JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – III Year / V Semester (2020-21)

Subject –Structural Analysis-I

Unit – II

Presented by – Akhil Maheshwari *(Asst. Prof., Department of Civil Engineering)*

VISSION AND MISSION OF INSTITUTE

Vision

To become a renowned center of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

Mission

M-1: Focus on evaluation of learning outcomes and motivate students to inculcate research Aptitude by project based learning.

M-2: Identify, based on informed perception of Indian, Regional and global needs, areas of focus and provide platform to gain knowledge and solutions.

M-3: Offer opportunities for interaction between academia and industry.

M-4: Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders can emerge in a range of professions.

VISSION AND MISSION OF DEPARTMENT

VISION

To become a role model in the field of Civil Engineering for the sustainable development of the society.

MISSION

To provide outcome base education.

To create a learning environment conducive for achieving academic excellence.

To prepare civil engineers for the society with high ethical values.

PROGRAMME OUTCOMES (PO)

1. **Engineering knowledge**: Apply the knowledge of mathematics, science, engineering Fundamentals and an engineering specialization to the solution of complex engineering problems.
2. **Problem analysis**: Identify, formulate, research literature, and analyze complex engineering problems reaching substantiated conclusions using first principles of mathematics, natural sciences, and engineering sciences.
3. **Design/development of solutions**: Design solutions for complex engineering problems and design system components or processes that meet the specified needs with appropriate consideration for the public health and safety, and the cultural, societal, and environmental considerations.
4. **Conduct investigations of complex problems**: Use research-based knowledge and research methods including design of experiments, analysis and interpretation of data, and synthesis of the information to provide valid conclusions.
5. **Modern tool usage**: Create, select, and apply appropriate techniques, resources, and modern engineering and IT tools including prediction and modeling to complex engineering activities with an understanding of the limitations.

6. **The engineer and society**: Apply reasoning informed by the contextual knowledge to assess societal, health, safety, legal and cultural issues and the consequent responsibilities relevant to the professional engineering practice.

7. **Environment and sustainability**: Understand the impact of the professional engineering solutions in societal and environmental contexts, and demonstrate the knowledge of, and need for sustainable development.

8. **Ethics**: Apply ethical principles and commit to professional ethics and responsibilities and norms of the engineering practice.

9. **Individual and team work**: Function effectively as an individual, and as a member or leader in diverse teams, and in multidisciplinary settings.

10. **Communication**: Communicate effectively on complex engineering activities with the engineering community and with society at large, such as, being able to comprehend and write effective reports and design documentation, make effective presentations, and give and receive clear instructions.

11. **Project management and finance**: Demonstrate knowledge and understanding of the engineering and management principles and apply these to one's own work, as a member and leader in a team, to manage projects and in multidisciplinary environments.

12. **Life-long learning**: Recognize the need for, and have the preparation and ability to engage in independent and life-long learning in the broadest context of technological change.

Course Outcomes (CO)

CO1. Students will be able to understand the Static and Kinematic Indeterminacy.

CO 2. Students will be able to understand the different types of Prop, Fixed and Continuous Beam.

CO 3. Students will be able to understand the Slope Deflection and Moment Distribution Method.

CO 4. Students will be able to understand Mechanical vibrations.

CO-PO MAPPING

CO/PO	PO1	PO2	PO3	PO4	PO5	PO6	PO7	PO8	PO9	PO10	PO11	PO12
CO1	3	2	3	2	2	1	-	-	1	1	1	2
CO2	3	3	3	2	2	1	-	-	2	1	1	2
CO3	3	3	3	2	2	1	-	-	1	1	2	2
CO4	3	2	2	2	3	2	-	-	2	1	3	3

Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
1	1.1	Introduction,Scope, and Coutcome of subject	July	1	PPT
2	2.1	Introduction to Indeterminate structures	July	1	PPT
3	2.2	Degrees of freedom per node		1	PPT
4	2.3	Static and Kinematic indeterminacy (i.e. for beams, frames & portal with & without sway etc.)		1	PPT
5	2.4	Releases in structures		1	PPT
6	2.5	Maxwell's reciprocal theorem and Betti's theorem.	August	1	PPT
7	2.6	Analysis of prop cantilever structures		1	PPT
8	2.7	Analysis of Indeterminate Structure (fixed and continues beams) using Area moment method		1	PPT
9	2.8	Conjugate beam method		1	PPT
10	2.9	Three moments Theorem.		1	PPT

Teaching Plan

Lect No.	Unit Code	Topic Discription	Expexcted Month	Expected week	Plan of teaching
11	3.1	Analysis of Statically Indeterminate Structures using Slope-deflection method	September	1	PPT
12	3.2	Moment-distribution method applied to continuous beams and portal frames with and without inclined members		1	PPT
13	4.1	Vibrations: Elementary concepts of structural vibration, Mathematical models, basic elements of vibratory system.		1	PPT
14	4.2	Degree of freedom. Equivalent Spring stiffness of springs in parallel and in series.		1	PPT
15	4.3	Simple Harmonic Motion: vector representation, characteristic, addition of harmonic motions, Angular oscillation.	October	1	PPT
16	4.4	Undamped free vibration of SDOF system: Newton's law of motion		1	PPT
17	4.5	D Almbert's principle, deriving equation of motions, solution of differential equation of motion, frequency & period of vibration, amplitude of motion; Introduction to damped and forced vibration.		1	PPT

Static and Kinematic Indeterminacy of Structure

Structure

- A *structure* refers to a system of connected parts used to support a load.

- A *structure* defined as an assembly of different members connected to each other which transfers load from space to ground.

- Mainly of two types :

  1. Load Bearing Structure

  2. Framed Structure

Support System

- Supports are used in structures to provide it stability and strength.
- Main types of support :

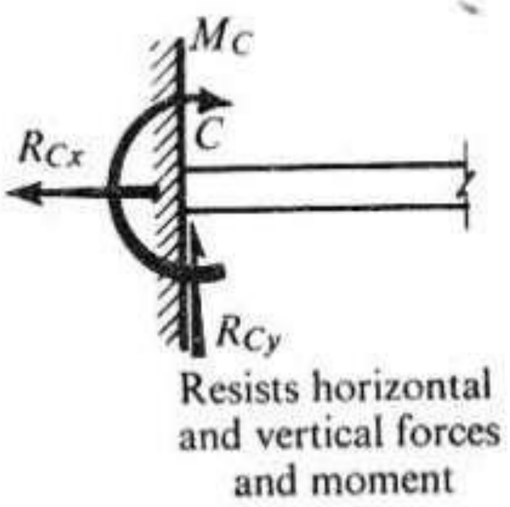
  1. Fixed Support
  2. Hinged or Pinned Support
  3. Roller Support
  4. Vertical Guided Roller Support
  5. Horizontal Guided Roller Support

2-D Support

2-D Support

Fixed Support :

- No. of Reaction - 3 (R_{CX}, R_{CY}, M_{CZ})
  - R_{CX} -Reaction at joint ' C ' in x-direction
  - R_{CY} -Reaction at joint ' C ' in y-direction
  - M_{CZ} -Moment at joint ' C ' about z-direction

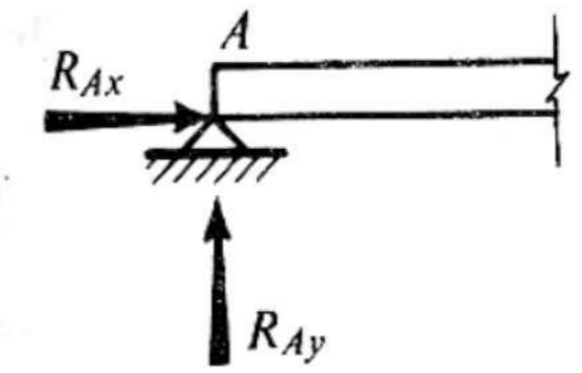


- Displacement in x-direction at joint ' C ' is zero (i.e y_{CX} = O)
- Displacement in y-direction at joint ' C ' is zero (i.e y_{CY} = O)
- Rotation about z-direction at joint ' C ' is zero (i.e θ_{CZ} = O)

2-D Support

Hinged or Pinned Support :

- No. of Reaction - 2 (R_{AX}, R_{AY})
  - R_{AX} -Reaction at joint ' A ' in x-direction
  - R_{AY} -Reaction at joint ' A ' in y-direction



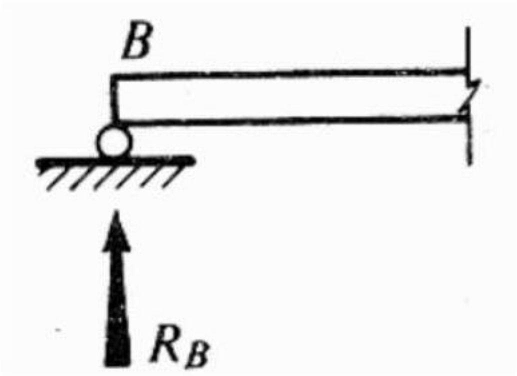
Resists horizontal and vertical forces

- Displacement in x-direction at joint ' A ' is zero (i.e $y_{AX} = O$)
- Displacement in y-direction at joint ' A ' is zero (i.e $y_{AY} = O$)
- Rotation about z-direction at joint ' A ' is not zero (i.e $\theta_{AZ} \neq O$)

2-D Support

Roller Support :

- No. of Reaction - 1 (R_{BY})
- R_{BY} -Reaction at joint ‘ B ’ in y-direction

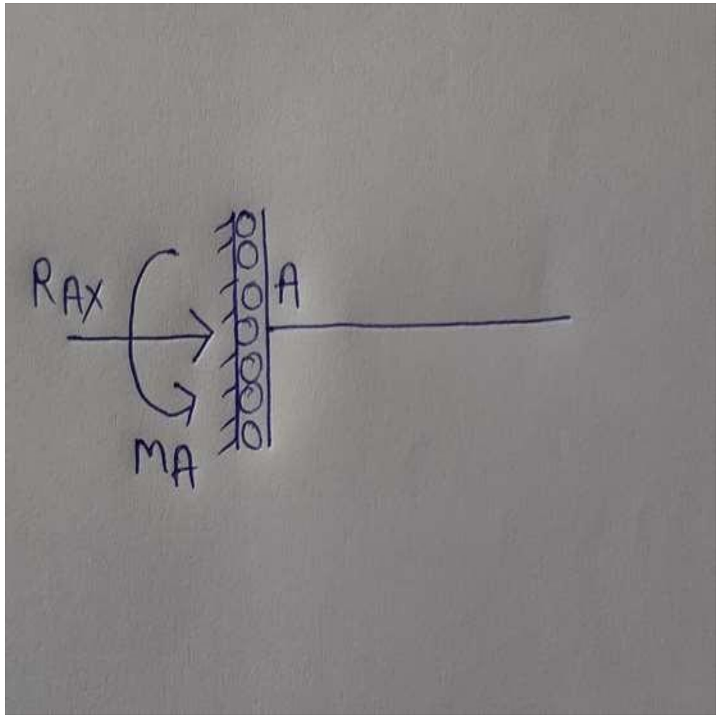


Resists vertical forces

- Displacement in x-direction at joint ‘ B ‘ is not zero (i.e $y_{BX} \neq O$)
- Displacement in y-direction at joint ‘ B ‘ is zero (i.e $y_{BY} = O$)
- Rotation about z-direction at joint ‘ B ‘ is not zero (i.e $\theta_{BZ} \neq O$)

2-D Support

Vertical Guided Roller Support :

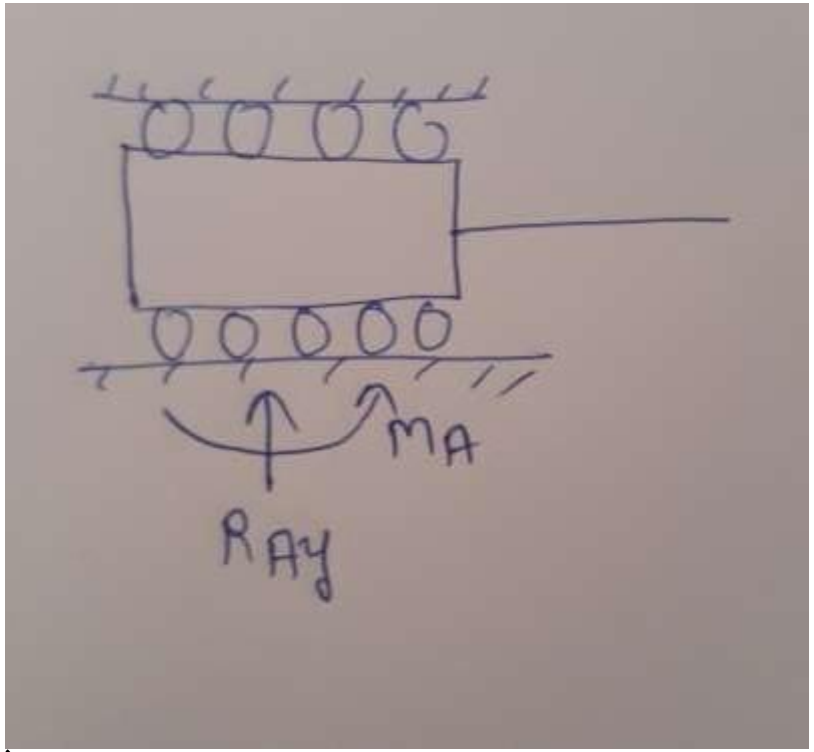


- No. of Reaction - 2 (R_{AX}, M_{AZ})
  - R_{AX} -Reaction at joint ' A ' in x-direction
  - M_{AZ} -Moment at joint ' A ' about z-direction

- Displacement in x-direction at joint ' A ' is zero (i.e $y_{AX} = O$)
- Displacement in y-direction at joint ' A ' is not zero (i.e $y_{AY} \neq O$)
- Rotation about z-direction at joint ' A ' is zero (i.e $\theta_{AZ} = O$)

2-D Support

Horizontal Guided Roller Support :

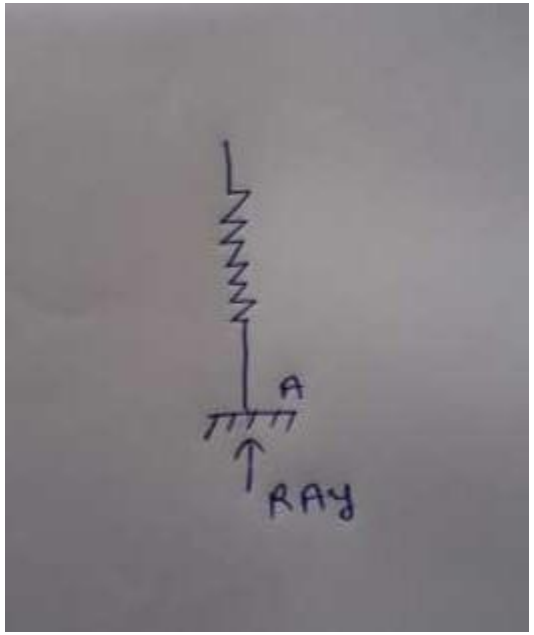


- No. of Reaction - 2 (R_{AY}, M_{AZ})
  - R_{AY} -Reaction at joint ' A ' in y-direction
  - M_{AZ} -Moment at joint ' A ' about z-direction

- Displacement in y-direction at joint ' A ' is zero (i.e $y_{AY} = O$)
- Displacement in x-direction at joint ' A ' is not zero (i.e $y_{AX} \neq O$)
- Rotation about z-direction at joint ' A ' is zero (i.e $\theta_{AZ} = O$)

2-D Support

Spring Support :

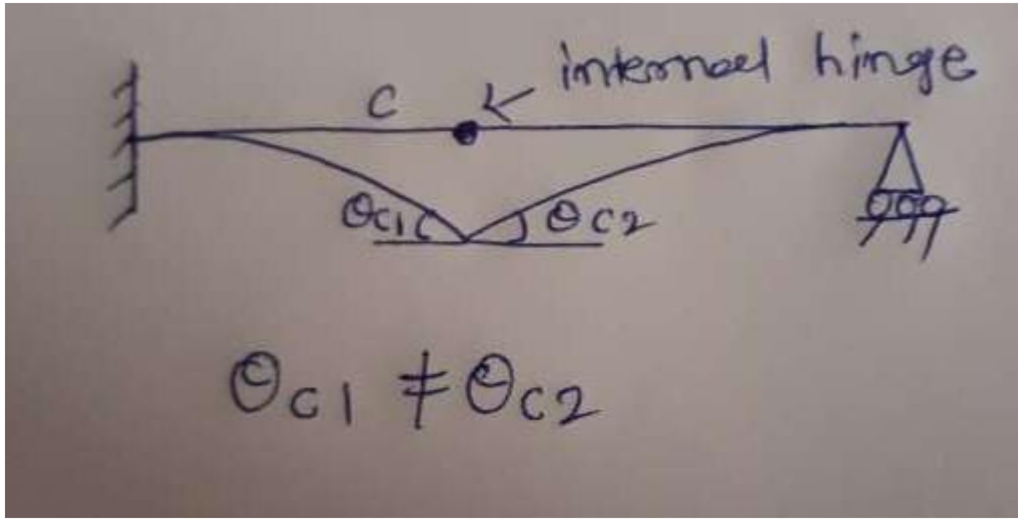


- No. of Reaction - 1 (R_{AY})
  - R_{AY} -Reaction at joint ' A ' in y-direction

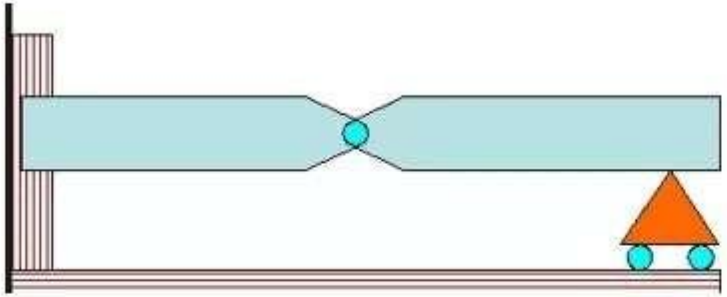
- Displacement in y-direction at joint ' A ' is zero (i.e $y_{AY} = O$)

2-D INTERNAL JOINTS

2-D INTERNAL JOINTS



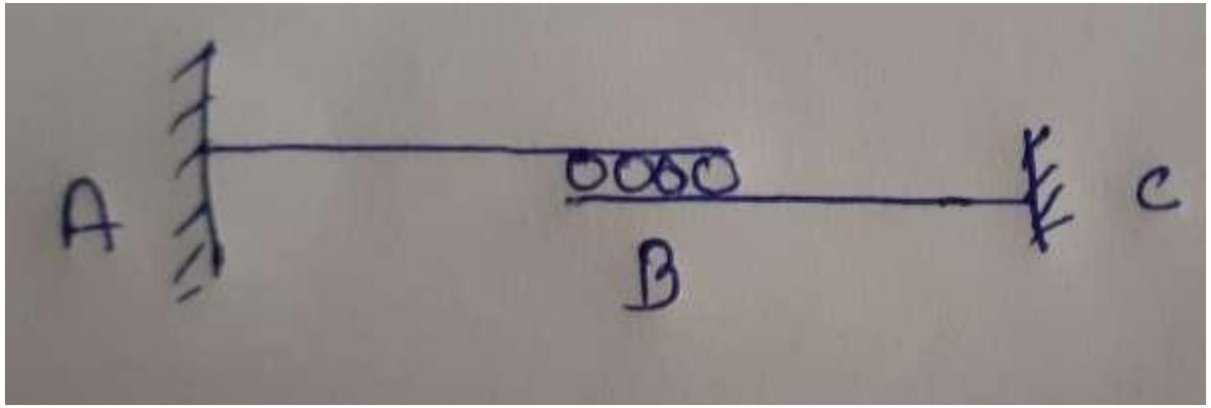
Internal Hinge or Pin :



- Characteristics :
  - Moment at ' C ' is zero (i.e. M@C = O)
- Displacement in y-direction at joint ' C ' is not zero (i.e $y_{CY} \neq O$)
- Displacement in x-direction at joint ' C ' is not zero (i.e $y_{CX} \neq O$)
- Rotation about z-direction at joint ' C ' are may be different at either side (i.e $\theta_{C_1} \neq \theta_{C_2}$)

2-D INTERNAL JOINTS

Internal Roller :



- Characteristics :

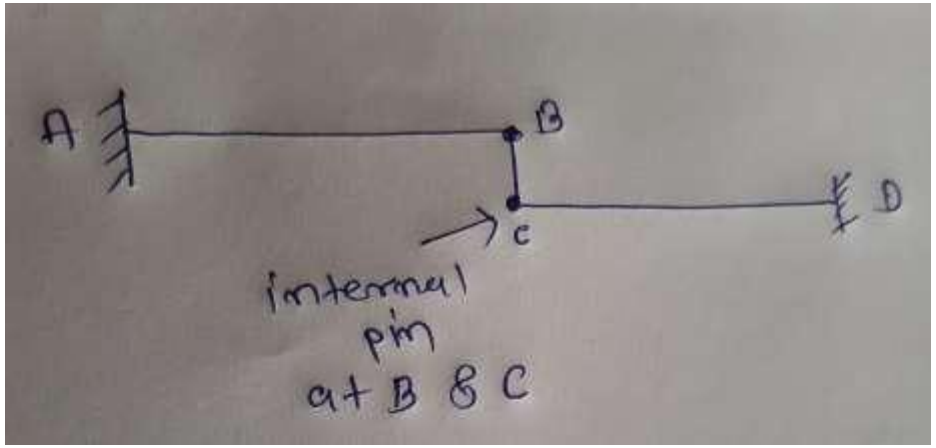
- Can't transfer horizontal reaction (axial thrust)(i.e. $F_{BX} = O$)

- Displacement in y-direction at joint ' B ' may not be zero (i.e $y_{BY} \neq O$)
- Displacement in x-direction at joint ' B ' is not zero (i.e $y_{BX} \neq O$)

2-D INTERNAL JOINTS

Internal Link :

- Portion ‛ BC ‛ is known as internal link.



