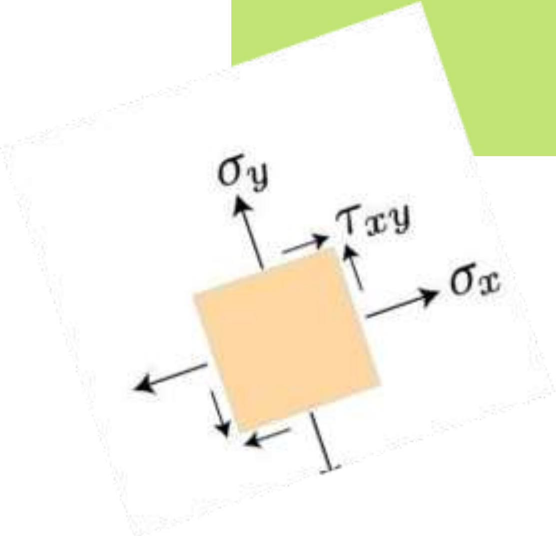
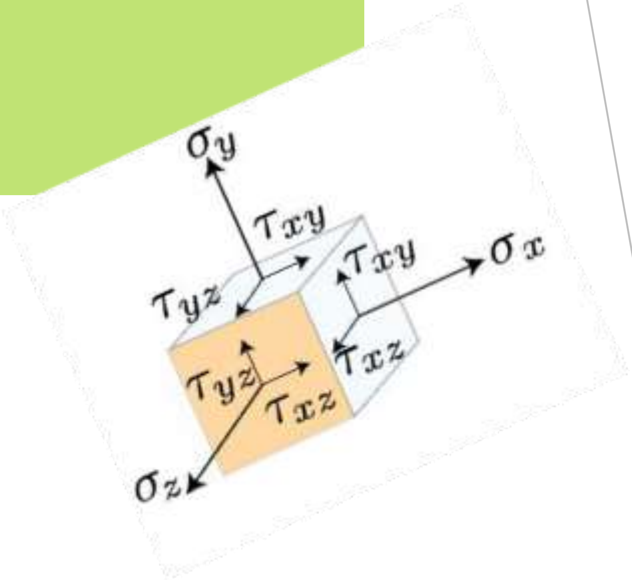
JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – II & IV
Branch – Civil Engineering
Subject –Strength of Material
Unit- 2
Name of Faculty – Sumit Saini (Assistant Professor)

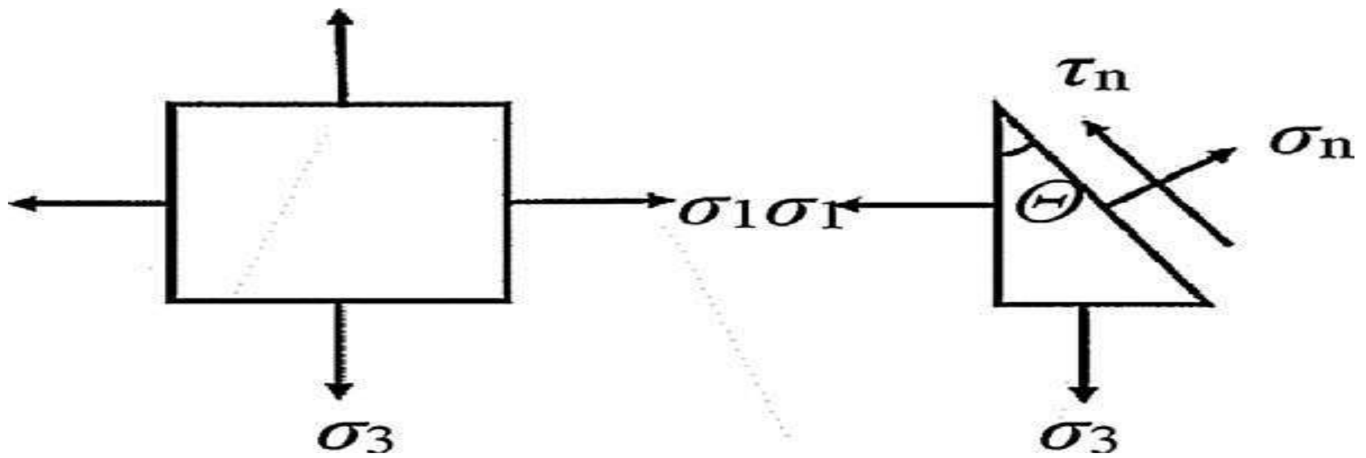
Stress Transformation





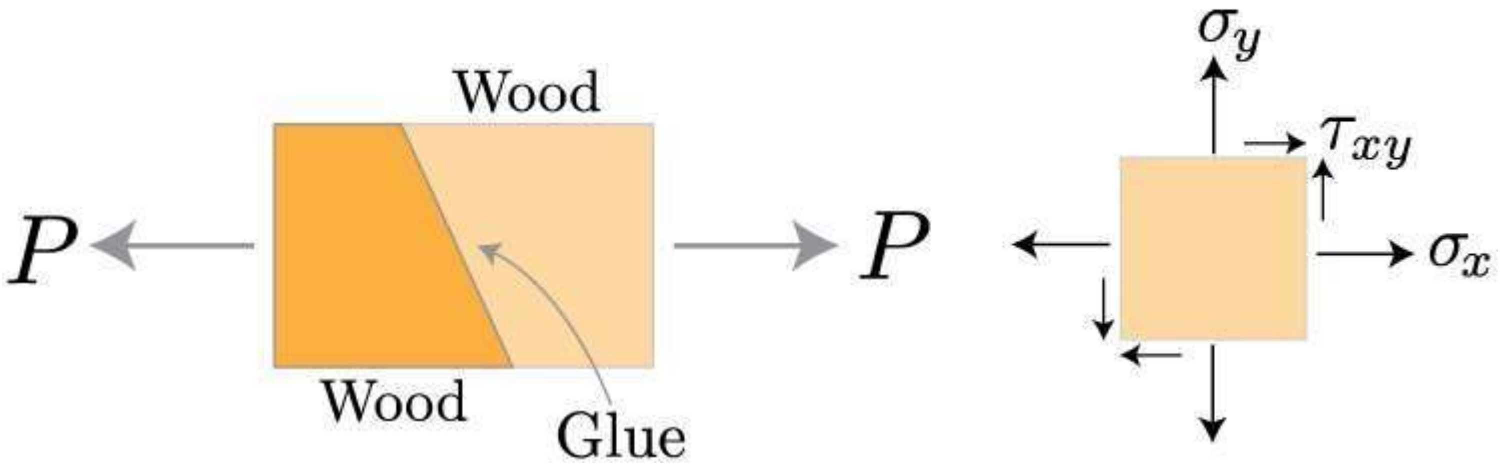
What is Stress Transformation Actually?

Stress transformation is a method of rotating the normal **stresses** and shear on a cross-section element to a new coordinate system. Typically this is to evaluate the **stress** and shear with the principal **stresses**.



What is Stress Transformation Actually?

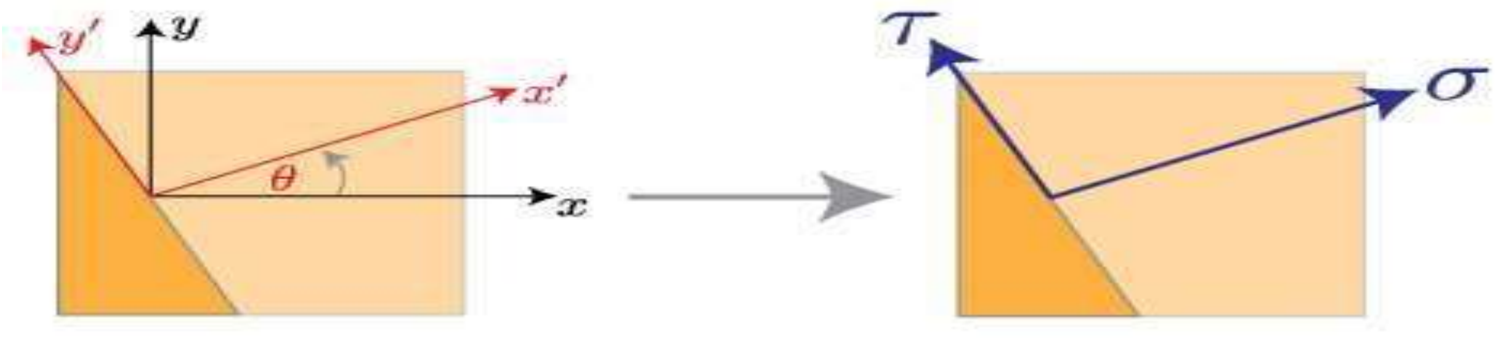
Take a look at the image below



Two pieces of wood, cut at an angle, and glued together. The wood is being pulled apart by a tensile force *P*. How do we know if the glued joint can sustain the resultant stress that this force produces? We need to calculate the normal and shear stresses perpendicular and parallel to the join

What is Stress Transformation Actually?

Therefore, we need to rotate, or transform, the coordinates associated with the force P to the direction associated with the angle of the glued joint. Then, we can evaluate the stresses along these new directions, x' and y'



Once we've rotated the coordinate system, we need to transform the forces acting in the old coordinate frame to this new coordinate frame

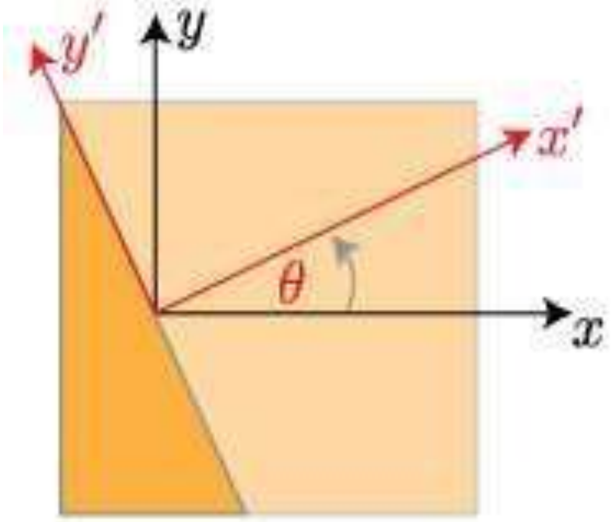
What is Stress Transformation Actually?

