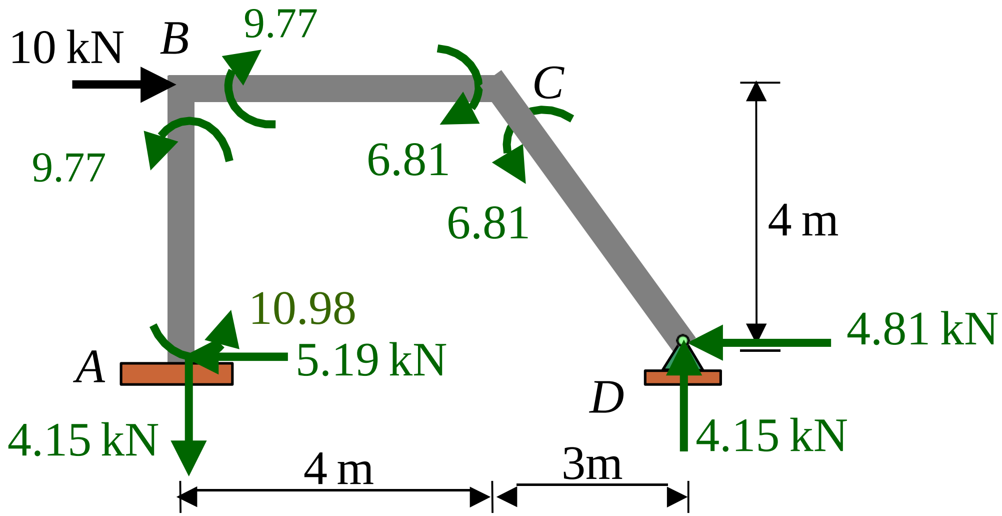

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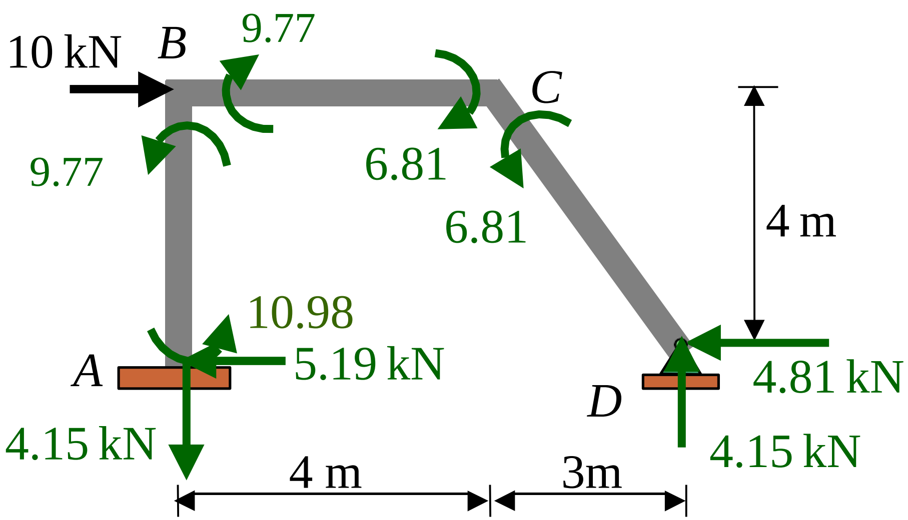
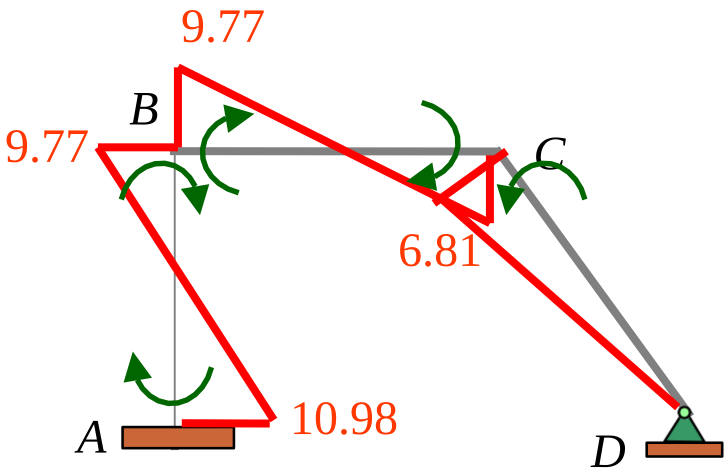
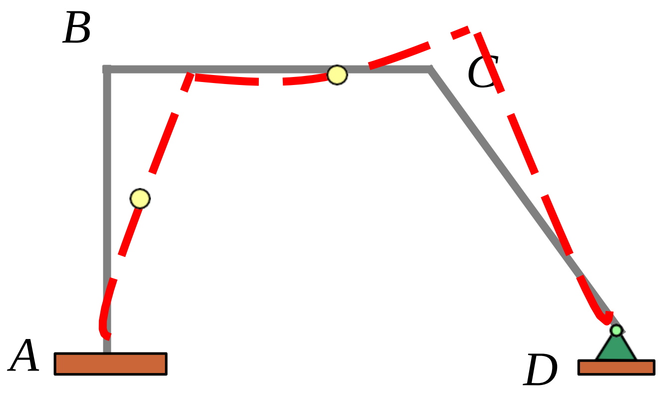
JECRC, JAIPUR





