# CIVIL ENGINEERING

## GEOTECHNICAL ENGINEERING

Department of Civil Engineering

# 8.5 Capillarity and Effective stress

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**Soil Moisture:**

Soil water is the water present in the void spaces of soil. Soil moisture is that part of sub surface water which occupies the voids in the soil above the ground water table.

Types of soil water

1. **Gravitational water :** is the water in excess of the moisture that can be retained by the soil
    - It is capable of transmitting hydraulic pressure
        a. Free water (or bulk water)
            - Moves in the pores of the soil under the influence of gravity or the difference in hydraulic pressure head.
                i. Free surface water    ii. Ground water
        a. Capillary water :
            - Water held in the voids of soil due to capillary forces (surface tension)

## 2. Held water :

- Retained in the pores of the soil due to forces of attraction
- It can not move under the influence of gravity

### a. Structural Water:

- It is chemically combined water in the crystalline structure of the soil.
- It cannot be removed by simple oven drying at $105^0$C-$110^0$C
- A temperature of more than $300^0$C is required for removing structural water
- In soil engineering the structural water is considered as an integral part of the soil solid.

### b. Adsorbed water:

- The water held by electrochemical forces existing on the soil surface.
- It imparts plasticity and cohesion to clayey soil
- For coarse grained soils it is negligible or zero
- It is also known as Hygroscopic water.
- The amount of water in an air dried soil is defined as hygroscopic water.

- Since air drying removes capillary water, the remaining water is approximately equal to the adsorbed water.
- Adsorbed water can be removed by oven drying

## 3. Capillarity

**Capillarity rise:** The phenomenon in which water rises above the ground water table against the pull of gravitational force

**Capillary moisture:** The water associated with capillary rise

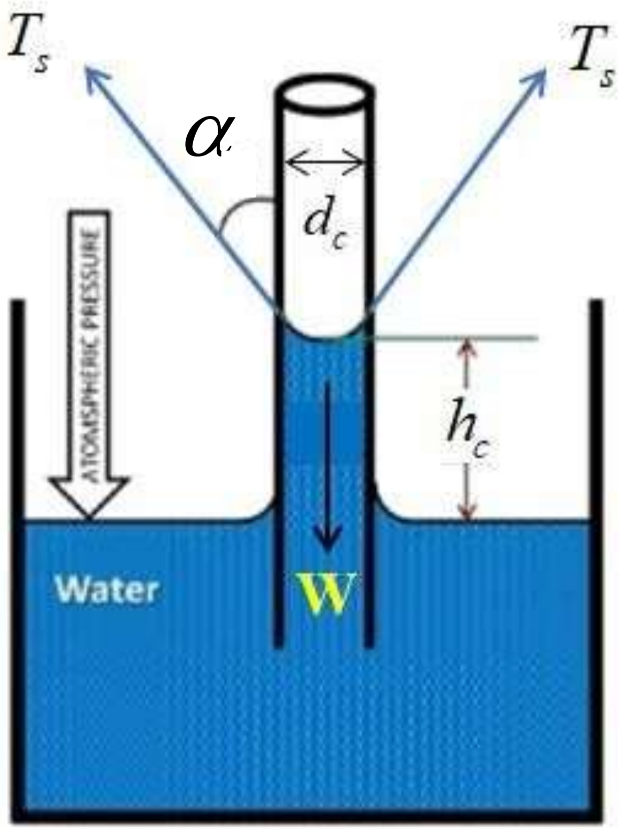
**Capillus:** The size of opening with which the phenomenon of capillarity is connected.

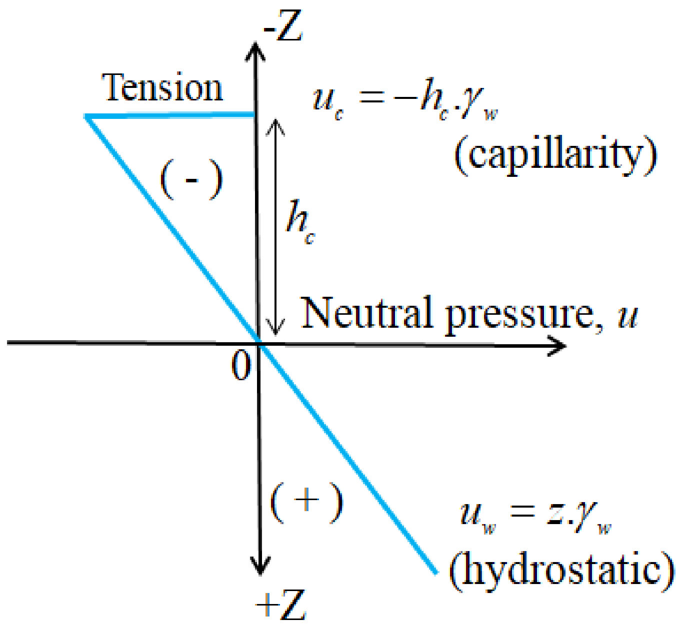
**Capillary fringe:** The height to which capillary water rises is soils above the water table.

- In clays the capillarity rise will be maximum and it may be even more than 30 m

**Meniscus :** The resulting liquid surface is the column

eg. For water and glass, the shape is concave from top. The water surface is lower at the center of the column than at the walls of the tube.




For water and glass, capillary rise depends on i. magnitude of surface ii. unit weight of water and iii. diameter of the capillarity tube

$T_s$ = 73x10$^{-6}$ N/m ; $\gamma_w$ = 9.81 10$^{-6}$ N/mm$^3$

**Contact angle($\alpha$) :** The angle of the surface for pure water meniscus is hemisphere and hence

$\alpha = 0$ Vertical component of surface tension $(T_s)$ is $\pi . d_c . T_s . \cos\alpha$

$\alpha$ = contact angle

$T_s$ = surface tension per unit length

$d_c$ = diameter of the capillary tube

Weight of water column is the capillary tube = $\dfrac{\pi d_c^2}{4} . h_c . \gamma_w$

$$\pi . d_c . T_s . \cos\alpha = \frac{\pi d_c^2}{4} . h_c . \gamma_w \quad ; \quad h_c = \frac{4 T_s \cos\alpha}{\gamma_w . d_c}$$

$$h_c = \frac{30}{d_c} \qquad \alpha = 0$$

$$h_c = \frac{0.3}{d_c} \qquad d_c = cm \quad h_c = cm$$

$d_c$ : Diameter of the glass capillary

$h_c$: Capillary rise of water in the glass tube

- If the temperature increases, surface tension decreases and hence decrease in capillarity
- Warm condition – capillarity decreases
- Cold condition – capillarity increases
- The height of capillarity rise is not dependent on

  i. the orientation of the capillary tube or

  ii. the variations is the shape and size of the tube at levels below the meniscus.