The final result for the normal and shear stresses in our new coordinate system (denoted by theta, which is a counterclockwise rotation from the *x* axis to the *x′* axis) is given by

$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



Principal and Shear stresses

Do You Know?
Brittle Materials Fail in Norman i.e. AT 0º

- Principal Stresses

There will be 2 normal stresses . Max and Min Stresses .These normal stresses will occur when the shear stress is zero, which means

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$$(\sigma - \sigma_{avg})^2 + \cancel{\tau^2} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2$$

Skipping to Final Equation of Normal Stress ……………………….

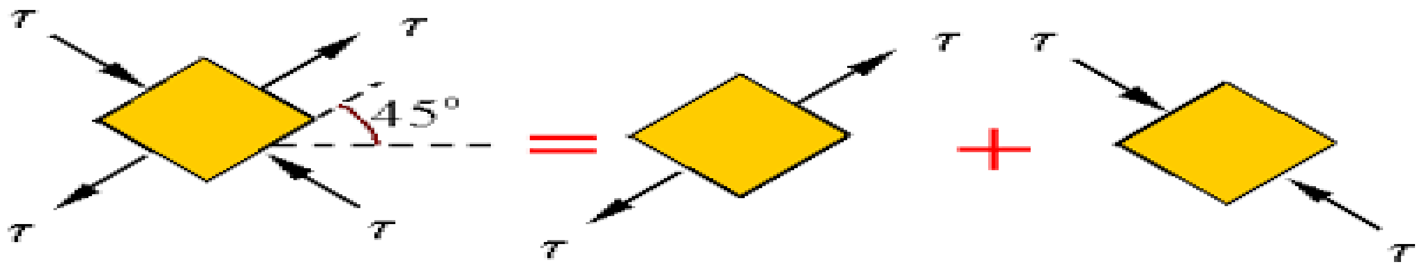
$$\boxed{\sigma_{1,2} = \frac{\sigma_x + \sigma_y}{2} \pm \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}}$$

Principal and Shear stresses

- Shear Stress

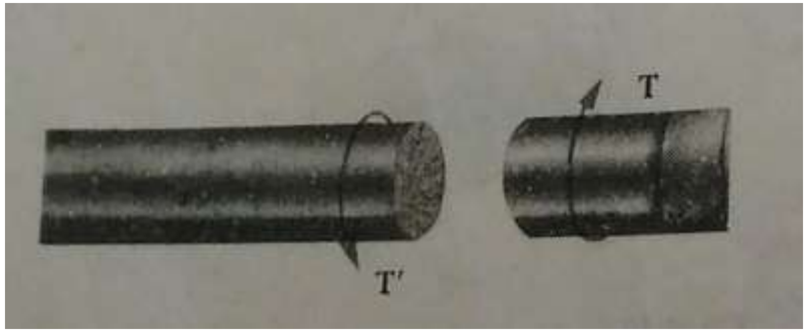
This Stress will have Max value at 45^0 . There will be 2 Shear Stresses both Equal and Opposite in direction.

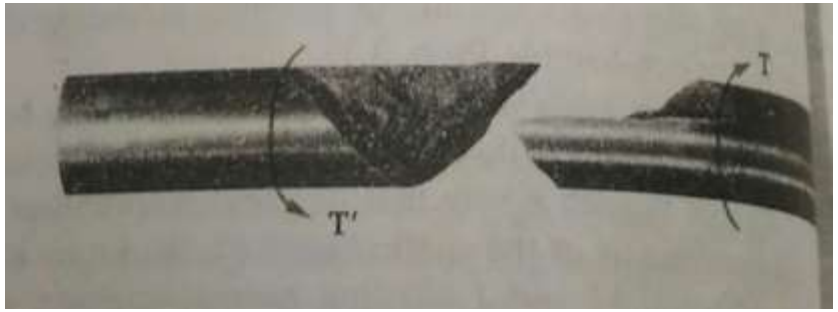
$$\tau_{\max} = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^2}$$



Do You Know?
Ductile Materials Fail in Shear i.e. Diagonally

Principal and Shear stresses





Principal Failure

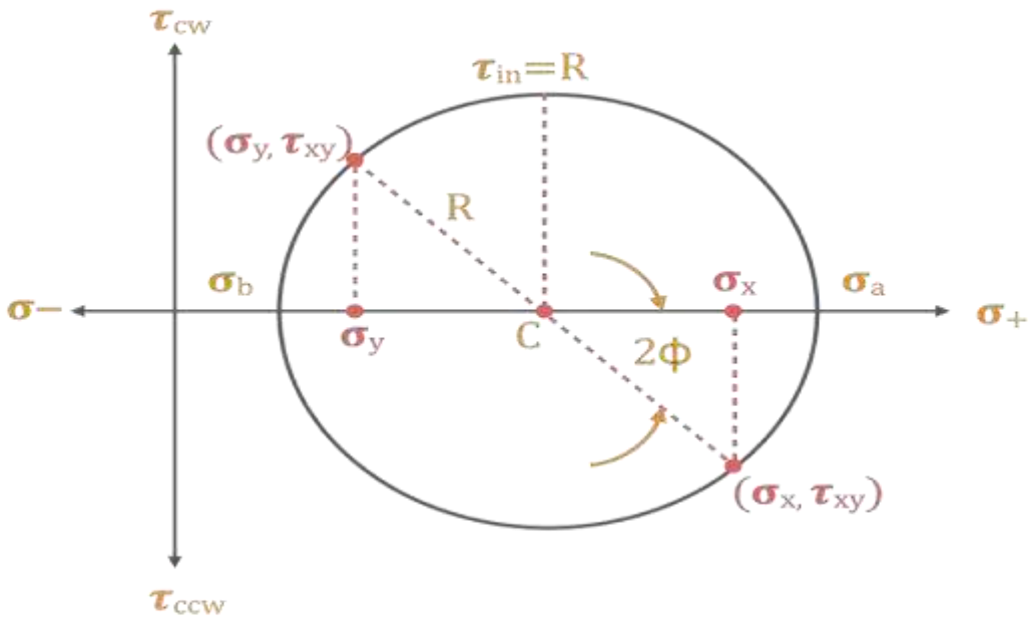
Cast Iron

Shear Failure

Ductile Steel Bar

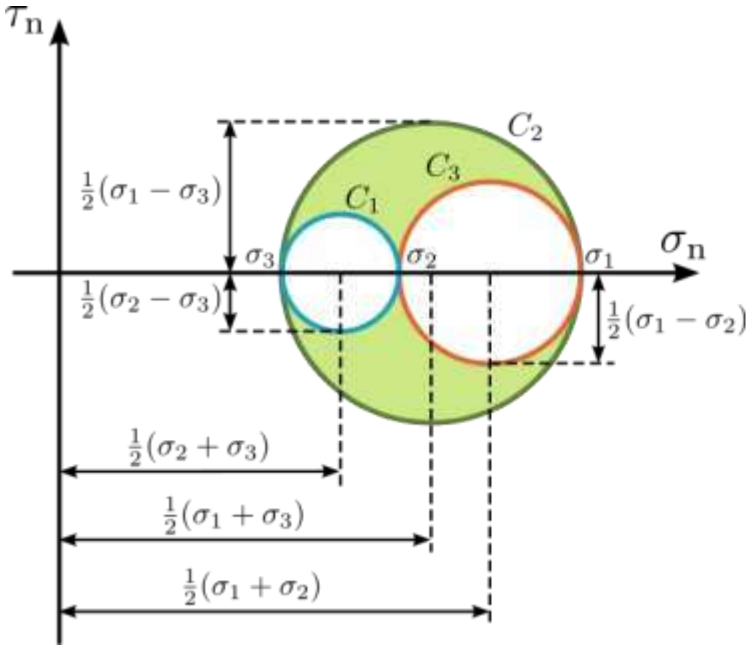
MOHR's Circle

A graphical method to represent the plane stress (also strain) relations. It's a very effective way to visualize a specific point's stress states, stress transformations for an angle, principal and maximum shear stresses .



MOHR's Circle

The Mohr circle is used to determine graphically the stress components acting on a rotated coordinate system, i.e., acting on a differently oriented plane passing through that point



MOHR's Circle

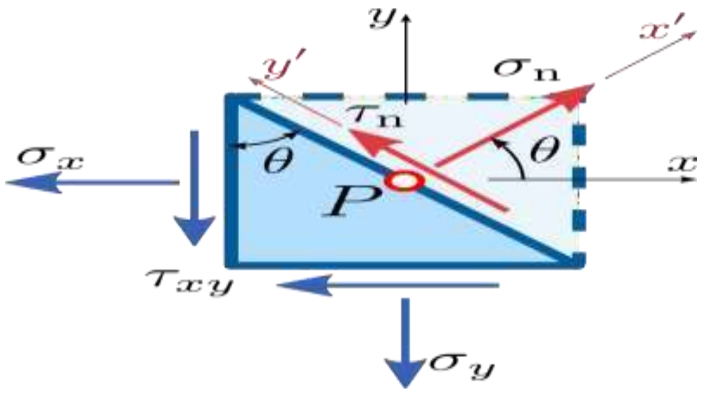
- Equation of the Mohr circle

From equilibrium of forces on the infinitesimal element, the magnitudes of the normal stress and the shear stress are given by:

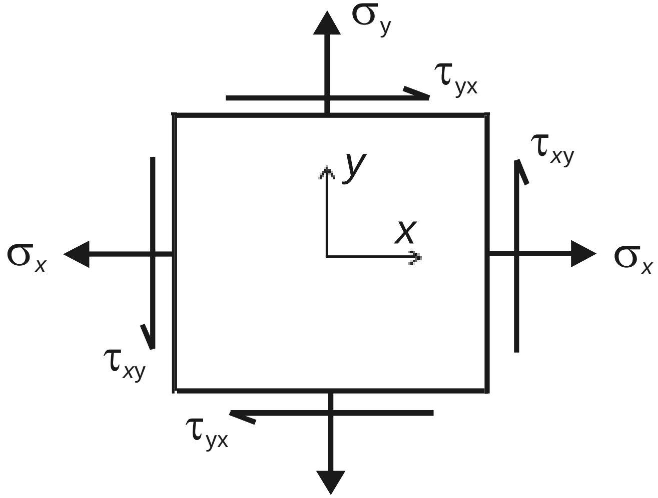
$$\sigma_{x'} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

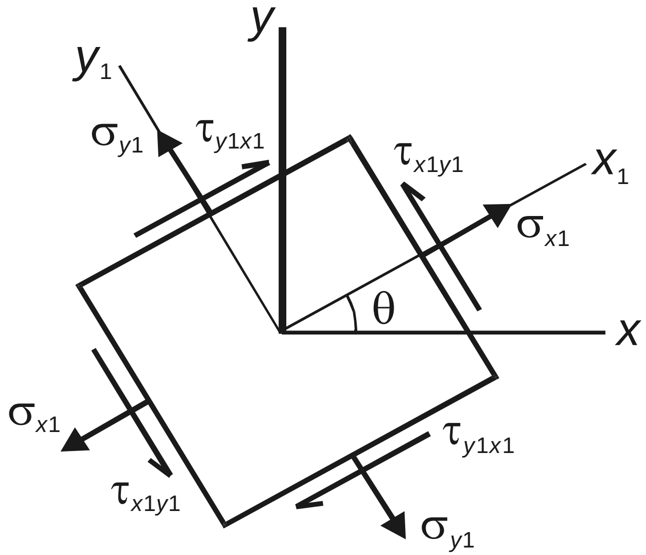
$$\sigma_{y'} = \frac{\sigma_x + \sigma_y}{2} - \frac{\sigma_x - \sigma_y}{2}\cos 2\theta - \tau_{xy}\sin 2\theta$$

$$\tau_{x'y'} = -\frac{\sigma_x - \sigma_y}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$



Stress Transformation Equations





$$\sigma_{x1} = \frac{\sigma_x + \sigma_y}{2} + \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x1y1} = -\frac{\left(\sigma_x - \sigma_y\right)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

If we vary θ from 0° to 360°, we will get all possible values of σ_{x1} and τ_{x1y1} for a given stress state. It would be useful to represent σ_{x1} and τ_{x1y1} as functions of θ in graphical form.

To do this, we must re-write the transformation equations.

$$\sigma_{x1} - \frac{\sigma_x + \sigma_y}{2} = \frac{\sigma_x - \sigma_y}{2}\cos 2\theta + \tau_{xy}\sin 2\theta$$

$$\tau_{x1y1} = -\frac{\left(\sigma_x - \sigma_y\right)}{2}\sin 2\theta + \tau_{xy}\cos 2\theta$$

Eliminate θ by squaring both sides of each equation and adding the two equations together.

$$\left(\sigma_{x1} - \frac{\sigma_x + \sigma_y}{2}\right)^2 + \tau_{x1y1}^{\;2} = \left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}$$

Define σ_{avg} and R

$$\sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} \qquad R = \sqrt{\left(\frac{\sigma_x - \sigma_y}{2}\right)^2 + \tau_{xy}^{\;2}}$$

Substitue for σ_{avg} and R to get

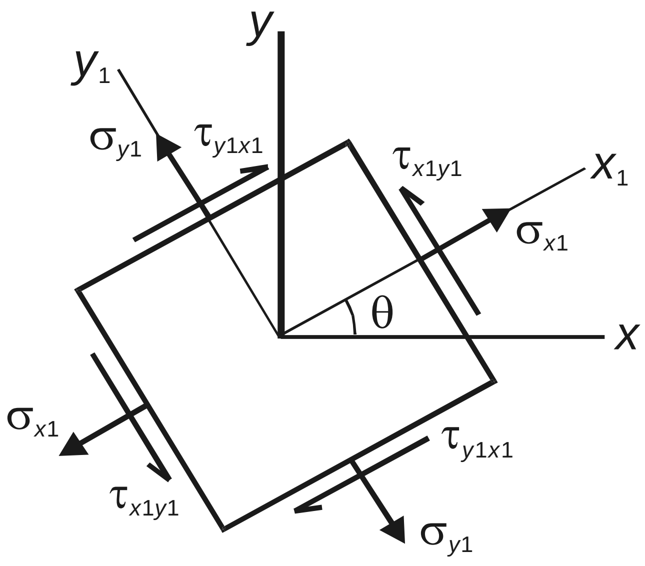
$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}^{\ 2} = R^2$$

which is the equation for a **circle** with centre (σ_{avg},0) and radius R.

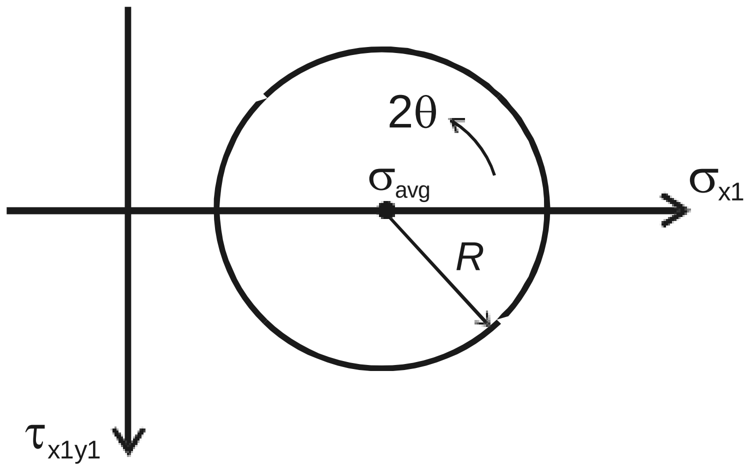
This circle is usually referred to as **Mohr's circle**, after the German civil engineer Otto Mohr (1835-1918). He developed the graphical technique for drawing the circle in 1882.

The construction of Mohr's circle is one of the few graphical techniques still used in engineering. It provides a simple and clear picture of an otherwise complicated analysis.

Sign Convention for Mohr's Circle



$$\left(\sigma_{x1} - \sigma_{avg}\right)^2 + \tau_{x1y1}^{\;2} = R^2$$



Notice that shear stress is plotted as positive downward.

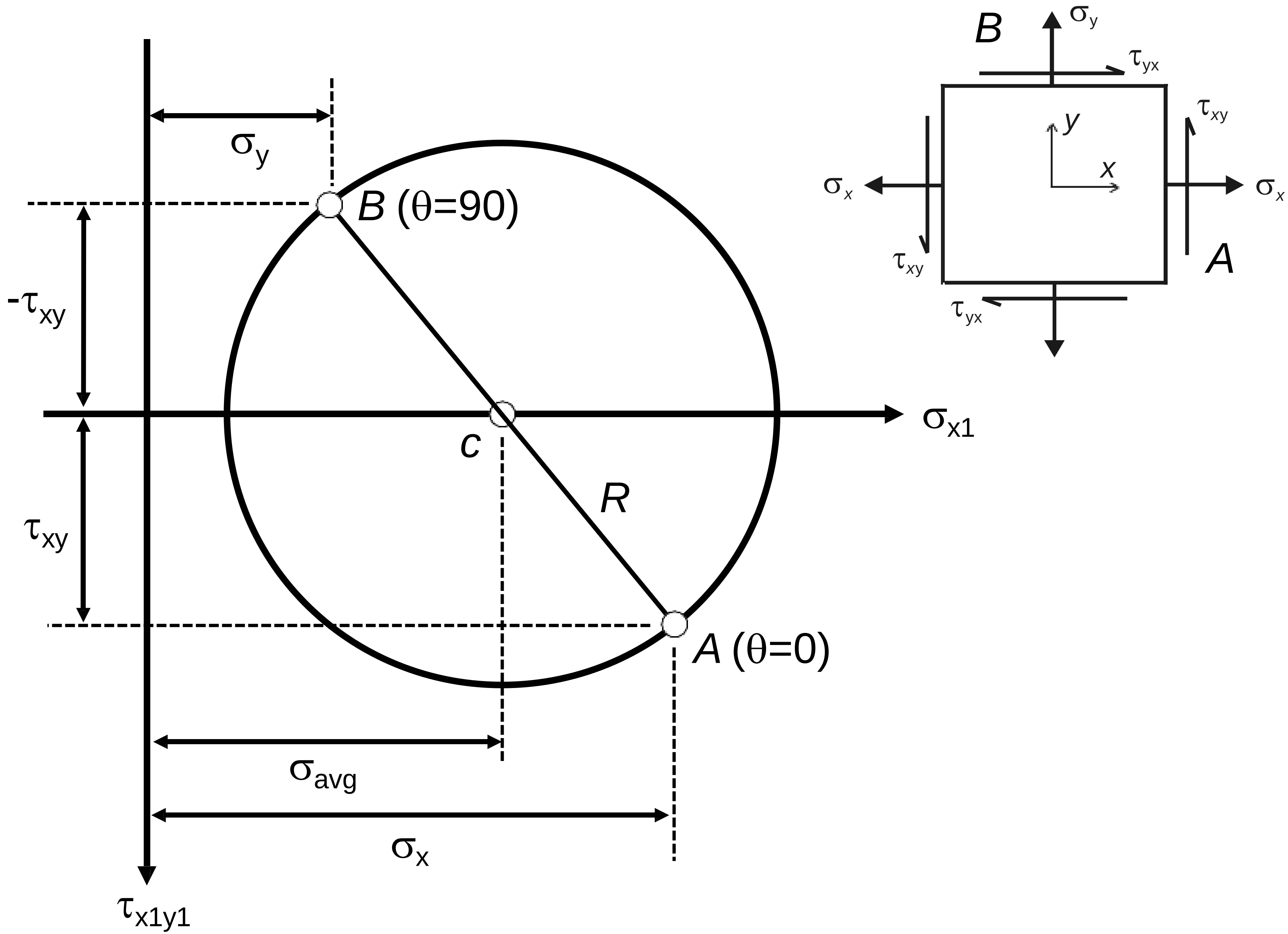
The reason for doing this is that 2θ is then positive counterclockwise, which agrees with the direction of 2θ used in the derivation of the tranformation equations and the direction of θ on the stress element.