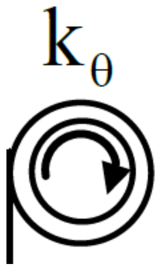
Characteristics :

- Two internal pins at B & C
- Portion BC contains only axial load because moment at B and C is zero.(i.e. M@B & M@C = O)

- Displacement in y-direction at joint ‛ B & C ‛ is not zero (i.e Y_{BY} , $y_{CY} \neq O$)
- Displacement in x-direction at joint ‛ B & C ‛ is not zero (i.e Y_{Bx} ,$Y_{CX} \neq O$)

2-D INTERNAL JOINTS

Torsional Spring Support :



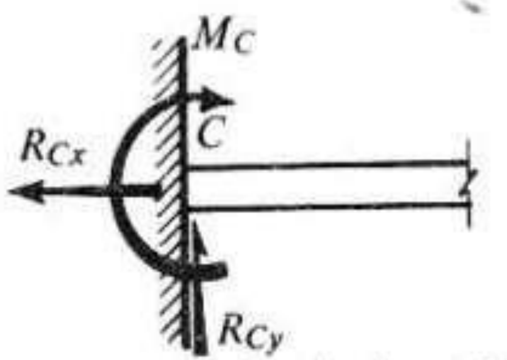
➢ Characteristics :

- θ_Z -Rotational resistance at joint in z-direction

3-D Support

3-D Support

Fixed Support :

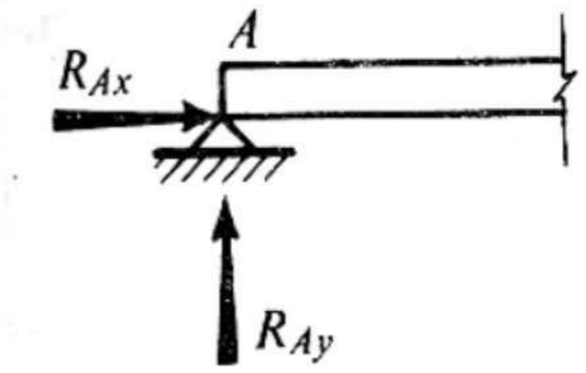


- No. of Reaction - 6 (R_{CX}, R_{CY}, R_{CZ}, M_{CX}, M_{CY}, M_{CZ})
  - R_{CX}, R_{CY}, R_{CZ} - Reaction at joint ' C ' in x,y,z-direction
  - M_{CX}, M_{CY}, M_{CZ} -Moment at joint ' C ' about x,y,z-direction

- Displacement in x,y,z-direction at joint ' C ' is zero (i.e y_{CX}, y_{CY}, y_{CZ} = O)
- Rotation about x,y,z-direction at joint ' C ' is zero (i.e θ_{CX}, θ_{CY}, θ_{CZ} = O)

3-D Support

Hinged or Pinned Support :

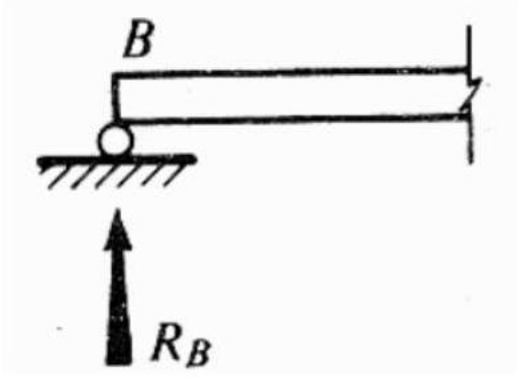


- No. of Reaction - 3 (R_{CX} ,R_{CY} ,R_{CZ})
  - R_{CX} ,R_{CY} ,R_{CZ} - Reaction at joint ' C ' in x,y,z-direction

- Displacement in x,y,z-direction at joint ' C ' is zero (i.e y_{CX} , y_{CY} ,y_{CZ} = O)
- Rotation about x,y,z-direction at joint ' C ' is not zero (i.e θ_{CX} , θ_{CY} , $\theta_{CZ} \neq$ O)

3-D Support

Roller Support :



Resists vertical forces

- No. of Reaction - 1 (R_{CY})

- R_{CY} - Reaction at joint ' C ' in y-direction

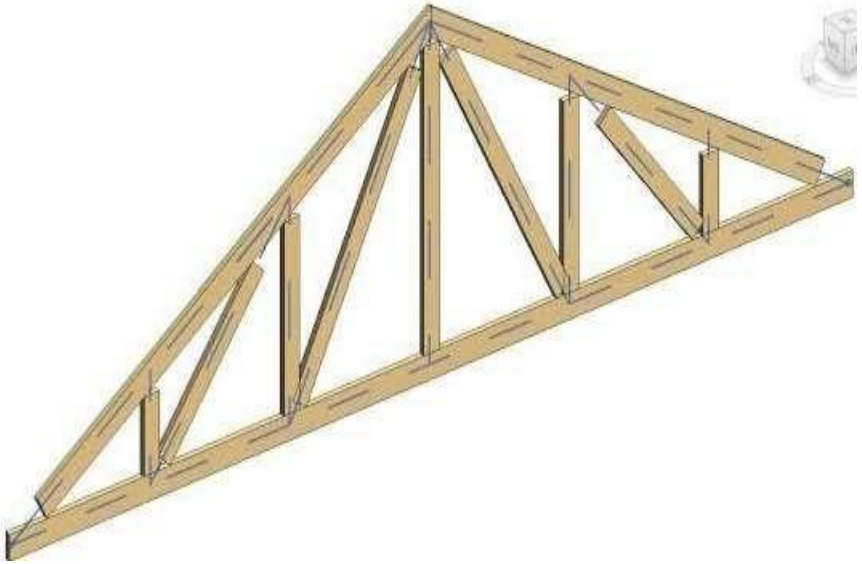
- Displacement in y-direction at joint ' C ' is zero (i.e y_{CY} = O)
- Displacement in x,z-direction at joint ' C ' is not zero (i.e y_{CX} , $y_{CZ} \neq$ O)
- Rotation about x,y,z-direction at joint ' C ' is not zero (i.e θ_{CX}, θ_{CY}, $\theta_{CZ} \neq$ O)

Equilibrium Equation

- When a body is in static equilibrium, no translation or rotation occurs in any direction.
- Since there is no translation, the sum of the forces acting on the body must be zero.
- Since there is no rotation, the sum of the moments about any point must be zero.

Equilibrium Equation

PIN JOINT PLANE FRAME (2-D Truss)

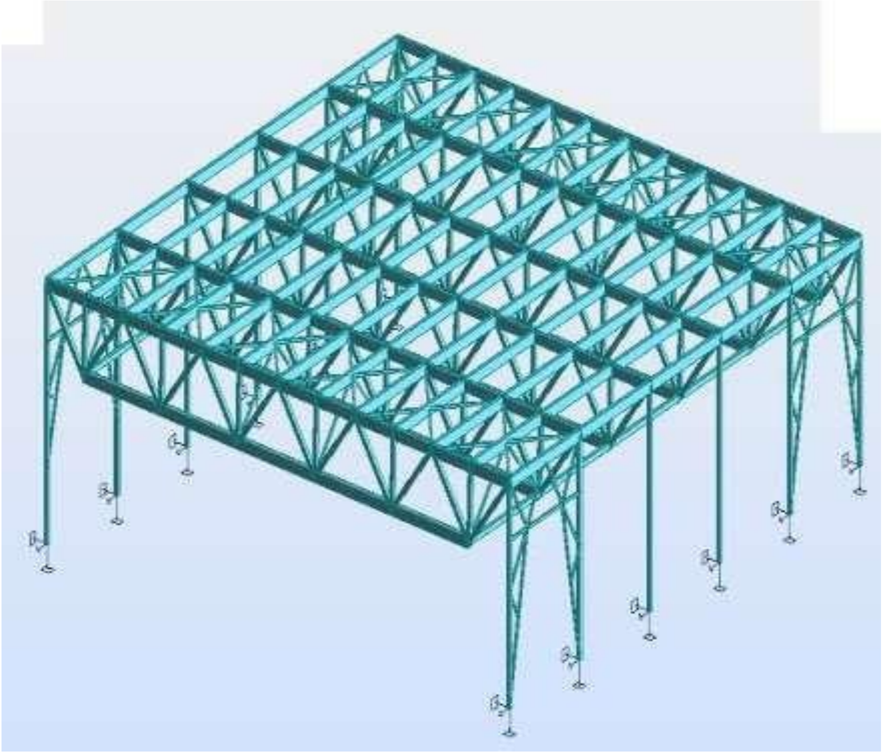


No. of Equilibrium Equation : 2

- $\sum Fx = 0$
- $\sum Fy = 0$

Equilibrium Equation

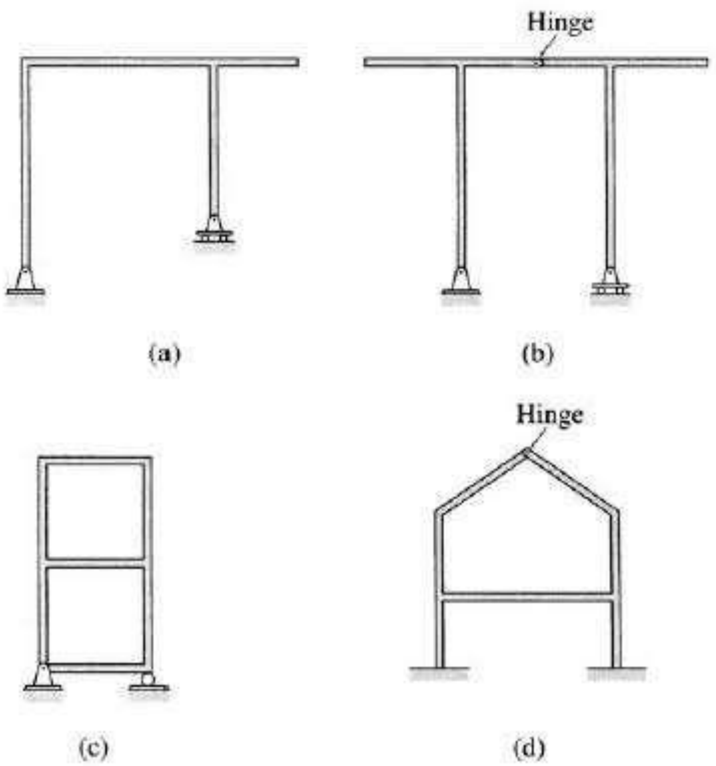
PIN JOINT SPACE FRAME (3-D Truss)



No. of Equilibrium Equation : 3

- $\sum F_x = 0$
- $\sum F_y = 0$
- $\sum F_z = 0$

Equilibrium Equation



RIGID JOINT PLANE FRAME (2-D Frame)

No. of Equilibrium Equation : 3

- $\sum Fx = 0$
- $\sum Fy = 0$
- $\sum Mz = 0$

Equilibrium Equation

RIGID JOINT SPACE FRAME (3-D Frame)

No. of Equilibrium Equation : 6

- $\sum Fx = 0$
- $\sum Fy = 0$
- $\sum Fz = 0$
- $\sum Mx = 0$
- $\sum My = 0$
- $\sum Mz = 0$

NOTE :Above equilibrium equations are used to find members forces and moments , To find out support reaction equilibrium equation for any type of structure always remains 3(i.e. $\sum Fx = 0$ $\sum Fy = 0$ $\sum Mz = 0$)for 2-D and 6 for 3-D structure.

Static Indeterminacy

Statical Determinant Structure :

- If condition of static equilibrium are sufficient to analyse the structure , it is called Statical Determinant Structure.
- Bending moment and Shear force are independent of material properties and cross section.
- Stresses are not induced due to temp. changes and support settlement.

Static Indeterminacy

Statical Indeterminant Structure :

- If condition of static equilibrium are not sufficient to analyse the structure , it is called Statical Indeterminant Structure.
- Bending moment and Shear force are dependent on material properties and cross section.
- Stresses are induced due to temp. changes and support settlement.

Static Indeterminacy

Static Indeterminacy = External Indeterminacy + Internal Indeterminacy Ds = Dse + Dsi

External Indeterminacy : If no. of reactions are more than equilibrium equation is known as Externally Indeterminant Structure.

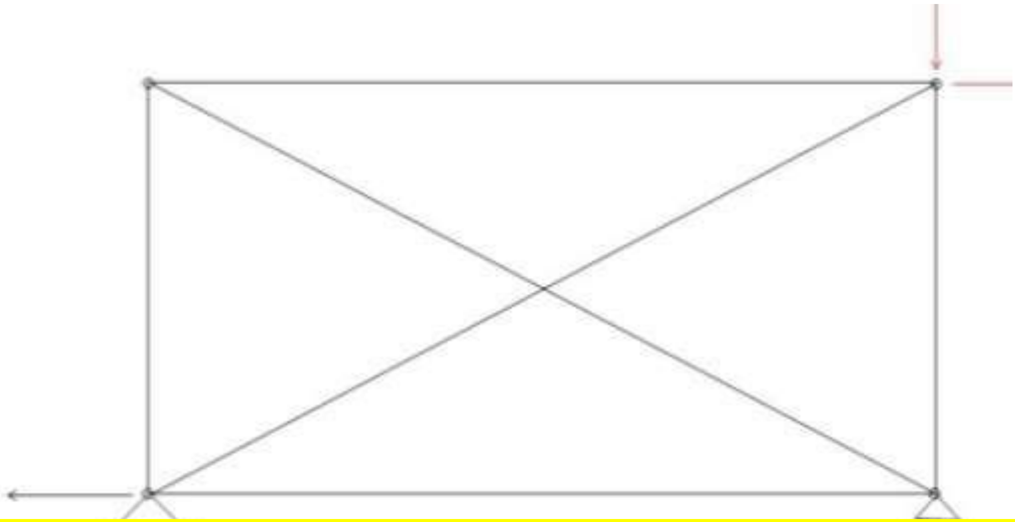
No of Reactions = 4   Equilibrium Equations=3 for 2-D and 6 for 3-D structure.

Beams is externally indeterminate to the first degree

Static Indeterminacy

Internal Indeterminacy : If no. of Internal forces or stresses can't evaluated based on equilibrium equation is known as Internally Indeterminant Structure.

- Member forces of Truss can not be determined based on statics alone, forces in the members can be calculated based on equations of equilibrium. Thus, structures is internally indeterminate to first degree.



Static Indeterminacy

(A) Rigid Jointed Plane Frame :

- External Indeterminacy, D_{se} : R-E
- Internal Indeterminacy, D_{si} : 3C-r'

OR

$D_s = 3m+R-3j-r'$

R = No. of external unknown reaction

E = No. of Equilibrium Equation = 3

m = No. of members , j = joints

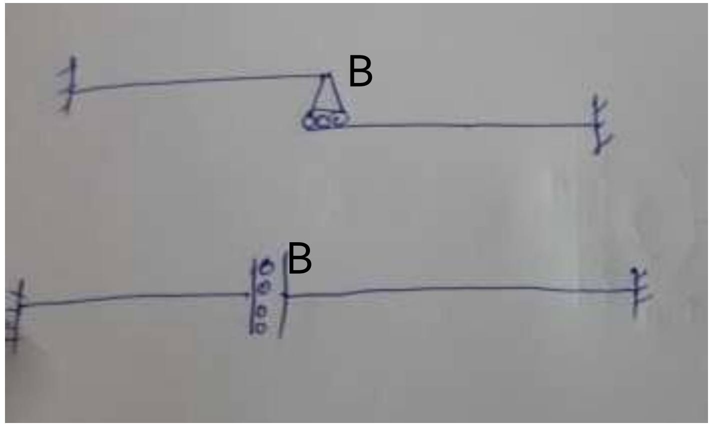
C = No. of close loop

r' = Total no. of internal released or

= No. of members connected with internal hinge -1

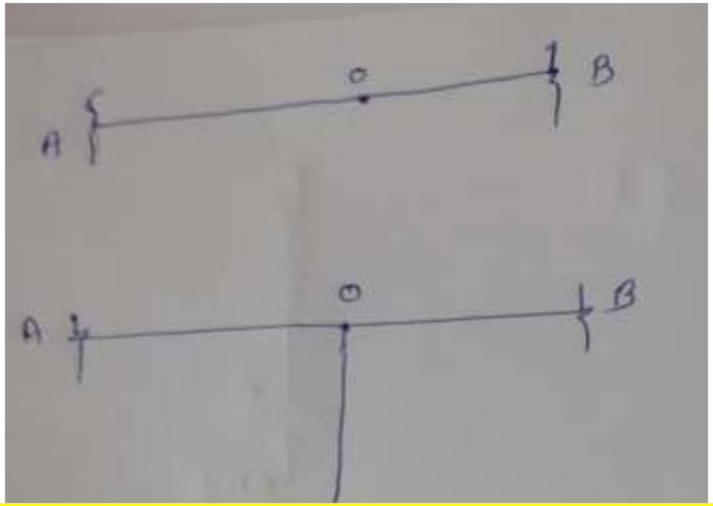
Static Indeterminacy

- Some of the example for the r′ :



r′ = 2 (Moment and Horizontal Reaction Released at joint ' B ')

r′ = 1 (Only Vertical Reaction Released at joint ' B ')



r′ = 2-1 =1 (i.e. member connected to hinges = 2)

r′ = 3-1 =2 (i.e. member connected to hinges = 3)

Static Indeterminacy

(B) Rigid Jointed Space Frame :

- External Indeterminacy, D_{se} : R-E
- Internal Indeterminacy, D_{si} : 6C-r′

OR

$D_s = 6m+R-6j-r'$

R = No. of external unknown reaction

E = No. of Equilibrium Equation = 6

m = No. of members , j = joints

C = No. of close loop

r′ = Total no. of internal released or

= 3 * (No. of members connected with internal hinge -1)

Static Indeterminacy

(C) Pinned Jointed Plane Frame :

- External Indeterminacy, D_{se} : R-E
- Internal Indeterminacy, D_{si} : m+E-2j

OR

D_s = m+R-2j

R = No. of external unknown reaction

E = No. of Equilibrium Equation = 3

m = No. of members

j = joints

Static Indeterminacy

(D) Pinned Jointed Space Frame :

- External Indeterminacy, D_{se} : R-E
- Internal Indeterminacy, D_{si} : m+E-3j

OR

D_s = m+R-3j

R = No. of external unknown reaction

E = No. of Equilibrium Equation = 6

m = No. of members

j = joints

Static Indeterminacy

- $Ds < 0$: Unstable & statically determinant structure
  Deficient Frame or Structure
- $Ds = 0$: Stable & statically determinant structure
  Perfect Frame or Structure
- $Ds > 0$: Stable & statically indeterminant structure
  Redundant Frame or Structure

Kinematic Indeterminacy

- Kinematic Indeterminacy = Degree of Freedom
- If the displacement component of joint can't be determined by Compatibility Equation , it is called Kinematic Indeterminant Structure.

