JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – II & IV

Branch – Civil Engineering

Subject –Strength of Material

Unit- 3

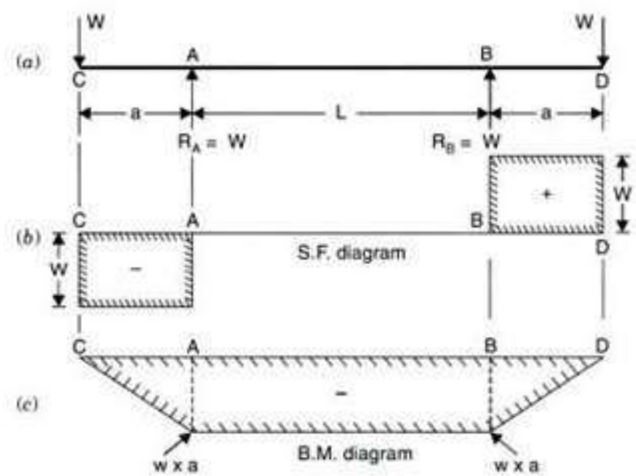
Name of Faculty – Sumit Saini (Assistant Professor)

4.1 SIMPLE BENDING OR PURE BENDING

- When some external force acts on a beam, the shear force and bending moments are set up at all the sections of the beam
- Due to shear force and bending moment, the beam undergoes deformation. The material of the beam offers resistance to deformation
- Stresses introduced by bending moment are known as bending stresses
- Bending stresses are indirect normal stresses

4.1 SIMPLE BENDING OR PURE BENDING

- When a length of a beam is subjected to zero shear force and constant bending moment, then that length of beam is subjected to pure bending or simple pending.
- The stress set up in that length of the beam due ... simple bending stresses



4.1 SIMPLE BENDING OR PURE BENDING

- Consider a simply supported beam with over hanging portions of equal lengths. Suppose the beam is subjected to equal loads of intensity W at either ends of the over hanging portion
- In the portion of beam of length l, the beam is subjected to constant bending moment of intensity w x a and shear force in this portion is zero
- Hence the portion AB is said to be subjected to pure bending or simple bending

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

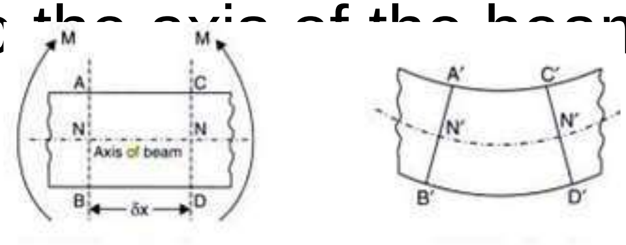
- The material of the beam is isotropic and homogeneous. Ie of same density and elastic properties throughout
- The beam is initially straight and unstressed and all the longitudinal filaments bend into a circular arc with a common centre of curvature
- The elastic limit is nowhere exceeded during the bending
- Young's modulus for the material is the same in tension and compression

4.2 ASSUMPTIONS FOR THE THEORY OF PURE BENDING

- The transverse sections which were plane before bending remain plane after bending also
- Radius of curvature is large compared to the dimensions of the cross section of the beam
- There is no resultant force perpendicular to any cross section
- All the layers of the beam are free to elongate and contract, independently of the layer, above or below it.

4.3 THEORY OF SIMPLE BENDING

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam and parallel to each other.



- Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The layers of the beam are not of the same length before bending and after bending .

4.3 THEORY OF SIMPLE BENDING

- The layer AC is shortened to A'C'. Hence it is subjected to compressive stress
- The layer BD is elongated to B'D'. Hence it is subjected to tensile stresses.
- Hence the amount of shortening decrease from the top layer towards bottom and the amount of elongation decreases from the bottom layer towards top
- Therefore there is a layer in between which neither elongates nor shortens. This layer is called neutral layer .

4.3 THEORY OF SIMPLE BENDING

- The filaments/ fibers of the material are subjected to neither compression nor to tension
- The line of intersection of the neutral layer with transverse section is called neutral axis (N-N).
- Hence the theory of pure bending states that the amount by which a layer in a beam subjected to pure bending, increases or decreases in length, depends upon the position of the layer w.r.t neutral axis N-N.

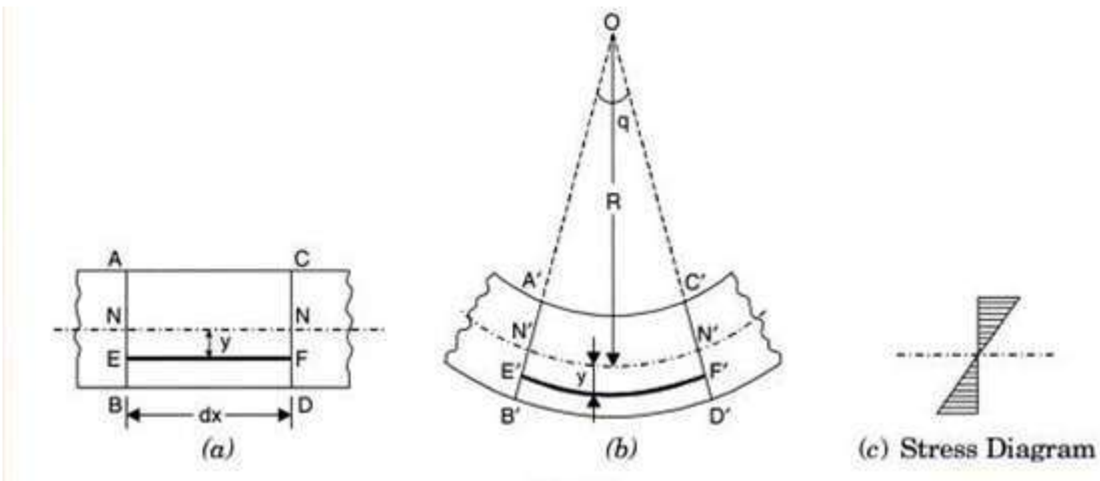
4.4 EXPRESSION FOR BENDING STRESS

- Consider a beam subjected to simple bending. Consider an infinitesimal element of length dx which is a part of this beam. Consider two transverse sections AB and CD which are normal to the axis of the beam. Due to the bending action the element ABCD is deformed to A'B'C'D' (concave curve).
- The lines B'A' and D'C' when extended meet at point O (which is the centre of curvature for the circular arc formed).
- Let R be the radius of curvature of the neutral axis.

4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Consider a layer EF at a distance y from the neutral axis. After bending this layer will be deformed to E'F'.
- Strain developed= (E'F'-EF)/EF

EF=NN=dx=R x θ



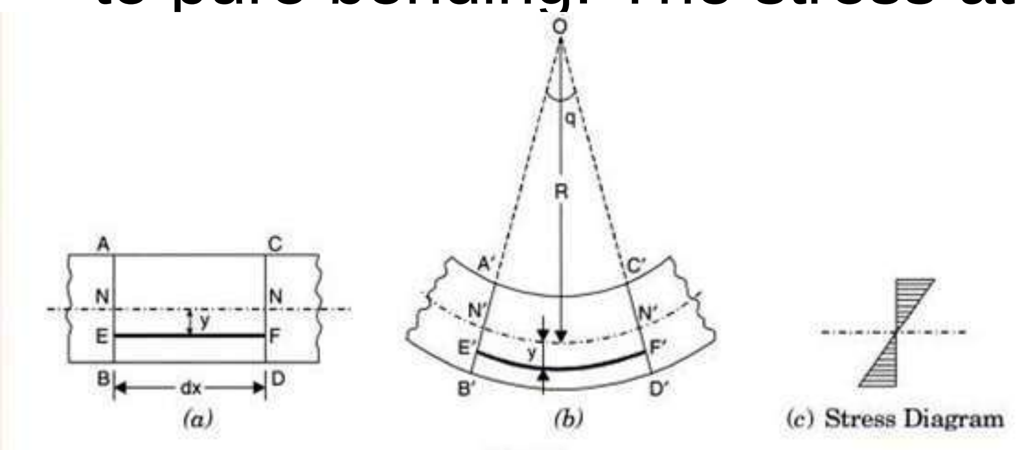
4.4.1 STRAIN VARIATION ALONG THE DEPTH OF BEAM

- Strain developed eb= { (R + y) x θ - R x θ)}/ R x θ=y/R
- STRESS VARIATION WITH DEPTH OF BEAM
- σ/E= y/R or σ= Ey/R or σ/y = E/R
- Hence σ varies linearly with y (distance from neutral axis)
- Therefore stress in any layer is directly proportional to the distance of the layer from the neutral layer

4.5 NEUTRAL AXIS

- For a beam subjected to a pure bending moment, the stresses generated on the neutral layer is zero.
- Neutral axis is the line of intersection of neutral layer with the transverse section
- Consider the cross section of a beam subjected to pure bending. The stress at a distance y from

=E/R



(a) (b) (c) Stress Diagram

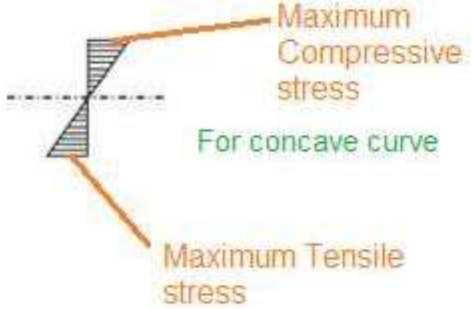
4.5 NEUTRAL AXIS

- $\sigma = E \times y/R$;
- The force acting perpendicular to this section, $dF = E \times y/R \times dA$, where dA is the cross sectional area of the strip/layer considered.
- Pure bending theory is based on an assumption that "There is no resultant force perpendicular to any cross section". Hence $F=0$;
- Hence, $E/R \times \int y\,dA = 0$

  $\Rightarrow \int y\,dA =$ Moment of area of the entire cross section w.r.t the neutral axis$=0$

4.5 NEUTRAL AXIS

- Moment of area of any surface w.r.t the centroidal axis is zero. Hence neutral axis and centroidal axis for a beam subjected to simple bending are the same.
- Neutral axis coincides with centrodial axis or the centroidal axis gives the position of neutral axis



Stress strain variation w.r.t the depth of the beam

4.6 MOMENT OF RESISTANCE

- Due to the tensile and compressive stresses, forces are exerted on the layers of a beam subjected to simple bending
- These forces will have moment about the neutral axis. The total moment of these forces about the neutral axis is known as moment of resistance of that section
- We have seen that force on a layer of cross sectional area dA at a distance y from the neutral axis,

  $dF = (E \times y \times dA)/R$

  Moment of force dF about the neutral axis= dF x y= (E x y x dA)/R x y= $E/R \times (y^2 dA)$

4.6 MOMENT OF RESISTANCE

- Hence the total moment of force about the neutral axis= Bending moment applied= $\int E/R \times (y^2 dA)$= $E/R \times I_{xx}$; I_{xx} is the moment of area about the neutral axis/centroidal axis.

  Hence $M=E/R \times I_{xx}$

  Or $M/I_{xx}=E/R$
- Hence $M/I_{xx}=E/R = \sigma b/y$;$\sigma b$ is also known as flexural stress (Fb)
- Hence **$M/I_{xx}=E/R=Fb/y$**
- The above equation is known as bending equation
- This can be remembered using the sentence "Elizabeth Rani May I Follow You"

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Bending equation is applicable to a beam subjected to pure/simple bending. Ie the bending moment acting on the beam is constant and the shear stress is zero
- However in practical applications, the bending moment varies from section to section and the shear force is not zero
- But in the section where bending moment is maximum, shear force (derivative of bending moment) is zero
- Hence the bending equation is valid for the section where bending moment is maximum

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- Or in other words, the condition of simple bending may be satisfied at a section where bending moment is maximum.
- Therefore beams and structures are designed using bending equation considering the section of maximum bending moment
- Flexural rigidity/Flexural resistance of a beam:-
- For pure bending of uniform sections, beam will deflect into circular arcs and for this reason the term circular bending is often used.