Maximum value of the capillary tension, $u_c = \gamma_w . h_c$

Capillary rise is more in fine grained soils and hence capillary pressure is more in fine grained soil than in coarse grained soils.

## Effective Stress

Karl Terzaghi was first enunciated the "Effective Stress" principle.

$$\text{Total stress, } \sigma = \frac{\text{Total load}}{\text{Crosssectional Area}} = \frac{A.h.\gamma}{A} = h.\gamma$$

Total stress is due to i. self weight of soil and ii. overburden on the soil

The parameters required to compute total stress are

- unit weight of soil
- thickness of soil layer
- position of water table

$$\text{Total stress, } \sigma = \gamma .z$$

$\sigma$ : Vertical geostatic stress

$\gamma$ : Unit weight of soil

$z$ : Depth under consideration

## Neutral pressure or Pore water pressure ($u$):

- It is the stress carried by the pore water.
- It is same is all the directions, also called pore water pressure $u = \gamma_w . z$

  $\gamma_w =$ unit weight of water
- It does not have any measurable influence on void ratio or any other mechanical property of soil such as Shearing Resistance.
- Neutral pressure (Pore pressure) is compressive below water table and tensile above water table.

## Effective stress:

- It is equal to the total vertical reaction (Normal force) transmitted at the points of contact of soil grains divided by the total area, including that occupied by water.

$$\text{Effective stress, } \sigma' = \frac{\Sigma N'}{A}$$

- It is the pressure transmitted from particle to particle through their points of contact through soil mass.

# Actual contact stress, $\sigma' = \frac{\Sigma N'}{A_c}$

$A_c$ : actual contact area

- It is also called "Inter Granular Pressure" or "Contact Stress".
- It has no physical meaning.
- It can only be computed knowing the σ and u.
- It is much smaller than the actual contact stress.
- The increase in effective stress causes the particles to pack more closely, decreases the void ratio.
- The permeability and consolidation depends on void ratio.
- With a change in effective stress the void ratio of the soil changes.
- To some extent permeability of a soil also governed by the effective stress.
- As effective stress increases, the compressibility of soil occurs. This causes settlement of soil structures built on soils

$$s = c + \sigma \tan\phi$$

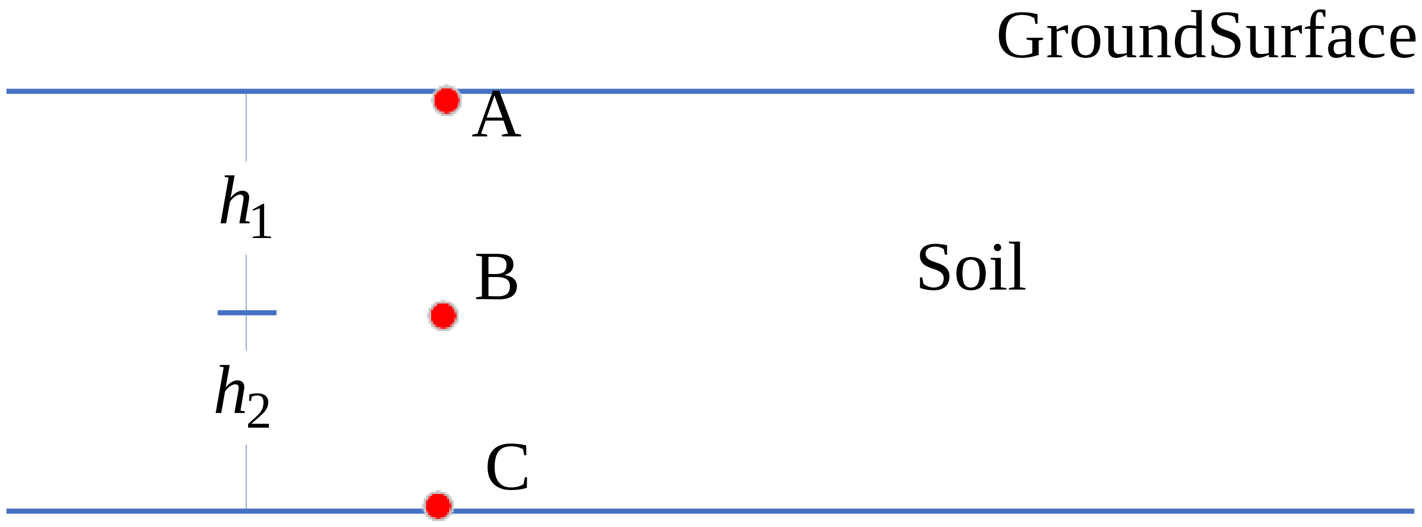
- The increase in effective stress increases the shear resistance of the soil.
- As the effective stress is changed, the shear stress changes.
- The stability of slopes, the earth pressure against retaining walls and the bearing capacity of soil depend upon the Shear strength of the soil and hence the effective stress.

$$\sigma' = \sigma - u = \gamma_{sat}.z - \gamma_w.z = z(\gamma_{sat} - \gamma_w) = \gamma'.z$$

$\gamma'$ = submerged unit weight of soil
$\sigma'$ = effective stress
$\sigma$ = total stress
$u$ = pore water pressure.

- Effective stress is computed with the value of the buoyant or effective unit weight.
- Effective stress is inversely proportional to pore water pressure.
- If the water table increases below the ground surface effective stress decreases or vice versa.
- The equation $\sigma' = \sigma - u$ is not strictly applicable to a partially saturated soil.