Degree of Kinematic Indeterminacy(D_k) :

- It is defined as total number of unrestrained displacement (translation and rotation) component at joint.

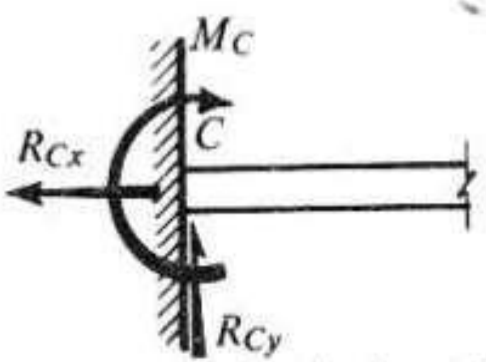
2-D Support

2-D Support

Fixed Support :

- Degree of Freedom - O

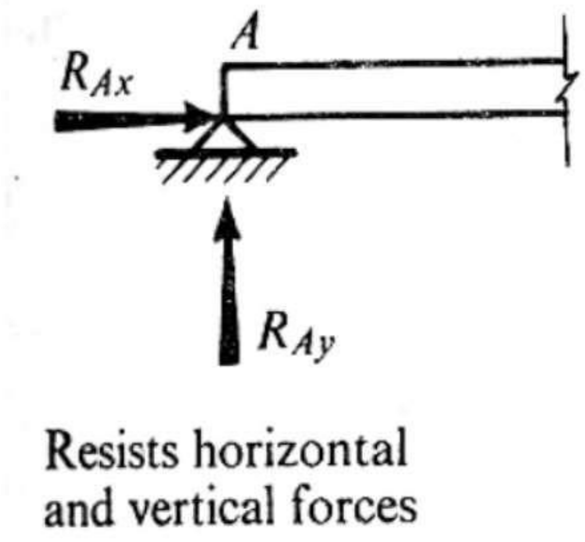
- Displacement in x-direction at joint ' C ' is zero (i.e $y_{CX} = 0$
- Displacement in y-direction at joint ' C ' is zero (i.e $y_{CY} = 0$
- Rotation about z-direction at joint ' C ' is zero (i.e $\theta_{CZ} = 0$)



Resists horizontal and vertical forces and moment

2-D Support

Hinged or Pinned Support :



- Degree of Freedom - 1 (θ_{AZ})

- θ_{AZ} -Rotation about z-direction at joint ' A '

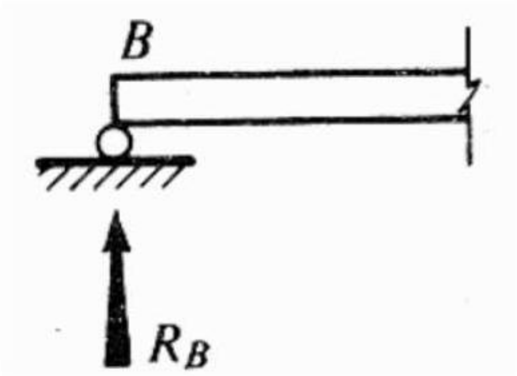
- Displacement in x-direction at joint ' A ' is zero (i.e y_{AX} = O)
- Displacement in y-direction at joint ' A ' is zero (i.e y_{AY} = O)
- Rotation about z-direction at joint ' A ' is not zero (i.e $\theta_{AZ} \neq$ O)

2-D Support

Roller Support :

- Degree of Freedom - 2(θ_{BZ} ,y_{BX})

- y_{BX} -Displacement in x-direction at joint ' B '
- θ_{BZ} -Rotation about z-direction at joint ' B'

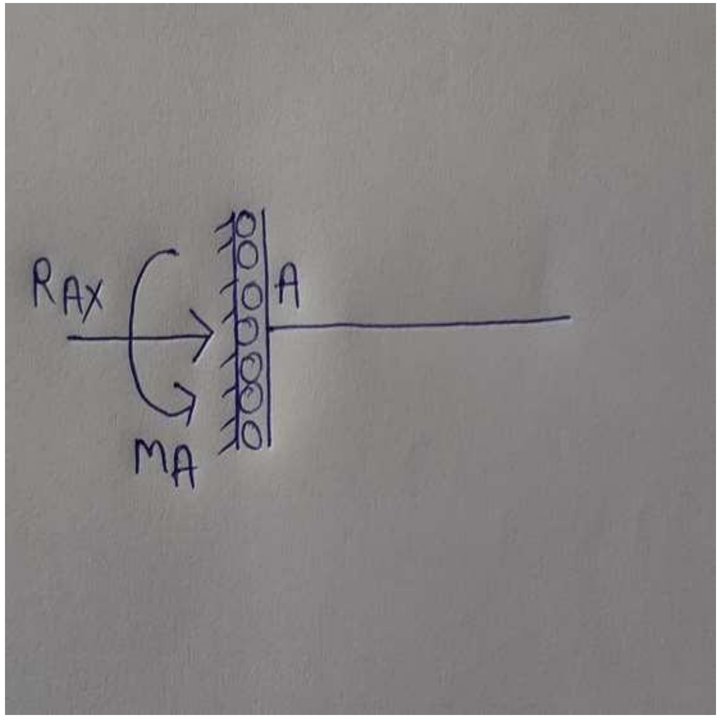


Resists vertical forces

- Displacement in x-direction at joint ' B ' is not zero (i.e $y_{BX} \neq O$)
- Displacement in y-direction at joint ' B ' is zero (i.e $y_{BY} = O$)
- Rotation about z-direction at joint ' B ' is not zero (i.e $\theta_{BZ} \neq O$)

2-D Support

Vertical Guided Roller Support :



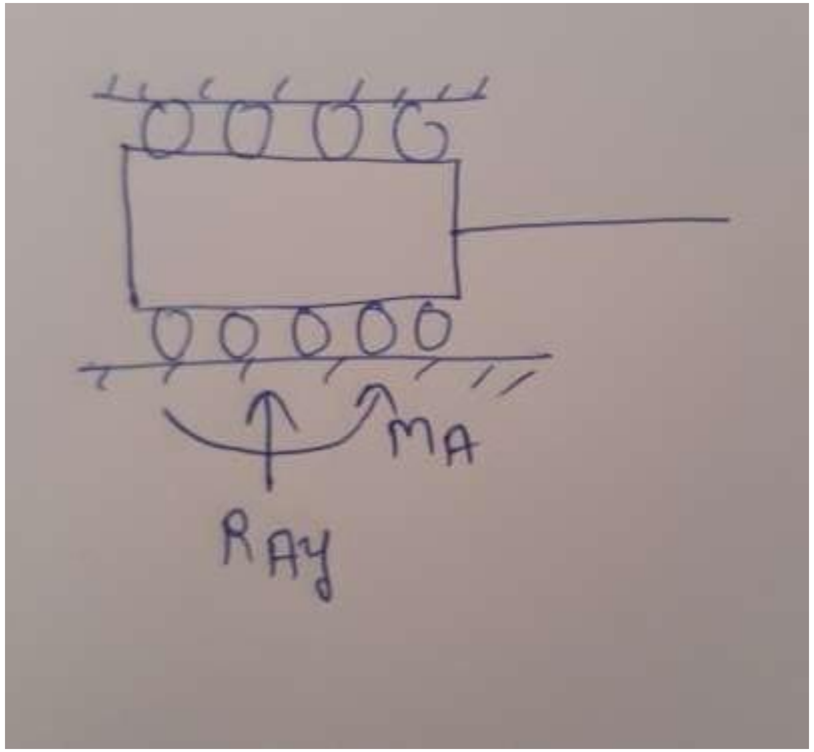
- Degree of Freedom - 1(y_{AY})

- y_{AY} -Displacement in y-direction at joint ' A '

- Displacement in x-direction at joint ' A ' is zero (i.e $y_{AX} = O$)
- Displacement in y-direction at joint ' A ' is not zero (i.e $y_{AY} \neq O$)
- Rotation about z-direction at joint ' A ' is zero (i.e $\theta_{AZ} = O$)

2-D Support

Horizontal Guided Roller Support :



- Degree of Freedom - 1(y_{AX})

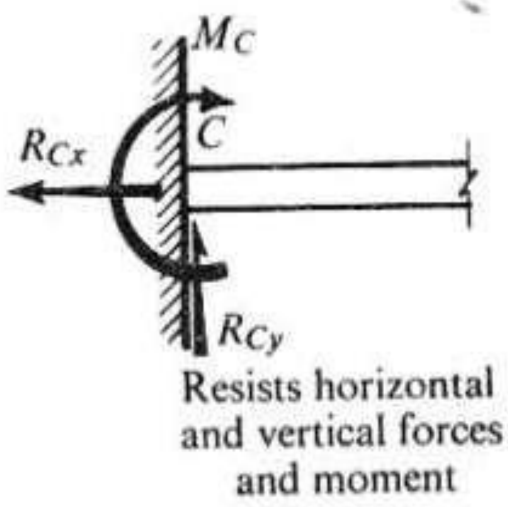
- y_{AX} -Displacement in x-direction at joint ' A '

- Displacement in y-direction at joint ' A ' is zero (i.e y_{AY} = O)
- Displacement in x-direction at joint ' A ' is not zero (i.e $y_{AX} \neq$ O
- Rotation about z-direction at joint ' A ' is zero (i.e θ_{AZ} =O)

3-D Support

3-D Support

Fixed Support :

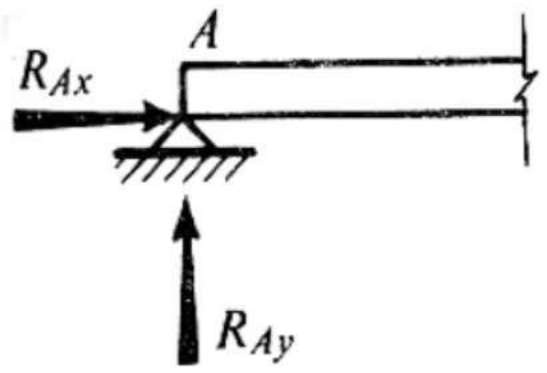


- Degree of Freedom - O

- Displacement in x,y,z-direction at joint ' C ' is zero (i.e y_{CX} , y_{CY} ,y_{CZ} = O)
- Rotation about x,y,z-direction at joint ' C ' is zero (i.e θ_{CX} , θ_{CY} , θ_{CZ} = O)

3-D Support

Hinged or Pinned Support :



- Degree of Freedom - 3 (θ_{CX}, θ_{CY}, θ_{CZ})

- θ_{CX}, θ_{CY}, θ_{CZ} -Rotation about x,y,z-direction at joint ' C '

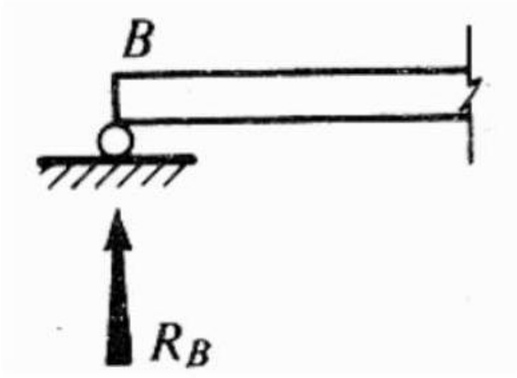
- Displacement in x,y,z-direction at joint ' C ' is zero (i.e y_{CX}, y_{CY} ,y_{CZ} = O)
- Rotation about x,y,z-direction at joint ' C ' is not zero (i.e θ_{CX}, θ_{CY}, $\theta_{CZ} \neq$ O)

3-D Support

Roller Support :

- Degree of Freedom - 5 (y_{CX}, y_{CZ}, θ_{CX}, θ_{CY}, θ_{CZ})

- y_{CX}, y_{CZ} -Displacement in x,z-direction at joint ' C'
- θ_{CX}, θ_{CY}, θ_{CZ} - Rotation about x,y,z-direction at joint ' C '

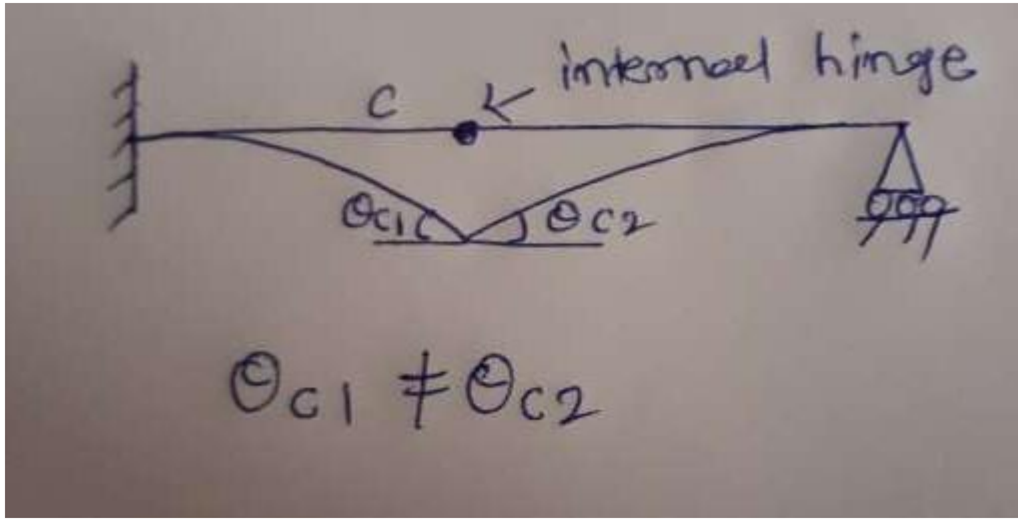


Resists vertical forces

- Displacement in y-direction at joint ' C ' is zero (i.e $y_{CY} = 0$)
- Displacement in x,z-direction at joint ' C ' is not zero (i.e $y_{CX}, y_{CZ} \neq 0$)
- Rotation about x,y,z-direction at joint ' C ' is not zero (i.e $\theta_{CX}, \theta_{CY}, \theta_{CZ} \neq 0$)

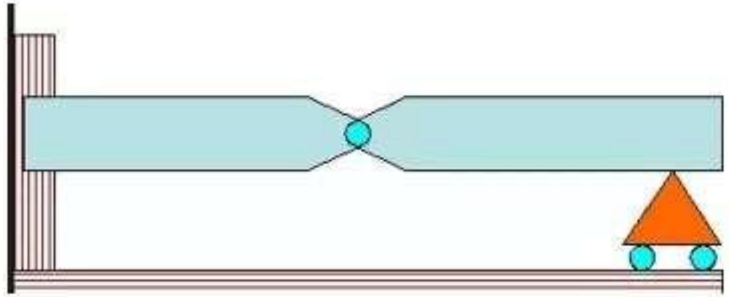
2-D INTERNAL JOINTS

2-D INTERNAL JOINTS



Internal Hinge or Pin :

- Degree of Freedom – 4(y_{CX}, y_{CY}, θ_{C1}, θ_{C2})



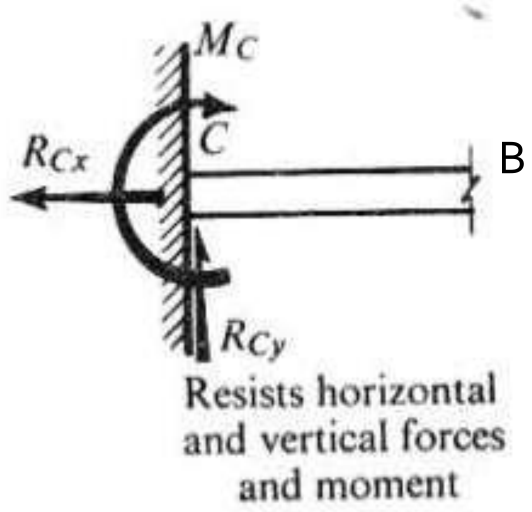
- Displacement in y-direction at joint ' C ' is not zero (i.e $y_{CY} \neq O$)
- Displacement in x-direction at joint ' C ' is not zero (i.e $y_{CX} \neq O$)
- Rotation about z-direction at joint ' C ' are may be different at eit
  i.e $\theta_{C1} \neq \theta_{C2}$)

2-D INTERNAL JOINTS

Free End :

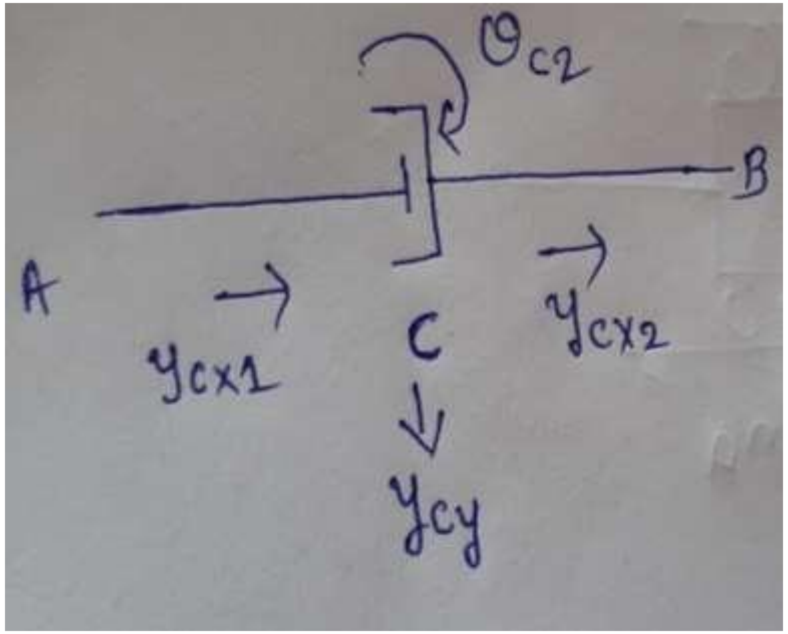
- Degree of Freedom – 3(y_{BX}, y_{BY}, θ_{BZ})

- Displacement in x-direction at joint ' B ' is not zero (i.e $y_{BX} \neq O$)
- Displacement in y-direction at joint ' B ' is not zero (i.e $y_{BY} \neq O$)
- Rotation about z-direction at joint ' B ' is not zero (i.e $\theta_{BZ} \neq O$)



2-D INTERNAL JOINTS

Axial Thrust Release:

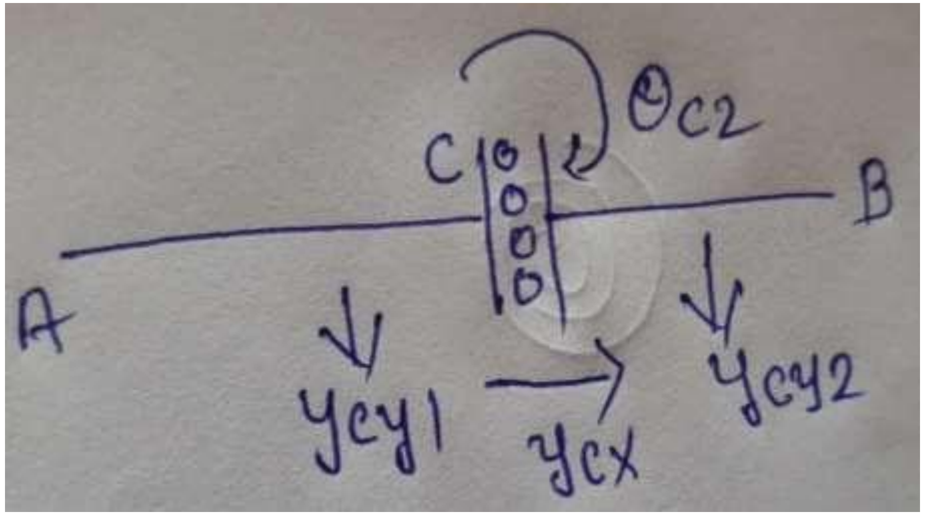


- Degree of Freedom – 4(y_{CX_1}, y_{CX_2}, y_{CY}, θ_{CZ})

- Displacement in x-direction at joint ' C ' is not zero (i.e y_{CX_1}, $y_{CX_2} \neq O$)
- Displacement in y-direction at joint ' C ' is not zero (i.e $y_{CY} \neq O$)
- Rotation about z-direction at joint ' C ' is not zero (i.e $\theta_{CZ} \neq O$)

2-D INTERNAL JOINTS

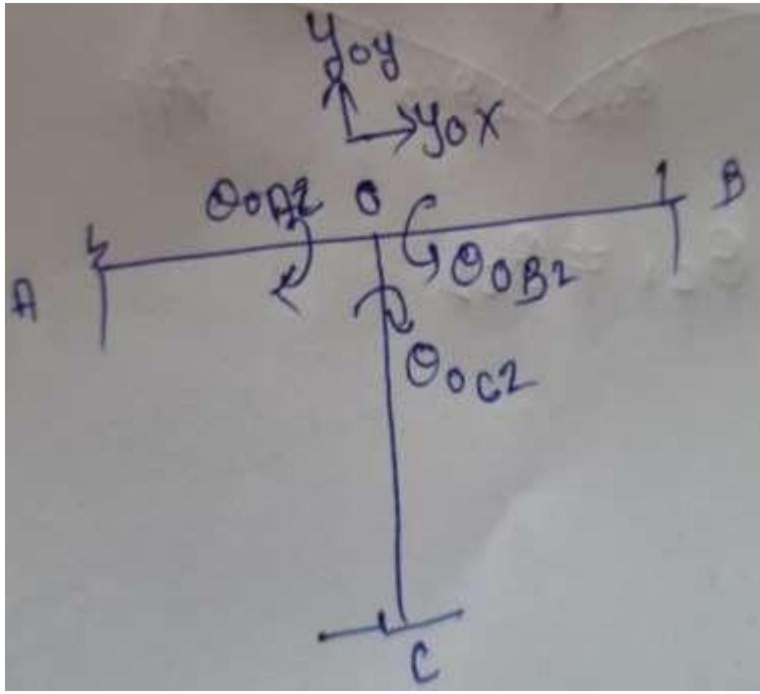
Shear Release:



- Degree of Freedom – 4(y_{CY1}, y_{CY2}, y_{CX}, θ_{CZ})

- Displacement in x-direction at joint ' C ' is not zero (i.e $y_{CX} \neq O$)
- Displacement in y-direction at joint ' C ' is not zero (i.e y_{CY1}, $y_{CY2} \neq O$)
- Rotation about z-direction at joint ' C ' is not zero (i.e $\theta_{CZ} \neq O$)

2-D INTERNAL JOINTS

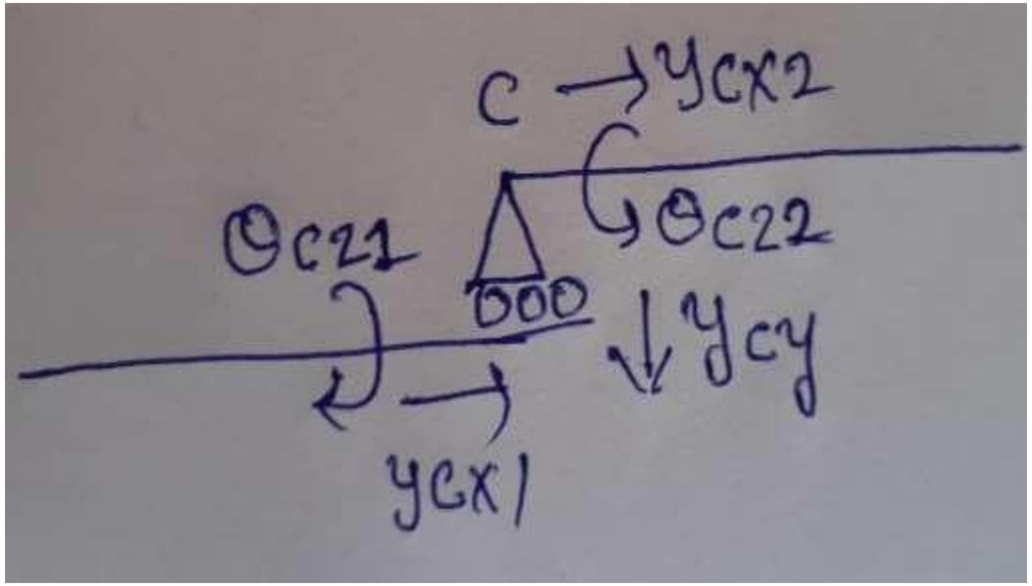


Frame Joint:

- Degree of Freedom – 5(y_{OY} ,y_{OX} , θ_{OAZ} , θ_{OBZ} , θ_{OCZ})

- Displacement in x-direction at joint ' O ' is not zero (i.e $y_{OX} \neq O$)
- Displacement in y-direction at joint ' O ' is not zero (i.e $y_{OY} \neq O$)
- Rotation about z-direction at joint ' O ' is not zero (i.e θ_{OAZ} , θ_{OBZ} , $\theta_{OCZ} \neq O$)

2-D INTERNAL JOINTS



Internal Roller:

- Degree of Freedom – 5(y_{CX_1}, y_{CX_2}, θ_{CZ_1}, θ_{CZ_2}, y_{CY})

- Displacement in x-direction at joint ' C ' is not zero (i.e $y_{CX_1}, y_{CX_2} \neq 0$)
- Displacement in y-direction at joint ' C ' is not zero (i.e $y_{CY} \neq 0$)
- Rotation about z-direction at joint ' C ' is not zero (i.e $\theta_{CZ_1}, \theta_{CZ_2} \neq 0$)

Kinematic Indeterminacy

(A) Rigid Jointed Plane Frame :

- Dk : 3j-R+r′
- Dk(NAD) : Dk-m′

R = No. of external unknown reaction

NAD=Neglecting Axial Deformations

m′ = No. of axially rigid members

(Beams are azially rigid or stiffness is infinite)

r′ = Total no. of internal released or = No. of members connected with internal hinge -1

Kinematic Indeterminacy

(B) Rigid Jointed Space Frame :

- Dk : 6j-R+r′
- Dk(NAD) : Dk-m′

R = No. of external unknown reaction

NAD=Neglecting Axial Deformations

m′ = No. of axially rigid members

r′ = Total no. of internal released or

= 3* (No. of members connected with internal hinge -1)

Kinematic Indeterminacy

- (C) Pinned Jointed Plane Frame :

- Dk : 2j-R
- Dk(NAD) : 0
-

R = No. of external unknown reaction

NAD=Neglecting Axial Deformations

j = No. of joints

Kinematic Indeterminacy

- (D) Pinned Jointed Space Frame :

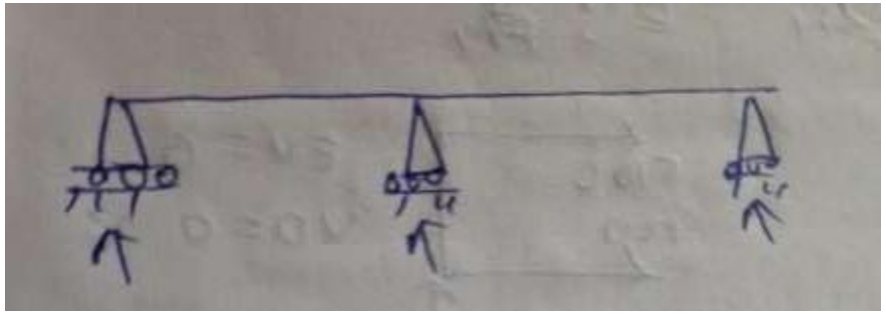
- Dk : 3j-R
- Dk(NAD) : 0
- R = No. of external unknown reaction

NAD=Neglecting Axial Deformations j = No. of joints

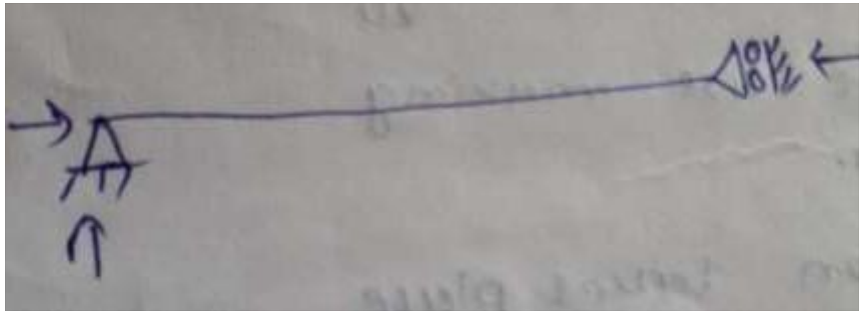
Stability of Structure

External stability :

- For any 2-D structure 3 no. of reactions and foe 3-D structure 6 no. of reactions are required to keep structure in stable condition.
- All reactions should not be Parallel.
- All reactions should not be Concurrent (line of action meets at one point).



Unstable Structure because all



Unstable Structure because all

Stability of Structure

Internal stability :

- No part of structure can move rigidly releative to other part.
- For geometric stability there should not be any condition of mechanism.
- Static Indeterminacy should not be less than zero.(i.e. Ds >=0)(But it is not mandatory, sometimes structure is not stable though this conditions satisfied)
- For internal stability following conditions should be satisfied :

(1) Pinned Jointed Plane Truss : $m \geq 2j-3$

(2) Pinned Jointed Space Truss : $m \geq 3j-6$

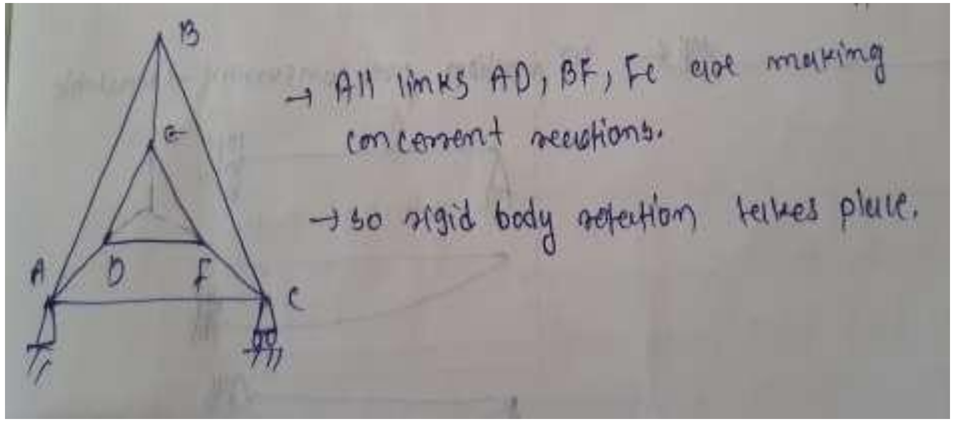
(3) Rigid Jointed Plane Frame : $3m \geq 3j-3$

(4) Rigid Jointed Space Frame : $6m \geq 6j-6$

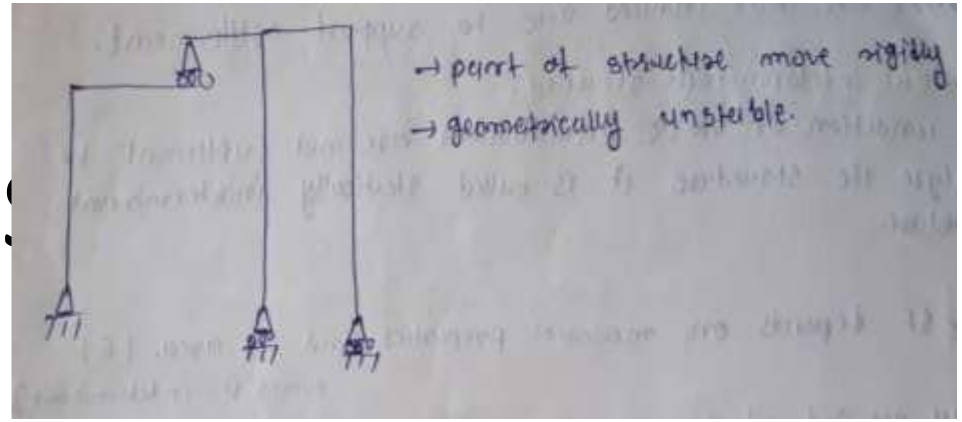
m = No. of members

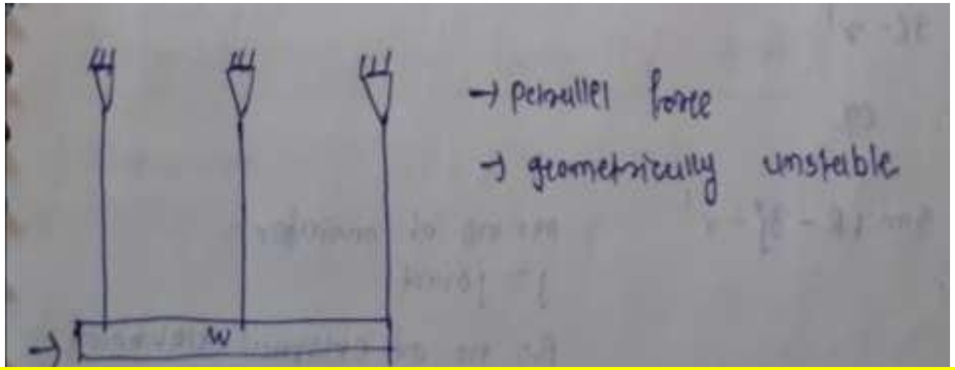
j =No. of joints

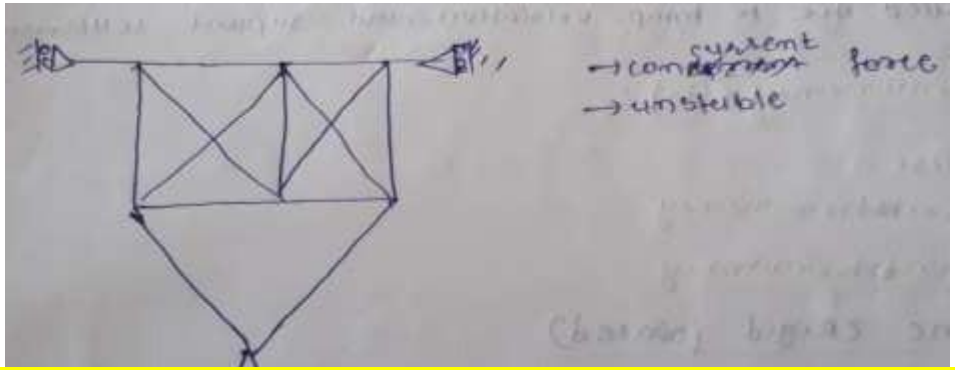
Example of unstable structure :



ity of

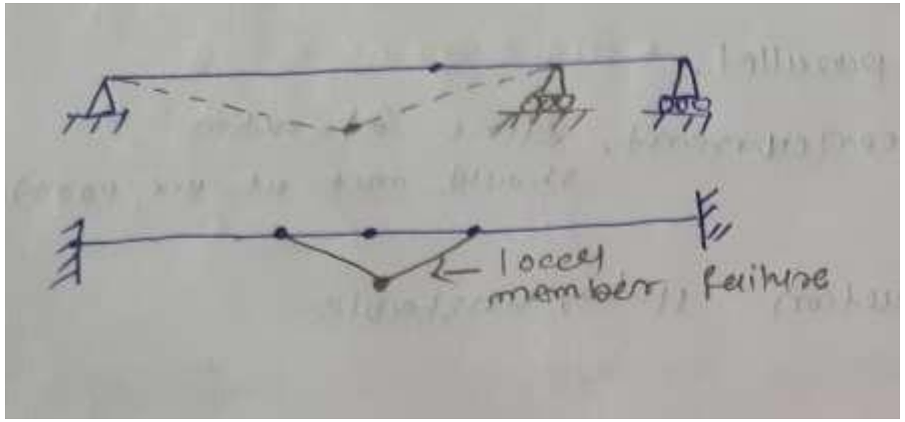




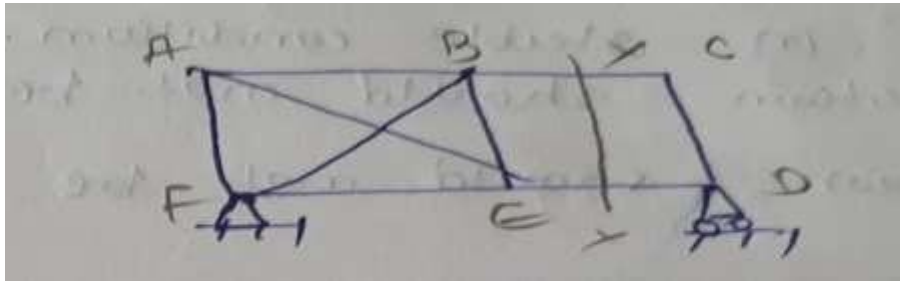


Stability of Structure

Example of unstable structure :



Unstable because of local member failure.



Geometric unstable because of no diagonal member.