4.7 CONDITION OF SIMPLE BENDING & FLEXURAL RIGIDITY

- The radius of curvature to which any beam is bent by an applied moment M is given by R=EI/M
- Hence for a given bending moment, the radius of curvature is directly related to "EI"
- Since radius of curvature is a direct indication of the degree of flexibility of the beam (larger the value of R, less flexible the beam is, more rigid the beam is), EI is known as flexural rigidity of flexural stiffness of the beam.
- The relative stiffnesses of beam sections can then easily be compared by their EI value

4.8 SECTIONAL MODULUS (Z)

- Section modulus is defined as the ratio of moment of area about the centroidal axis/neutral axis of a beam subjected to bending to the distance of outermost layer/fibre/filament from the centroidal axis
- **$Z = I_{xx}/y_{max}$**
- From the bending equation, $M/I_{xx} = \sigma_{bmax}/y_{max}$

  Hence $I_{xx}/y_{max} = M/\sigma_{bmax}$

  $M = \sigma_{bmax} \times Z$
- Higher the Z value for a section, the higher the BM which it can withstand for a given maximum stress

VARIOUS SHAPES OR BEAM SECTIONS

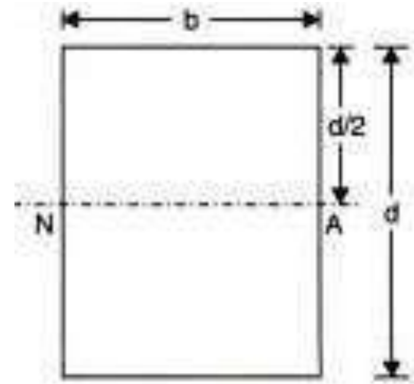
- 1) For a Rectangular section

  $Z = I_{xx}/y_{max}$

  $I_{xx} = I_{NA} = bd^3/12$

  $y_{max} = d/2$

  $\boxed{Z = bd^2/6}$

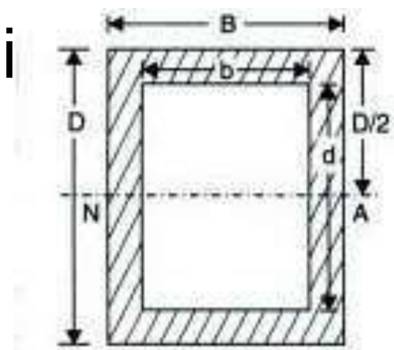


- 2) For a Rectangular hollow secti

  $I_{xx} = 1/12 \times (BD^3/12 - bd^3/12)$

  $Y_{max} = D/2$

  $\boxed{Z = (BD^3 - bd^3)/6D}$



VARIOUS SHAPES OR BEAM SECTIONS

- 3) For a circular section of diameter D,

  $I_{xx} = \pi D^4/64$

  $y_{max} = D/2$

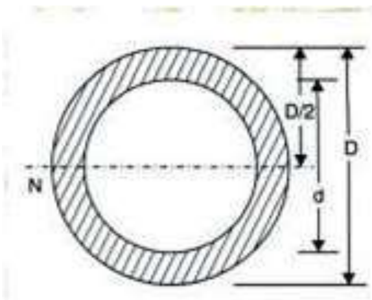
  $\boxed{Z = \pi D^3/32}$

- 4) For a hollow circular section of outer diameter D and inner diameter d,

  $I_{na} = (\pi D^4 - d^4)/64$

  $y_{max} = D/2$

  $\boxed{Z = (\pi D^4 - d^4)/32D}$



4.9 BENDING OF FLITCHED BEAMS

- A beam made up of two or more different materials assumed to be rigidly connected together and behaving like a single piece is called a flitched beam or a composite beam.
- Consider a wooden beam re-inforced by steel plates. Let

  E_1= Modulus of elasticity of steel plate

  E_2= Modulus of elasticity of wooden beam

  M_1= Moment of resistance of steel plate

  M_2= Moment of resistance of wooden beam

4.9 BENDING OF FLITCHED BEAMS

I1= Moment of inertia of steel plate about neutral axis

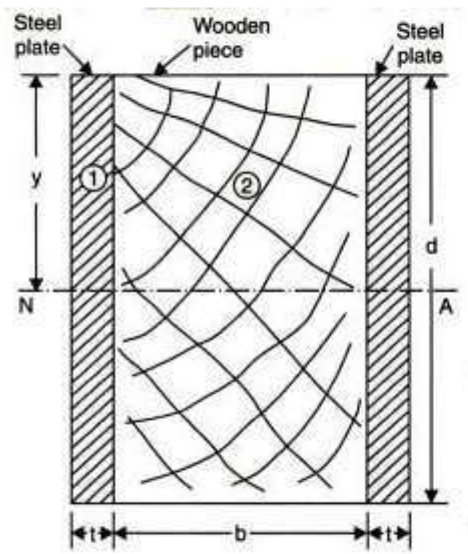
I2= Moment of inertia of wooden beam about neutral axis.

The bending stresses can be calculated using two conditions.

- Strain developed on a layer at a particular distance from the neutral axis is the same for both the materials
- Moment of resistance of composite beam is equal to the sum of individual moment of resistance of the members

4.9 BENDING OF FLITCHED BEAMS

- Let σ1 be the bending stress developed in steel at a distance y from the neutral axis and σ2 be the bending stress developed in wooden beam at th… n neutral axis.



4.9 BENDING OF FLITCHED BEAMS

- Using condition-1:

  $\sigma 1/E1 = \sigma\ 2/E2$;

  $\sigma 1 = \sigma\ 2 \times (E1/E2)$ or $\sigma 1 = \sigma 2 \times m$; where $m = E1/E2$ is the modular ratio between steel and wood

- Using condition-2:

  $M = M1 + M2$;

  $M1 = \sigma 1 \times I1/y$

  $M1 = \sigma 2 \times I2/y$

4.9 BENDING OF FLITCHED BEAMS

- Hence $M= \sigma 1 x\ I1/y + \sigma 2 x\ I2/y$

  $M= \sigma 2/\ y\ x\ (I2 + I1 x\ \sigma 1/\ \sigma 2)$ $M=$

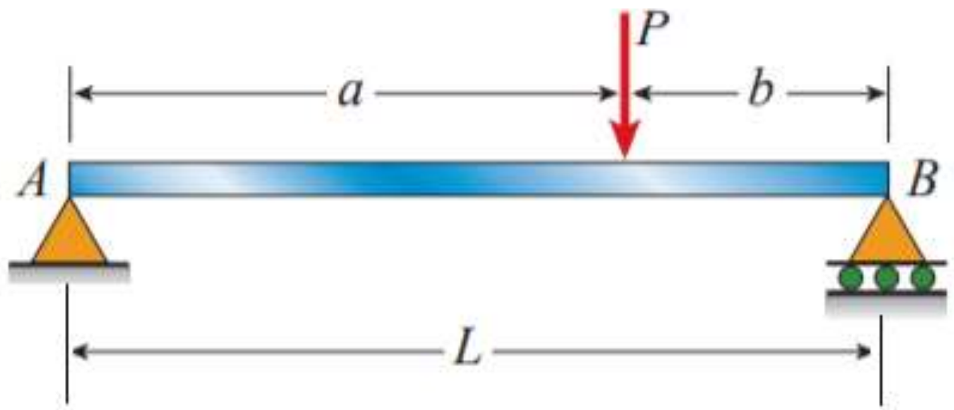
  $\sigma 2/y\ x\ (\ I2 + I1\ x\ m)$
- $(I2 + I1\ x\ m)= I =$ equivalent moment of inertia of the cross section;
- Hence $M= \sigma 2/y\ x\ I$

SHEAR FORCE AND BENDING MOMENT DIAGRAM

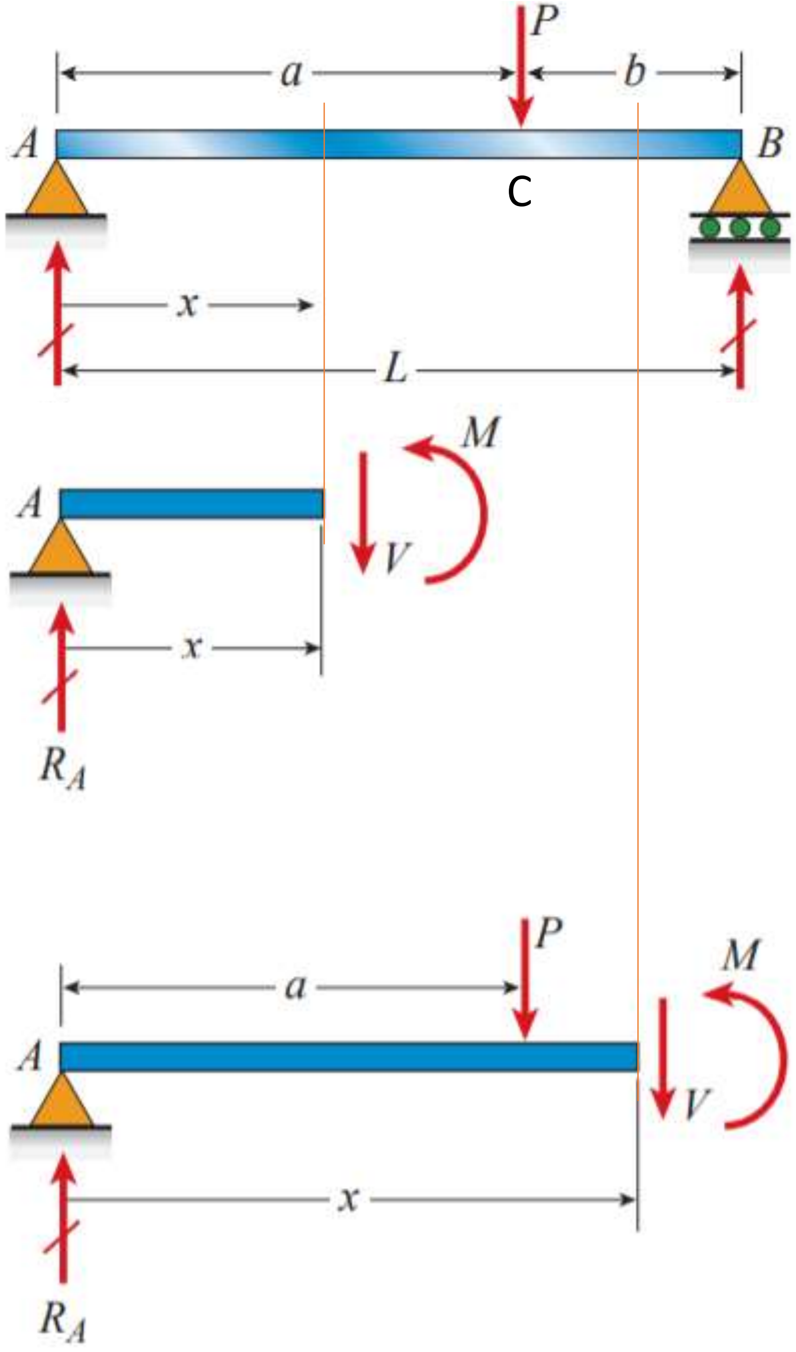
Shear force and Bending moment diagram

When designing a beam, we usually need to know how the shear forces and bending moments vary throughout the length of the beam. Of special importance are the maximum and minimum values of these quantities. Information of this kind is usually provided by graphs in which the shear force and bending moment are plotted as ordinates and the distance x along the axis of the beam is plotted as the abscissa. Such graphs are called shear-force and bending-moment diagrams.

Q1.Draw the shear force and bending moment diagram for a simple beam AB supporting a concentrated load P. The load P acts at distance ‘a’ from the left-hand support and distance ‘b’ from the right-hand support.