Bending moment diagram (kN•m)



Deflected shape

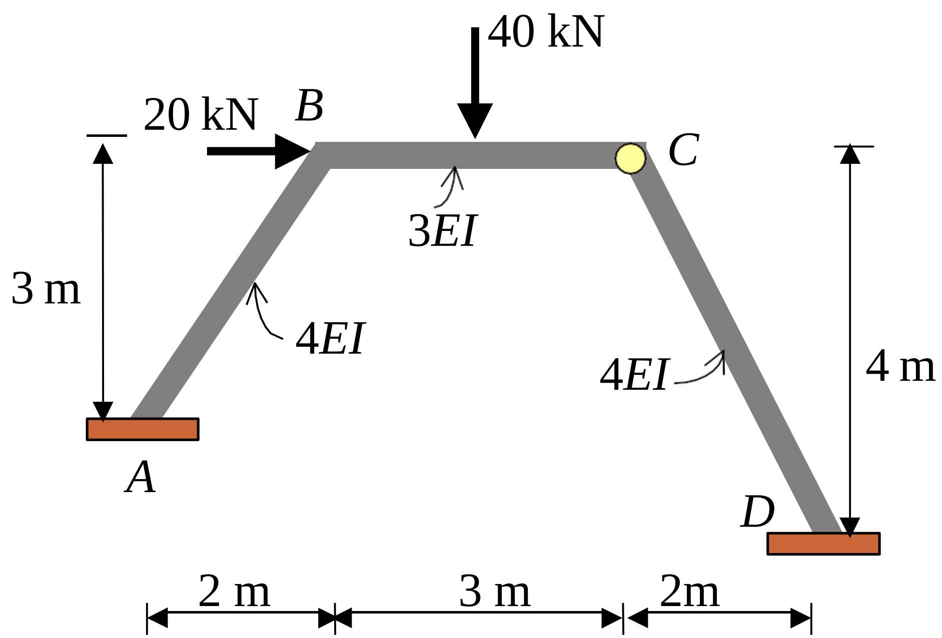
Example 9

From the frame shown use the moment distribution method to:

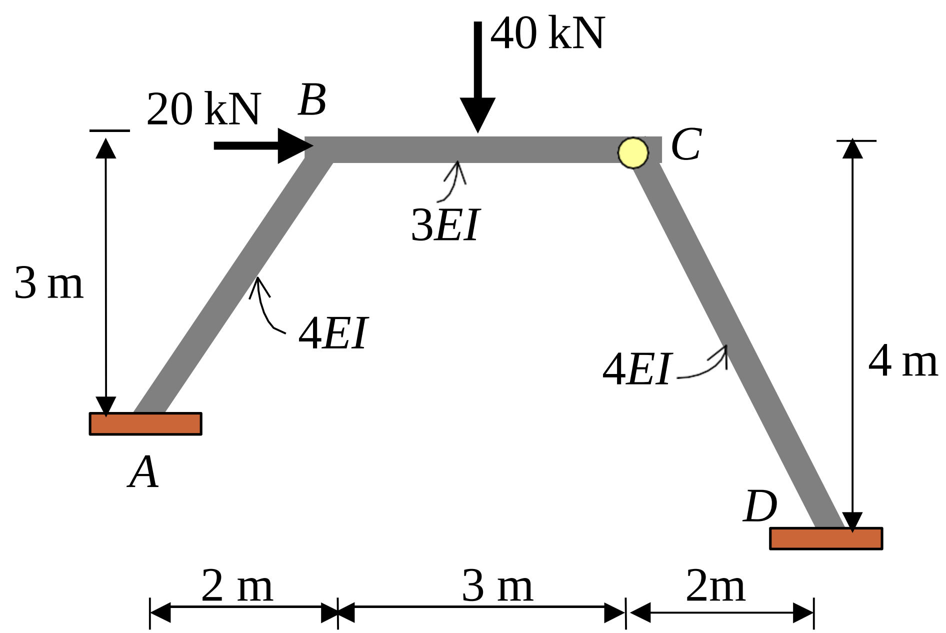
(a) Determine all the reactions at supports, and also

(b) Draw its **quantitative shear** and **bending moment diagrams**,and **qualitative deflected shape.**