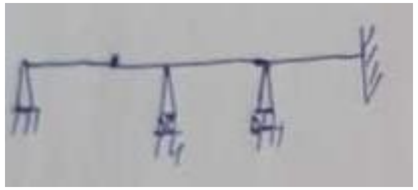
Examples

* Calculate static indeterminacy and comment on stability of structure :

1) 

Dse =R-E =3-3=0 Ds=0

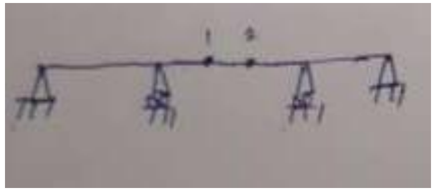
Dsi =3C-r'= 0 (no close loop) Stable and Determinate Structure

2) 

Dse =R-E =7-3=4 Ds=3 (internal hinge is part of internal indeterminacy)

Dsi =3C-r'=3*0-(2-1)=-1 (no close loop)

Stable and Indeterminate Structure

3) 

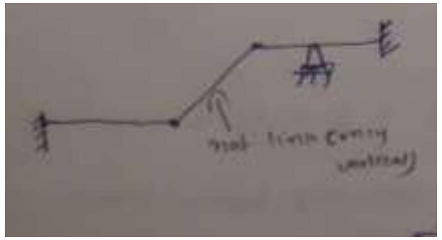
Dse=R-E=6-3=3 Ds=1

Dsi=3C-r'=3*0-(2*(2-1))=-2 (no close loop)

Stable and Indeterminate Structure

Examples

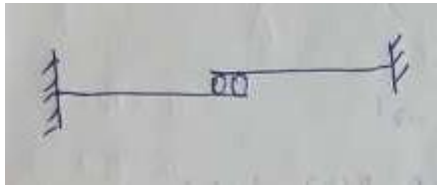
* Calculate static indeterminacy and comment on stability of structure :

4) 

Dse =R-E =7-3=4 Ds=2

Dsi =3C-r'=3*0-2*(2-1)=-2 (no close loop, two hinges & not link,link is only vertical)

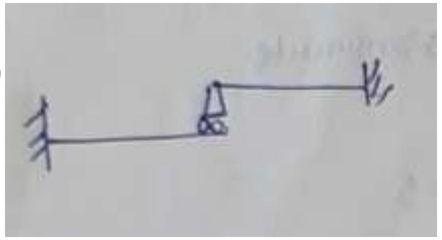
Stable and Indeterminate Structure

5) 

Dse =R-E =6-3=3 Ds=2 (Guided roller)

Dsi =3C-r'=3*0-(2-1)=-1 (no close loop and one releases)

Stable and Indeterminate Structure

6) 

Dse=R-E=6-3=3 Ds=1

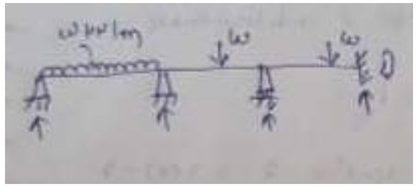
Dsi=3C-r'=3*0-2=-2 (no close loop and two releases)

Stable and Indeterminate Structure

Examples

* Calculate static indeterminacy and comment on stability of structure :

7)

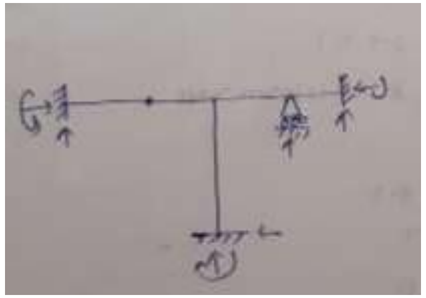


Dse =R-E =5-2=3(no axial load) Ds=3

Dsi =3C-r'=0 (no close loop)

Stable and Indeterminate Structure

8)

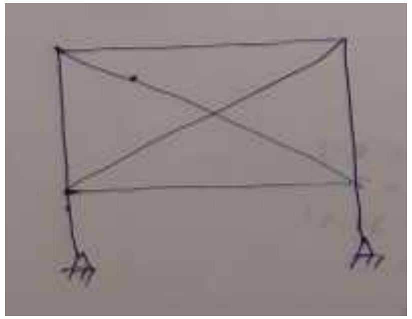


Dse =R-E =10-3=7 Ds=6

Dsi =3C-r'=3*0-(2-1)=-1 (no close loop)

Stable and Indeterminate Structure

9)



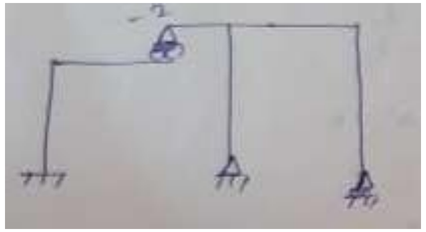
Dse=R-E=4-3=1 Ds=12

Dsi=3C-r'=3*4-(2-1)=11 (4 close loop and one hinge)

Stable and Indeterminate Structure

Examples

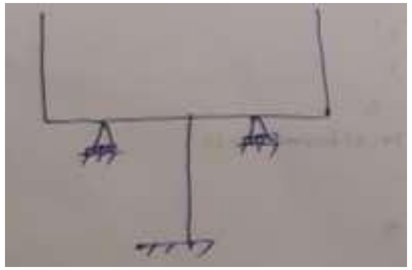
* Calculate static indeterminacy and comment on stability of structure :

10) 

Dse =R-E =3-3=3 Ds=1

Dsi =3C-r'=-2 (no close loop and two releases)

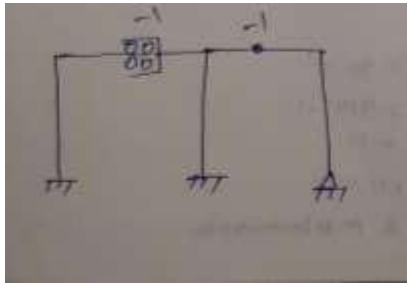
Stable and Indeterminate Structure

11) 

Dse =R-E =5-3=2 Ds=2

Dsi =3C-r'=3*0-0=0 (no close loop)

Stable and Indeterminate Structure

12) 

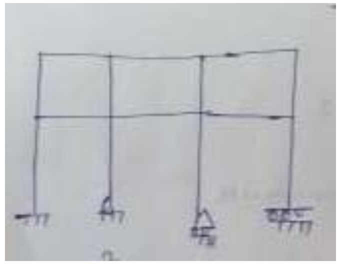
Dse=R-E=8-3=5 Ds=3

Dsi=3C-r'=3*0-2=-2 (0 close loop and two releases)

Stable and Indeterminate Structure

Examples

* Calculate static indeterminacy and comment on stability of structure :

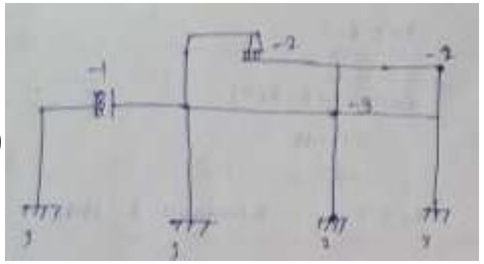
13) 

Dse =R-E =8-3=5

Ds=14

Dsi =3C-r'=3*3-0=9 (3close loop and no releases because guided roller is support not internal joint)

Stable and Indeterminate Structure

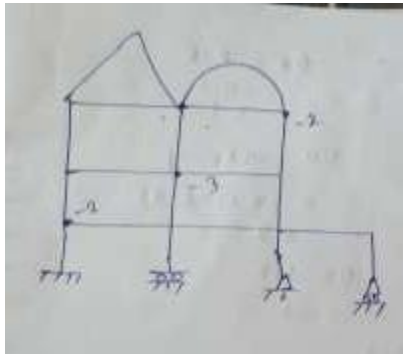
14) 

Dse =R-E =11-3=8

Ds=7

Dsi =3C-r'=3*2-7=-1 (two close loop and 7 releases)

Stable and Indeterminate Structure

15) 

Dse=R-E=8-3=5

Ds=16

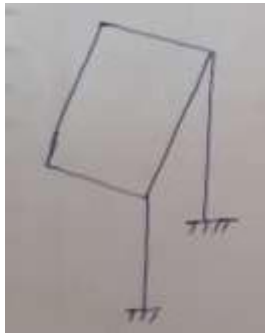
Dsi=3C-r'=3*6-7=11 (6 close loop and 7 releases)

Stable and Indeterminate Structure

Examples

* Calculate static indeterminacy and comment on stability of structure :

16)



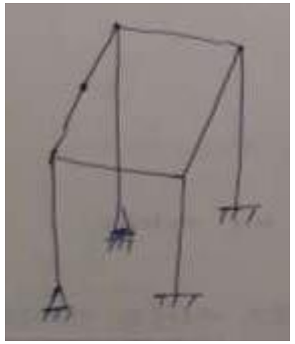
Dse =R-E =12-6=6

Ds=12

Dsi =6C-r'=6*1-0=6 (one close loop and zero releases)

Stable and Indeterminate Structure

17)



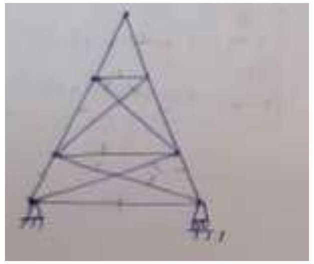
Dse =R-E =16-6=10

Ds=13

Dsi =6C-r'=6*1-3(2-1)=3 (one close loop and 3 releases)

Stable and Indeterminate Structure

18)



Dse=R-E=3-3=0

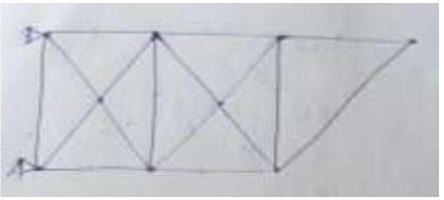
Ds=2

Dsi=m+E-2j=13+3-2*7=2 (13 members and 7 joints)

Stable and Indeterminate Structure

Examples

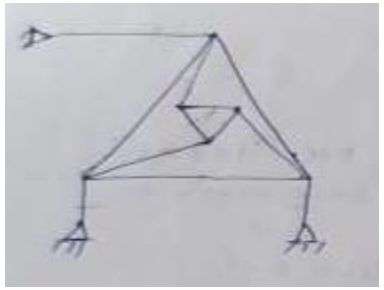
* Calculate static indeterminacy and comment on stability of structure :

19) 

Dse =R-E =4-3=1 Ds=3

Dsi =m+E-2j=17+3-2*9=2 (17 members and 9 joints)

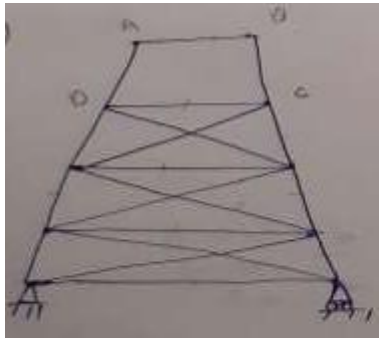
Stable and Indeterminate Structure

20) 

Dse =R-E =6-3=3 Ds=0

Dsi =m+E-2j=12+3-2*9=-3 (12 members and 9 joints)

Stable and Determinate Structure

21) 

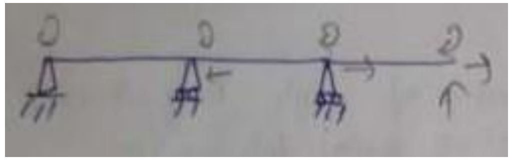
Dse=R-E=3-3=0 Ds=2

Dsi=m+E-2j=19+3-2*10=2 (19 members and 10 joints)

Stable and Indeterminate Structure

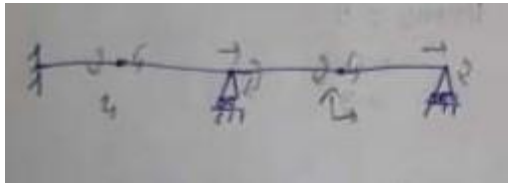
Examples

* Calculate kinematic indeterminacy structure :

1) 

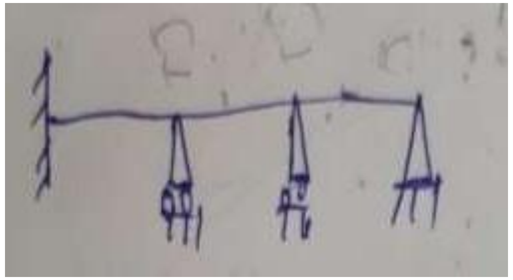
Dk =3j-R+r' =3*4-4=8

Dk(NAD)=Dk-m=8-3=5

2) 

Dk =3j-R+r' =3*5-5+2*(2-1)=12

Dk(NAD)=Dk-m=12-4=8

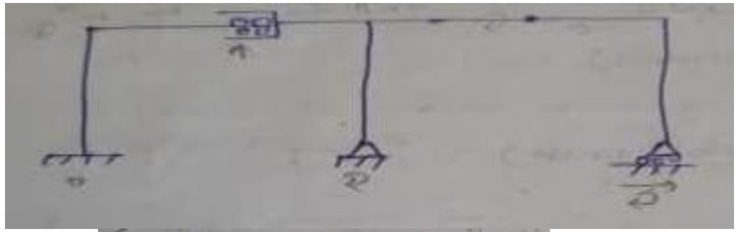
3) 

Dk =3j-R+r' =3*4-7=5

Dk(NAD)=Dk-m=5-3=2

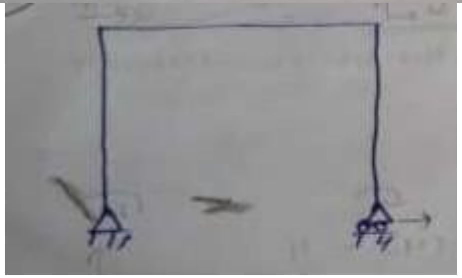
Examples

* Calculate kinematic indeterminacy structure :

4) 

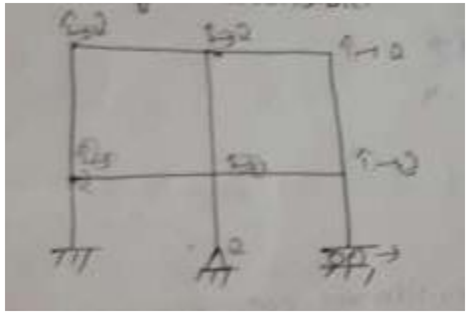
Dk =3j-R+r' =3*8-6+2=20

Dk(NAD)=Dk-m=20-7=13

5) 

Dk =3j-R+r' =3*4-3=9

Dk(NAD)=Dk-m=9-3=6

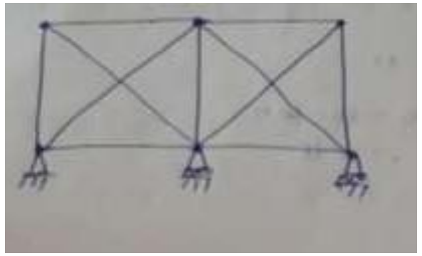
6) 

Dk =3j-R+r' =3*9-7=20

Dk(NAD)=Dk-m=20-10=10

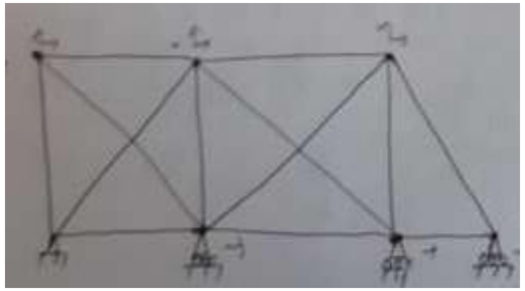
Examples

* Calculate kinematic indeterminacy structure :

7) 

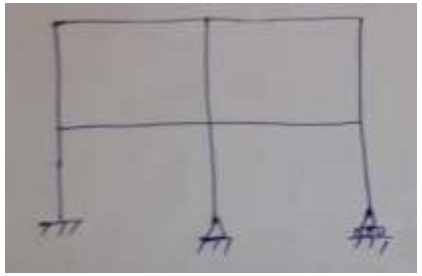
Dk =2j-R =2*6-4=8

Dk(NAD)=0

8) 

Dk =2j-R =2*7-3=9

Dk(NAD)=0

9) 

Dk =3j-R+r′ =3*9-6=21

Dk(NAD)=Dk-m=21-10=11

10) 

Dk =3j-R+r′ =3*3-5=4

Dk(NAD)=Dk-m=4-2=2

Fixed and continuous beam

NEED FOR SUPPORT

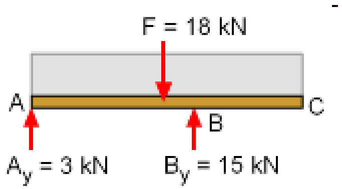
- **THE LOAD CARRYING STRUCTURES NEED SUPPORTS TO AVOID**