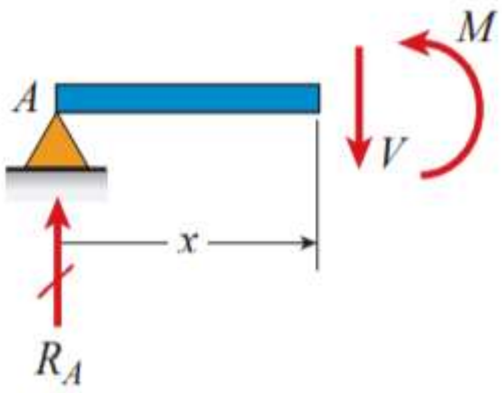
Free body diagram of given simple beam:



$$\boxed{R_A = \frac{Pb}{L}}$$

$$\boxed{R_B = \frac{Pa}{L}}$$

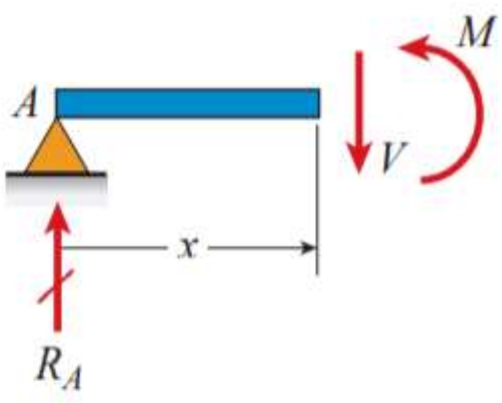
Free body diagram of section1:



$(0 < x < a)$

Free body diagram of section1: $(0 < x < a)$

when x=0 section coincides with A, when x=a section coincides with C



$$\Sigma \mathbf{F_y} = \mathbf{0}$$

R_A-V =0

$$V = R_A = \frac{Pb}{L}$$

$$\Sigma \mathbf{M_{1-1}} = \mathbf{0}$$

$- \quad R_A \;.\; x + M = 0$

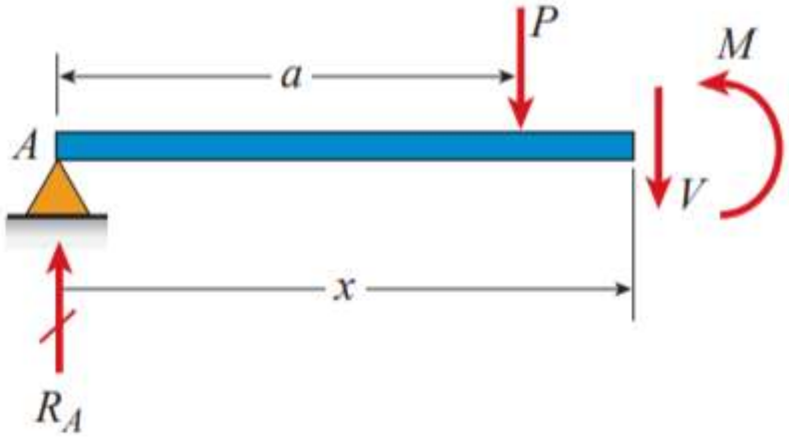
$M = R_A \;.\; x = \frac{Pb}{L} \;.\; x$

0< x<a Section1-1	Shear force $V = \frac{Pb}{L}$	Bending Moment $M = \frac{Pb}{L} \;.\; x$
x=0 (at A)	$V_A = \frac{Pb}{L}$	$M_A = \frac{Pb}{L} .\ 0 = 0$
x= a (at C)	$V_C = \frac{Pb}{L}$	$M_C = \frac{Pb}{L} .\ a = \frac{Pab}{L}$

Free body diagram of section2: $(a < x < L)$

when x=a section coincides with C, when x=L section coincides with B

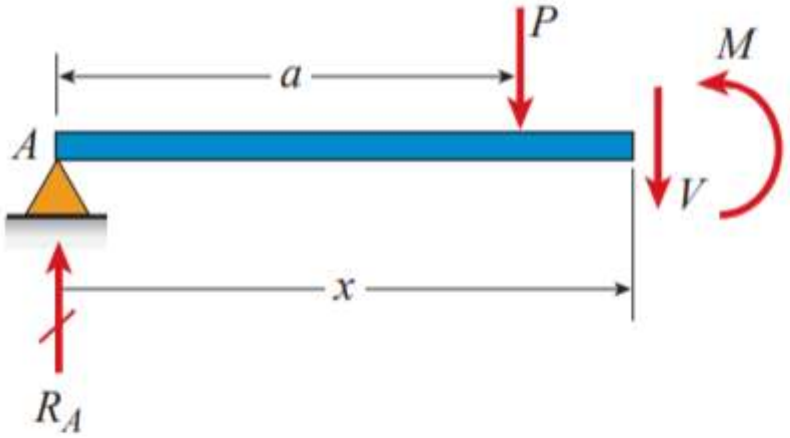
$$\Sigma F_y = 0$$



$$\Sigma M_{2-2} = 0$$

Free body diagram of section2: $(a < x < L)$

when x=a section coincides with C, when x=L section coincides with B



$\Sigma F_y = 0$ $\quad R_A - P - V = 0$

$$V = R_A - P$$
$$= \frac{Pb}{L} - P = -\frac{Pa}{L}$$

$\Sigma M_{2-2} = 0$

$-R_A \cdot x + P(x - a) + M = 0$

$M = R_A \cdot x - P(x - a)$

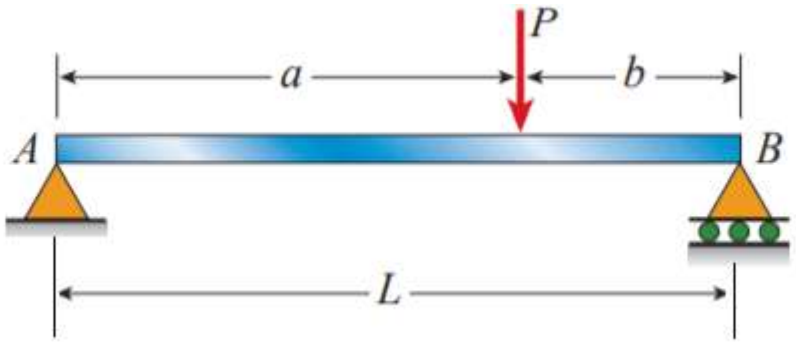
$M = \frac{Pa}{L} \cdot x - P(x - a)$

$M = \frac{Pa}{L}(L - x)$

$a < x < L$ Section2-2	Shear force $V = -\frac{Pa}{L}$	Bending Moment $M = \frac{Pa}{L}(L - x)$
x=a (at C)	$V_C = -\frac{Pa}{L}$	$M_C = \frac{Pa}{L}(L - a) = \frac{pab}{L}$
x= L (at B)	$V_B = -\frac{pa}{L}$	$M_B = \frac{Pa}{L}(L - L) = 0$

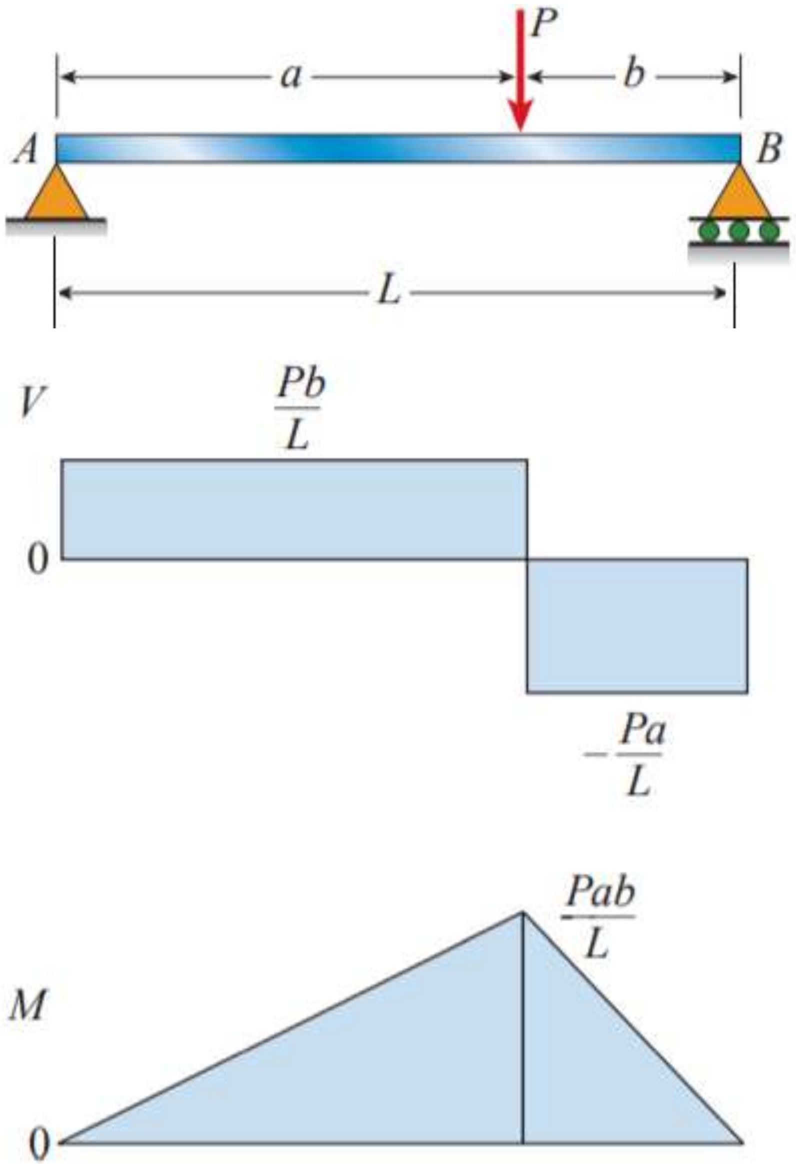
0< x<a Section1-1	Shear force	Bending Moment
x=0 (at A)	$V_A = \frac{Pb}{L}$	$M_A = 0$
x= a (at C)	$V_C = \frac{Pb}{L}$	$M_C = \frac{Pab}{L}$

a< x<L Section2-2	Shear force	Bending Moment
x=a (at C)	$V_C = -\frac{Pa}{L}$	$M_C = \frac{pab}{L}$
x= L (at B)	$V_B = -\frac{pa}{L}$	$M_B = 0$



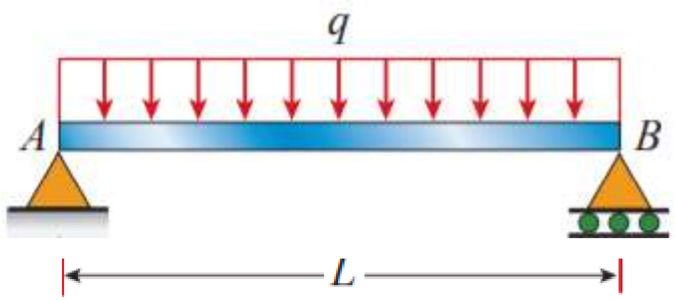
$0< x<a$ Section1-1	Shear force	Bending Moment
$x=0$ (at A)	$V_A= \frac{Pb}{L}$	$M_A= 0$
$x= a$ (at C)	$V_C= \frac{Pb}{L}$	$M_C= \frac{Pab}{L}$

$a< x<L$ Section2-2	Shear force	Bending Moment
$x=a$ (at C)	$V_C= -\frac{Pa}{L}$	$M_C = \frac{pab}{L}$
$x= L$ (at B)	$V_B= -\frac{pa}{L}$	$M_B= 0$



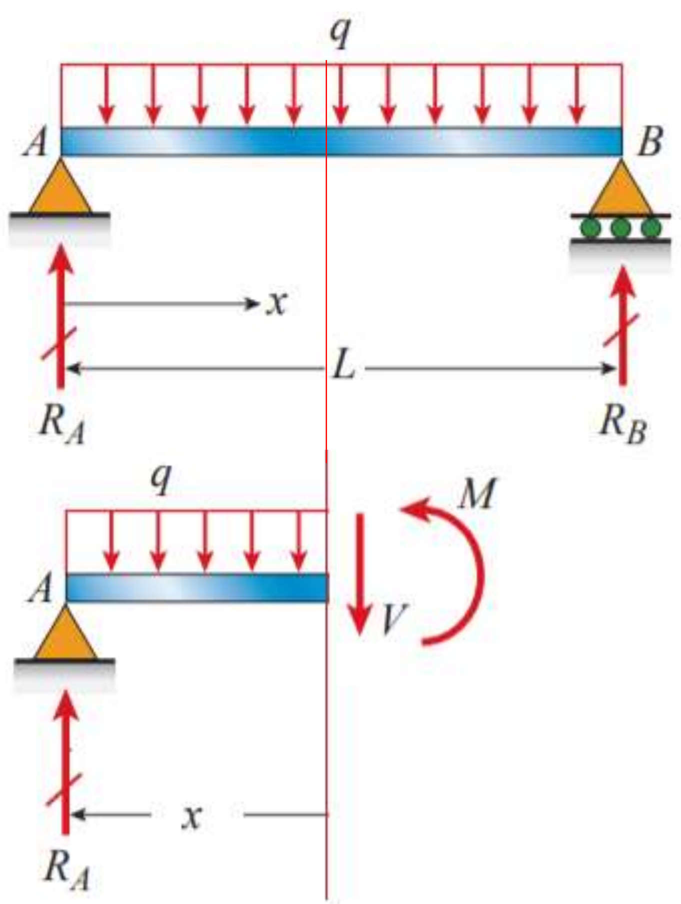
Shear-force and bending moment diagrams for a simple beam with a concentrated load

Q2. Draw the shear force and bending moment diagram for a simple beam AB supporting a uniformly distributed load of intensity ‘q’ through out the length of the beam.