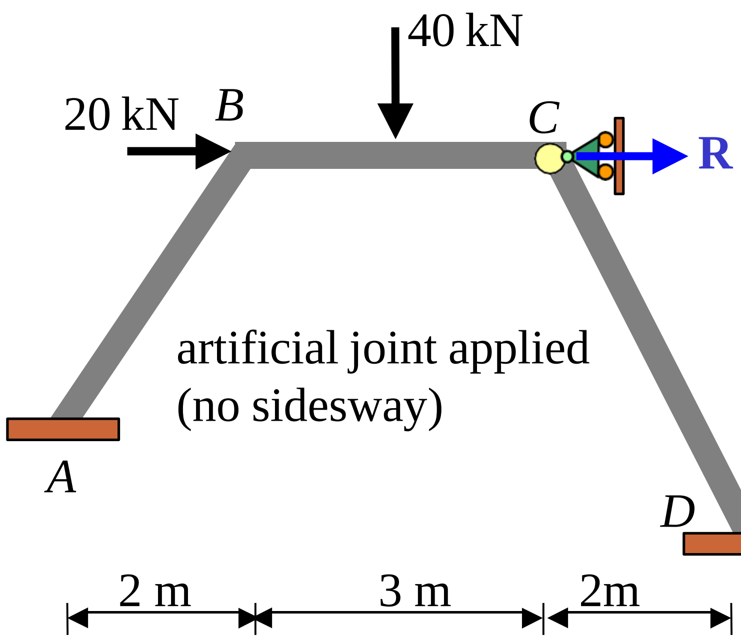
EI is constant.



- **Overview**



$$R + C_1 R' = 0 \qquad \text{----------(1)}$$



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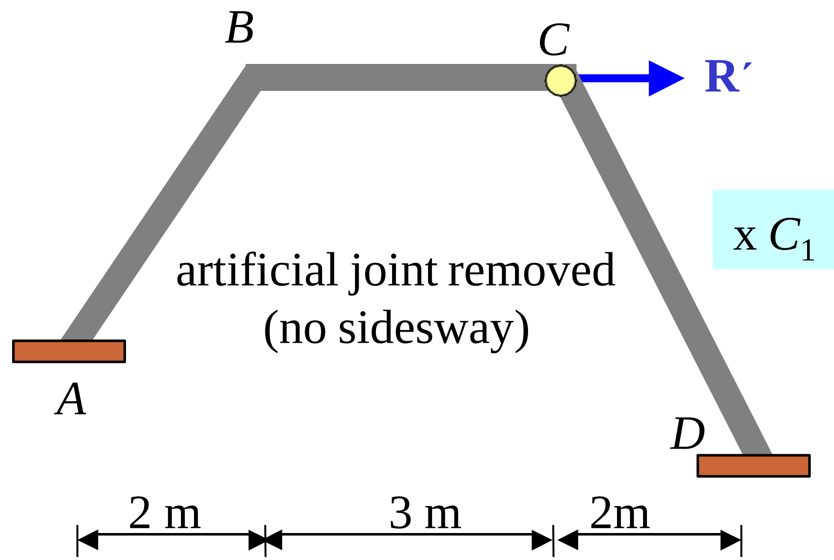
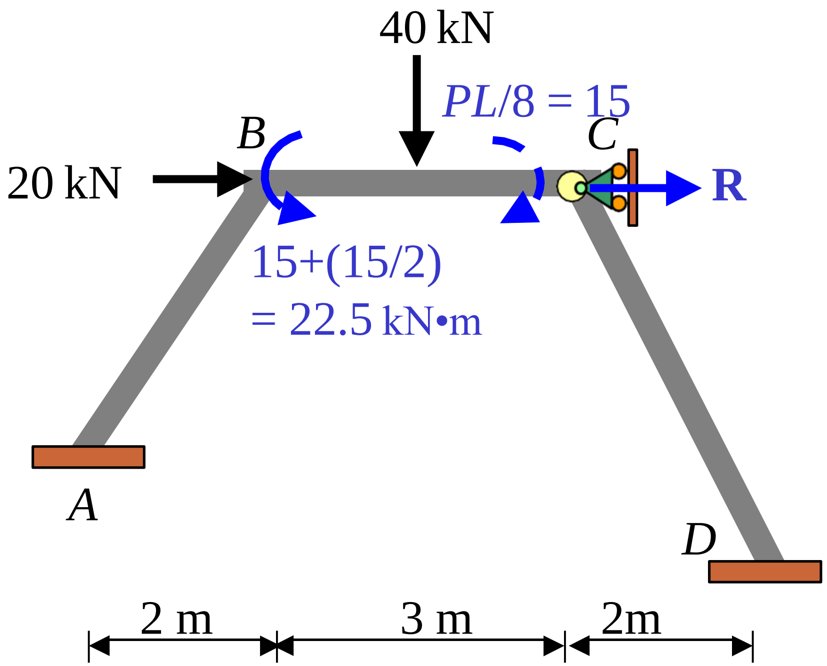