## Case 1:




At A, $\sigma = 0$

$u = 0$

$\sigma' = 0$

At B, $\sigma = \gamma . h_1$

$u = 0$

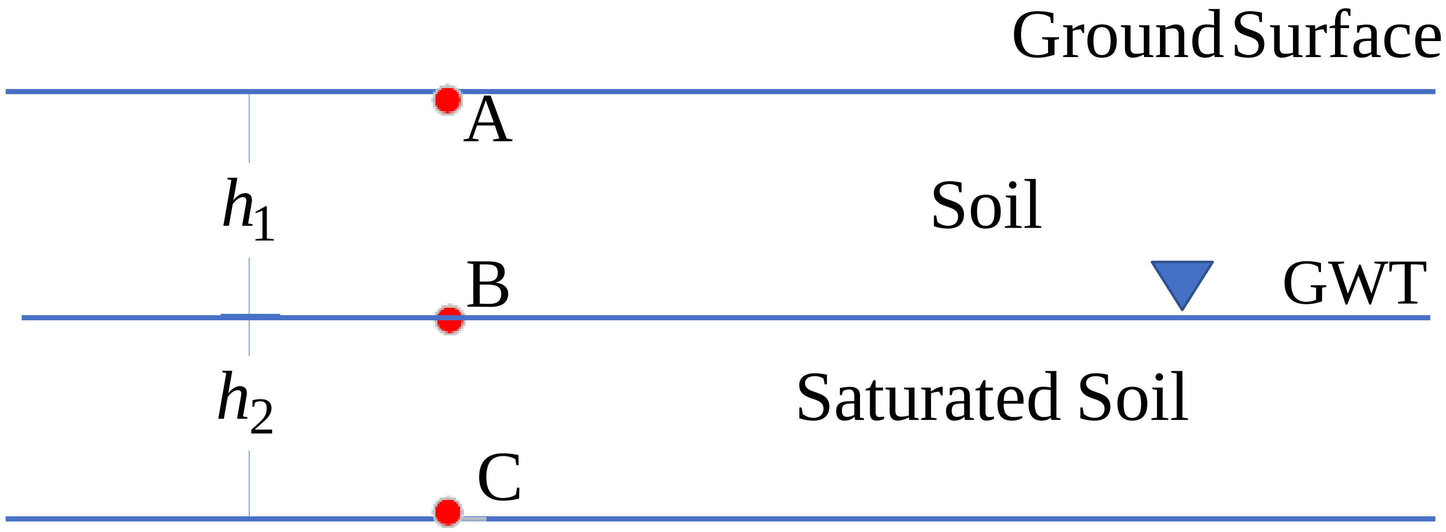
$\sigma' = \gamma . h_1$

At C, $\sigma = \gamma (h_1 + h_2)$

$u = 0$

$\sigma' = \gamma (h_1 + h_2)$

## Case 2:




At A, $\sigma = 0$ $\quad u = 0$

$\sigma' = 0$

At B, $\sigma = \gamma . h_1$

$u = 0$

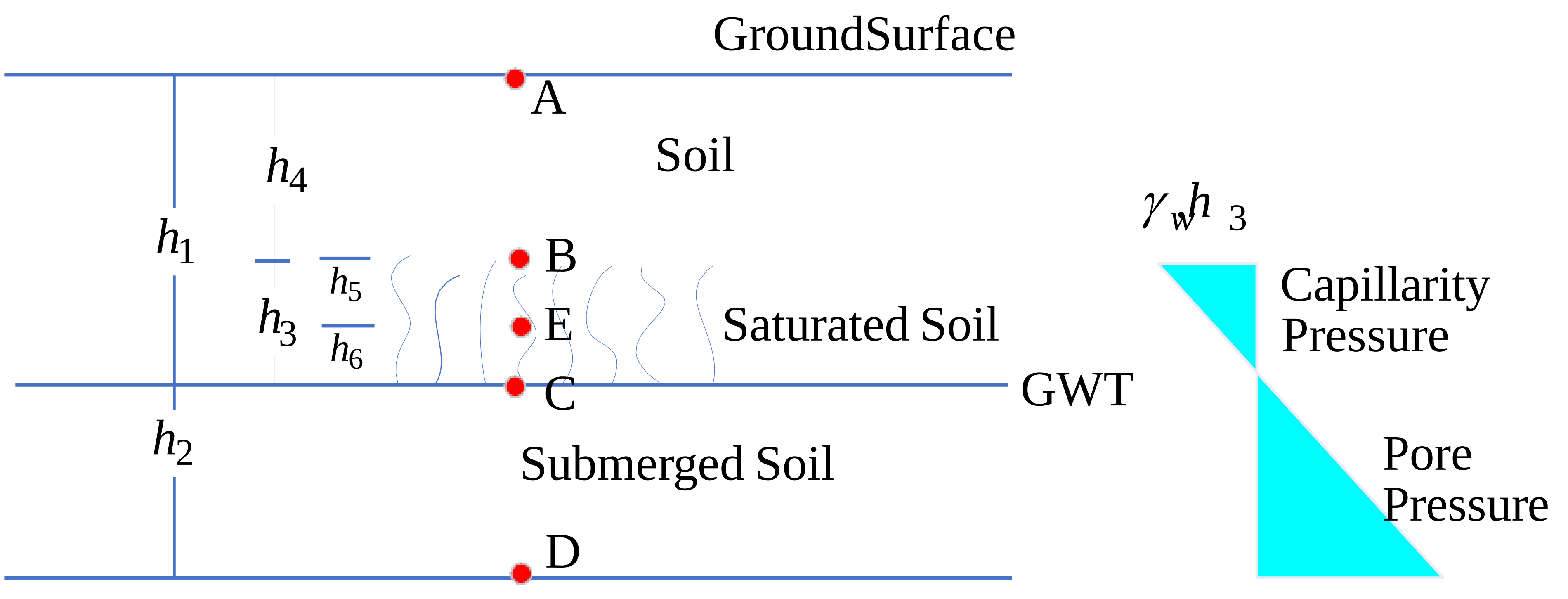
$\sigma' = \gamma . h_1$

At C, $\sigma = \gamma . h_1 + \gamma_{sat} . h_2$

$u = \gamma_w . h_2$

$$\sigma' = \sigma - u = \gamma . h_1 + \gamma_{sat} . h_2 - \gamma_w . h_2 = \gamma . h_1 + (\gamma_{sat} - \gamma_w) . h_2 = \gamma . h_1 + \gamma' . h_2$$

## Case 3:




At A, $\sigma = 0$

$u = 0$

$\sigma' = 0$

At B, $\sigma = \gamma . h_4$

$u = -\gamma_w . h_3$

$\sigma' = \sigma - u = \gamma . h_4 - (-\gamma_w . h_3) = \gamma . h_4 + \gamma_w . h_3$

