JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – II & IV
Branch – Civil Engineering
Subject –Strength of Material
Unit- 1
Name of Faculty – Sumit Saini (Assistant Professor)

1.1 LOAD

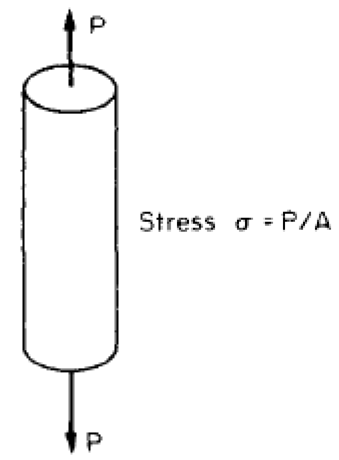
- Load is defined as the set of external forces acting on a mechanism or engineering structure which arise from service conditions in which the components work
- Common loads in engineering applications are tension and compression
- Tension:- Direct pull. Eg:Force present in lifting hoist
- Compression:- Direct push. Eg:- Force acting on the pillar of a building
- Sign convention followed: Tensile forces are positive and compressive negative

1.1.1TYPES OF LOAD

- There are a number of different ways in which load can be applied to a member. Typical loading types are:
- A) **Dead/ Static load**- Non fluctuating forces generally caused by gravity
- B) **Live load-** Load due to dynamic effect. Load exerted by a lorry on a bridge
- C) **Impact load or shock load**- Due to sudden blows
- D) **Fatigue or fluctuating or alternating loads**: Magnitude and sign of the forces changing with time

1.2 STRESS

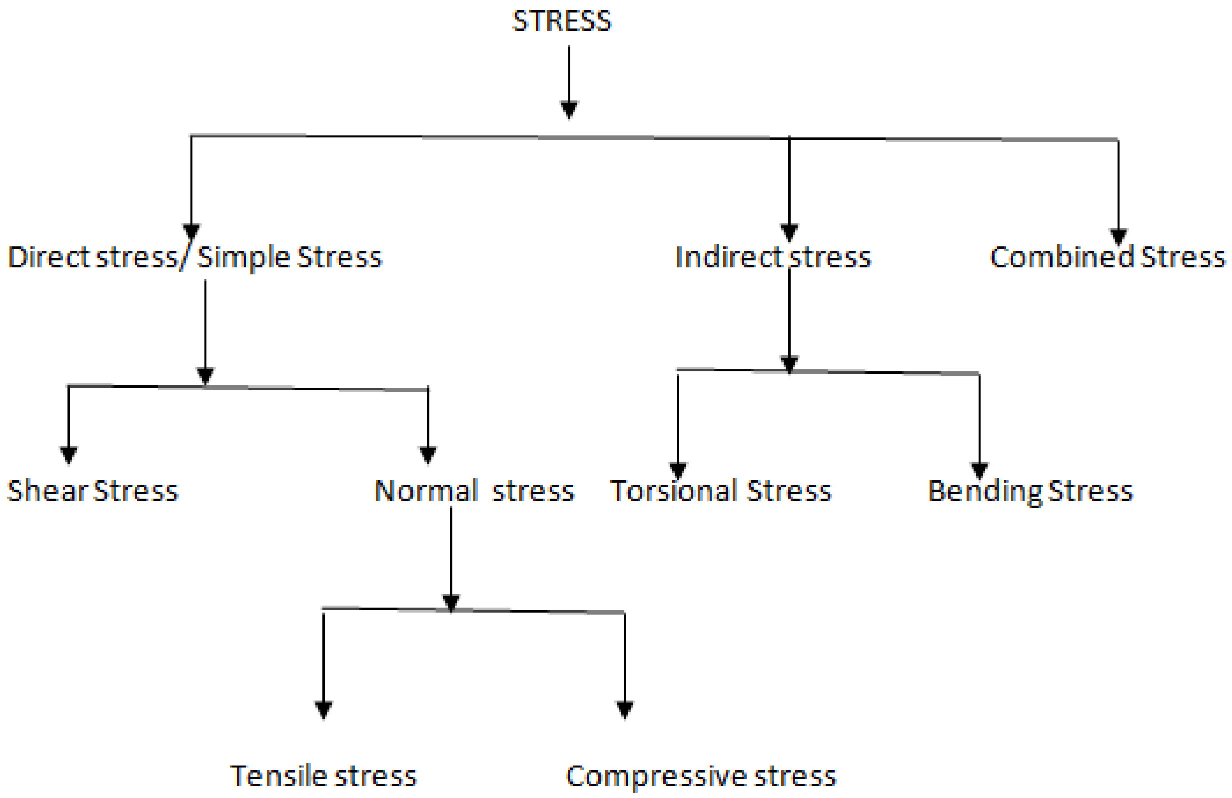
- When a material is subjected to an external force, a resisting force is set up within the component, this internal resistance force per unit area is called stress. SI unit is N/m^2(Pa). 1kPa=1000Pa, 1MPa=10^6 Pa, 1 Gpa Terra Pascal=10^12 Pa
- In engineering applications, we use th the original cross section area of the s and it is known as conventional stress Engineering stress