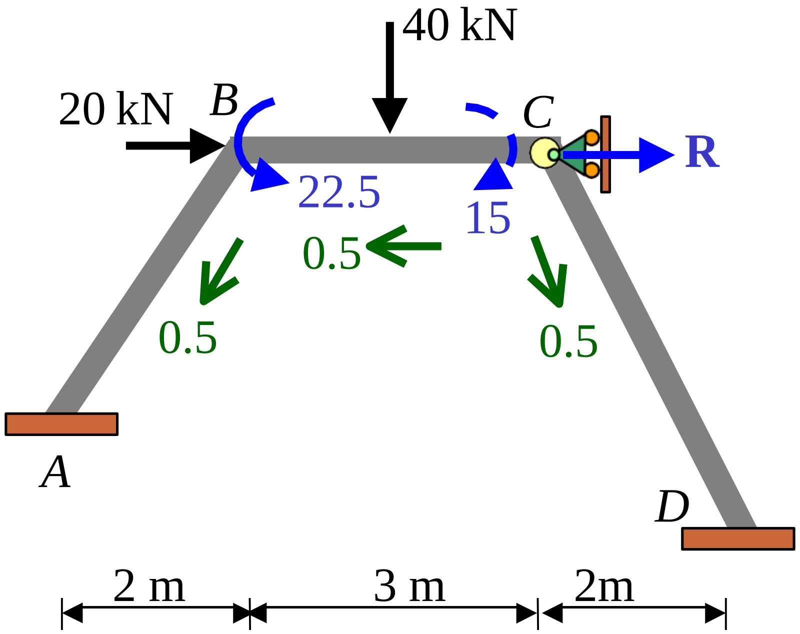
- **Artificial joint applied (no sidesway)**

Fixed end moments:



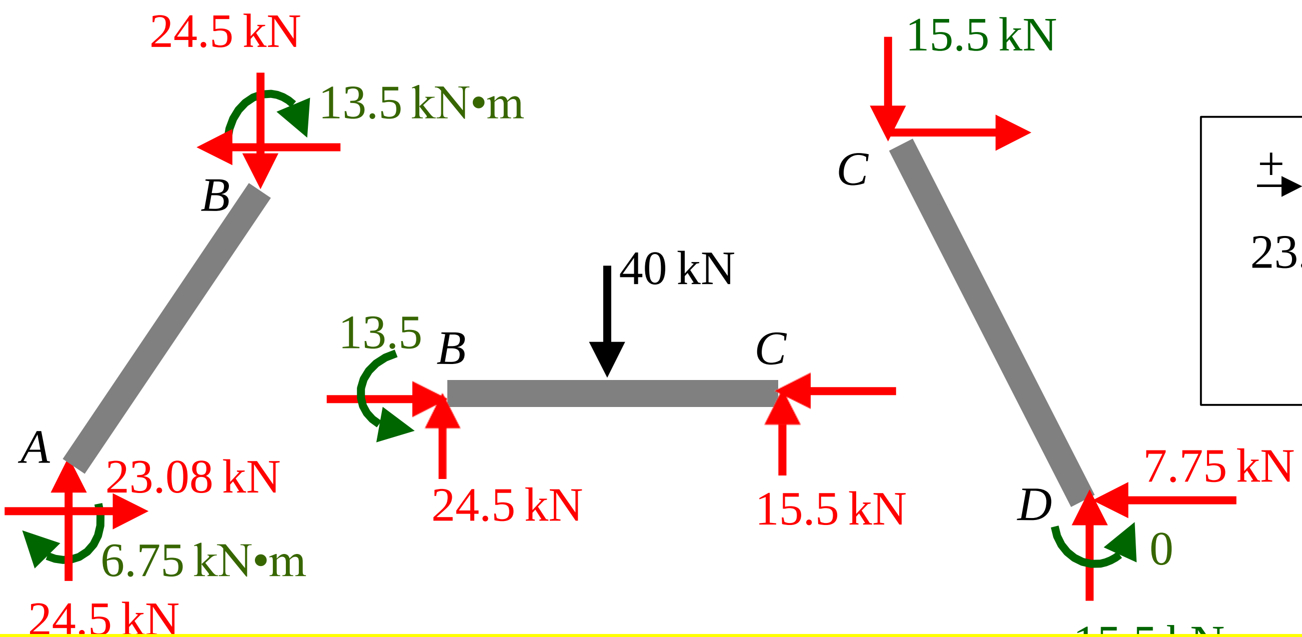
Equilibrium condition :