Notice that although 2θ appears in Mohr's circle, θ appears on the stress element.

Procedure for Constructing Mohr's Circle

1. Draw a set of coordinate axes with σ_{x1} as abscissa (positive to the right) and τ_{x1y1} as ordinate (positive downward).
2. Locate the centre of the circle *c* at the point having coordinates $\sigma_{x1} = \sigma_{avg}$ and $\tau_{x1y1} = 0$.
3. Locate point *A*, representing the stress conditions on the *x* face of the element by plotting its coordinates $\sigma_{x1} = \sigma_x$ and $\tau_{x1y1} = \tau_{xy}$. Note that point *A* on the circle corresponds to $\theta = 0°$.
4. Locate point *B*, representing the stress conditions on the *y* face of the element by plotting its coordinates $\sigma_{x1} = \sigma_y$ and $\tau_{x1y1} = -\tau_{xy}$. Note that point *B* on the circle corresponds to $\theta = 90°$.
5. Draw a line from point *A* to point *B*, a diameter of the circle passing through point *c*. Points *A* and *B* (representing stresses on planes at 90° to each other) are at opposite ends of the diameter (and therefore 180° apart on the circle).
6. Using point *c* as the centre, draw Mohr's circle through points *A* and *B*. This circle has radius *R.*

(based on Gere)



σy
B (θ=90)
-τxy
τxy
σx1
c
R
A (θ=0)
σavg
σx
τx1y1
B
σy
τyx
τxy
y
x
σx
σx
τxy
A
τyx

Stresses on an Inclined Element

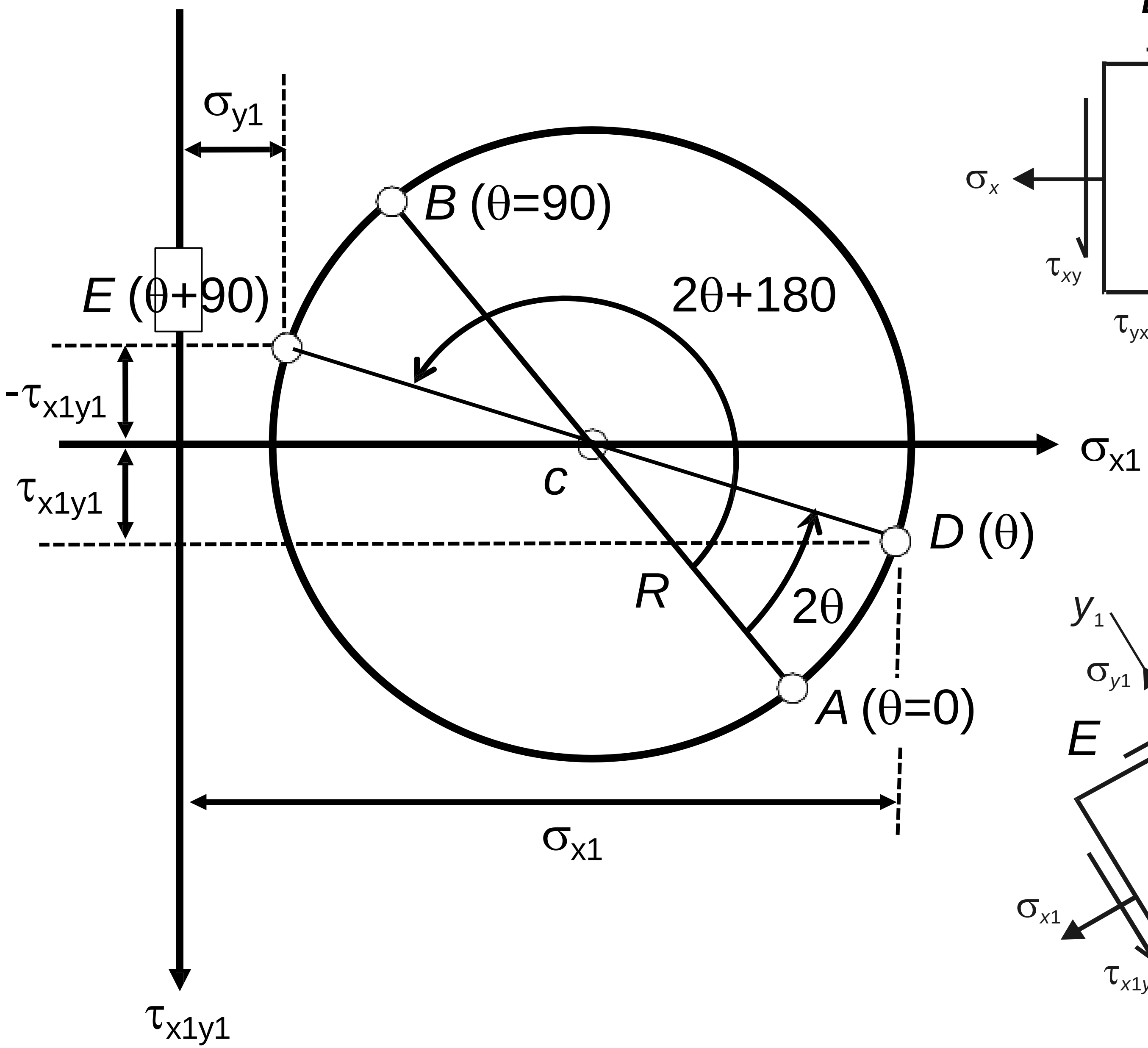
1. On Mohr's circle, measure an angle 2θ counterclockwise from radius *cA*, because point *A* corresponds to $\theta = 0$ and hence is the reference point from which angles are measured.

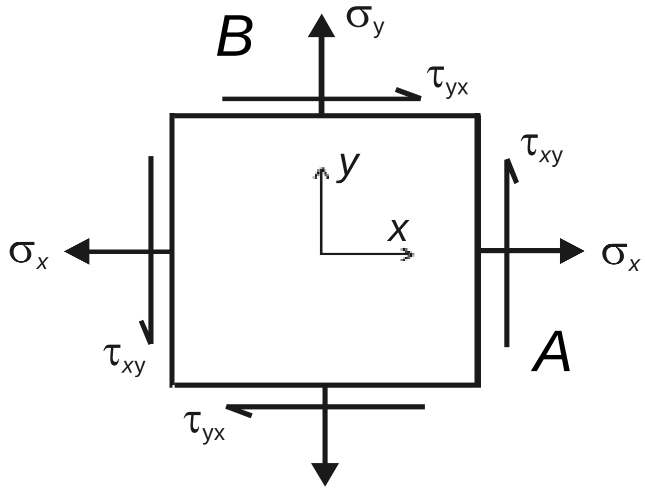
2. The angle 2θ locates the point *D* on the circle, which has coordinates σ_{x1} and τ_{x1y1}. Point *D* represents the stresses on the *x1* face of the inclined element.

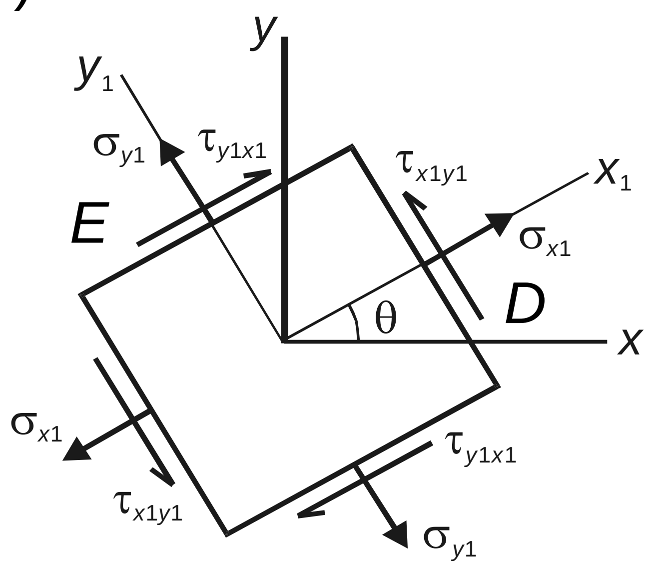
3. Point *E*, which is diametrically opposite point *D* on the circle, is located at an angle $2\theta + 180°$ from *cA* (and 180° from *cD*). Thus point *E* gives the stress on the *y1* face of the inclined element.

4. So, as we rotate the *x1y1* axes counterclockwise by an angle θ, the point on Mohr's circle corresponding to the *x1* face moves counterclockwise through an angle 2θ.