  -DEFORMATION

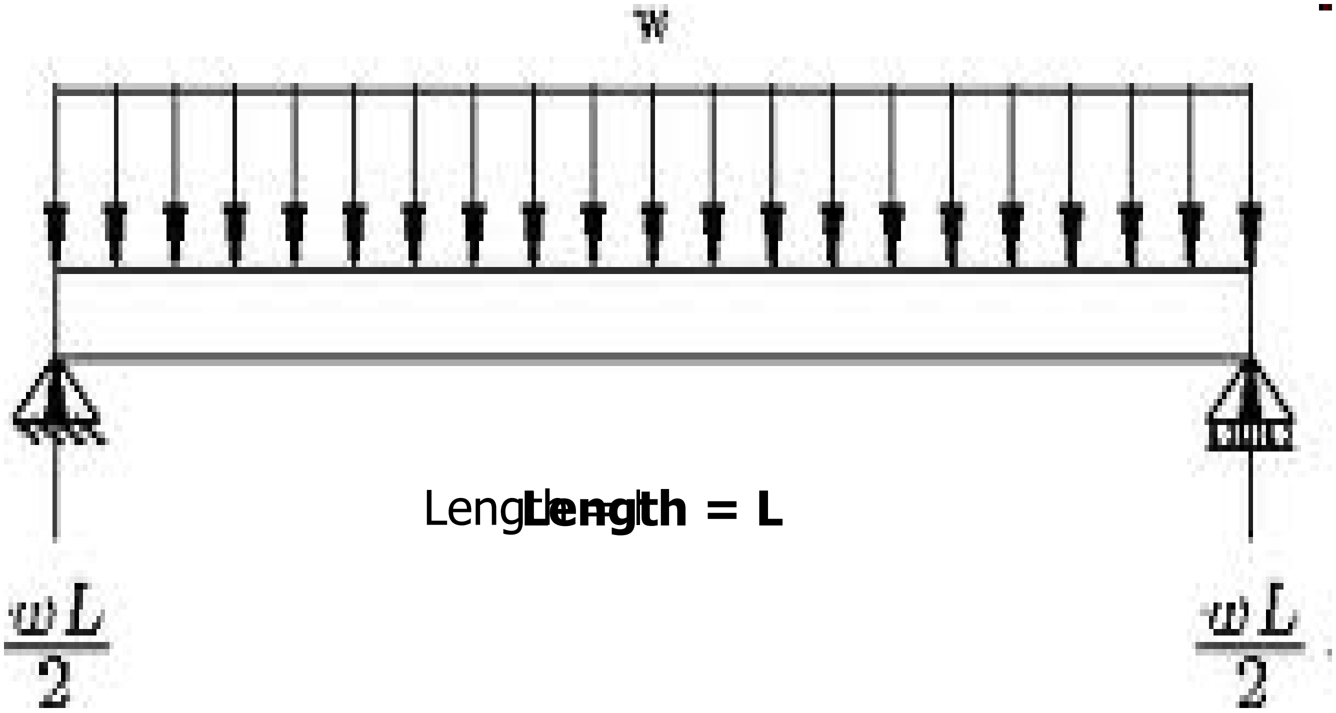
  -BENDING

  -INSTABILITY

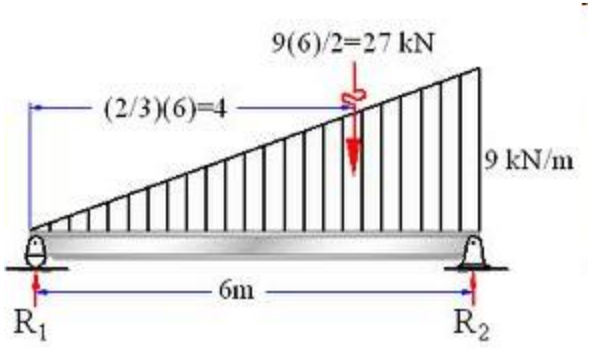
POINT LOAD



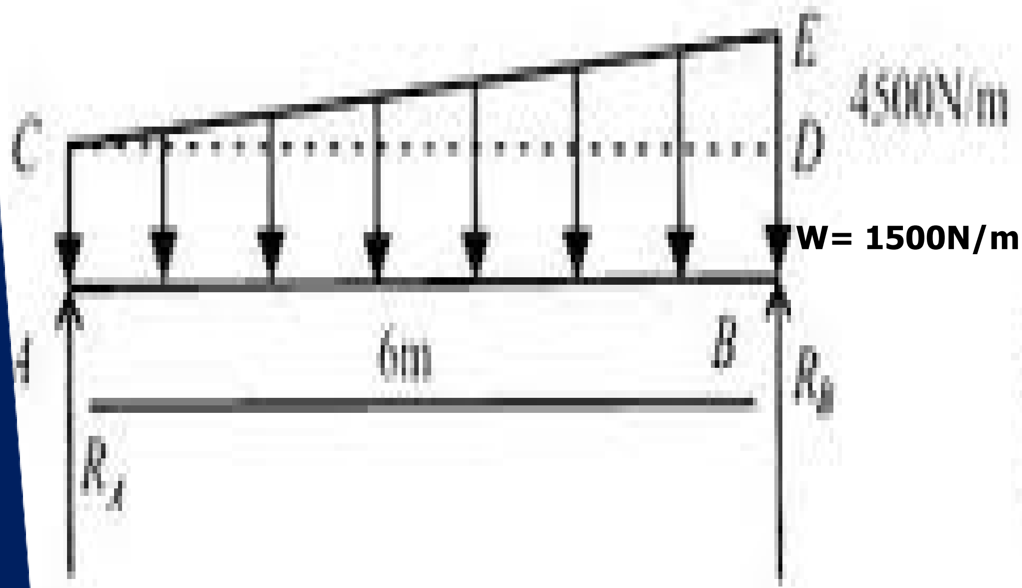
UDL

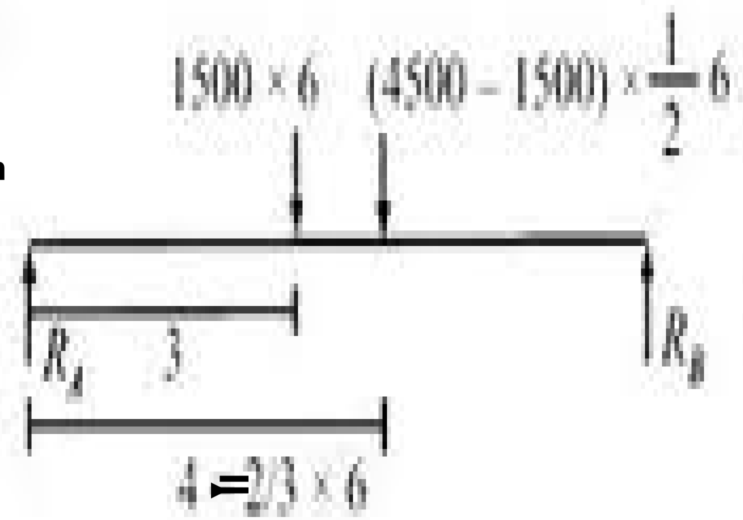


UNIFORMLY VARYING LOAD



COMBINED UDL AND UVL





TYPES OF SUPPORT

SUPPORTS

- SIMPLE
- ROLLER
- HINGED

SIMPLE

- 2 OR MORE VERTICAL SUPPORTS
- JUST PIVOTS
- TAKES ONLY VERTICAL LOADS

ROLLER

- 2 (USUALLY ONE) ROLLER SUPPORTS
- SUPPORTS ALLOW FREE EXPANSION
- TAKES VERTICAL LOADS NORMAL TO ROLLER PLANE

HINGED

- 2 (USUALLY ONE) HINGED SUPPORTS
- SUPPORTS TAKE VERTICAL AND HORI…LOAD
- USUALLY DESIGNED WITH A ROLLER SUPPORT FOR FREE EXPANSION OF ONE END
- VERTICAL AND HORI… LOADS DETERMINE REACTION AND LINE

Types of Support

- In order for loaded parts to remain in equilibrium, the balancing forces are the reaction forces at the supports
- Most real life products have support geometries which differ from the idealized case
- Designer must select the conservative case

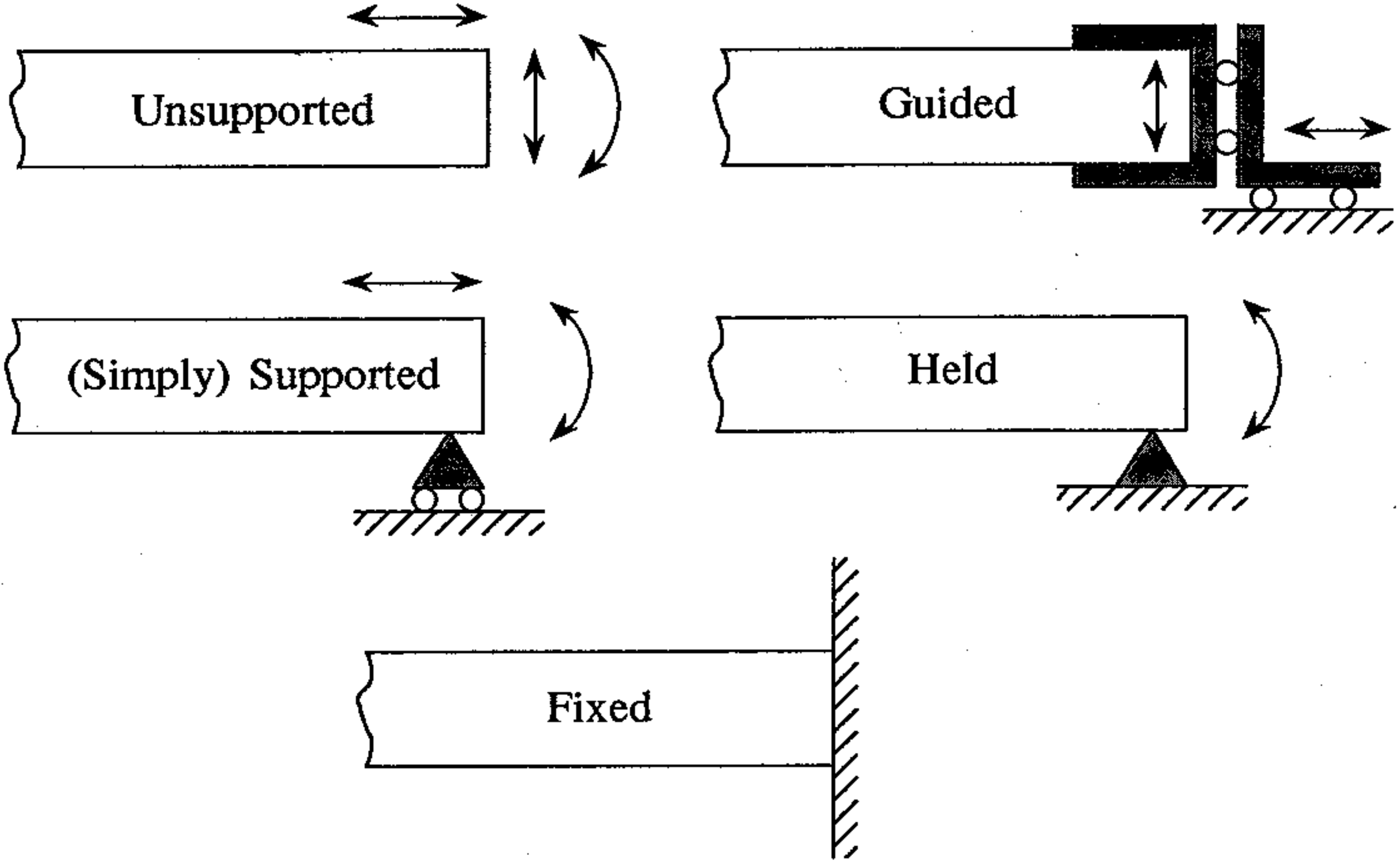
Types of Support

- Guided is support at the end of the beams that prevent rotation, but permits longitudinal and transverse displacement
- Free or unsupported is when the beam is totally free to rotate in any direction
- Held is support at the end of the beam that prevents longitudinal and transverse displacement but permits rotation

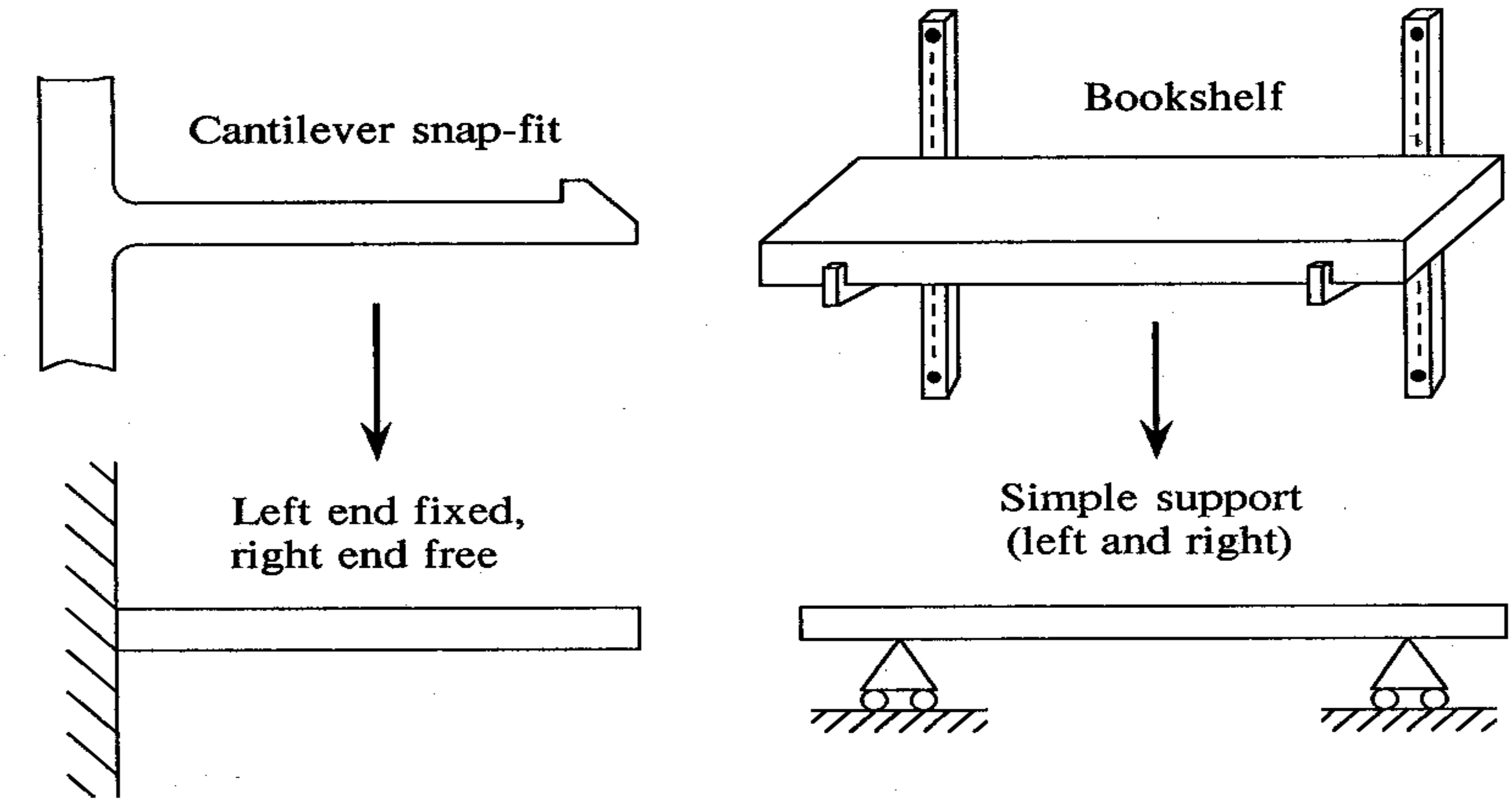
Types of Support

- Simply Supported is support at the end of the beam that prevents transverse displacement, but permits rotation and longitudinal displacement
- Fixed is support at the ends of the beam that prevents rotation and transverse displacement, but permits longitudinal displacement