$$\overset{+}{\rightarrow} \Sigma F_x = 0: \quad A_x + D_x + R = 0$$



	A	B	B	C	C	D
DF	0	0.60	0.40		1.00	0
FEM			22.5			
Dist.		-13.5	-9.0			
CO	-6.75					
Σ	**-6.75**	**-13.5**	**13.5**			

$K_{BA} = 4(4EI)/3.6 = 4.444EI$, $K_{BC} = 3(3EI)/3 = 3EI$,



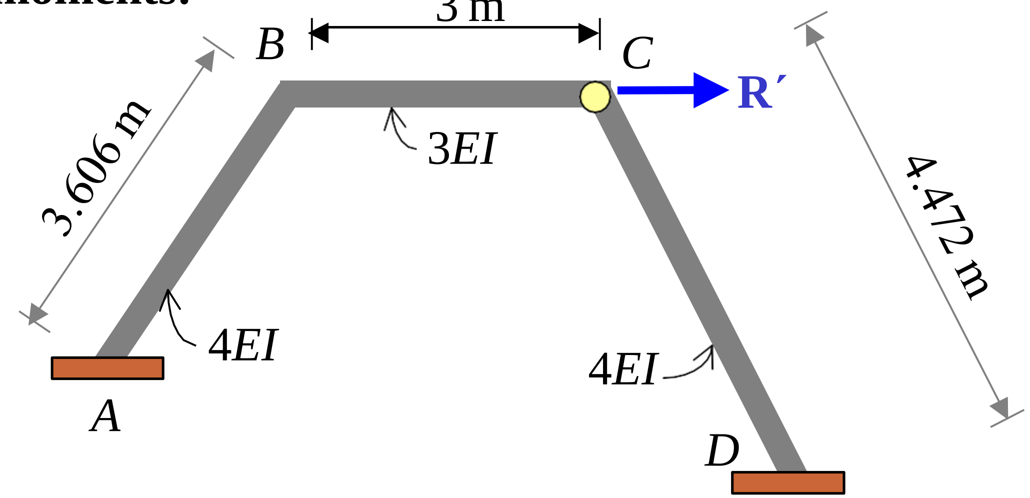
$\xrightarrow{+} \Sigma F_x = 0$:

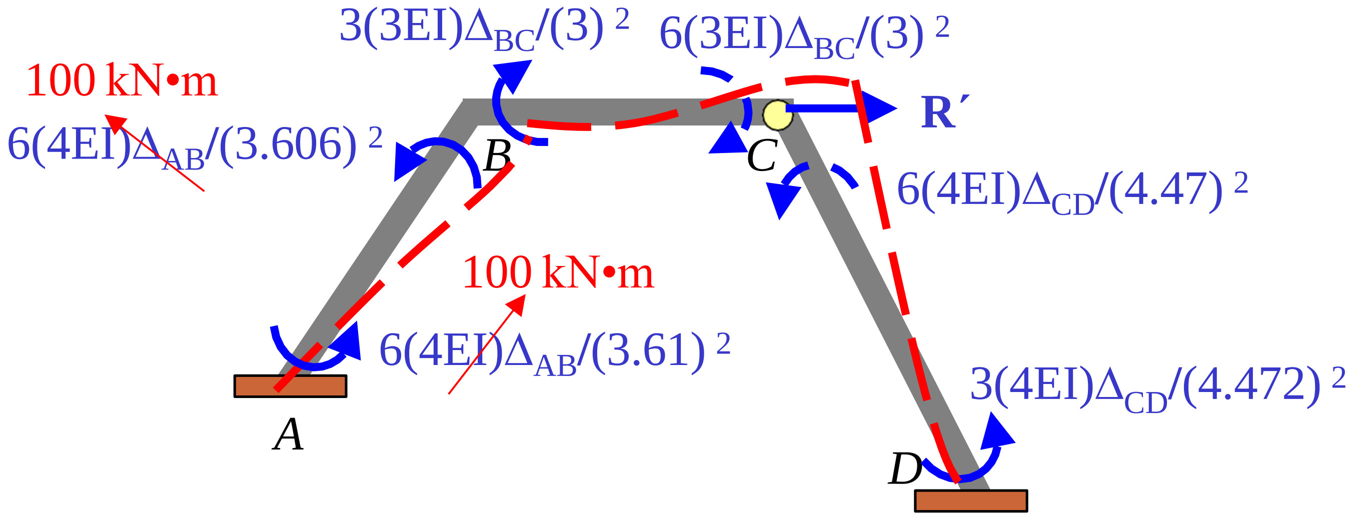
$23.08 + 20 - 7.75 + R' = 0$

$R' = -35.33$ kN ←

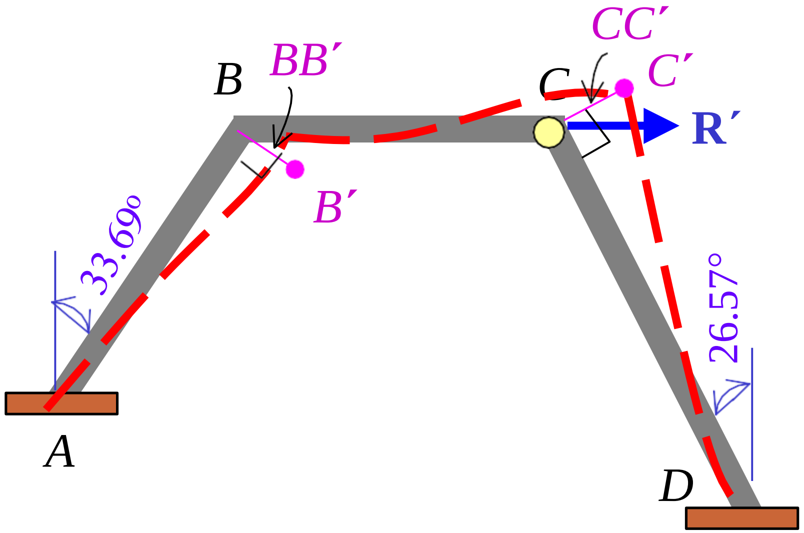
- **Artificial joint removed (sidesway)**

Fixed end moments:

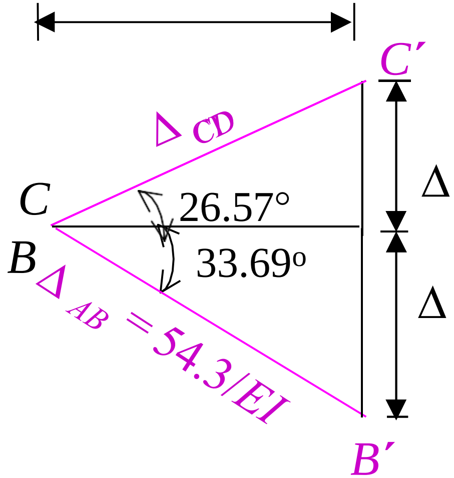




Assign a value of $(FEM)_{AB} = (FEM)_{BA} = 100$ kN•m : $\dfrac{6(4EI)\Delta_{AB}}{3.61^2} = 100, \quad \Delta_{AB} = 54.18/EI$