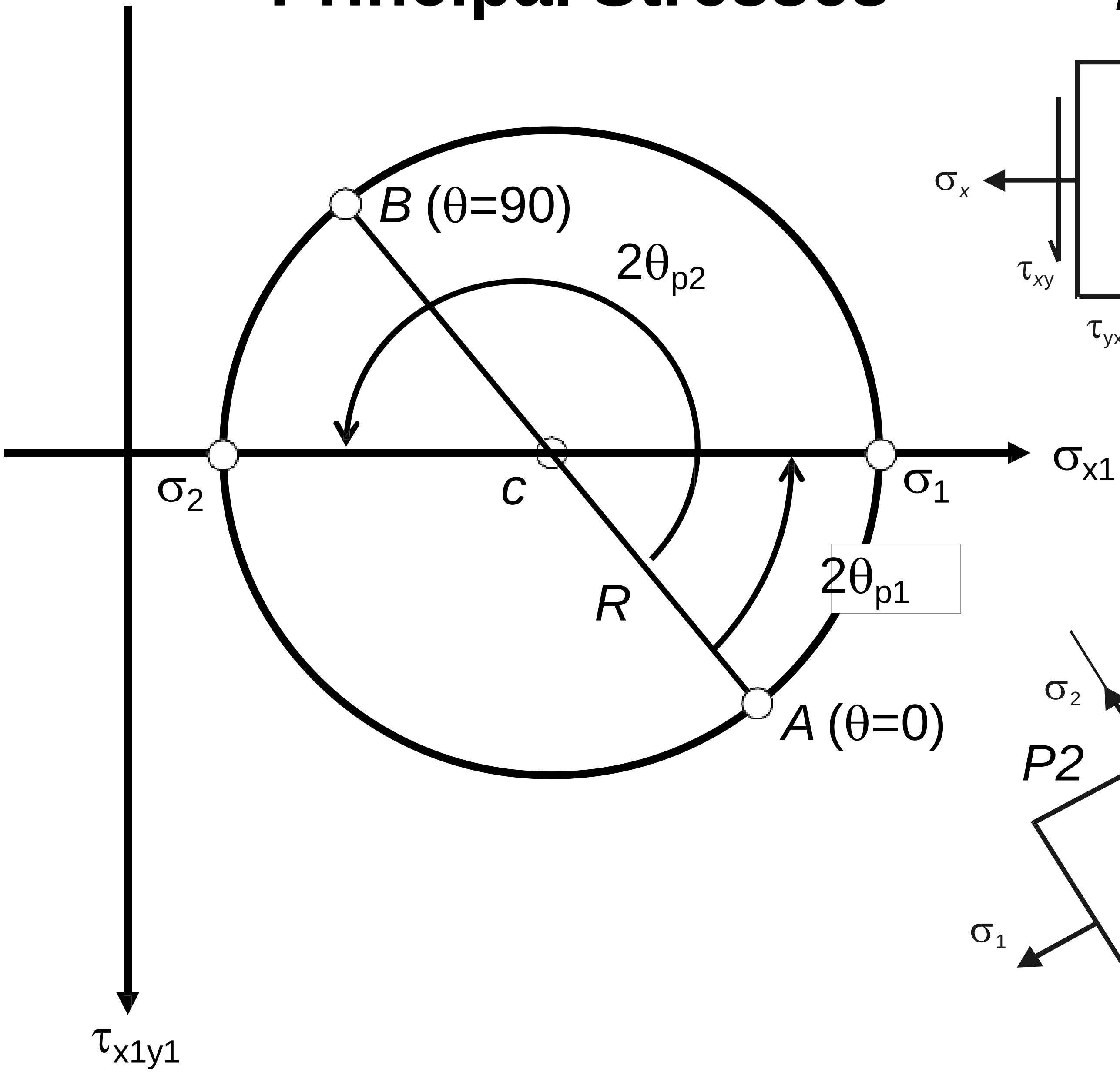
(based on Gere)