Increase in effective stress due to capillarity at B $= \gamma_w . h_3$

At C, $\sigma = \gamma . h_4 + \gamma_{sat} . h_3$

$$u = 0$$

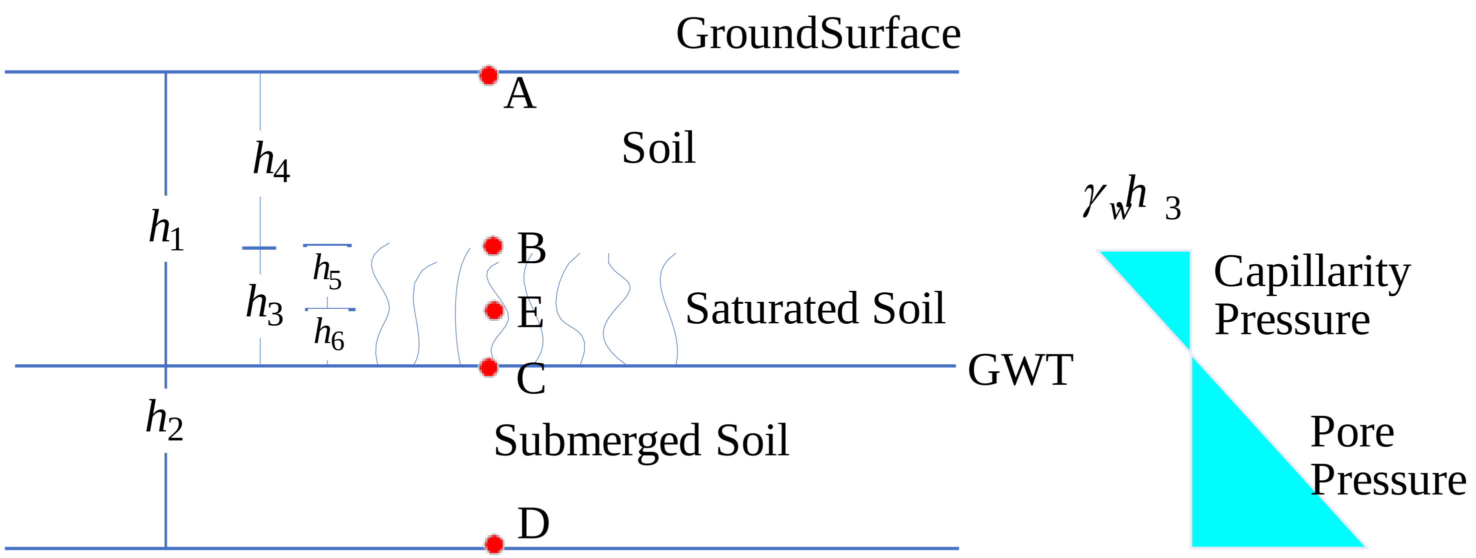
$$\sigma' = \sigma - u = \gamma . h_4 + \gamma_{sat} . h_3$$

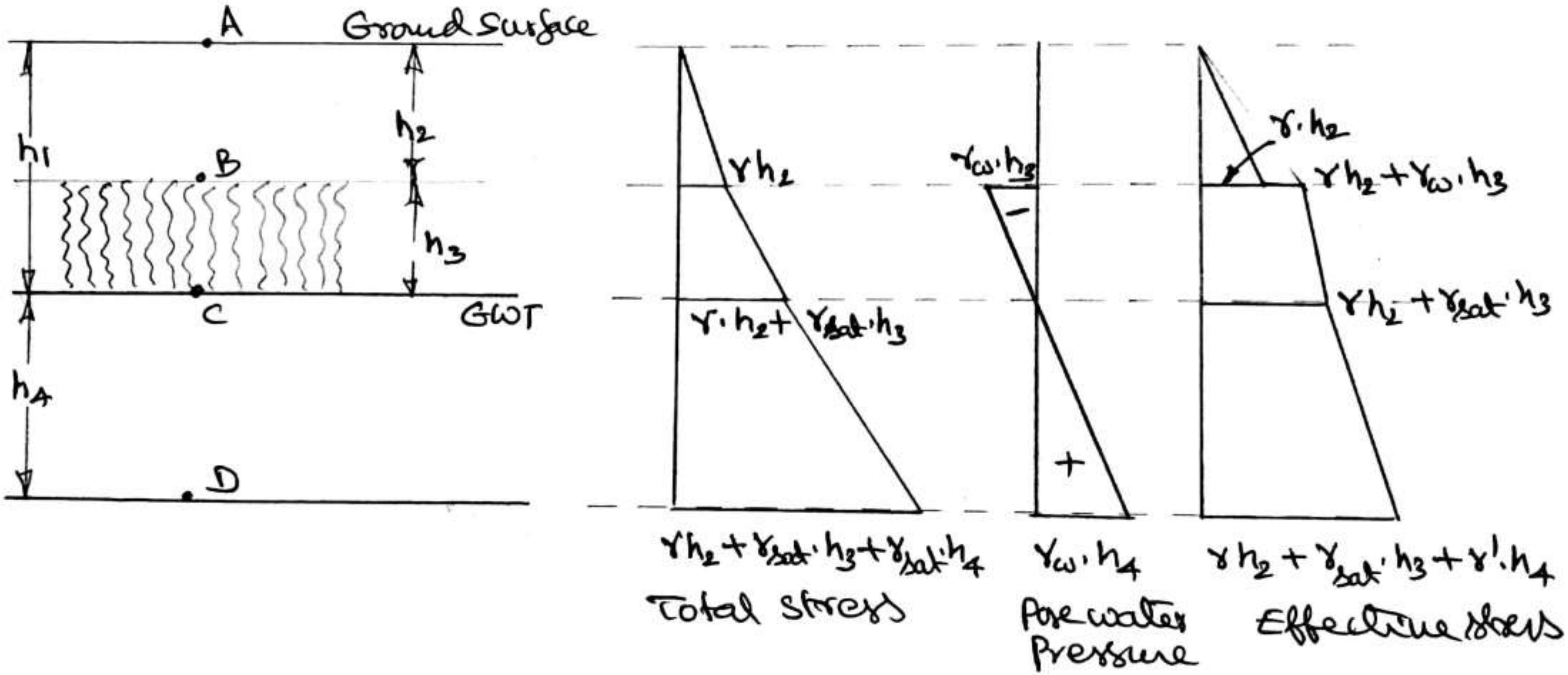
If there is no capillarity fringe, $\sigma' = \sigma - u = \gamma . (h_4 + h_3)$