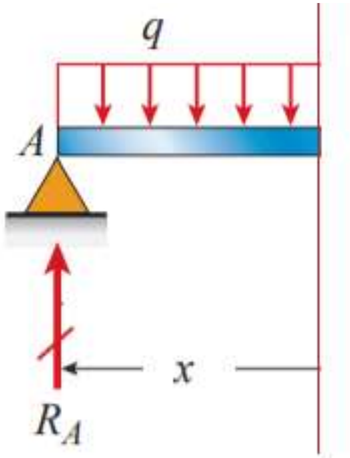


Free body diagram of given simple beam:

Because the beam and its loading are symmetric, we see immediately that each of the reactions (R_A and R_B) is equal to $qL/2$.

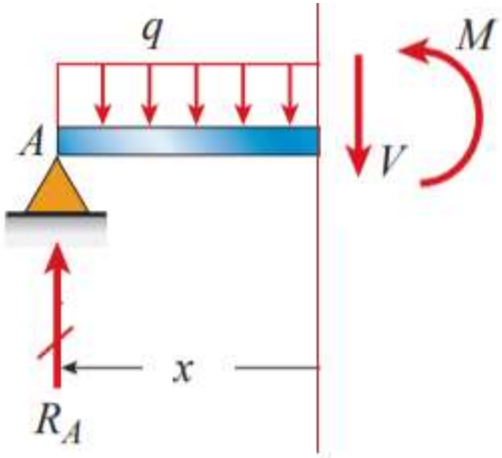


Free body diagram of given simple beam:



Free body diagram of section1: 0< x < L

when x=0 section coincides with A, when x=L section coincides with B



$$\Sigma F_y = 0$$

$$R_A - q \,.\, x - V = 0$$

$$V = R_A - q \,.\, x$$

$$V = \frac{q.L}{2} - q.\, x$$

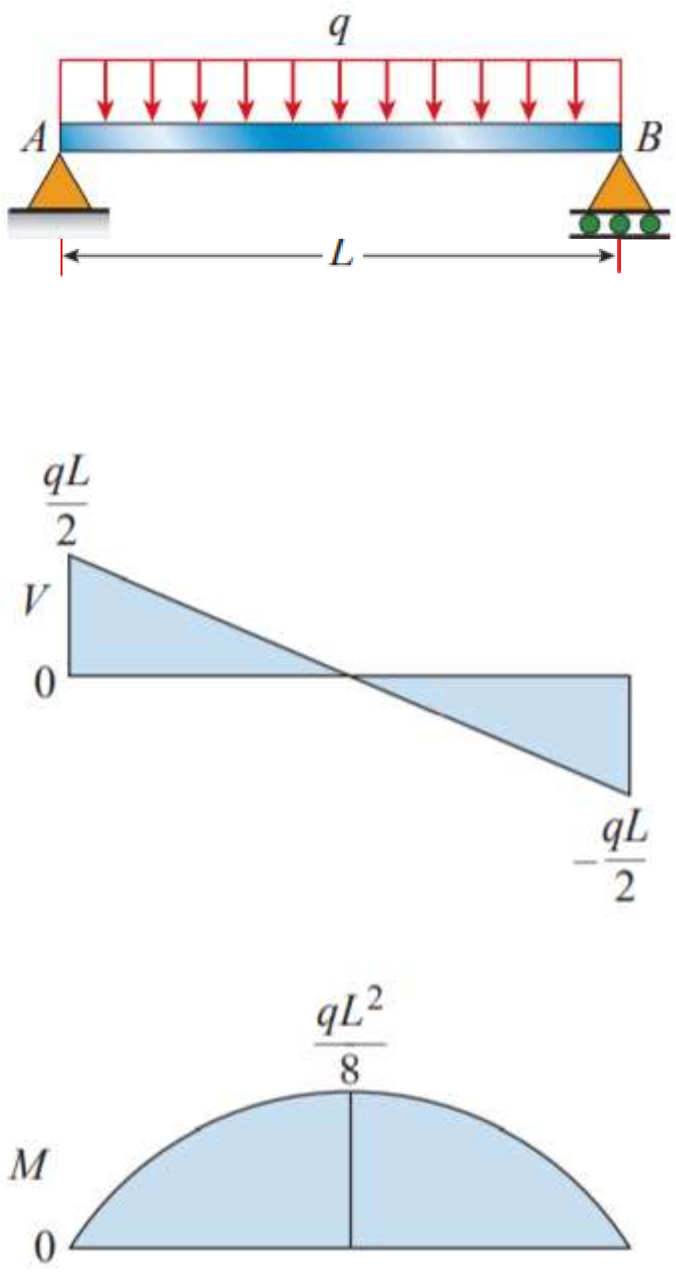
0< x< L Section1-1	Shear force $V = \frac{q.L}{2} - q.\, x$	Bending Moment $M = \frac{q.L.x}{2} - \frac{q.x^2}{2}$
x=0 (at A)	$V_A = \frac{q.L}{2}$	$M_A = \frac{q.L.0}{2} - \frac{q.02}{2} = 0$
x= L (at B)	$V_B = \frac{q.L}{2} - qL = -\frac{q.L}{2}$	$M_B = \frac{q.L.L}{2} - \frac{q.L^2}{2} = 0$
x= L/2 (at C)	$V_C = \frac{q.L}{2} - \frac{q.L}{2} = 0$	$M_C = \frac{q.L.\frac{L}{2}}{2} - \frac{q.(\frac{L}{2})^2}{2} = \frac{qL^2}{8}$

$$\Sigma M_{1-1} = 0$$

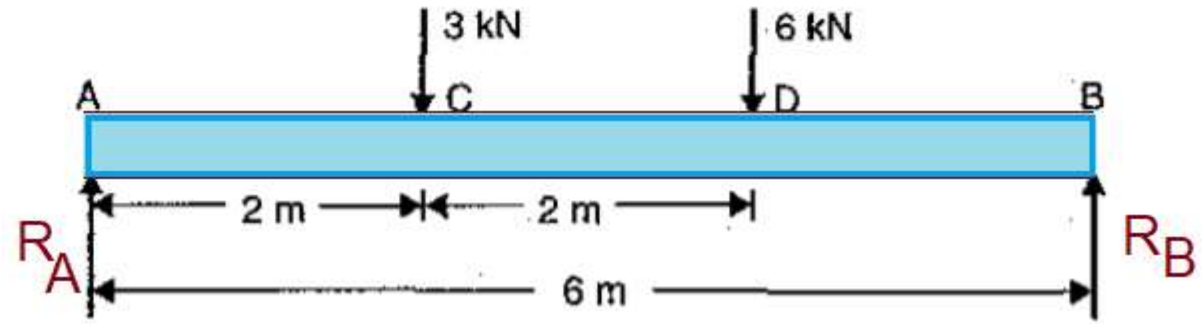
$$- \quad R_A \,.\, x + q \,.\, x.\frac{x}{2} + M = 0$$

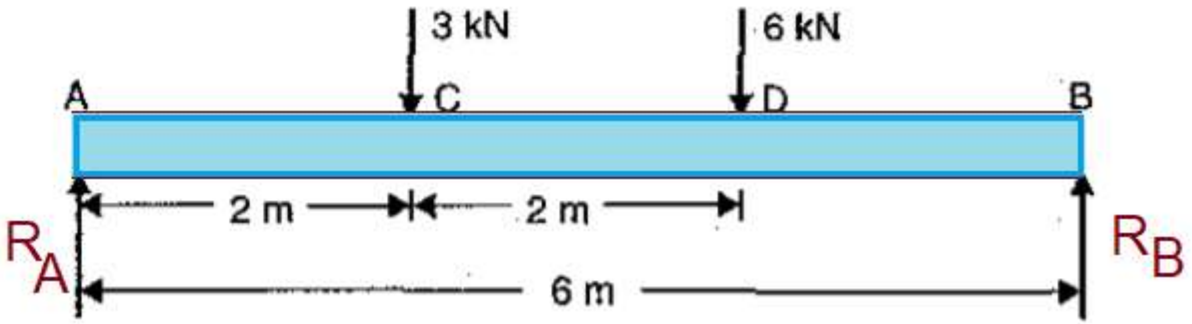
$$M = R_A \,.\, x - q \,.\, x.\frac{x}{2} = \frac{q.L}{2}\,.\, x - q \,.\, x.\frac{x}{2} = \frac{q.L.x}{2} - \frac{q.x^2}{2}$$

0< x< L Section1-1	Shear force $V = \frac{q.L}{2} - q.\,x$	Bending Moment $M= \frac{q.L.x}{2} - \frac{q.x^2}{2}$
x=0 (at A)	$V_A=\frac{q.L}{2}$	$M_A= 0$
x= L (at B)	$V_B=-\frac{q.L}{2}$	$M_B= 0$
x= L/2 (at C)	$V_C=0$	$M_C=\frac{qL^2}{8}$



Q3.A simply supported beam of length 6 m, carries point load of 3 kN and 6 kN at distances of 2 m and 4 m from the left end. Draw the shear force and bending moment diagrams for the beam.