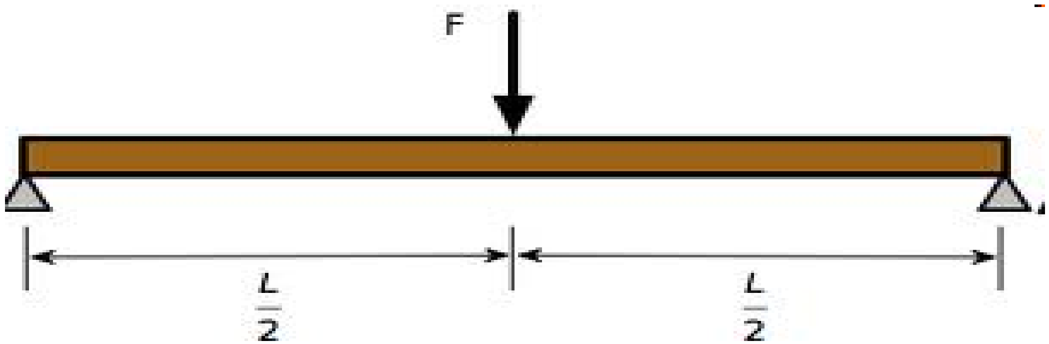
Idealized Supports



Idealized Supports



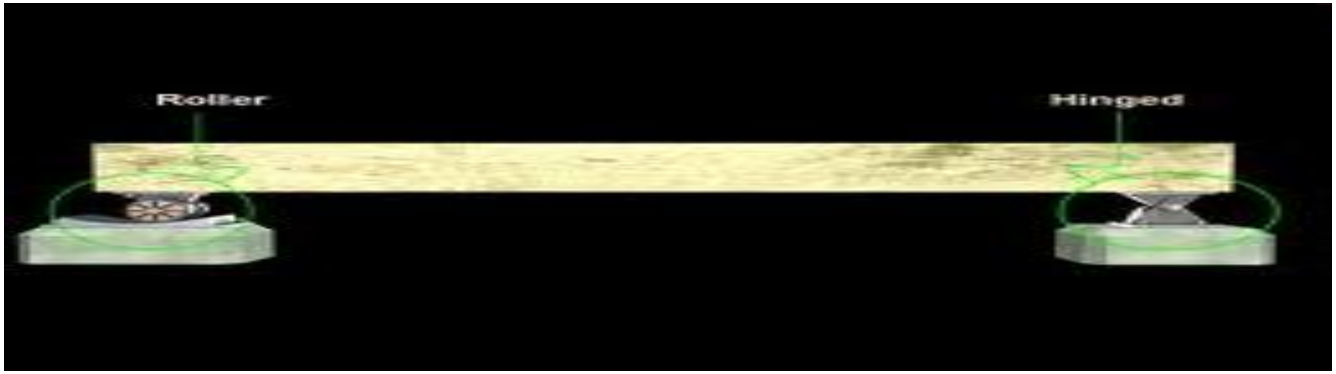
SIMPLE SUPPORT

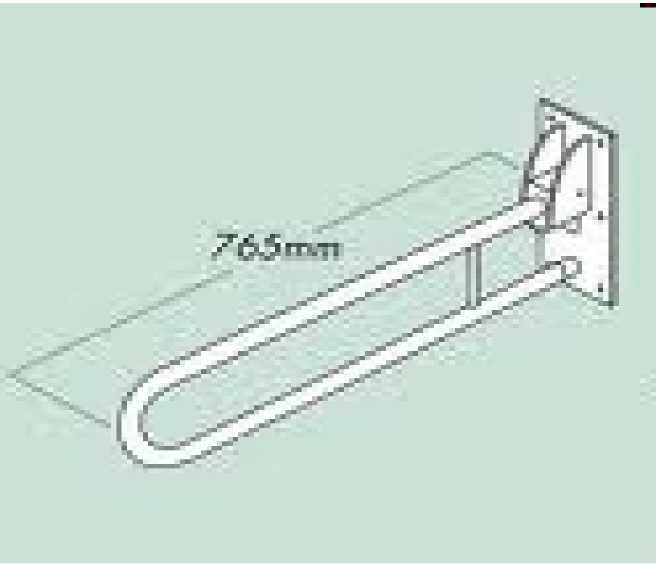


ROLLER SUPPORT

LOCATION OF ROLLER BEARING TO SUPPORT JET

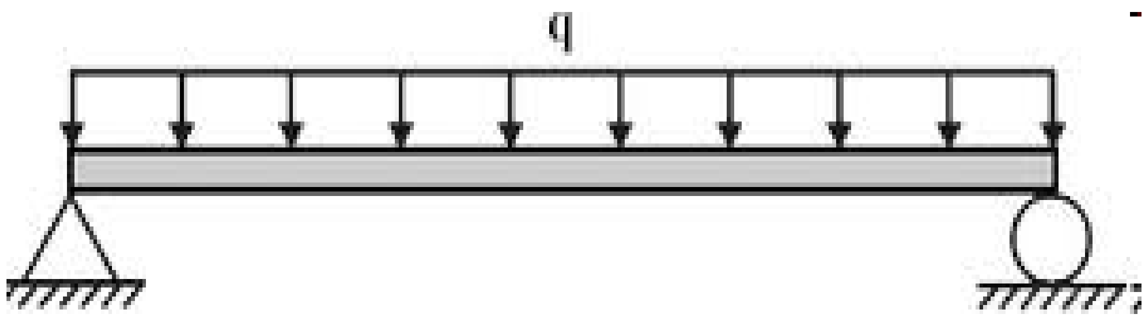
HINGED SUPPORT





COMBINED SUPPORT

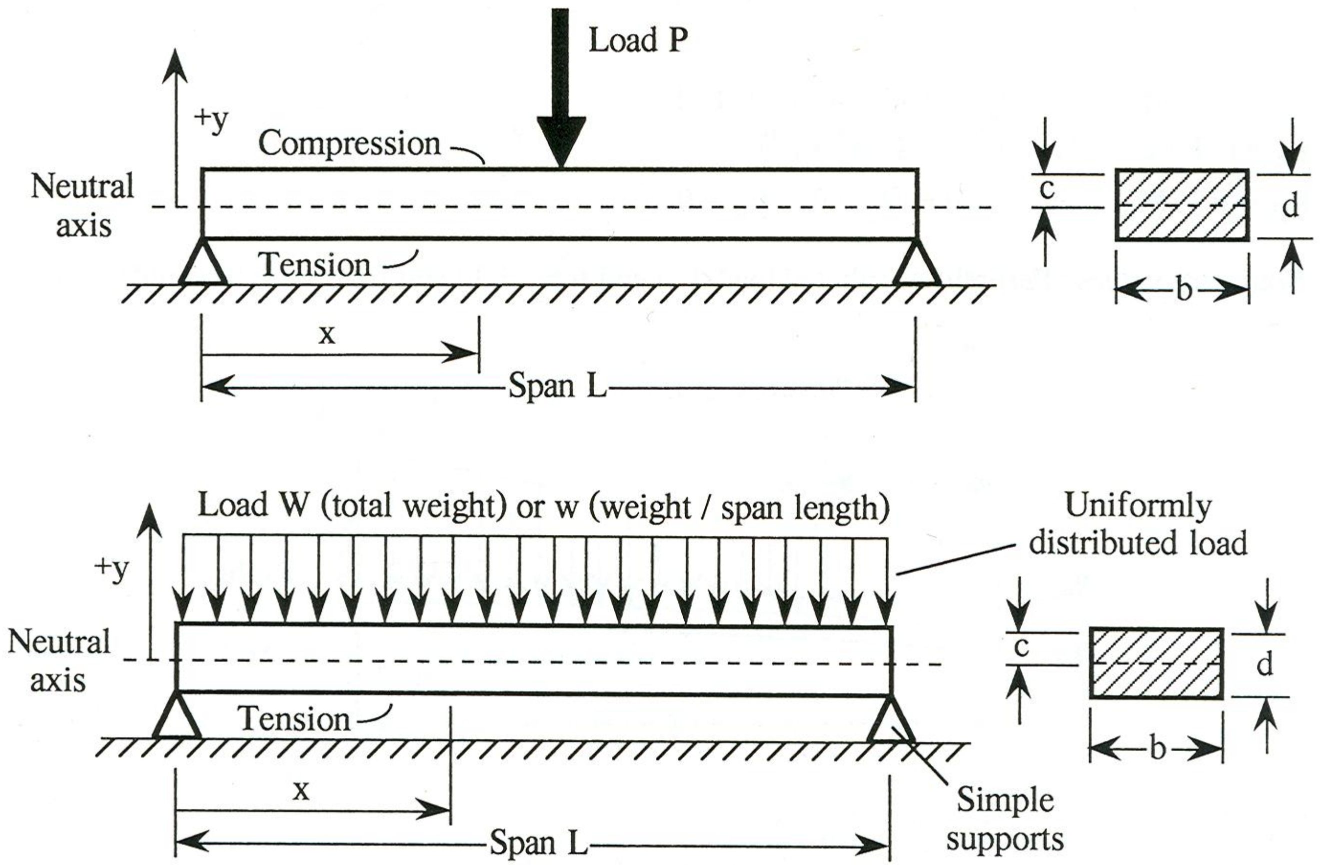
DISTRITIBUTED LOAD = w



HINGED SUPPORT

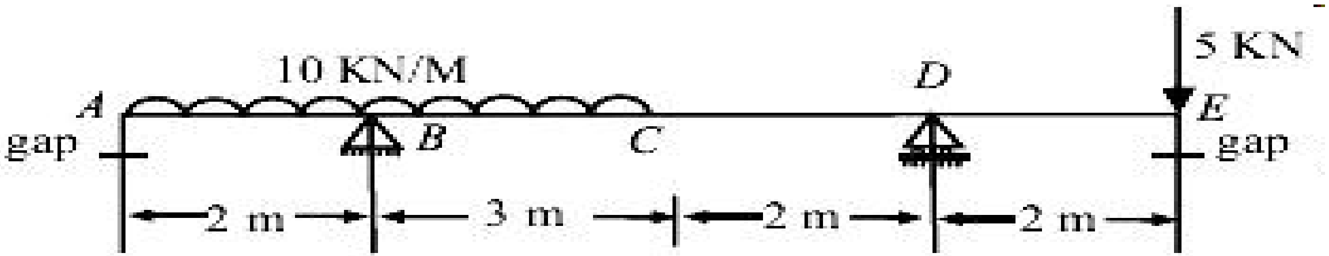
ROLLER SUPPORT

Concentrated and Distributed Loads



Representation of simply supported rectangular beams with concentrated and uniformly distributed loads.

Calculate the support reactions



•

Solution:

First change UDL in to point load.

Resolved all the forces in horizontal and vertical direction. Since roller at B (only one vertical reaction) and hinged at point B (one vertical and one horizontal reaction).

Let reaction at hinged i.e., point B is RBH and RBV, and reaction at roller support i.e. point D is RDV Let ΣH & ΣV is the sum of horizontal and vertical component of the forces ,The supported beam is in equilibrium, hence

$\Sigma H = \Sigma V = 0$
$RH = RBH = 0$
$RBH = 0$ ...(i)
$\Sigma V = RBV - 50 - 5 - RDV = 0$
$RBV + RDV = 55$ ...(ii)

Taking moment about point B

$50 \times 0.5 - RBV \times 0 - RDV \times 5 + 5 \times 7 = 0$
$RDV = 12$ KN ....ANS

Putting the value of RBV in equation (ii)

$RBV = 43$KN ....ANS

Hence RBH = 0, RDV = 12KN, RBV = 43KN

Types of loads

- *Concentrated loads* (eg. P_1, P_2, P_3, P_4)
- When a load is spread along the axis of a beam is a *distributed load*. Distributed loads are measured by their *intensity q (force per unit distance)*
- *Uniformly distributed load* has constant intensity q (fig 4-2a)
- A varying load has an intensity q that changes with distance along the axis. *Linearly varying load from* q_1 - q_2 (fig 4-2b)
- Another kind of load is a *couple of moment* M_1 acting on the overhanging beam (fig 4-2c)

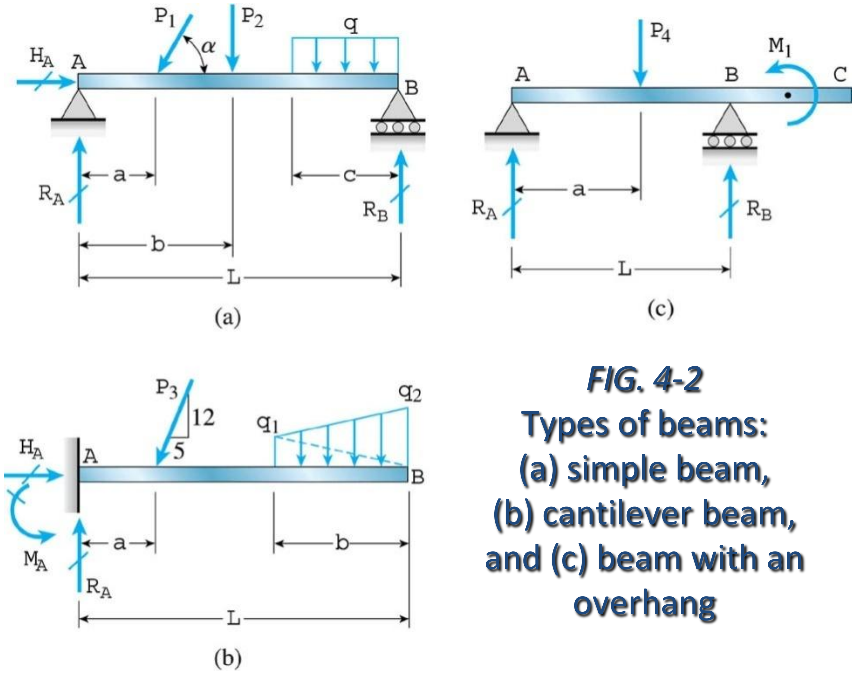
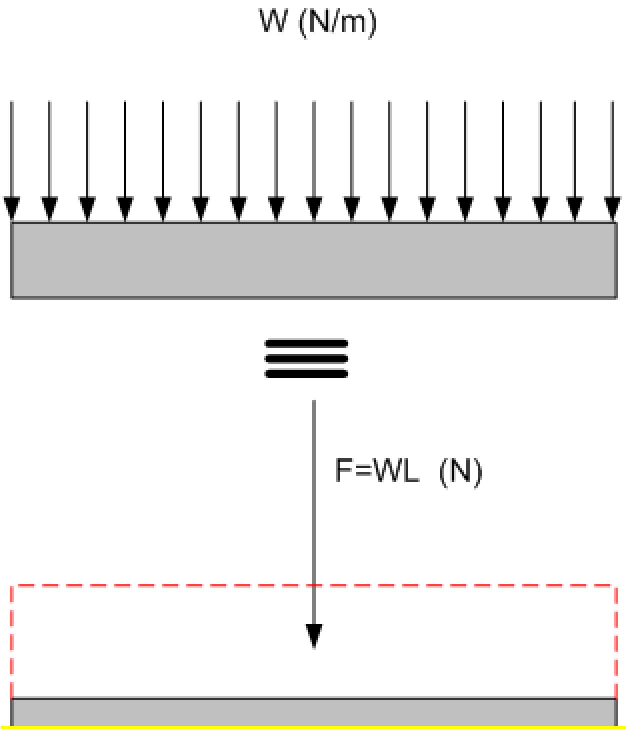


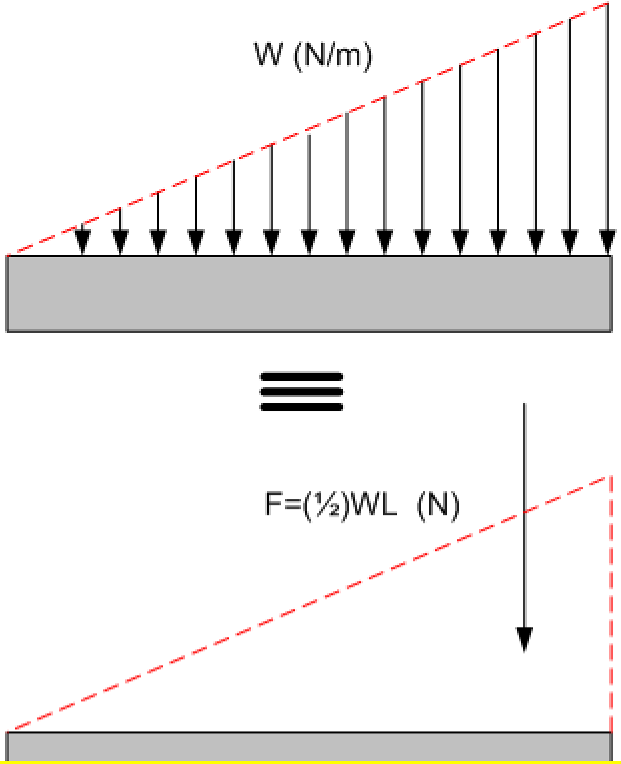
FIG. 4-2
Types of beams:
(a) simple beam,
(b) cantilever beam,
and (c) beam with an overhang

Distributed Load

For calculation purposes, distributed load can be represented as a single load acting on the center point of the distributed area.

Total force = area of distributed load (W : height and L: length)
Point of action: center point of the area





Example

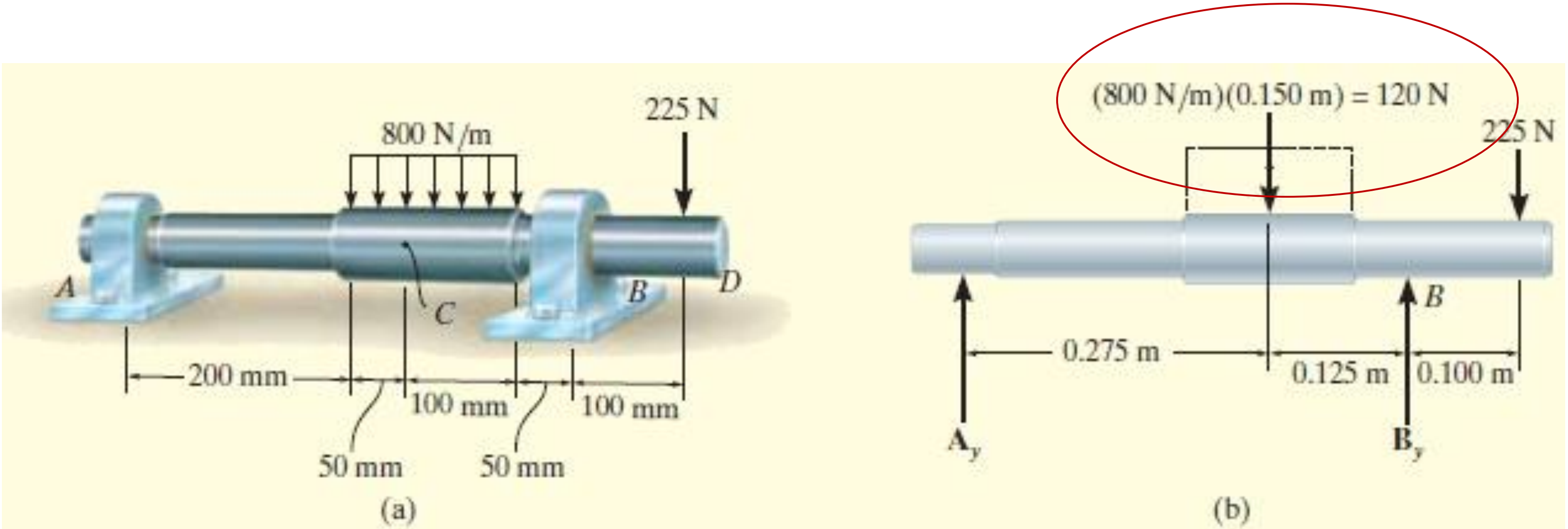


Fig. 1–5

Example

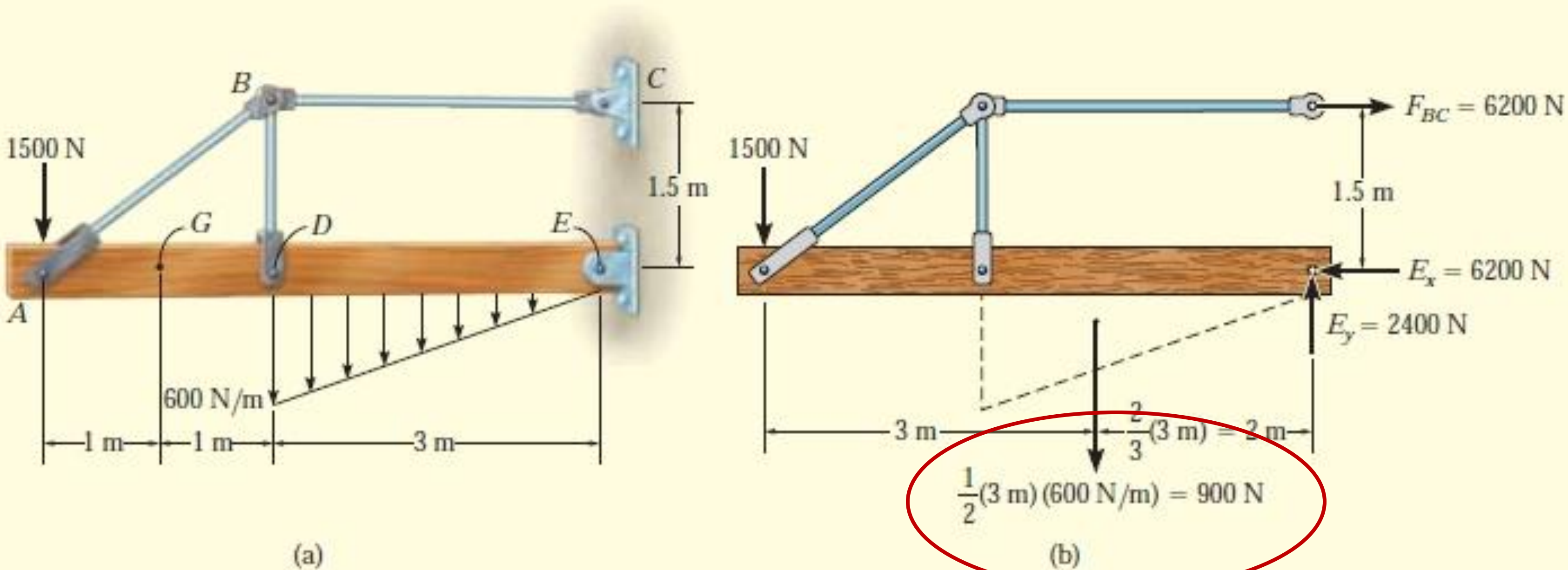
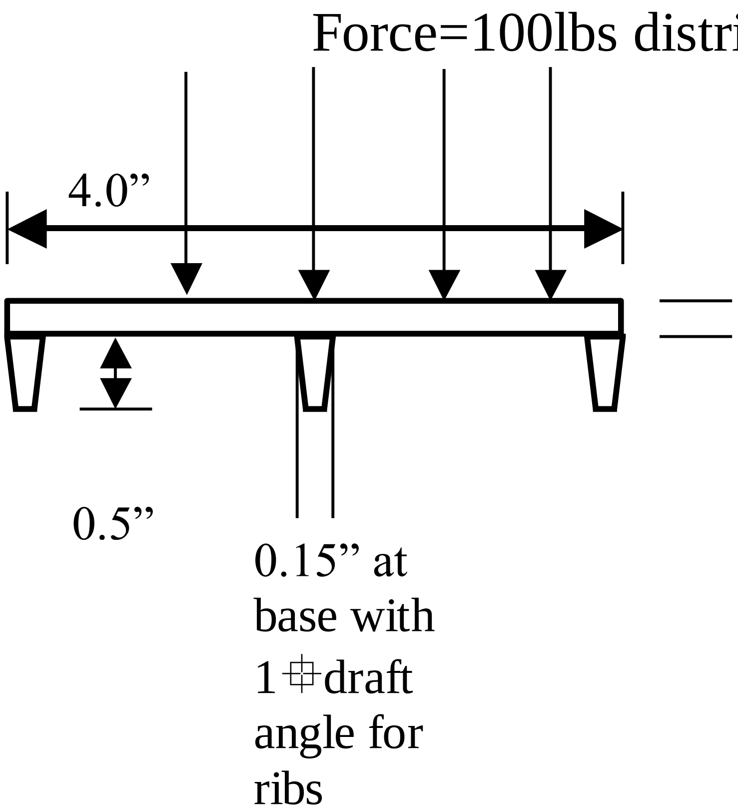