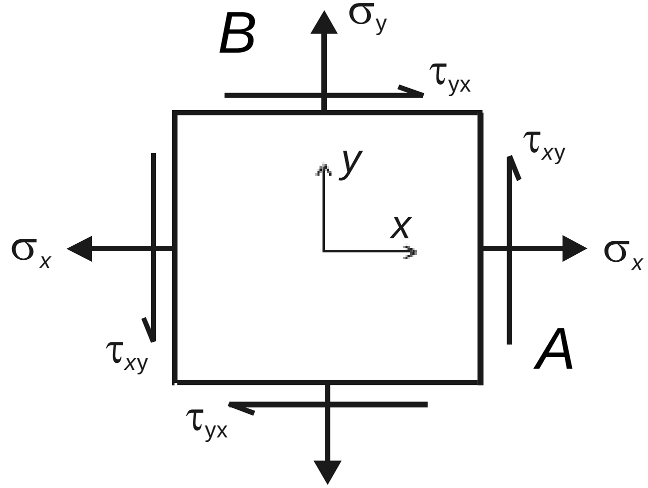

σy1
B (θ=90)
E (θ+90)
2θ+180
-τx1y1
σx1
c
τx1y1
D (θ)
R
2θ
A (θ=0)
σx1
τx1y1

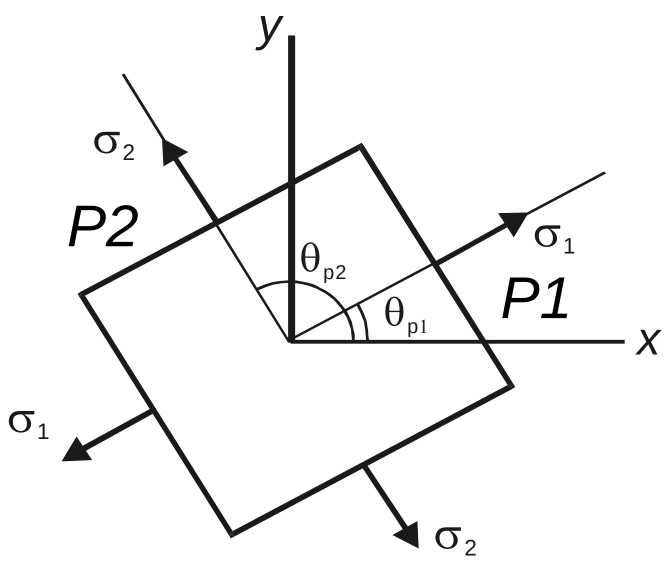

B
σy
τyx
τxy
y
x
σx
σx
τxy
A
τyx


y
y1
σy1
τy1x1
τx1y1
x1
E
σx1
D
θ
x
σx1
τy1x1
τx1y1
σy1

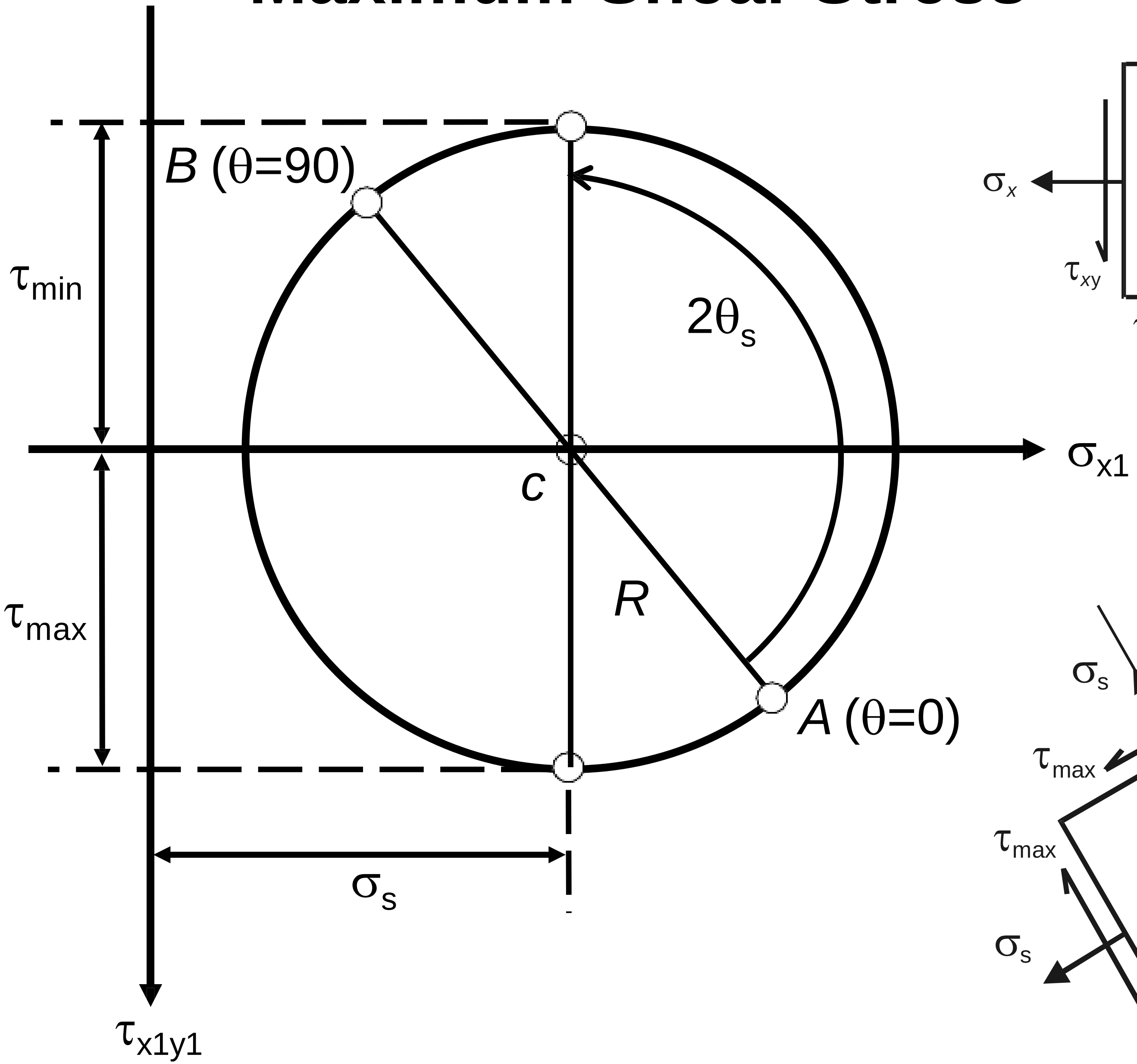
Principal Stresses

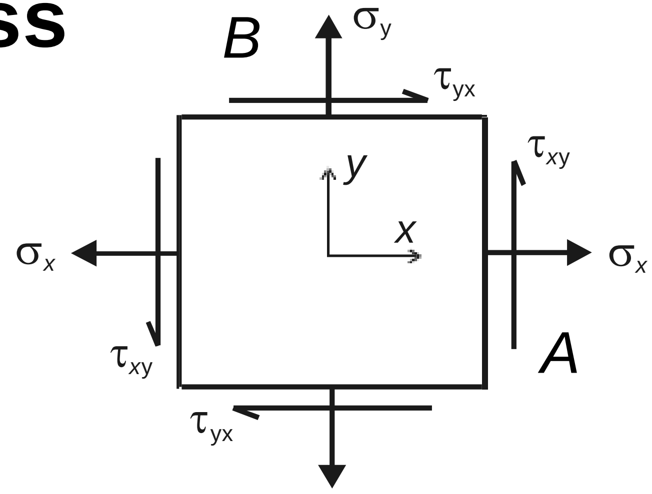




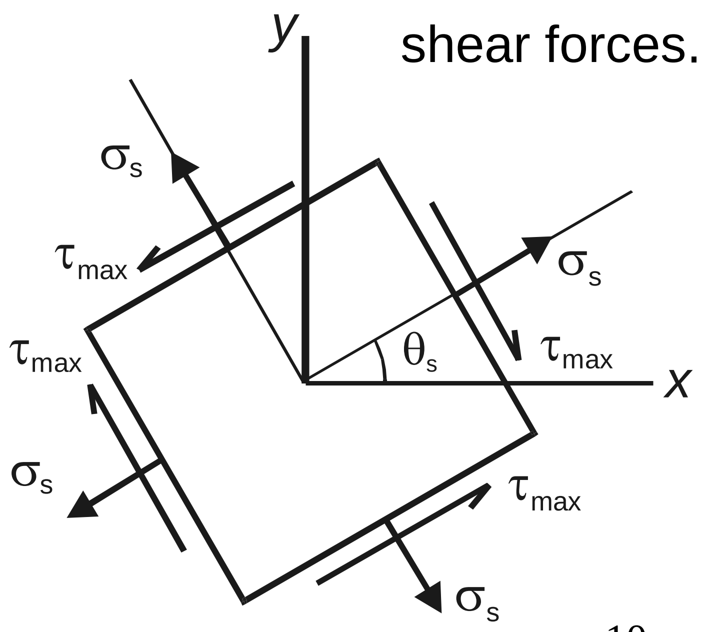


Maximum Shear Stress





Note carefully the directions of the shear forces.



Example: The state of plane stress at a point is represented by the stress element below. Draw the Mohr's circle, determine the principal stresses and the maximum shear stresses, and draw the corresponding stress elements.