1.3 STRAIN

- When a body is subjected to some external force, there is some change of dimension of the body. The ratio of change of dimension of the body to its original dimension is known as strain
- Strain is a dimensionless quantity
- Strain may be:- a) Tensile strain b) Compressive strain c) Volumetric strain d) Shear strain
- **Tensile strain-** Ratio of increase in length to original length of the body when it is subjected to a pull force
- **Compressive strain**- Ratio of decrease in length to original length of the body when it is subjected to a push force
- **Volumetric strain-** Ratio of change of volume of the body to the original volume
- **Shear strain-**Strain due to shear stress

1.4 TYPE OF STRESSES



1.4.1TYPES OF DIRECT STRESS

- Direct stress may be normal stress or shear stress
- **Normal stress (σ)** is the stress which acts in direction perpendicular to the area. Normal stress is further classified into tensile stress
- **Tensile stress** is the stress induced in a body, when it is subjected to two equal and opposite pulls (tensile forces) as a result of which there is a tendency in increase in length
- It acts normal to the area and pulls on the area

1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

- Consider a bar subjected to a tensile force P at its ends. Let

  A= Cross sectional area of the body

  L=Original length of the body

  dL= Increase in length of the body due to its pull P

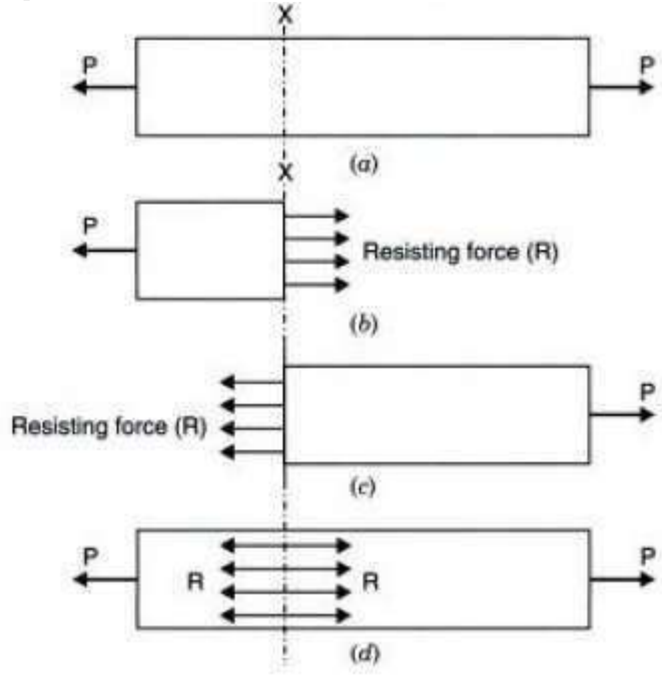
  ς= Stress induced in the body

  e= Tensile strain

Consider a section X-X which divides the body into two halves

1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

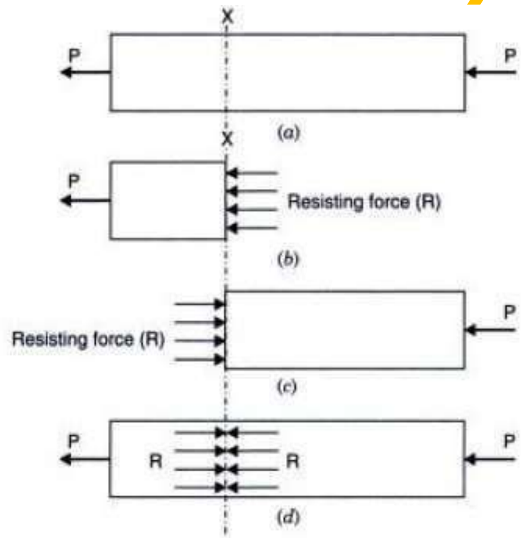
- The left part of the section x-x, will be in equilibrium if P=R (Resisting force). Similarly the right part of the section x-x will be in equilibrium if P=I



1.4.1 TYPES OF DIRECT STRESS (Tensile stress)

- Tensile stress (ς)= Resisting force/ Cross sectional area= Applied force/Cross sectional area=P/A
- Tensile strain= Increase in length/Original length= dL/L
- Compressive stress:- Stress induced in a body, when subjected to two equal and opposite pushes as a result of which there is a tendency of decrease in length of the body
- It acts normal to the area and it pushes on the area
- In some cases the loading situation is such that the stress will vary across any given section. In such cases the stress at any given point is given by
- ς= Lt ΔA→ 0 ΔP/ ΔA= dP/dA= derivative of force w.r.t area