Estimate the reactions at supports:

$\Sigma Fy = 0$

$\Rightarrow R_A - 3 - 6 + R_B = 0$

$\Rightarrow R_A + R_B = 9 \quad \rightarrow (1)$

$\Sigma MA = 0$

$\Rightarrow -3 * 2 - 6 * 4 + R_B * 6 = 0$

$\Rightarrow R_B * 6 = 3 * 2 + 6 * 4$

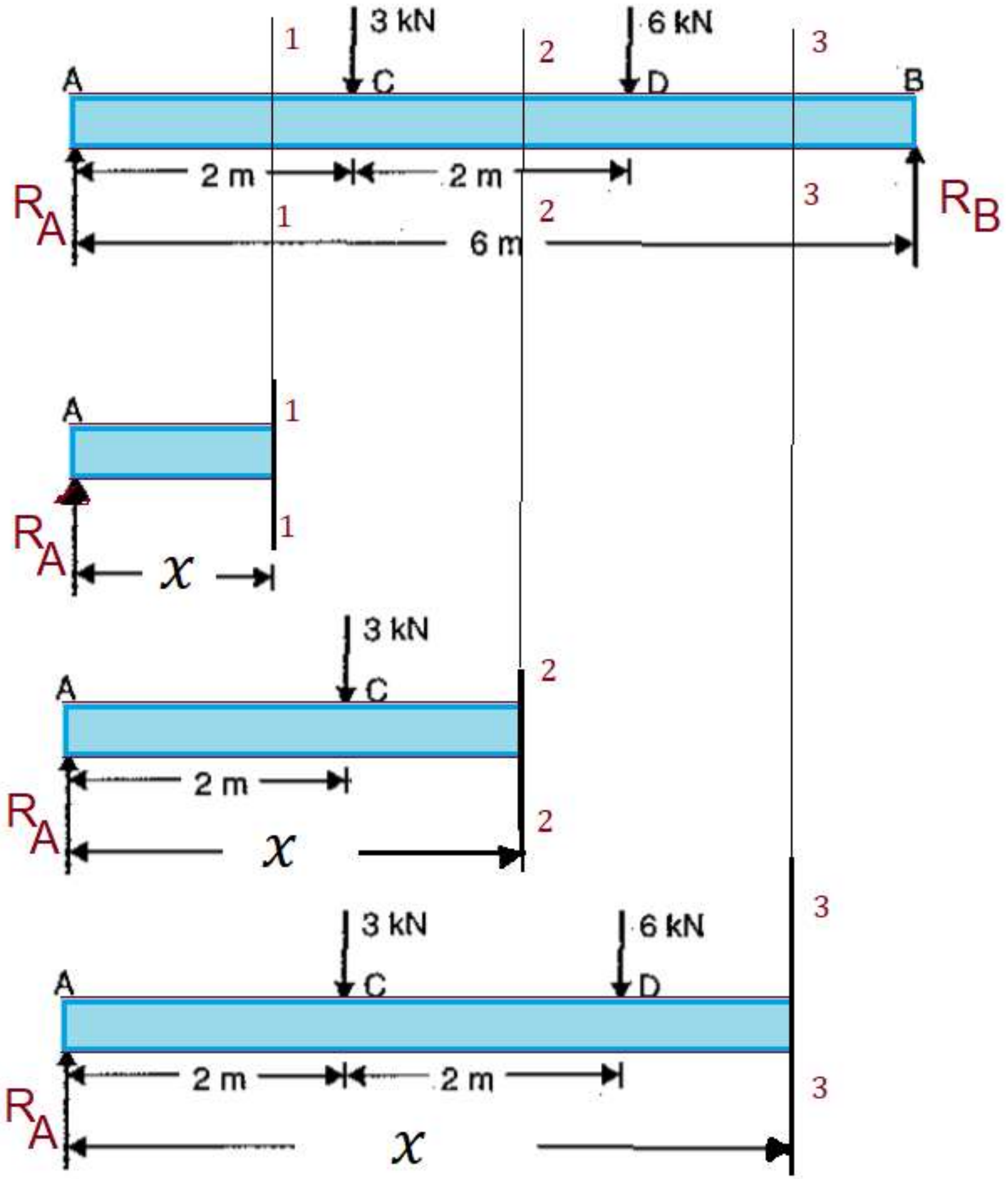
$\Rightarrow R_B = 5 \text{ kN} \quad \rightarrow (2)$

Substitute (2) in (1)

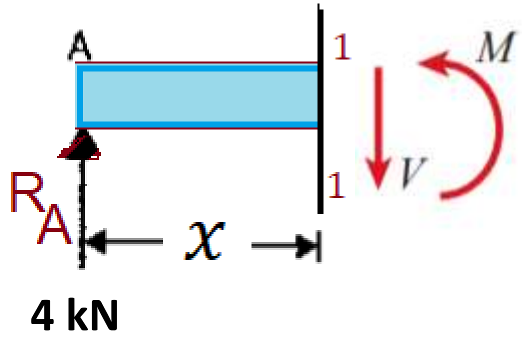
$\Rightarrow R_A = 9 - R_B$

$\Rightarrow R_A = 9 - 5$

$\Rightarrow R_A = 4 \text{ kN}$

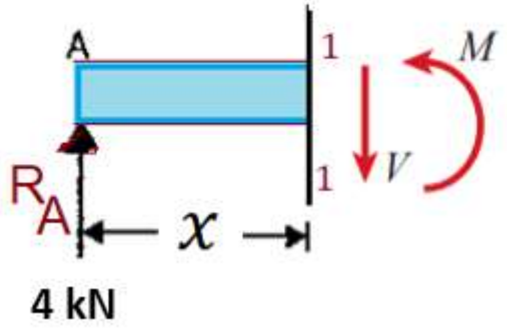


3 kN
6 kN
A
C
D
B
1
2
3
2 m
2 m
6 m
R_A
R_B
x


A
1
M
V
1
RA
x
4 kN

Free body diagram of section1: 0< x< 2 m

when x=0 section coincides with A, when x=a section coincides with C



$$\Sigma F_y = 0$$

R_A-V =0

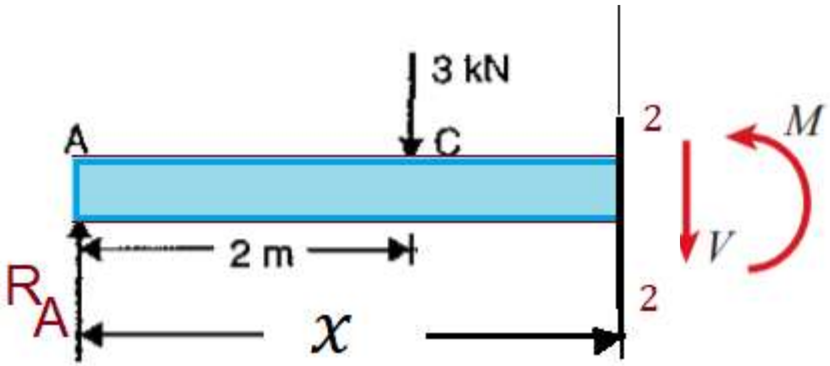
V =R_A = 4 kN

$$\Sigma M_{1-1} = 0$$

- R_A * x + M =0

M= R_A * x = 4. x

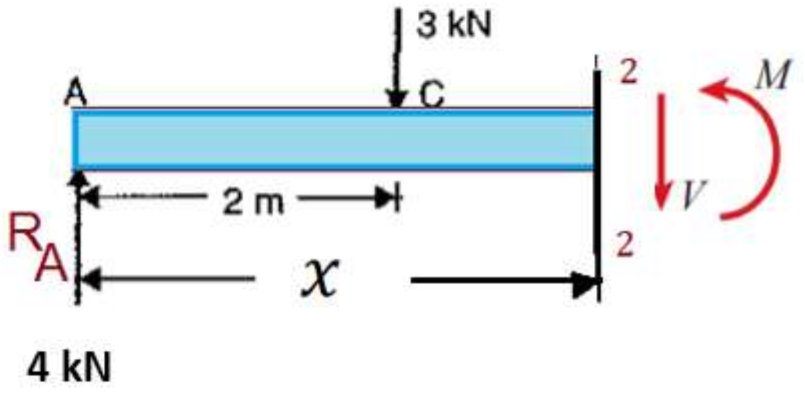
0< x<2 Section1-1	Shear force (kN) V= 4	Bending Moment (kN-m) M= 4 * x
x=0 (at A)	V_A= 4	M_A= 4 * 0= 0
x= 2 m (at C)	V_C=4	M_C=4 * 2= 8



4 kN

Free body diagram of section2: $2\text{ m} < x < 4\text{ m}$

when x=2 m section coincides with C, when x=4 m section coincides with D



$$\Sigma F_y = 0$$

$R_A - 3 - V = 0$

$\Rightarrow V = R_A - 3$

$\Rightarrow V = 4 - 3 = 1\text{kN}$

$$\Sigma M_{2-2} = 0$$

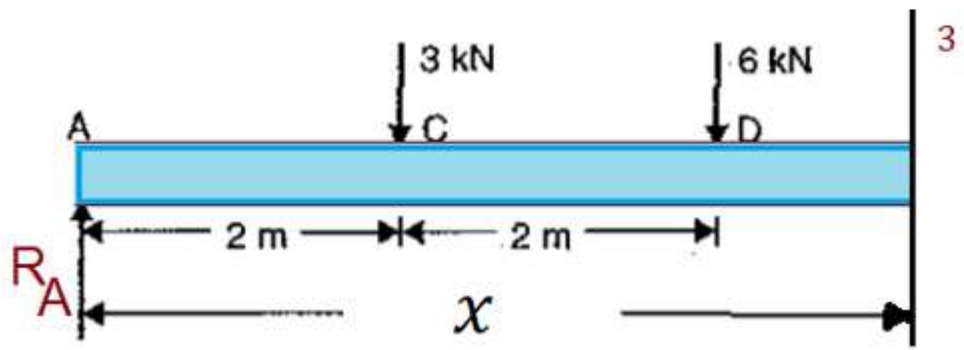
$-R_A \cdot x + 3 * (x - 2) + M = 0$

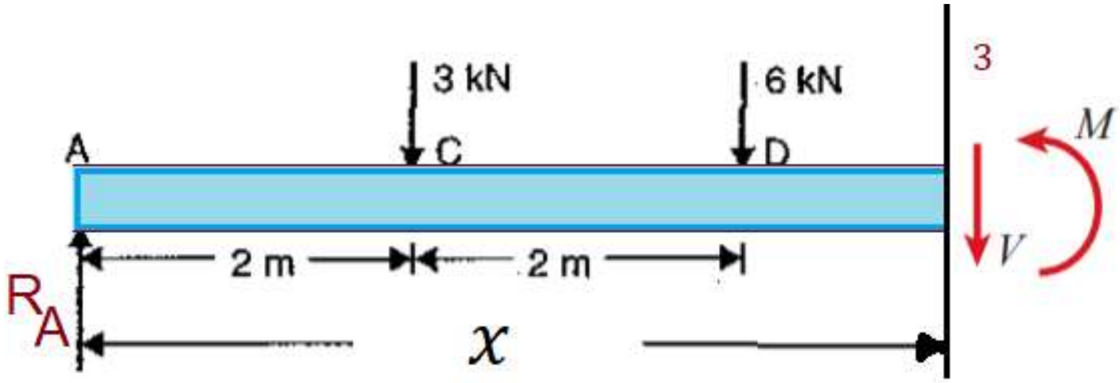
$M = 4 \cdot x - 3 * (x - 2)$

$M = 4 \cdot x - 3 * x + 6$

$M = x + 6$

$2 < x < 4$ Section2-2	Shear force $V = 1$ kN	Bending Moment(kN-m) $M = x + 6$
x=2 (at C)	$V_C = 1$	$M_C = 2 + 6 = 8$
x= 4 (at D)	$V_D = 1$	$M_D = 4 + 6 = 10$