$$c = \sigma_{avg} = \frac{\sigma_x + \sigma_y}{2} = \frac{-80+50}{2} = -15$$

$$R = \sqrt{\left(50 - (-15)\right)^2 + (25)^2}$$

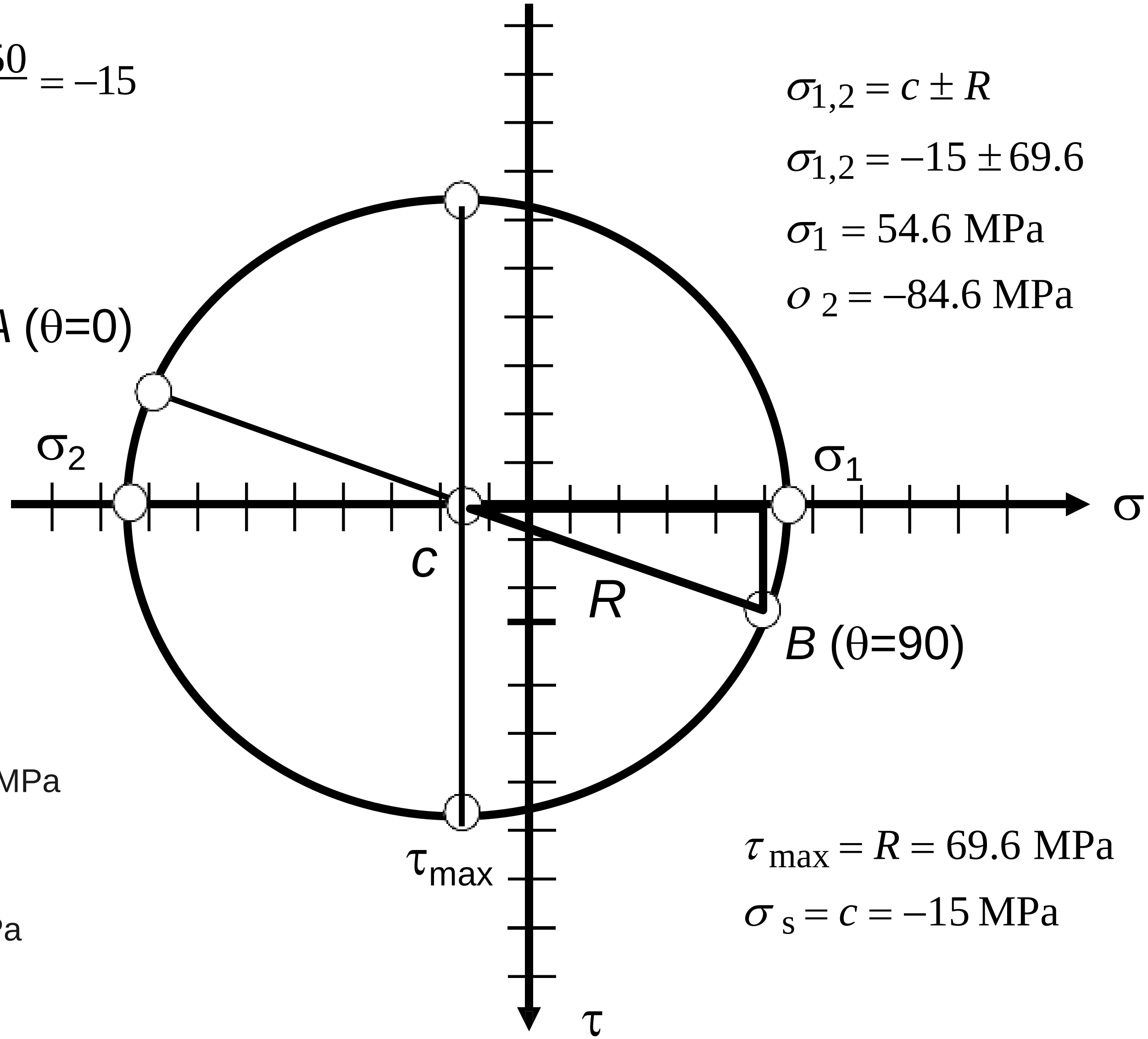
$$R = \sqrt{65^2 + 25^2} = 69.6$$

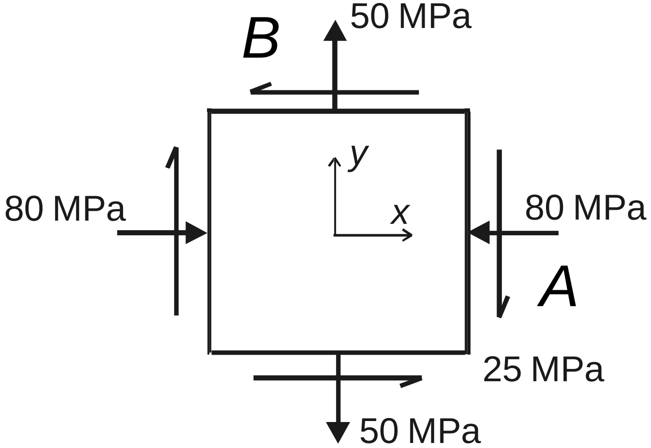
$$\sigma_{1,2} = c \pm R$$

$$\sigma_{1,2} = -15 \pm 69.6$$

$$\sigma_1 = 54.6 \text{ MPa}$$

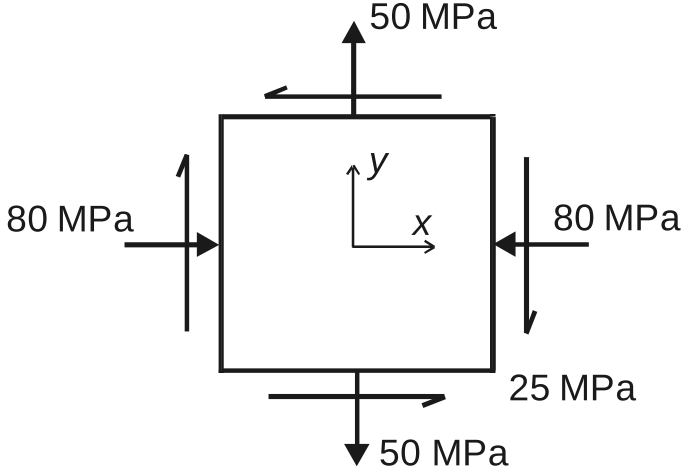
$$\sigma_2 = -84.6 \text{ MPa}$$





$$\tau_{max} = R = 69.6 \text{ MPa}$$

$$\sigma_s = c = -15 \text{ MPa}$$

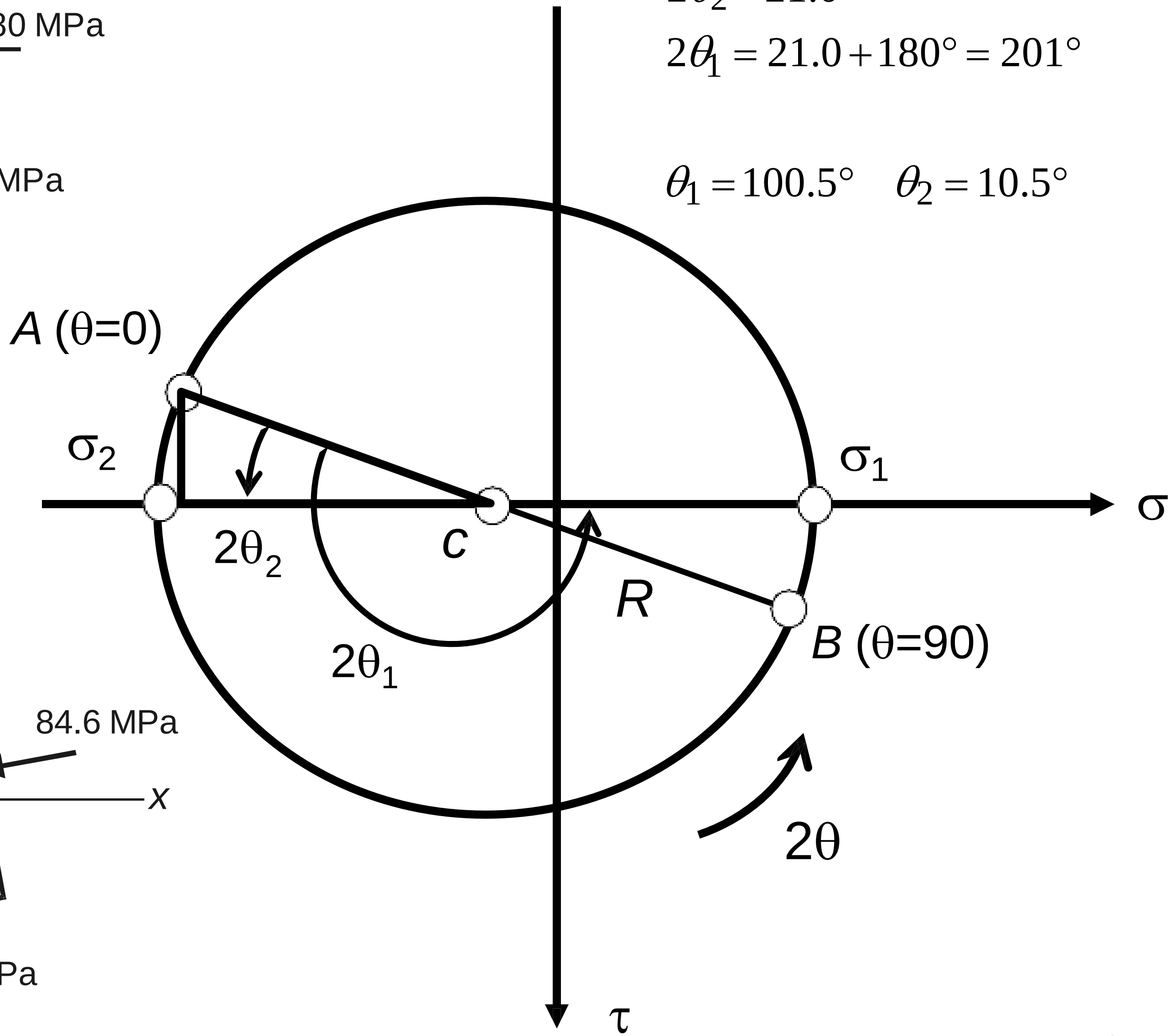


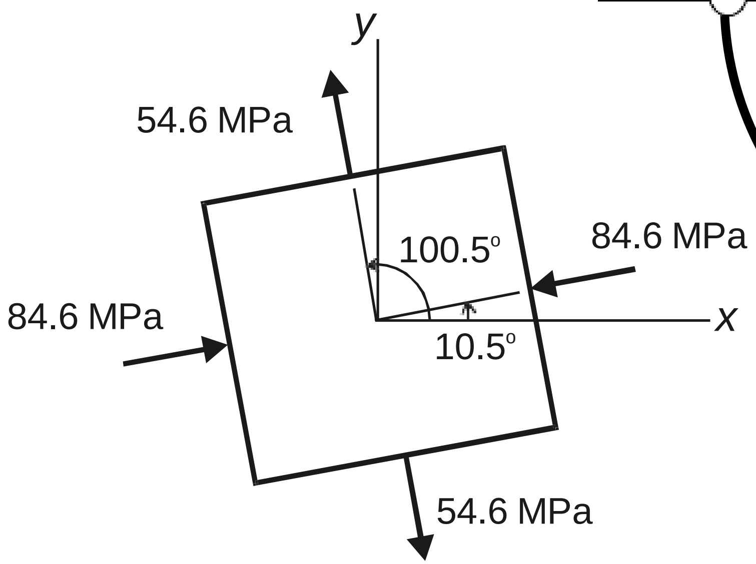
$$\tan 2\theta_2 = \frac{25}{80-15} = 0.3846$$