At D, $\sigma = \gamma . h_4 + \gamma_{sat} . h_3 + \gamma_{sat} . h_2$

$$u = \gamma_w . h_2$$

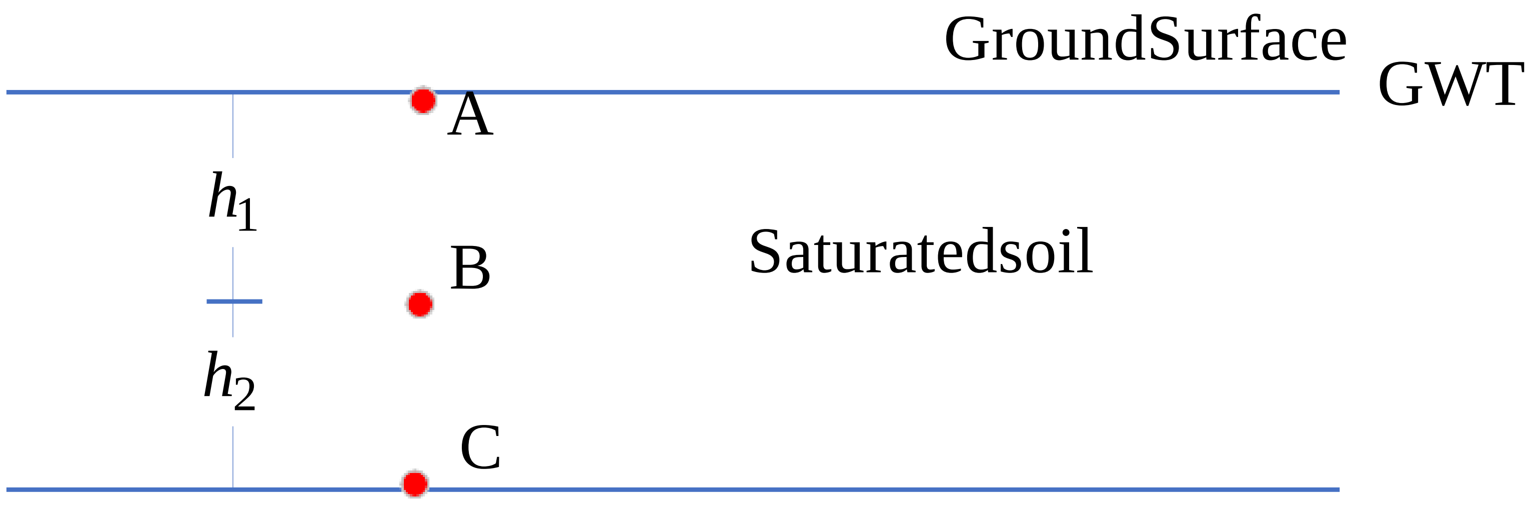
$$\sigma' = \gamma . h_4 + \gamma_{sat} . h_3 + \gamma' . h_2$$





A
Ground Surface
h1
h2
B
h3
C
GWT
h4
D
γh2
γ·h2+γsat·h3
γh2+γsat·h3+γsat·h4
Total stress
γw·h3
−
+
γw·h4
Pore water Pressure
γ·h2
γh2+γw·h3
γh2+γsat·h3
γh2+γsat·h3+γ'·h4
Effective stress