$\Delta = \Delta_{AB}\cos 33.69° = 45.08/EI$



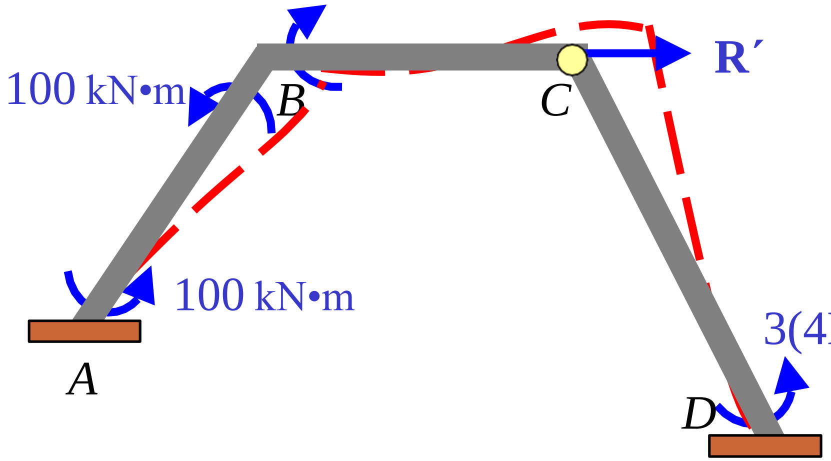
$\Delta \tan 26.57 = 22.54/EI$

$\Delta \tan 33.69 = 30.05/EI$

$$\Delta_{BC} = B'C' = 22.54/EI + 30.05/EI = 52.59/EI$$

$$\Delta_{CD} = \Delta/\cos 26.57° = 50.4/EI$$

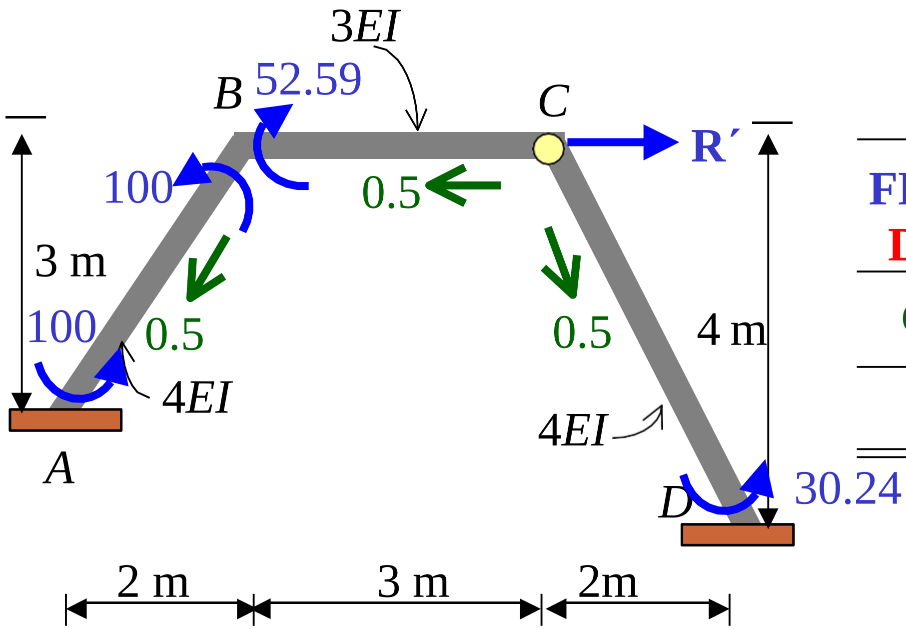
$3(3EI)\Delta_{BC}/(3)^2 = 3(3EI)(52.59/EI)/(3)^2 = 52.59$ kN•m



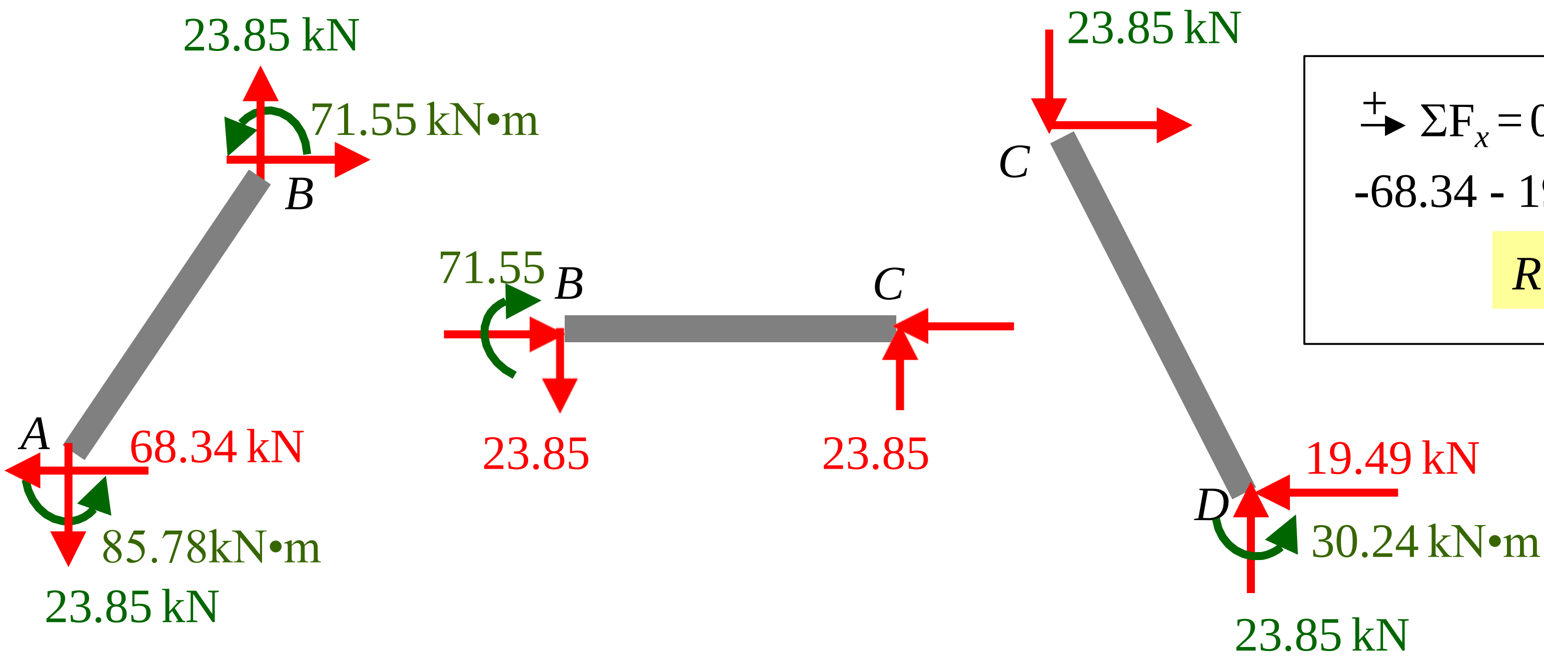
$3(4EI)\Delta_{CD}/(4.472)^2$

$= 3(4EI)(50.4/EI)/(4.472)^2$

$= 30.24$ kN•m



	A	B	B	C	C	D
DF	0	0.60	0.40	1.00	1.00	0
FEM	100	100	-52.59			30.24
Dist.		-28.45	-18.96			
CO	-14.223					
Σ	**85.78**	**71.55**	**-71.55**			**30.24**