3 kN
6 kN
A
C
D
2 m
2 m
R_A
x
3



Free body diagram of section 3: 4 m < x< 6 m

when x=4 m section coincides with D, when x=6 m section coincides with B

$$\mathbf{\Sigma F_y = 0}$$

$R_A - 3 - 6 - V = 0$

$\Rightarrow V = R_A - 9$

$\Rightarrow V = 4 - 9 = -5$ kN

$$\mathbf{\Sigma M_{2-2} = 0}$$

$-R_A \,.\, x + 3 * (x - 2) + 6 * (x - 4) + M = 0$

$M = 4 \,.\, x - 3 * (x - 2) - 6 * (x - 4)$

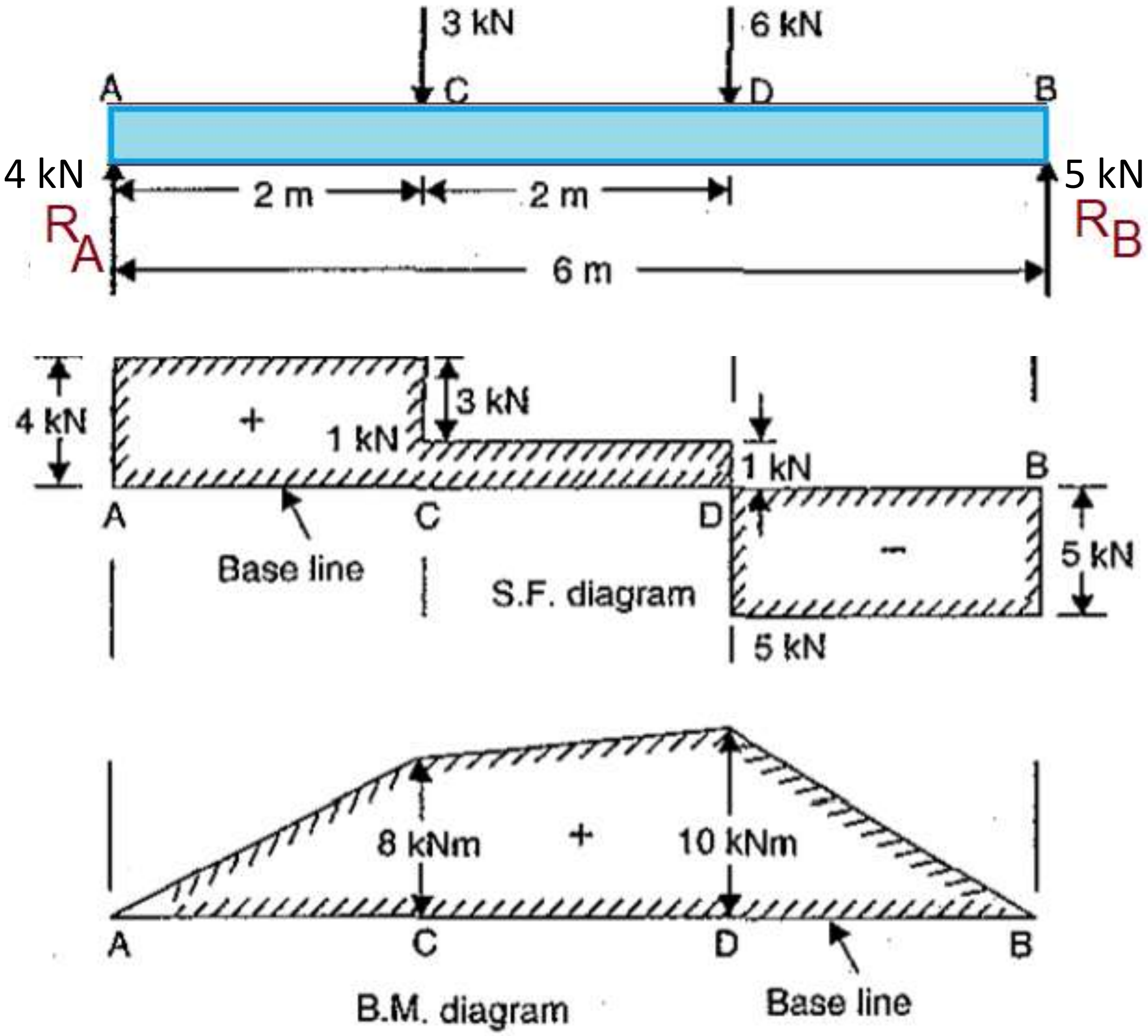
$M = -5 * x + 30$

4 < x< 6 Section 3-3	Shear force V= -5 kN	Bending Moment(kN-m) $M = -5 * x + 30$
x=4 (at D)	V_D= -5	$M_D = -5 * 4 + 30 = 10$
x=6 (at B)	V_B= -5	$M_B = -5 * 6 + 30 = 0$

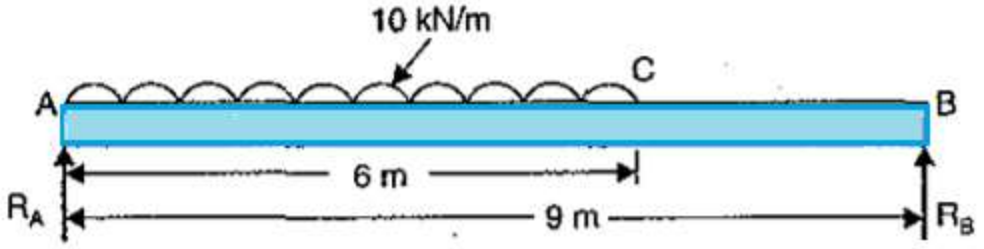
$0< x<2$ Section1-1	Shear force (kN) $V= 4$	Bending Moment $M= 4 * x$
x=0 (at A)	$V_A= 4$	$M_A= 0$
x= 2 m (at C)	$V_C=4$	$M_C= 8$

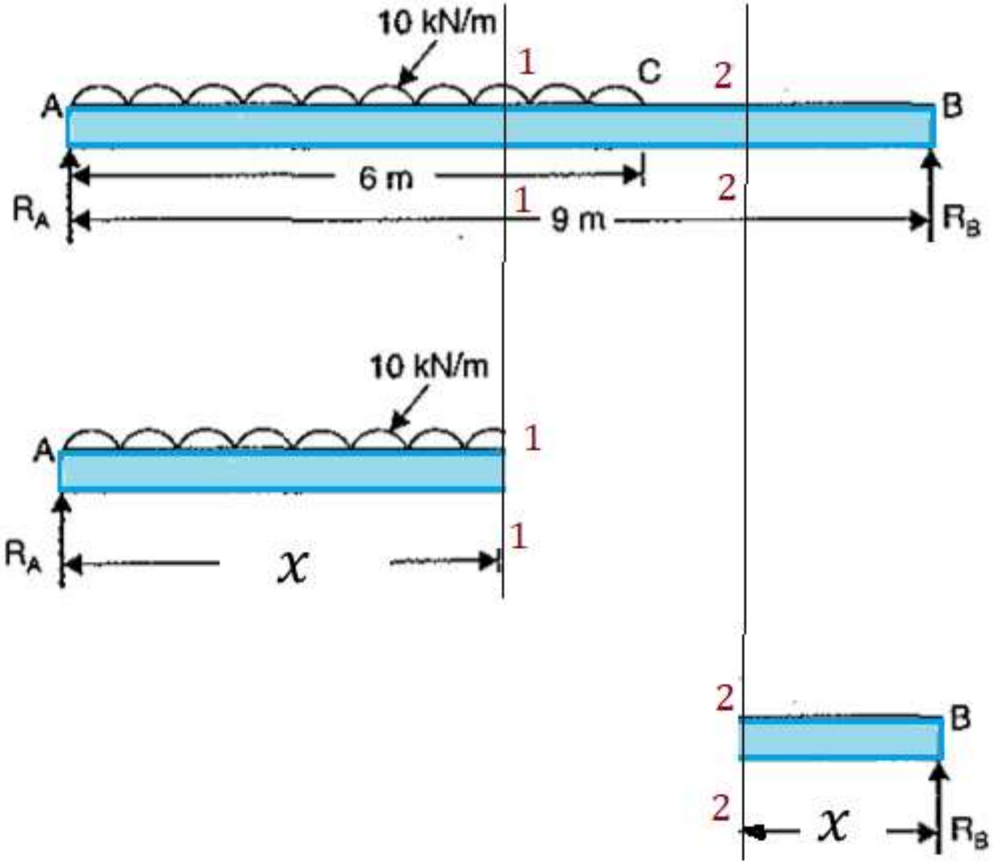
$2< x< 4$ Section2-2	Shear force $V=$ 1 kN	Bending Moment $M= x + 6$
x=2 (at C)	$V_C= 1$	$M_C= 8$
x= 4 (at D)	$V_D= 1$	$M_D= 10$

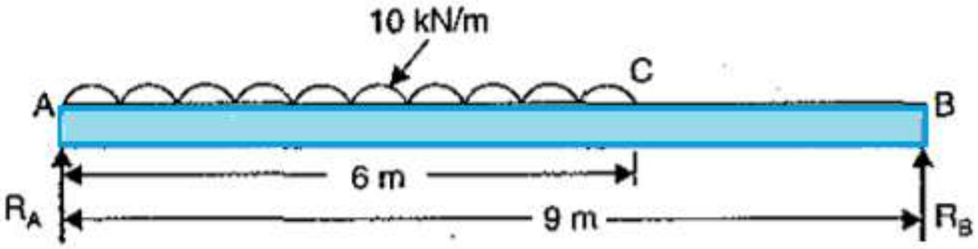
$4 < x< 6$ Section 3-3	Shear force $V=$ -5 kN	Bending Moment $M= - 5 * x + 30$
x=4 (at D)	$V_D= -5$	$M_D=10$
x=6 (at B)	$V_B= -5$	$M_B= 0$



Q5. Draw the shear force and bending moment diagram for a simply supported beam of length 9 m and carrying a uniformly distributed load of 10 kN/m for a distance of 6 m from the left end. Also calculate the maximum B.M. on the section.




10 kN/m
1
C
2
A
B
6 m
R_A
9 m
R_B
10 kN/m
A
1
R_A
x
2
B
2
x
R_B



Estimate the reactions at supports:

$\Sigma Fy = 0$

$\Rightarrow R_A - 10 * 6 + R_B = 0$

$\Rightarrow R_A + R_B = 60 \rightarrow (1)$

$\Sigma MA = 0$

$\Rightarrow - 10 * 6 * 3 + R_B * 9 = 0$

$\Rightarrow R_B * 9 = 180$

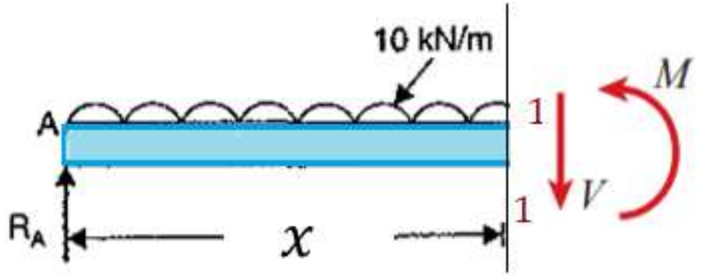
$\Rightarrow R_B = 20 \text{ kN} \rightarrow (2)$

Substitute (2) in (1)

$\Rightarrow R_A = 60 - R_B$

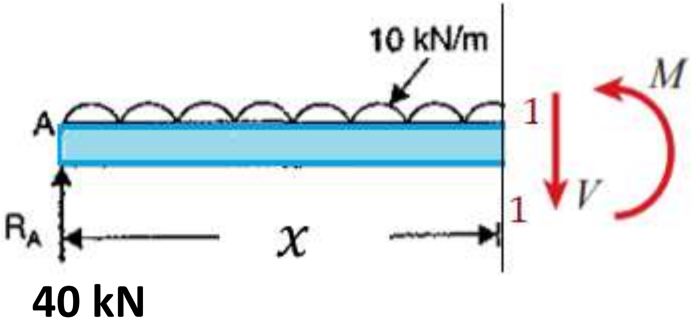
$\Rightarrow R_A = 60 – 20$

$\Rightarrow R_A = 40 \text{ kN}$


10 kN/m
A
1
1
M
V
R_A
x

Free body diagram of section1: 0< x< 6 m

when x=0 section coincides with A, when x=6 section coincides with C



$\Sigma F_y = 0$

$R_A - 10 * x - V = 0$

$V = R_A - 10 * x$

$V = 40 - 10 * x$

$\Sigma M_{1-1} = 0$

$- R_A * x + 10. * x . \frac{x}{2} + M = 0$

$M = 40 * x - 10 * x * \frac{x}{2} = 40 * x - 10 * \frac{x^2}{2}$

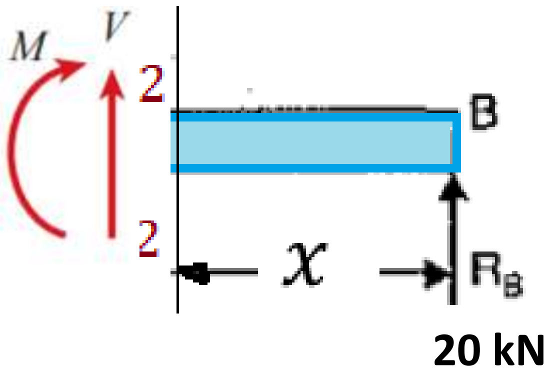
0< x<2 Section1-1	Shear force (kN) $V = 40 - 10 * x$	Bending Moment (kN-m) $M = 40 * x - 10 * \frac{x^2}{2}$
x=0 (at A)	$V_A = 40 - 10 * 0 = 40$	$M_A = 40 * 0 - 10 * \frac{0^2}{2} = 0$
x= 6 m (at C)	$V_C = 40 - 10 * 6 = -20$	$M_C = 40 * 6 - 10 * \frac{6^2}{2} = 60$
x= 4 m (at D)	V=0	B.M is maximum $M_D = 40 * 4 - 10 * \frac{4^2}{2} = 80$