## Case 4:




At A, $\sigma = 0$ $\quad u = 0$

$\sigma' = 0$

At B, $\sigma = \gamma_{sat}.h_1$

$u = \gamma_w.h_1$

$\sigma' = \gamma'.h_1$

At C, $\sigma = \gamma_{sat}.(h_1 + h_2)$

$u = \gamma_w.(h_1 + h_2)$

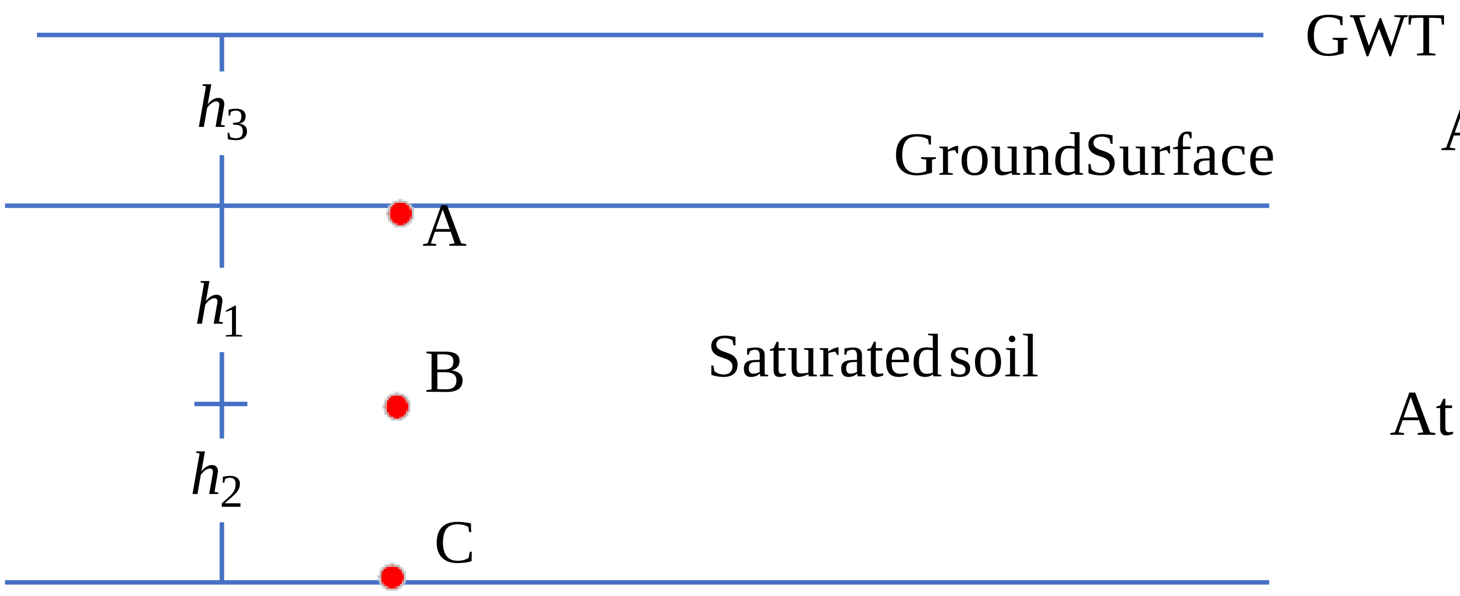
$\sigma' = \gamma'.(h_1 + h_2)$

At D, $\sigma = 0$

$u = 0$

$\sigma' = 0$

## Case 5:




At A, $\sigma = \gamma_w . h_3$

$$u = \gamma_w . h_3$$

$$\sigma' = 0$$

At C, $\sigma = \gamma_w . h_3 + \gamma_{sat} . (h_1 + h_2)$

$$u = \gamma_w . (h_3 + h_1 + h_2)$$

$$\sigma' = \gamma' . (h_1 + h_2)$$

At B, $\sigma = \gamma_w . h_3 + \gamma_{sat} . h_1$

$$u = \gamma_w . (h_3 + h_1)$$

$$\sigma' = \sigma - u = \gamma_w . h_3 + \gamma_{sat} . h_1 - \gamma_w . (h_3 + h_1) = \gamma_{sat} . h_1 - \gamma_w . h_1 = (\gamma_{sat} - \gamma_w) h_1 = \gamma' . h_1$$

total pressure and pore water pressure increases by the same amount and hence no change in effective stress at any point below the ground.

# CAPILLARITY

## Numerical Questions

01. The capillary rise difference in fine sand and silt was found to be 3.6 m. Surface tension is $75 \times 10^{-6}$ kN/m and unit weight of water is 10 kN/m$^3$. If the capillary rise in fine sand is 0.4 m, the difference in size of voids of the two soils is

a. $8.33 \times 10^{-3}$ mm b. $33.3 \times 10^{-3}$ mm **c.** $67.5 \times 10^{-3}$ mm d. $75 \times 10^{-3}$ mm

Ans. c

Capillary rise in fine sand, $h_1 = 0.4$ m

Capillary rise in silt, $h_2 = 0.4 + 3.6 = 4.0$ m

Let $d_1$ : size of voids in fine sand

$d_2$ : size of voids in silt

$$\text{capillary rise, } h_c = \frac{4T_s \cos\alpha}{d.\gamma_w}$$

$$d_1 = \frac{4\times 75\times 10^{-6}}{0.4\times 10} = 75\times 10^{-6} \text{ m}$$

$$d_2 = \frac{4 \times 75 \times 10^{-6}}{4 \times 10} = 7.5 \times 10^{-6} \text{ m}$$

Difference in size of voids $d = d_1 - d_2$ = (75-7.5) 10-6 m = $67.5 \times 10^{-6}$ m

= 0.0675 mm.

02. The soil 1 consists of average particle size 0.04 mm with capillary rise of 0.9 m. The capillary rise in soil 2 having same void ratio with average particle size 0.08 mm, is

**a.** 0.45 m b. 0.6 m c. 0.64 m d. 0.9 m

Ans. a

$D$ : average particle size

$D_1 = 0.04\,\text{mm} \quad D_2 = 0.08\,\text{mm}$

$d$ : average void size

void ratio, $e = \dfrac{V_v}{V_s} = \dfrac{d_1^3}{D_1^3} = \dfrac{d_2^3}{D_2^3} \Rightarrow \begin{matrix} d_1 \alpha D_2 \\ d_2 \alpha D_2 \end{matrix}$

For soil 1, $d_1 = \dfrac{4T_s \cos\alpha}{h_1 . \gamma_w}$

$$d_2 = \frac{4T_s \cos\alpha}{h_2.\gamma_w}$$

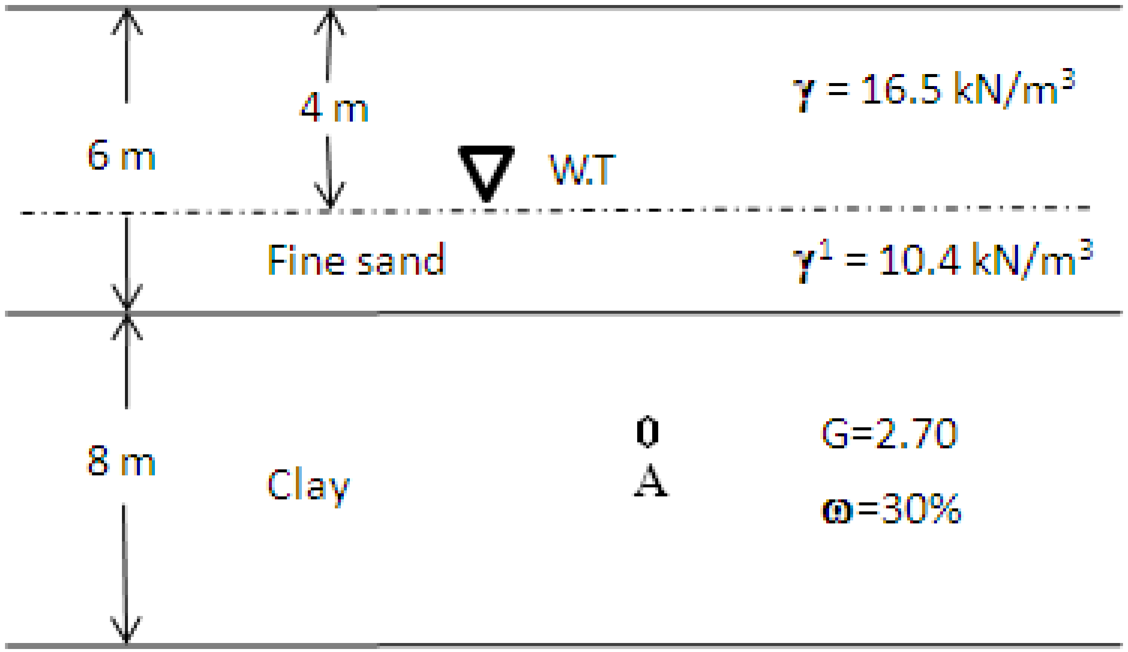
$$\frac{d_1}{d_2} = \frac{h_2}{h_1} \Rightarrow \frac{D_1}{D_2} = \frac{h_2}{h_1}; h_2 = \frac{0.04}{0.08} \times 0.9 = 0.45 \text{ m}$$