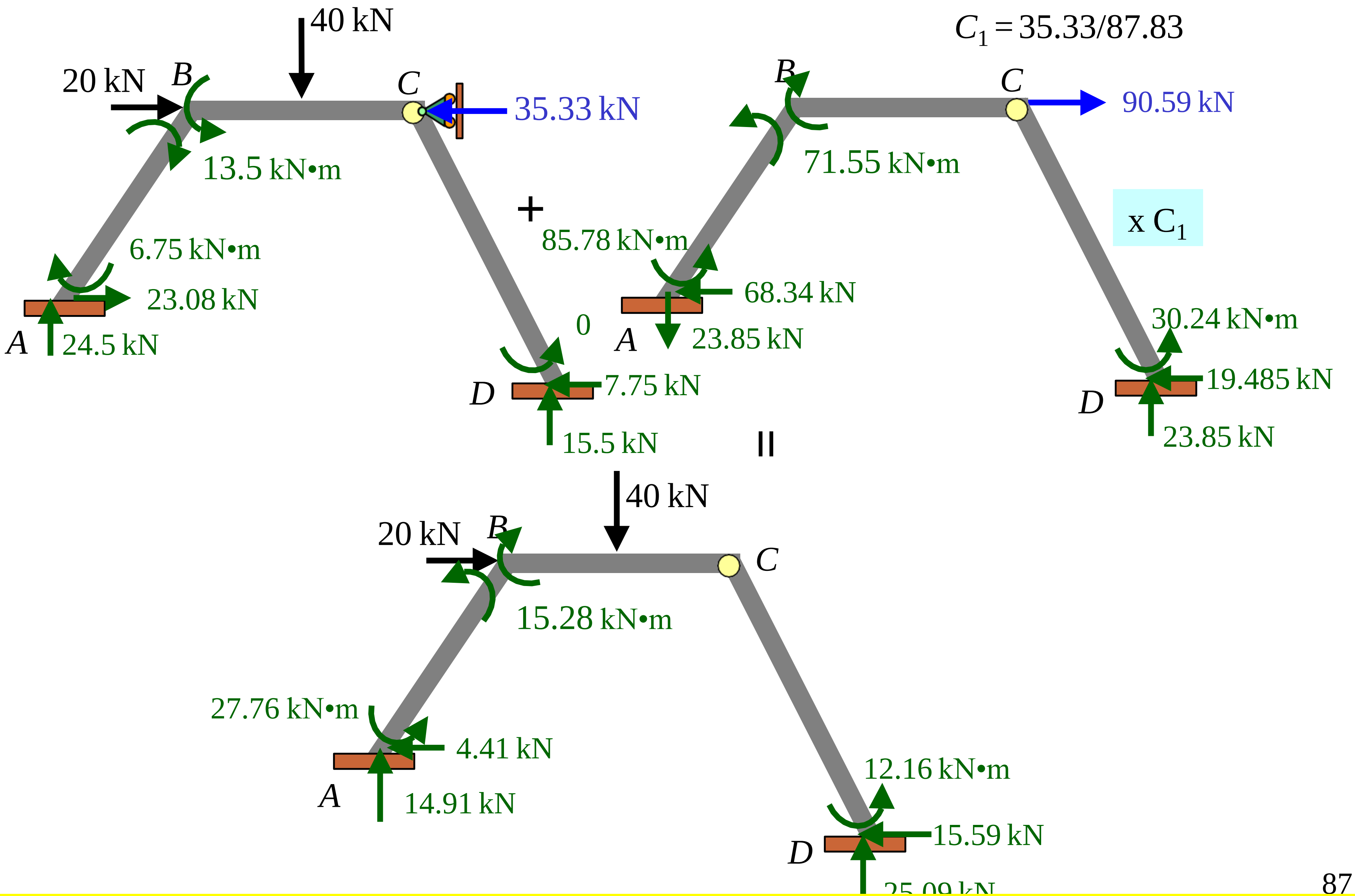
$\xrightarrow{+} \Sigma F_x = 0$:

$-68.34 - 19.49 + R' = 0$

$R' = 87.83$ kN

Substitute R = -35.33 and R' = 87.83 in (1) : $-35.33 + C_1(87.83) = 0$

$C_1 = 35.33/87.83$



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https://nptel.ac.in/courses/105/101/105101085/

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(4) Books: B.C. Punmia
(5) Books: R.K. Bansal
(6) Books: G.K. Grover

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Thank you!