V = 0

$40 - 10 * x = 0$

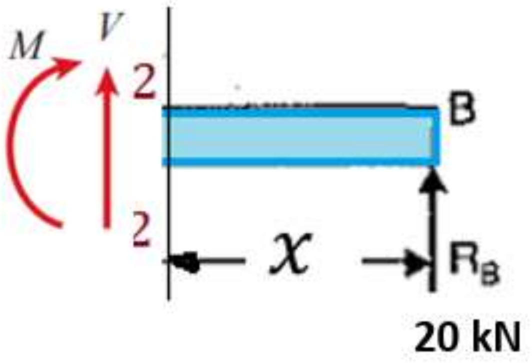
$10 * x = 40$

x = 4 m


M
V
2
2
B
x
R_B
20 kN

Free body diagram of section2: 0< x< 3 m

when x=0 section coincides with B, when x=3 section coincides with C



$$\Sigma F_y = 0$$

$R_B + V = 0$

$V = -R_B$

$V = -20$ kN

$$\Sigma M_{2-2} = 0$$

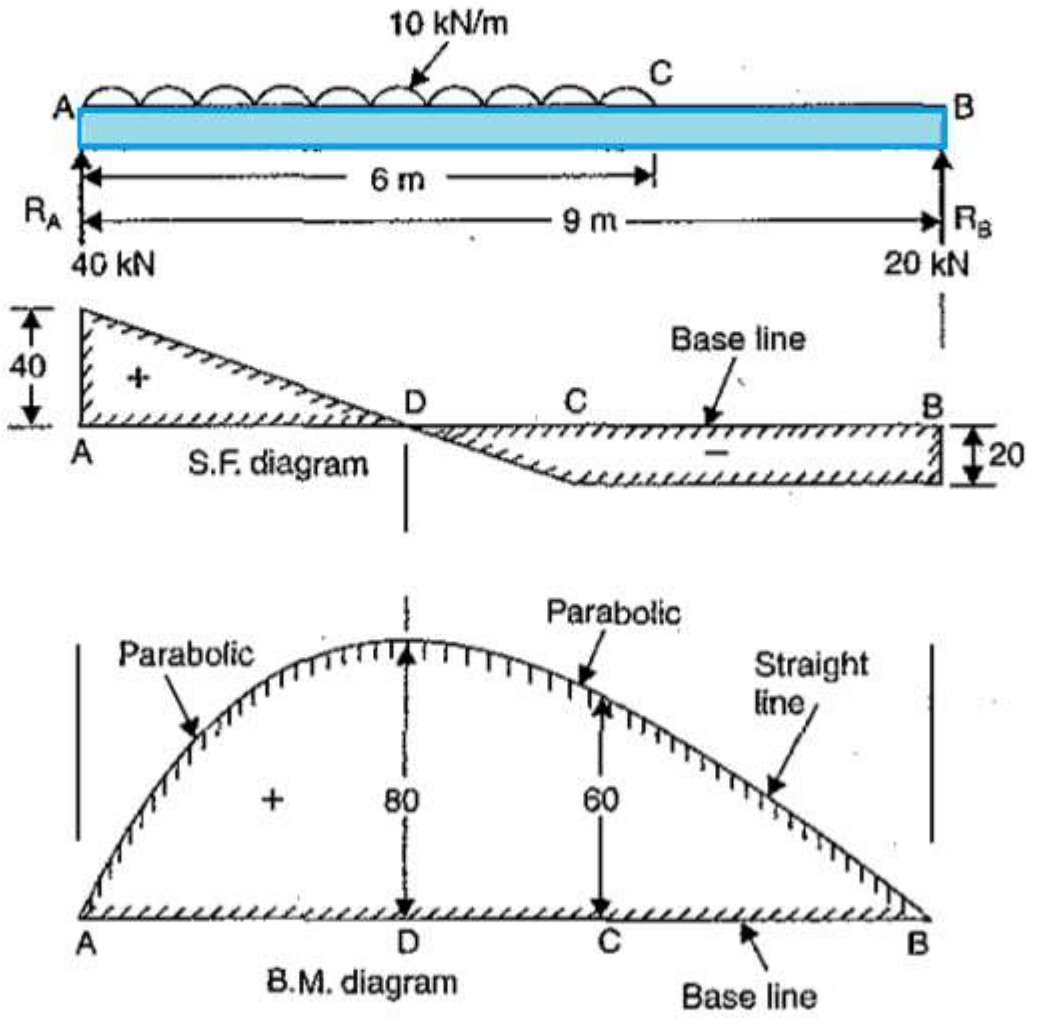
$-M + R_B * x = 0$

$M = 20 * x$

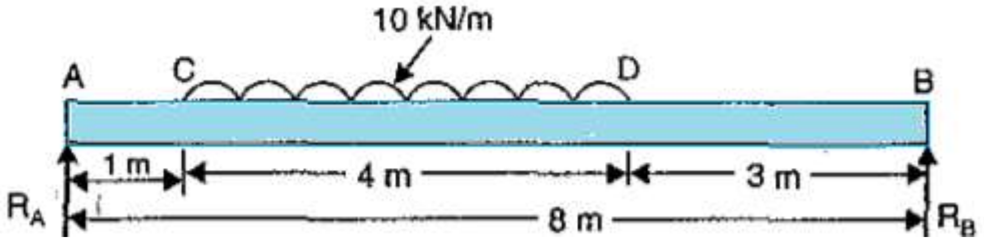
0< x< 3 Section2-2	Shear force (kN) V=-20	Bending Moment (kN-m) $M = 20 * x$
x=0 (at B)	V_B=-20	M_B= 20 *0 = 0
x= 3 m (at C)	V_C=−20	M_C=20 *3 = 60

0< x<2 Section1-1	Shear force (kN) V=40 -10 *x	Bending Moment (kN-m) M= 40 *x - 10 * $\frac{x^2}{2}$
x=0 (at A)	V_A=40	M_A= 0
x= 6 m (at C)	V_C=−20	M_C= 60
x= 4 m (at D)	V=0	B.M is maximum M_D= 80

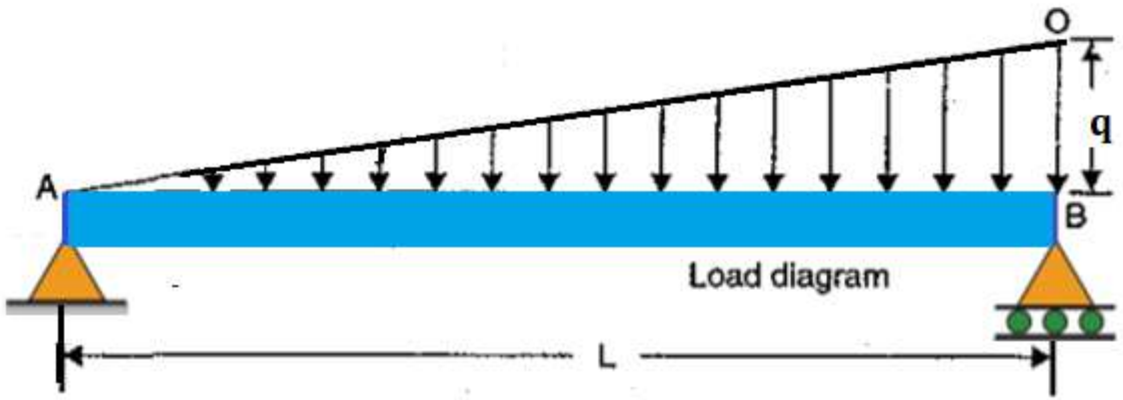
0< x< 3 Section2-2	Shear force (kN) V=-20	Bending Moment (kN-m) M= 20 * x
x=0 (at B)	V_B= -20	M_B= 0
x= 3 m (at C)	V_C=−20	M_C= 60

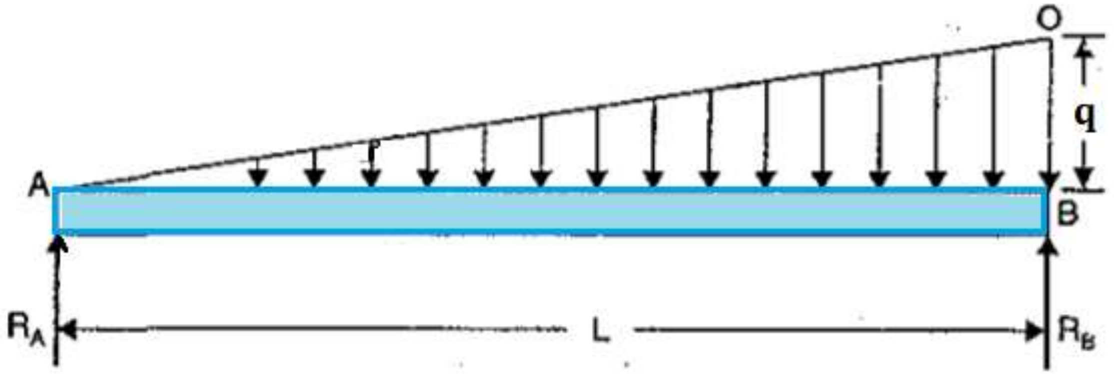


Q6. Draw the shear force and B.M. diagrams for a simply supported beam of length 8 m and carrying a uniformly distributed load of10 kN/m for a distance of 4 m as shown in Fig.



Q7. A beam AB of length L simply supported at the ends A and B and carrying a uniformly varying load from zero at end A to ‘q’ per unit length at B. Draw shear force and bending moment diagram.





Estimate the reactions at supports:

$\Sigma Fy = 0$

$\Rightarrow R_A - \frac{1}{2} qL + R_B = 0$

$\Rightarrow R_A + R_B = \frac{qL}{2} \quad \rightarrow (1)$

$\Sigma MA = 0$

$\Rightarrow -\frac{1}{2} qL \left(\frac{2L}{3}\right) + R_B * L = 0$

$\Rightarrow R_B * L = \left(\frac{qL^2}{3}\right)$

$\Rightarrow R_B = \frac{qL}{3} \quad \rightarrow (2)$

Substitute (2) in (1)