# EFFECTIVE STRESS

01. A soil profile consists of a surface layer of fine sand 6m thick with unit weight of 16.5 kN/m$^3$ and clay layer of 8m thick beneath the sand layer. The water table is located at a depth of 4m below the ground surface. The submerged unit weight of fine sand is 10.4 kN/m$^2$. For clay layer the specific gravity is 2.70 and water content is 30%. Effective stress at the middle of clay layer is

a. 99.96 kN/m$^2$ **b.** 124.36 kN/m$^2$ c. 161.92 kN/m$^2$ d. 174.12 kN/m$^2$

Ans. b



For clay layer, $e = \dfrac{wG}{S} = \dfrac{0.3 \times 2.7}{1} = 0.81$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.70+0.81}{1+0.81}\right)\times 10 = 19.39 \text{ kN/m}^3$$

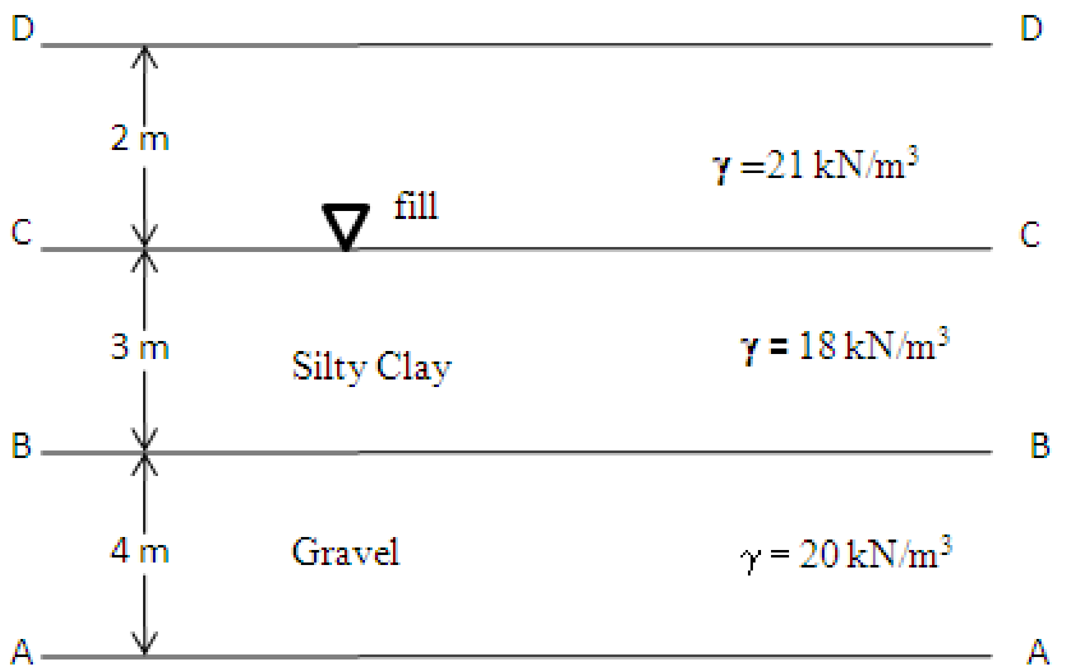
$$\gamma' = \gamma_{sat} - \gamma_w = 19.39 - 10 = 9.39 \text{ kN/m}^3$$

Effective stress at the middle of clay layer

$$\sigma'_A = 4\times 16.5 + 2\times 10.4 + 4\times 9.39 = 124.36 \text{ kN/m}^2$$

fill with bulk density of 21 kN/m were laid in compacted layers over an existing layer of silty clay 3 m thick having bulk density of 18 KN/$m^3$. It was underlain by a 4m thick layer of gravel having bulk density of 20 kN/m ... of the silty clay. The change in effective

a.0




d. 106 kN/$m^2$

Ans. c

Before fill, effective stress at A, $\sigma'_A = 3\times(18-10) + 4(20-10) = 64\ \text{kN/m}^2$

After the fill, effective stress at A, $\sigma'_A = 2\times 21 + 3(18-10) + 4(20-10) = 106\ \text{kN/m}^2$

- Change in effective stress at A = 106-64 = 42 kN/m$^2$.

  - **(OR)**

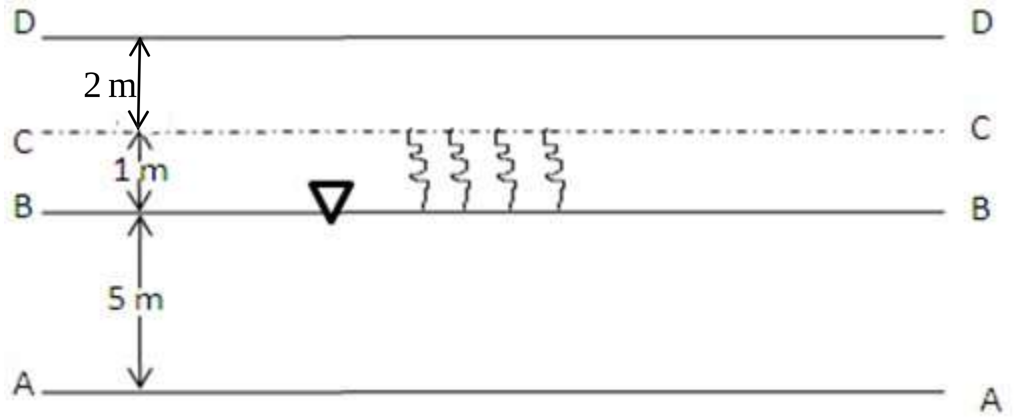
- Change in effective stress due to fill over the existing soil strata is equal to the effective stress due to fill.

- Change in effective stress = 2×21 = 42 kN/m$^2$.