Fig. 1–7

Deflection Calculation



$$E_{pe} = 250{,}000\,psi$$

$$I = 0.0192 in^4$$

$$w = 100 lbs / 4in$$

$$L = 4in$$

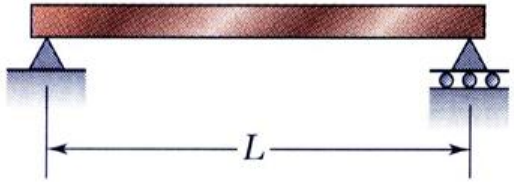
$$y_{\max} = \frac{-5 * w * L^4}{384 * E * I}$$

$$y_{\max} = \frac{-5 * 25\frac{lb}{in} * (4in)^4}{384 * 250{,}000\frac{lb}{in^2} * 0.0192 in^4}$$

$$y_{\max} = -0.0174 inches$$

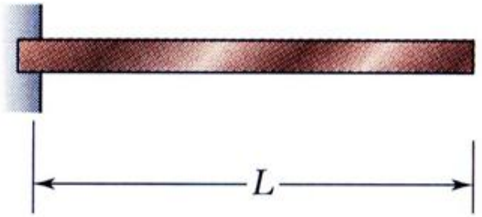
Type of Beams

Statically Determinate



Simply Supported Beam

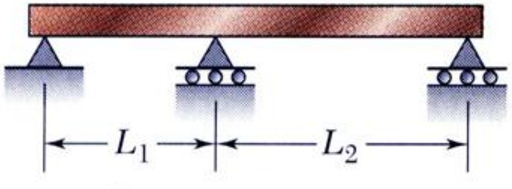
Overhanging Beam



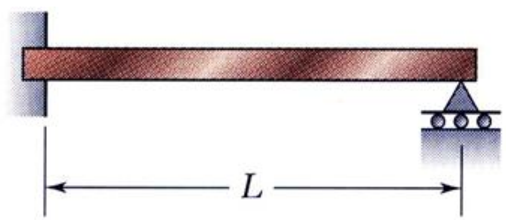
Cantilever Beam

Type of Beams

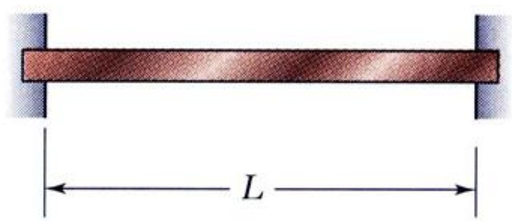
Statically Indeterminate



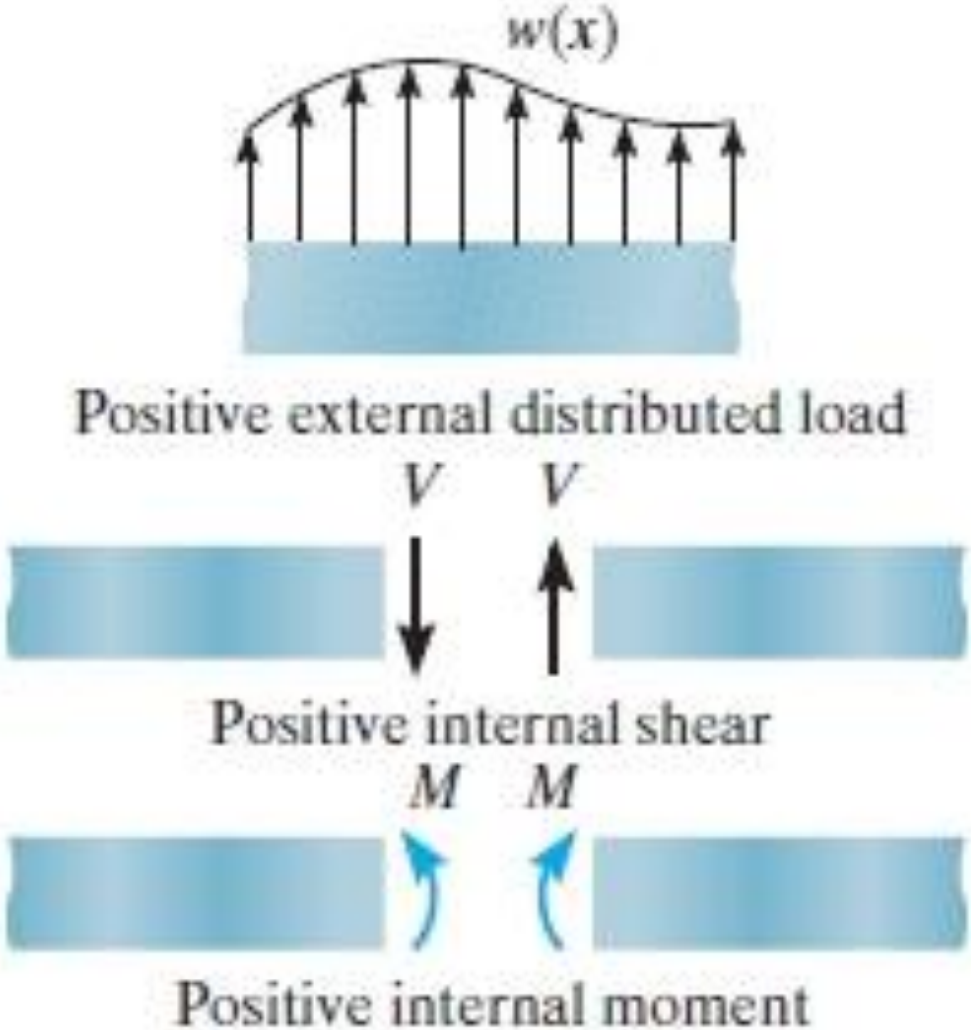
Continuous Beam



Propped Cantilever Beam



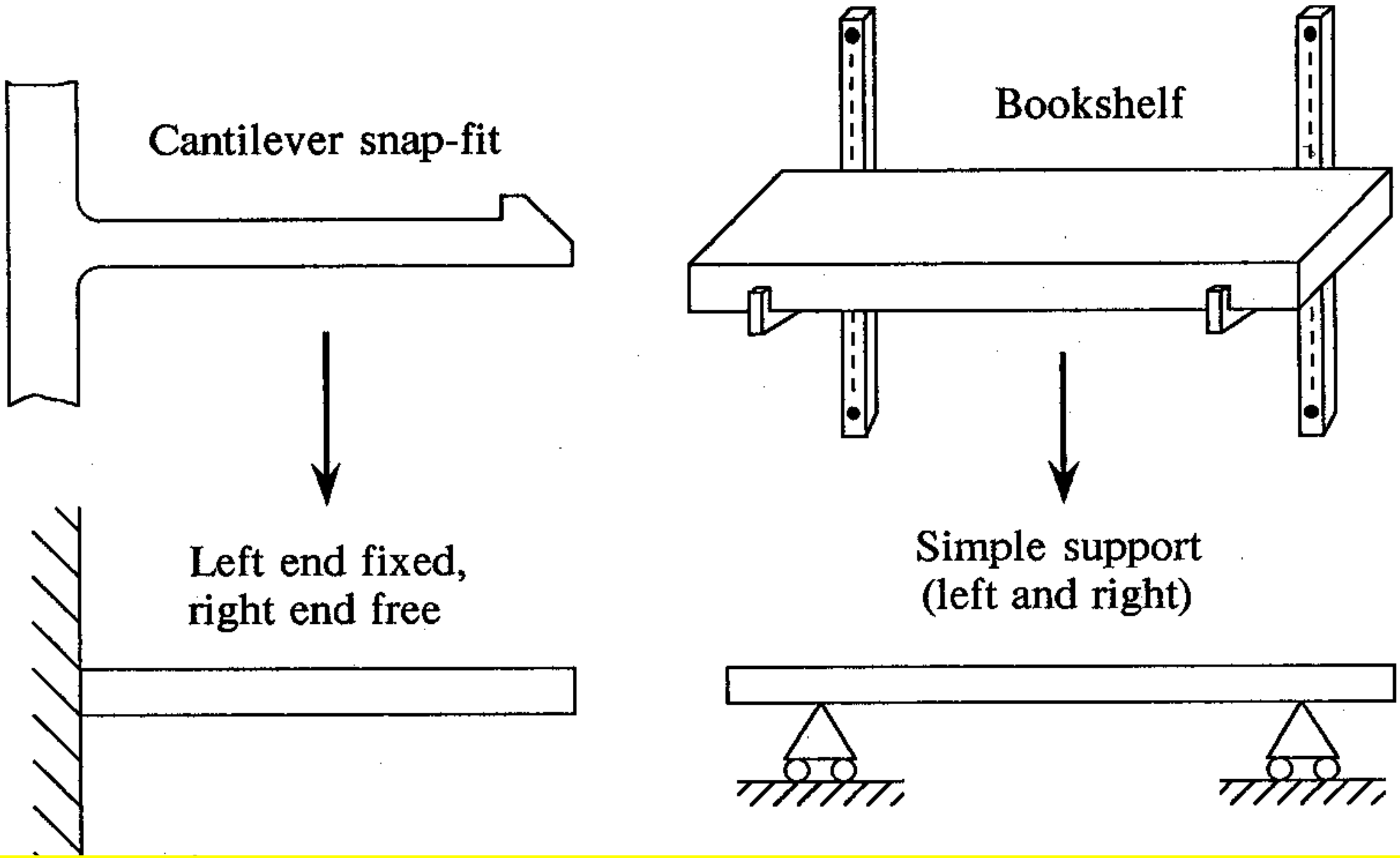
Fixed Beam



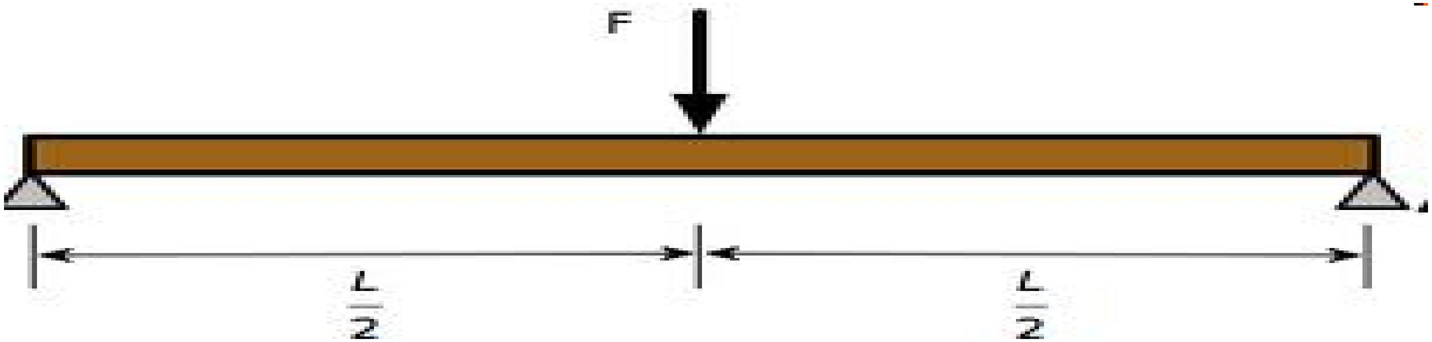
Beam sign convention

PROPPED CANTILEVER BEAM

Idealized Supports



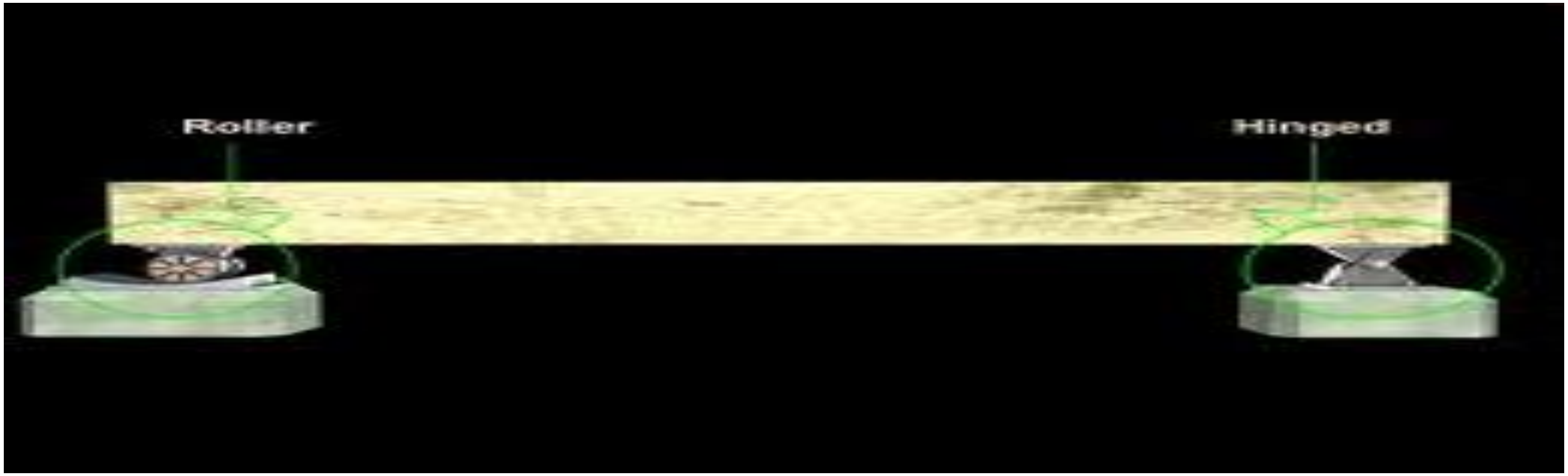
SIMPLE SUPPORT

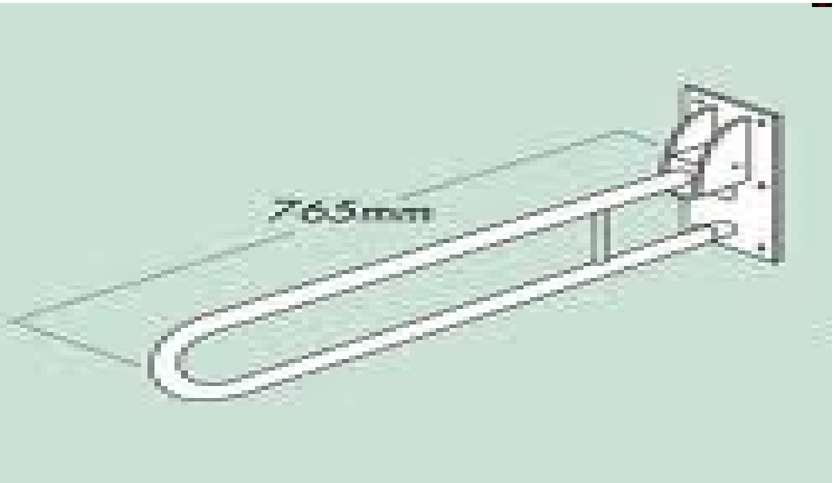


ROLLER SUPPORT

LOCATION OF ROLLER BEARING TO SUPPORT JET ENGINE ROTOR

HINGED SUPPORT





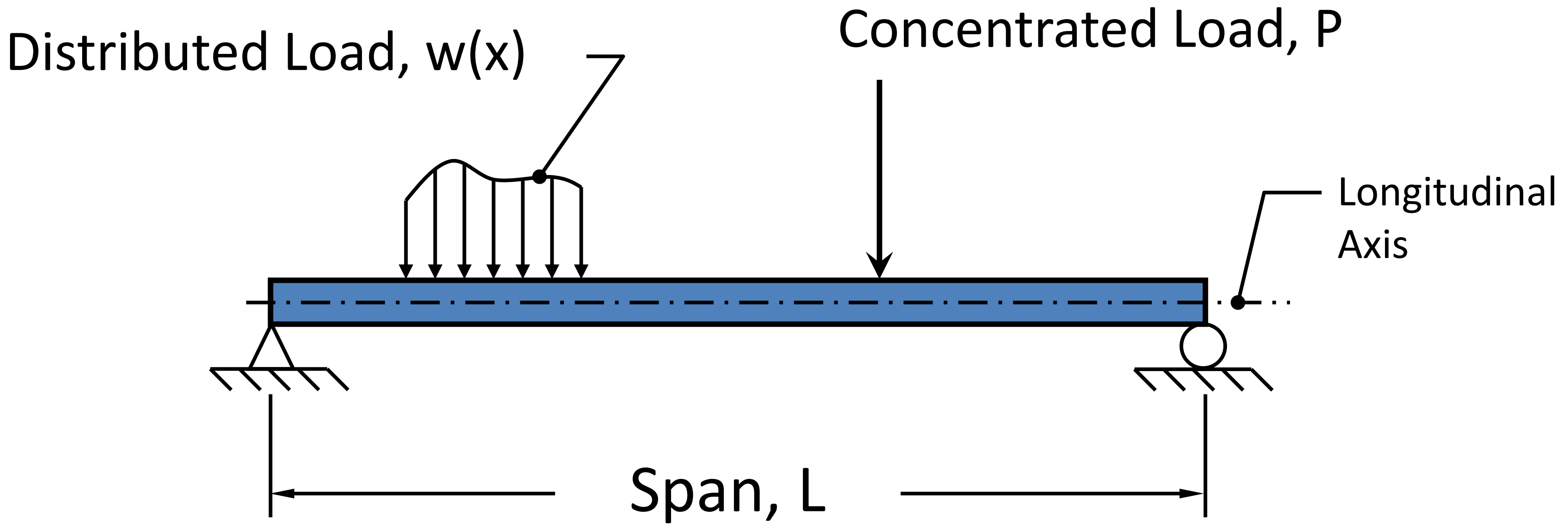
KNEE HINGE

PROPPED CANTILEVER

- A propped cantilever beam AB supports a uniform load q determine the reactions, shear forces, bending moments, slopes, and deflections.

Beams

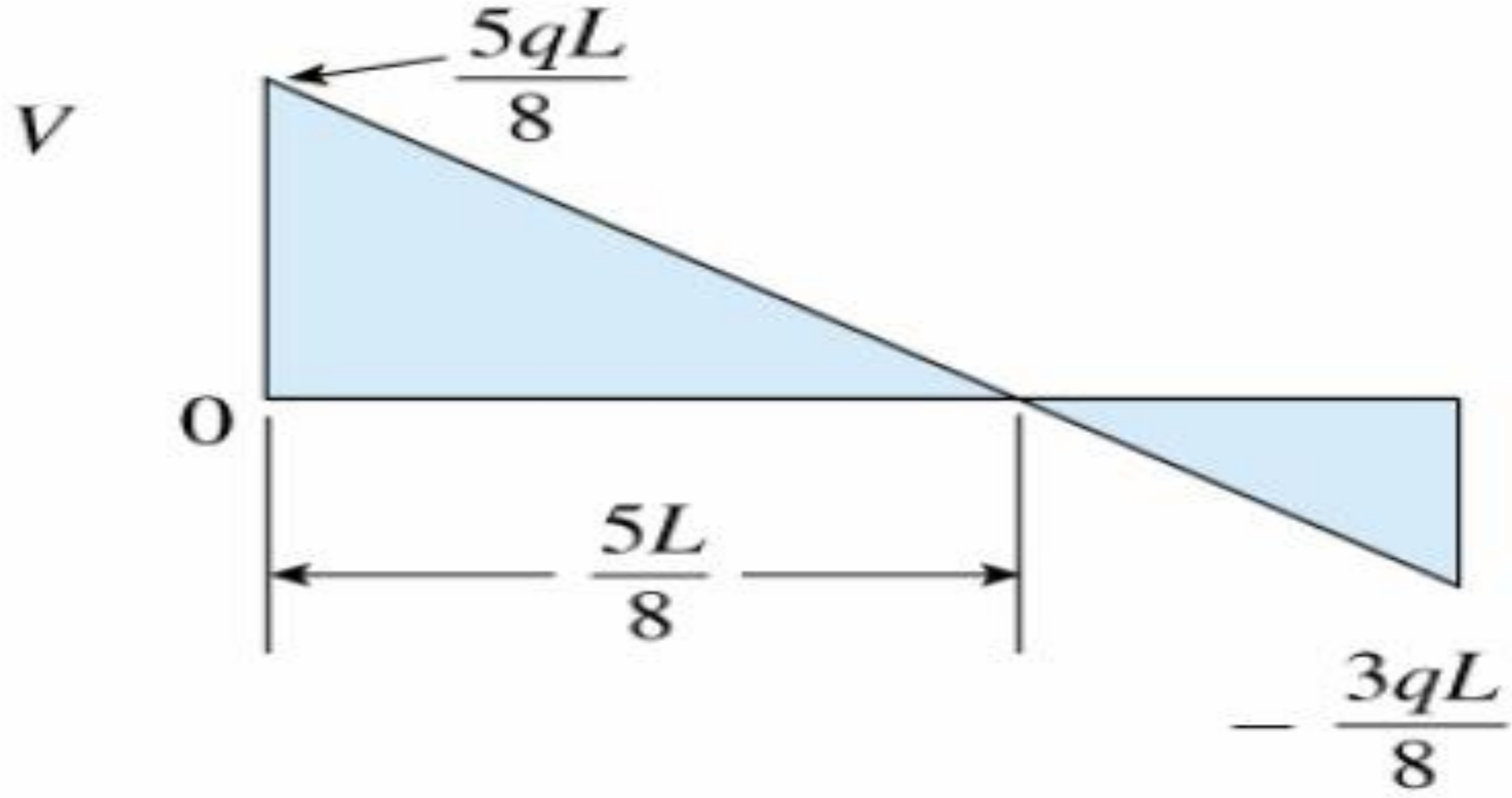
- Members that are slender and support loads applied perpendicular to their longitudinal axis.



Statically Indeterminate Beams

- Can you guess how we find the “extra” reactions?

SFD



LECTURE CONTENTS WITH A BLEND OF NPTEL CONTENTS

https://nptel.ac.in/courses/105/101/105101085/

REFERENCES/BIBLOGRAPHY

(1) Wikipedia
(2) NPTEL
(3) Books: Structural Engineering (by Gupta and Pundit)
(4) Books: B.C. Punmia
(5) Books: R.K. Bansal
(6) Books: G.K. Grover

JECRC Foundation

JAIPUR ENGINEERING COLLEGE
AND RESEARCH CENTRE

Thank you!