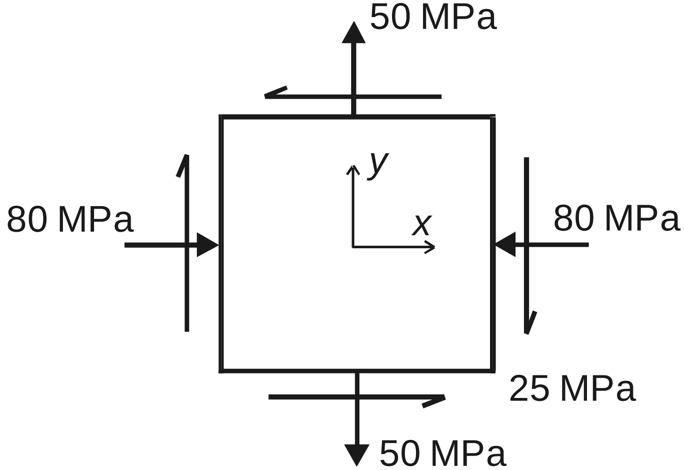
$$2\theta_2 = 21.0^\circ$$

$$2\theta_1 = 21.0 + 180^\circ = 201^\circ$$

$$\theta_1 = 100.5^\circ \quad \theta_2 = 10.5^\circ$$





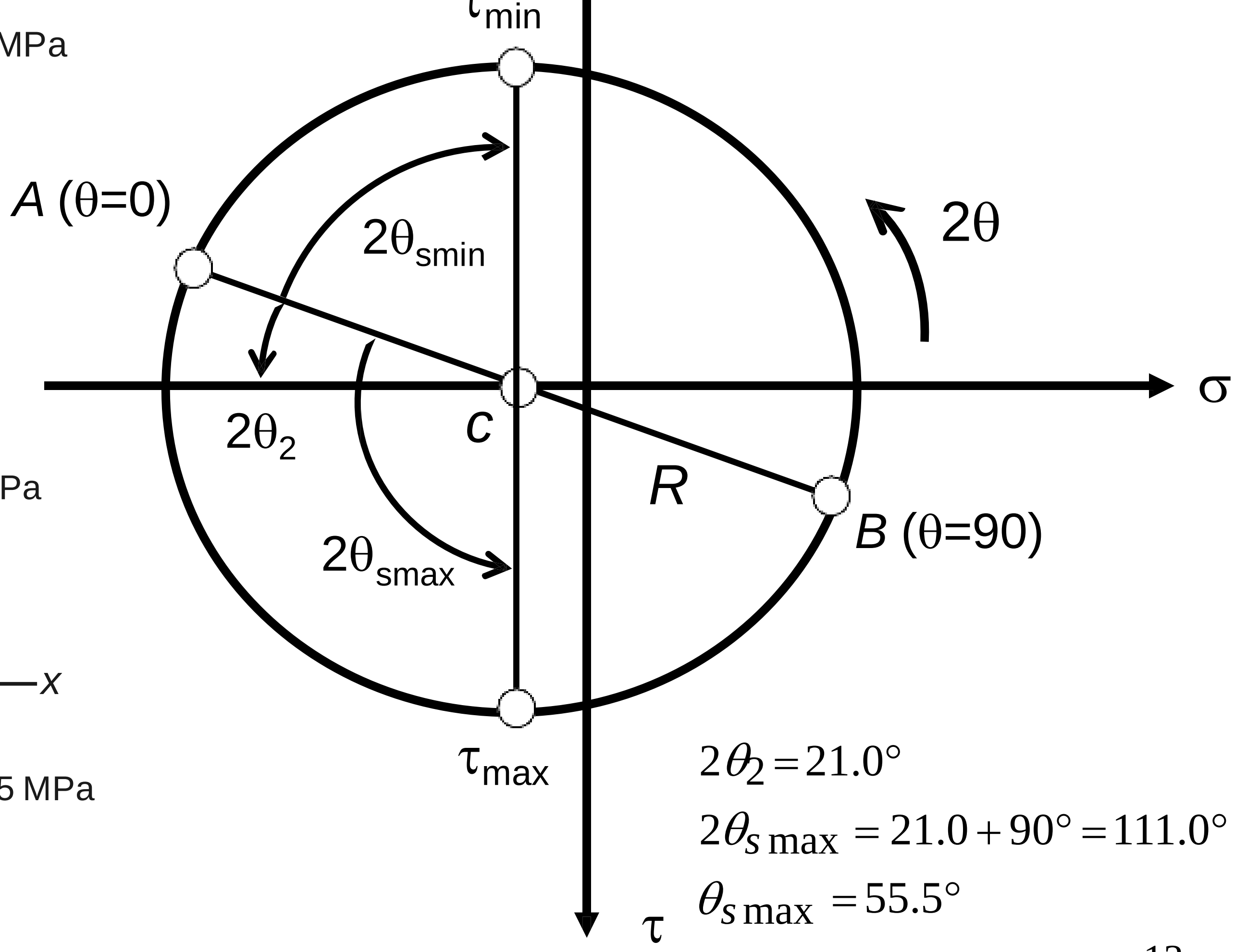


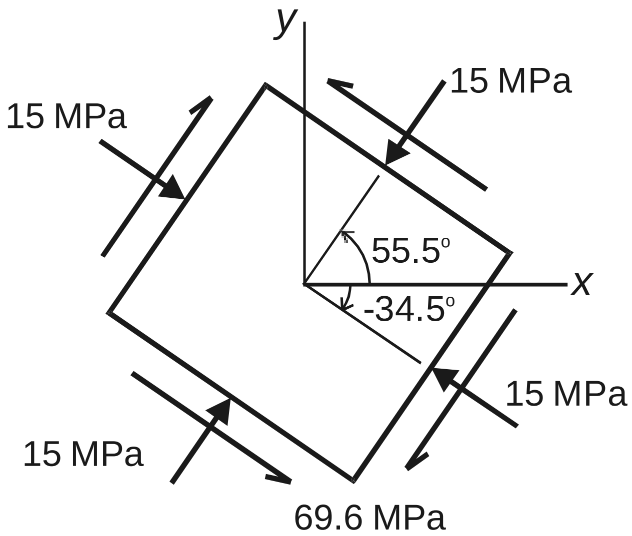
$$2\theta_2 = 21.0^\circ$$

$$2\theta_{s\,\min} = -(90 - 21.0) = -69.0^\circ$$

$$\theta_{s\,\min} = -34.5^\circ$$

taking sign convention into account



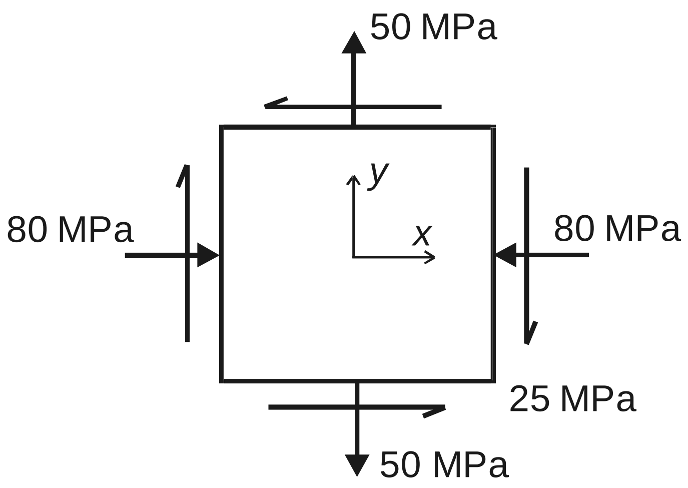


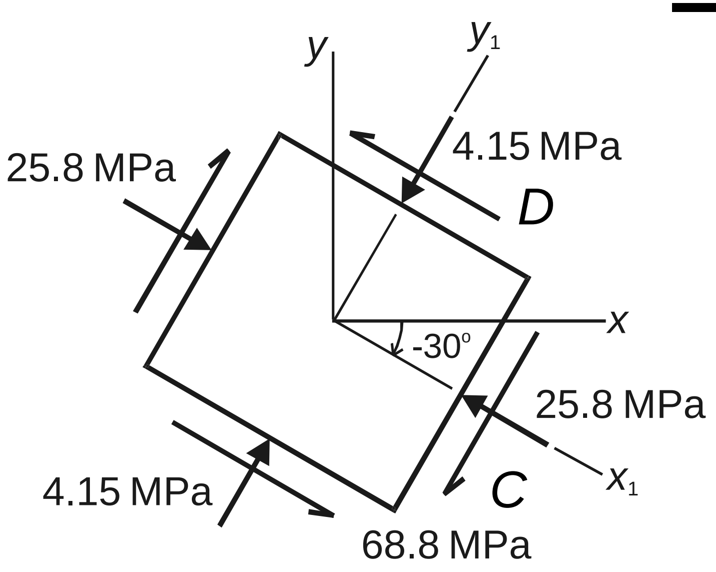
$$2\theta_2 = 21.0^\circ$$

$$2\theta_{s\,\max} = 21.0 + 90^\circ = 111.0^\circ$$

$$\theta_{s\,\max} = 55.5^\circ$$

Example: The state of plane stress at a point is represented by the stress element below. Find the stresses on an element inclined at 30° clockwise and draw the corresponding stress elements.





$\sigma_{x1} = c - R\cos(2\theta_2+60)$

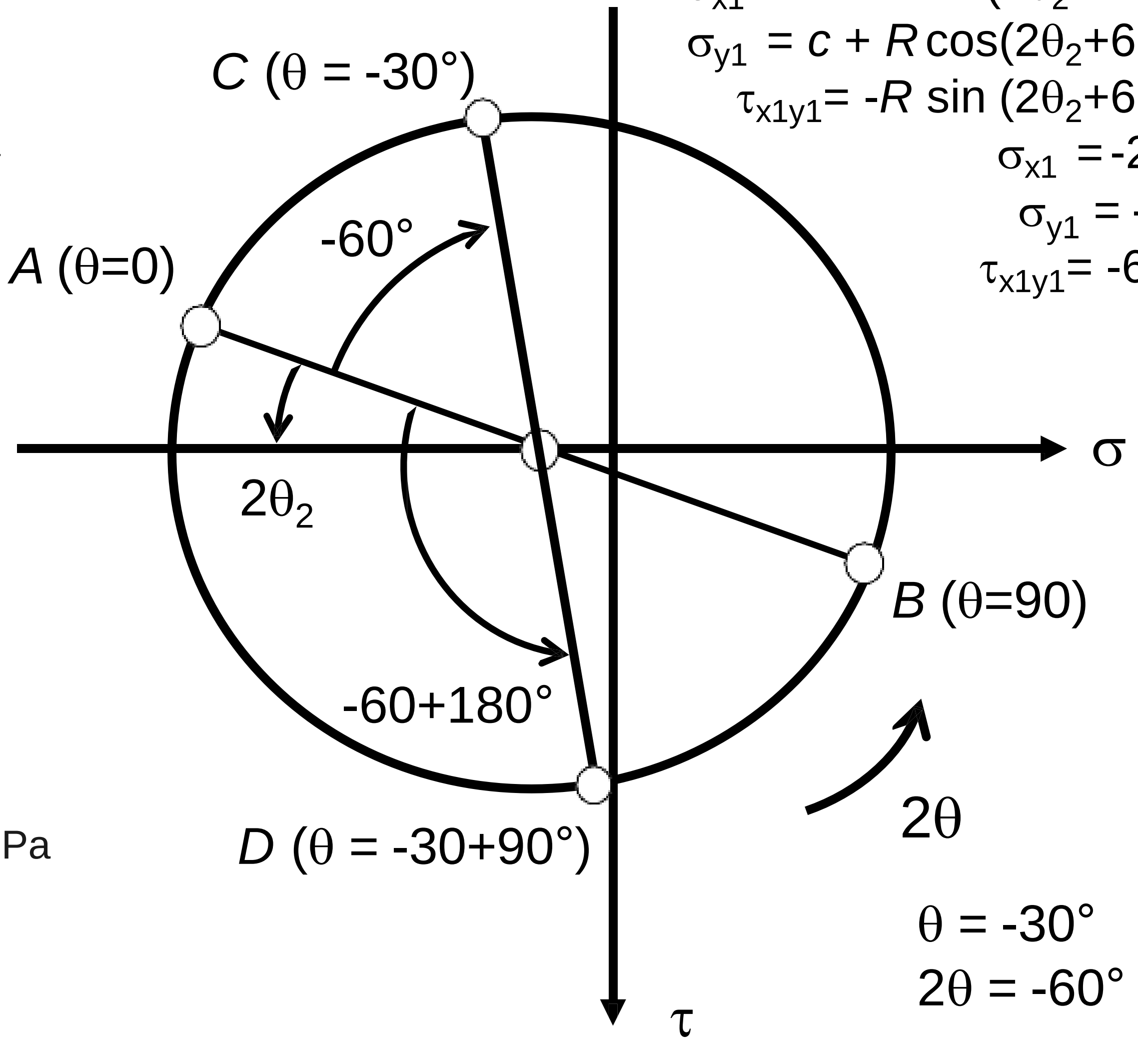
$\sigma_{y1} = c + R\cos(2\theta_2+60)$

$\tau_{x1y1} = -R\sin(2\theta_2+60)$

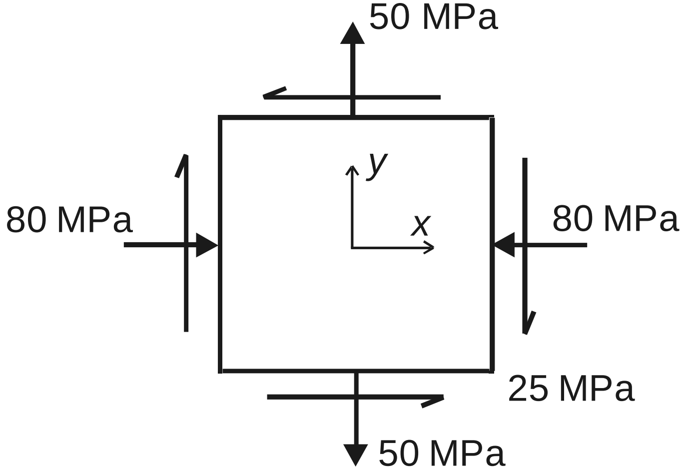
$\sigma_{x1} = -26$

$\sigma_{y1} = -4$

$\tau_{x1y1} = -69$



Example: The state of plane stress at a point is represented by the stress element below. Find the principal stresses.



$$M = \begin{bmatrix} \sigma_x & \tau_{xy} \\ \tau_{yx} & \sigma_y \end{bmatrix} = \begin{bmatrix} -80 & -25 \\ -25 & 50 \end{bmatrix}$$

We must find the eigenvalues of this matrix.

Remember the general idea of eigenvalues. We are looking for values of λ such that:

$A\mathbf{r} = \lambda\mathbf{r}$ where $\mathbf{r}$ is a vector, and A is a matrix.

$A\mathbf{r} - \lambda\mathbf{r} = \mathbf{0}$ or $(A - \lambda I)\,\mathbf{r} = \mathbf{0}$ where I is the identity matrix.

For this equation to be true, either $\mathbf{r} = \mathbf{0}$ or $\det(A - \lambda I) = 0$. Solving the latter equation (the “characteristic equation”) gives us the eigenvalues λ_1 and λ_2.