03. A sand stratum is 8m thick has a porosity of 43% and specific gravity of particle 2.70. The ground water table is 3m below the ground surface and the capillary rise above water table is 1m. The effective stress at the bottom of sand stratum is

a. 88.64 kN/m² b. 94.32 kN/m² **c.** 98.64 kN/m² d. 148.64 kN/m²

Ans. c




Porosity $n = 43\%$

Specific gravity, $G = 2.7$

Void ratio, $e = \dfrac{n}{1-n} = \dfrac{0.43}{1-0.43} = 0.76$

The soil is dry upto 2 m below the ground surface

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7\times 10}{1+0.76} = 15.34\,\text{kN/m}^3$$

For the soil in the fully saturated zones

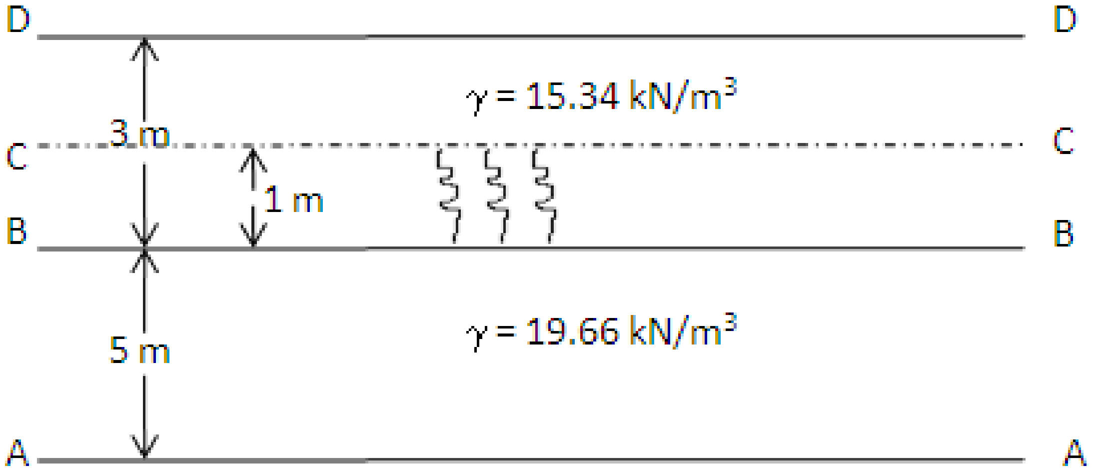
$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.7+0.76}{1+0.76}\right)10 = 19.66\,\text{kN/m}^3$$

Effective stress at A, $\sigma'_A = 2\times 15.34 + 1\times 19.66 + 5\times(19.66-10) = 98.64\ \text{kN/m}^2$.

04. A sandstratum is 8m thick having ground water table 3m below the ground surface and the capillary rise above water table is 1m. The unit weights of sand layer above and below water table are 15.34 kN/m$^3$ and 19.66 kN/m$^3$ respectively. The effective stress at 2m below the ground surface is

a.10.68 kN/m$^2$ b. 20.68 kN/m$^2$ c. 30.68 kN/m$^2$ **d.** 40.68 kN/m$^2$.

Ans. d




Total stress at C, $\sigma_C = 2\times15.34$

Pore water pressure at C, $u = -1\times10 = -10\text{kN/m}^2$

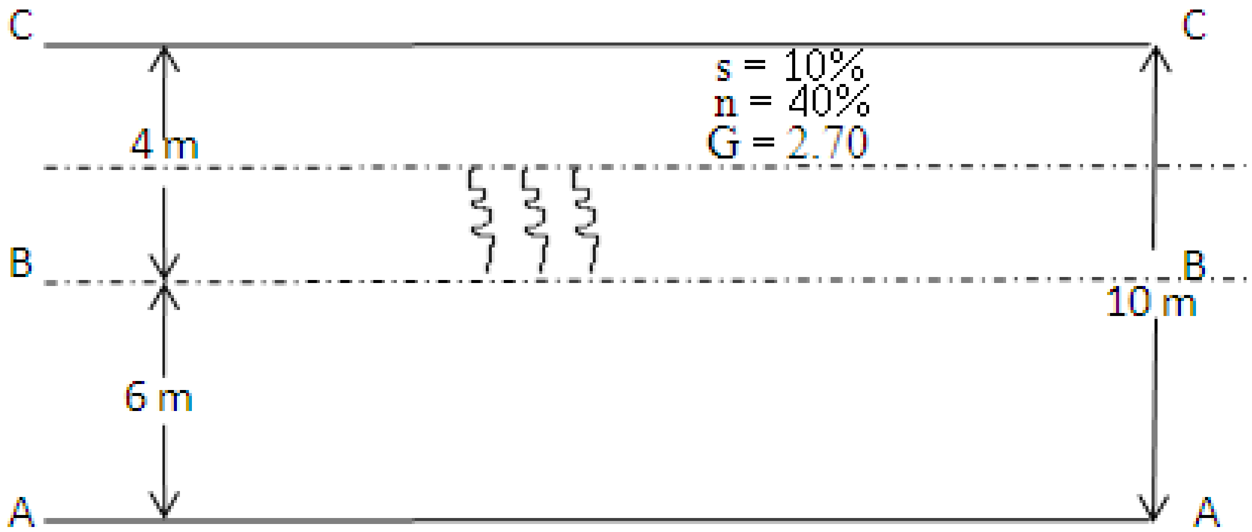
Effective stress at C, $\sigma'_C = \sigma_C - u$ = 30.68 -(-10) = 40.68 kN/m$^2$.

05. A stratum of fine sand has porosity of 40% and specific gravity of 2.70. The ground water table is 4 m below the ground surface and the sand is saturated by capillary water upto a height of 1m due to the water table. The degree of saturation of the sand upto 3 m below the ground surface is 10% . The total stress pore water pressure and effective stress respectively at a depth of 8m below the ground surface is

**a.** 189.8 kN/m², 60 kN/m², 129.8 kN/m² b. 185.8 kN/m², 70 kN/m², 115.8 kN/m²

c. 189.5 kN/m², 70 kN/m², 119.5 kN/m² d. 185.8 kN/m², 60 kN/m², 125.8 kN/m²

Ans. a



Porosity, $n$ = 40%

Specific gravity, $G$ = 2.70

Degree of saturation, $S$ = 10%

Void ratio, $e = \dfrac{n}{1-n} = \dfrac{0.40}{1-0.4} = 0.67$

$$s = \frac{wG}{e} \Rightarrow w = \frac{S.e}{G} \Rightarrow w = \frac{0.1 \times 0.67}{2.70} = 0.0248 = 2.48\%$$

$$\gamma_d = \frac{G\gamma_w}{1+e} = \frac{2.7 \times 10}{1+0.67} = 16.17\,\text{kN/m}^3$$

$$\gamma_{sat} = \left(\frac{G+e}{1+e}\right)\gamma_w = \left(\frac{2.7+0.67}{1+0.67}\right) \times 10 = 20.18\,\text{kN/m}^3$$

Total stress, $\sigma_A = 7 \times 20.18 + 3 \times 16.17 = 189.8\,\text{kN/m}^2$

Neutral stress, $u = 6 \times 10 = 60\,\text{kN/m}^2$

Effective stress = 189.77 – 60 = 129.8 kN/m$^2$.