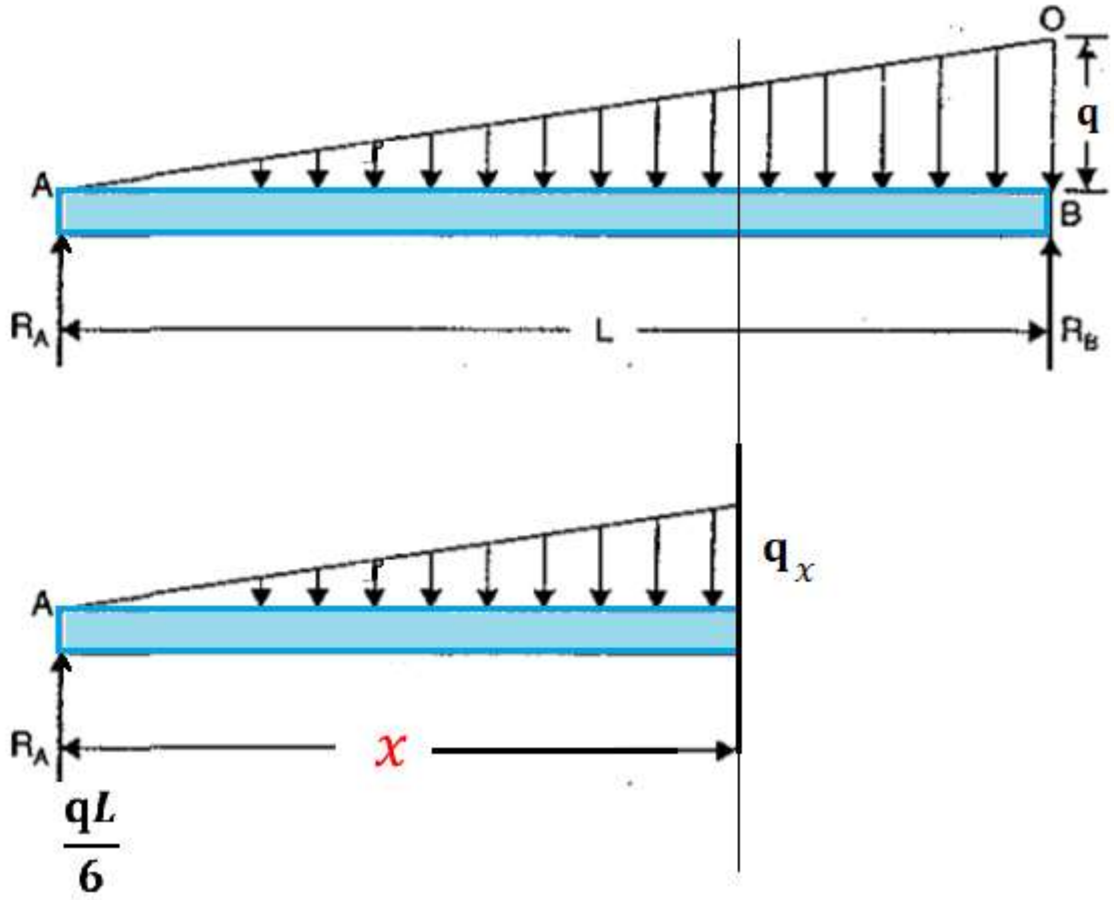
$\Rightarrow R_A = \frac{qL}{2} - R_B$

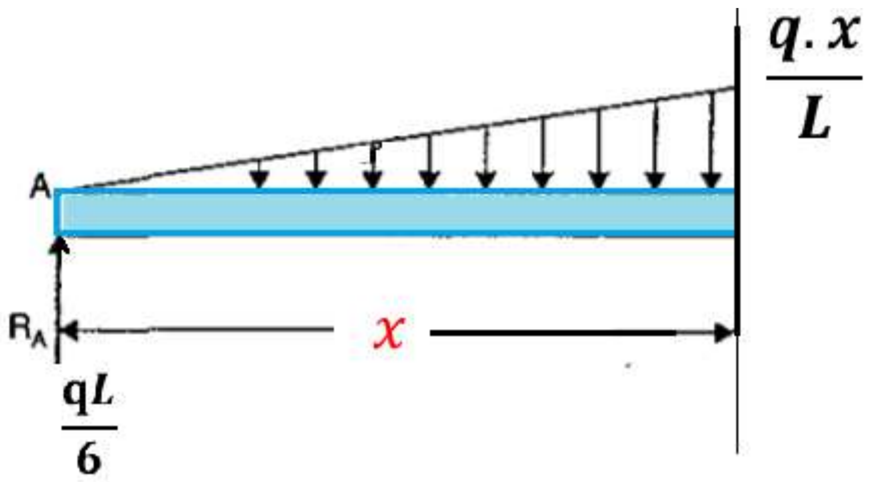
$\Rightarrow R_A = \frac{qL}{2} - \frac{qL}{3}$

$\Rightarrow R_A = \frac{qL}{6}$

$$\frac{q}{L} = \frac{q_x}{x}$$

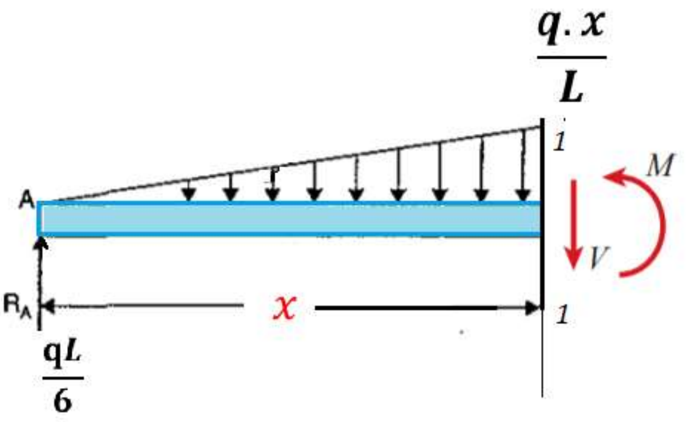
$$\Rightarrow q_x = \frac{q.x}{L}$$




$\frac{q.x}{L}$
A
R_A
x
$\frac{qL}{6}$

Free body diagram of section1: $0< x < L$

when x=0 section coincides with A, when x=a section coincides with C



$\Sigma F_y = 0$

$$R_A - \frac{1}{2} . \frac{q.x}{L} x - V = 0$$

$$V = R_A - \frac{1}{2} . \frac{q.x}{L} x$$

$$V = \frac{qL}{6} - \frac{q.x^2}{2L}$$

$0< x< L$ Section1-1	Shear force $V= \frac{qL}{6} - \frac{q.x^2}{2L}$	Bending Moment $M = \frac{qL}{6} . x - \frac{q.x^3}{6L}$
$x=0$ (at A)	$V_A=\frac{qL}{6}$	$M_A= \frac{qL}{6} . 0 - \frac{q.0^3}{6L} = 0$
$x= L$ (at B)	$V_B=\frac{qL}{6} - \frac{q.L^2}{2L} = - \frac{q.L}{3}$	$M_B=\frac{qL}{6} . L - \frac{q.L^3}{6L} = 0$

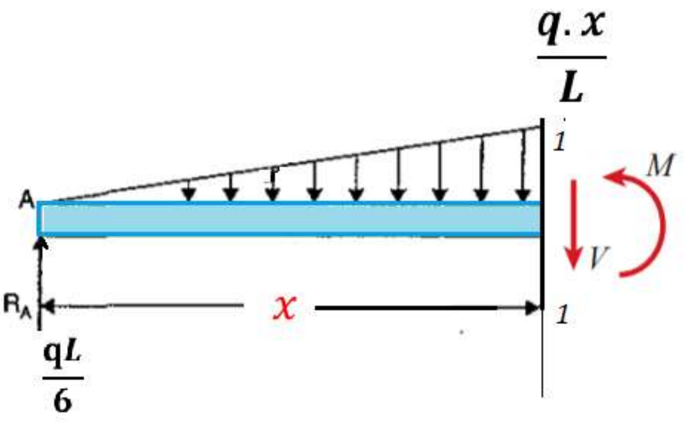
$\Sigma M_{1-1} = 0$

$$- \frac{qL}{6} . x + \frac{1}{2} . \frac{q.x}{L} x . \frac{x}{3} + M = 0$$

$$M= \frac{qL}{6} . x - \frac{q.x^3}{6L}$$

Free body diagram of section1: 0< x < L

when x=0 section coincides with A, when x=a section coincides with C



$\Sigma F_y = 0$

$R_A - \frac{1}{2} . \frac{q.x}{L} x - V = 0$

$V = R_A - \frac{1}{2} . \frac{q.x}{L} x$

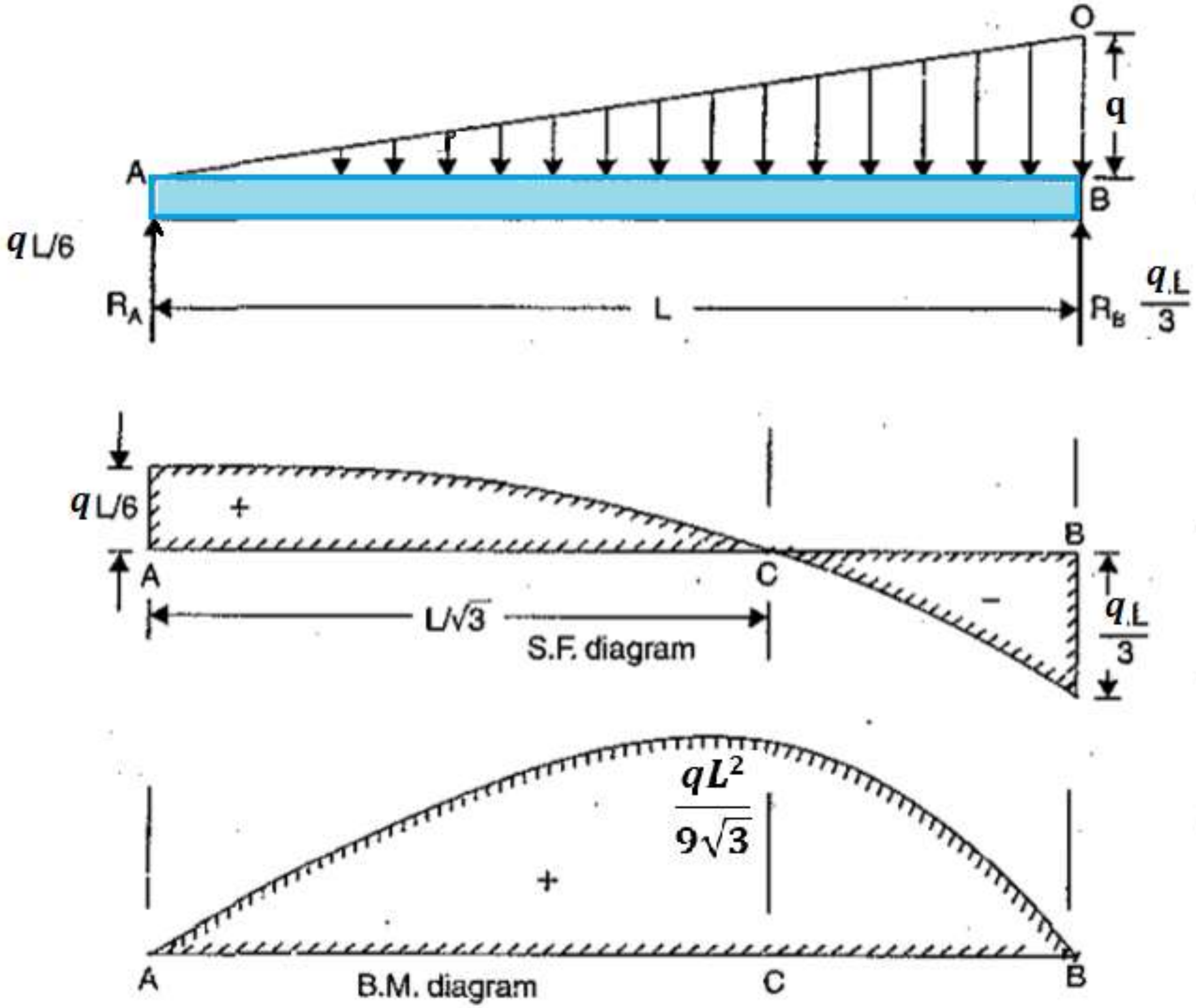
$V = \frac{qL}{6} - \frac{q.x^2}{2L}$

$\Sigma M_{1-1} = 0$

$- \frac{qL}{6} . x + \frac{1}{2} . \frac{q.x}{L} x . \frac{x}{3} + M = 0$

$M = \frac{qL}{6} . x - \frac{q.x^3}{6L}$

0< x< L Section1-1	Shear force $V = \frac{qL}{6} - \frac{q.x^2}{2L}$	Bending Moment $M = \frac{qL}{6} . x - \frac{q.x^3}{6L}$
x=0 (at A)	$V_A = \frac{qL}{6}$	$M_A = \frac{qL}{6} . 0 - \frac{q.0^3}{6L} = 0$
x= L (at B)	$V_B = \frac{qL}{6} - \frac{q.L^2}{2L} = -\frac{q.L}{3}$	$M_B = \frac{qL}{6} . L - \frac{q.L^3}{6L} = 0$
x= L/$\sqrt{3}$ (at C)	$V_C = 0$	$M_C = \frac{qL}{6} . \frac{L}{\sqrt{3}} - \frac{q.(\frac{L}{\sqrt{3}})^3}{6L} = \frac{qL^2}{9\sqrt{3}}$



O
q
A
B
q L/6
R_A
L
R_B $\frac{q L}{3}$
q L/6
+
B
A
C
−
$\frac{q L}{3}$
L/√3
S.F. diagram
$\frac{qL^2}{9\sqrt{3}}$
+
A
B.M. diagram
C
B