1.4.1 TYPES OF DIRECT STRESS (Compressive stress)



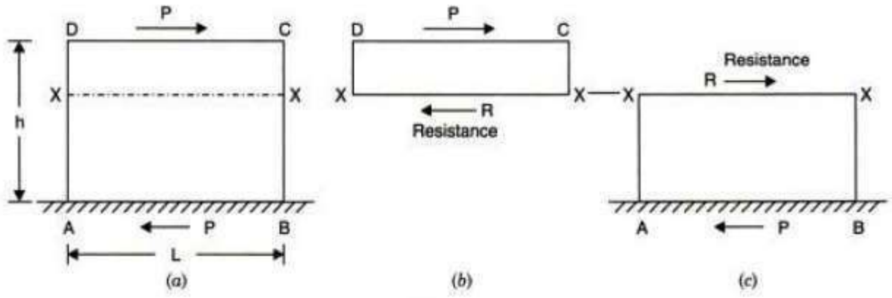
- **Compressive stress**=Resisting force/ cross sectional area= Applied force/ cross sectional area
- Compressive strain= Decrease in length/ Original length= -dL/L
- Sign convention for direct stress and strain:- Tensile stresses and strains are considered positive in sense producing an increase in length. Compressive stresses and strains are considered negative in sense producing decrease in length

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

- **Shear stress** :- Stress Induced in a body, when subjected to two equal and opposite forces which are acting tangentially across the resisting section as a result of which the body tends to shear off across that section
- Consider a rectangular block of height h, length L and width unity. Let the bottom face AB of the block be fixed to the surface as shown. Let P be the tangential force applied along top face CD of the block. For the equilibrium of the block, the surface AB will offer a tangential reaction force R which is equal in magnitude and opposite in direction to the applied tangential force P

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

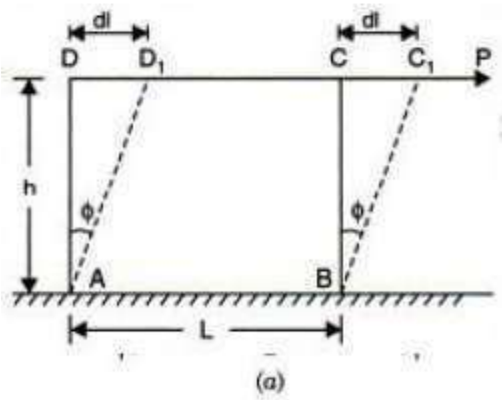
- Consider a section X-X cut parallel to the applied force which splits rectangle into two parts



- For the upper part to be in equilibrium; Applied force P=Resisting force R
- For the lower part to be in equilibrium; Applied force P=Resisting force R
- Hence, shear stress τ= Resisting force/Resisting area=P/L x 1=P/L
- Shear stress is tangential to the area on which it acts

1.4.1 TYPES OF DIRECT STRESS (Shear stress)

- As the face AB is fixed, the rectangular section ABCD will be distorted to ABC1D1, such that new vertical face AD1 makes an angle φ with the initial face AD



- Angle φ is called shear strain. As φ is very small,
- φ=tan φ=DD1/AD=dl/h
- Hence shear strain=dl/h

1.5 ELASTICITY & ELASTIC LIMIT

- The property of a body by virtue of which it undergoes deformation when subjected to an external force and regains its original configuration (size and shape) upon the removal of the deforming external force is called elasticity.
- The stress corresponding to the limiting value of external force upto and within which the deformation disappears completely upon the removal of external force is called elastic limit
- A material is said to be elastic if it returns to its original, unloaded dimensions when load is removed.
- If the external force is so large that the stress exceeds the elastic limit, the material loses to some extent its property of elasticity. If now the force is removed, the material will not return to its original shape and size and there will be a residual deformation in the material

1.6 HOOKE'S LAW & ELASTIC MODULI

- Hooke's law states that: " When a body is loaded within elastic limit, the stress is proportional to strain developed" or "Within the elastic limit the ratio of stress applied to strain developed is a constant"
- The constant is known as Modulus of elasticity or Elastic modulus or Young's modulus
- Mathematically within elastic limit

Stress/Strain=ς/e=E

ς= P/A; e =ΔL/L

E=PL/A ΔL

1.7 HOOKE'S LAW & ELASTIC MODULI

- Young's modulus (E) is generally assumed to be the same in tension or compression and for most of engineering applications has a high numerical value. Typically, $E=210 \times 10^9\ N/m^2$ (=210 GPa) for steel
- Modulus of rigidity, $G= \tau/\varphi$= Shear stress/ shear strain
- Factor of safety= Ultimate stress/Permissible stress
- In most engineering applications strains donot often exceed 0.003 so that the assumption that deformations are small in relation to orinal dimensions is generally valid

1.8 STRESS-STRAIN CURVE (TENSILE TEST)

- Standard tensile test involves subjecting a circular bar of uniform cross section to a gradually increasing tensile load until the failure occurs
- Tensile test is carried out to compare the strengths of various materials
- Change in length of a selected gauge length of bar is recorded by extensometers
- A graph is plotted with load vs extension or stress vs strain

1.8 STRESS-STRAIN CURVE (TENSILE TEST)

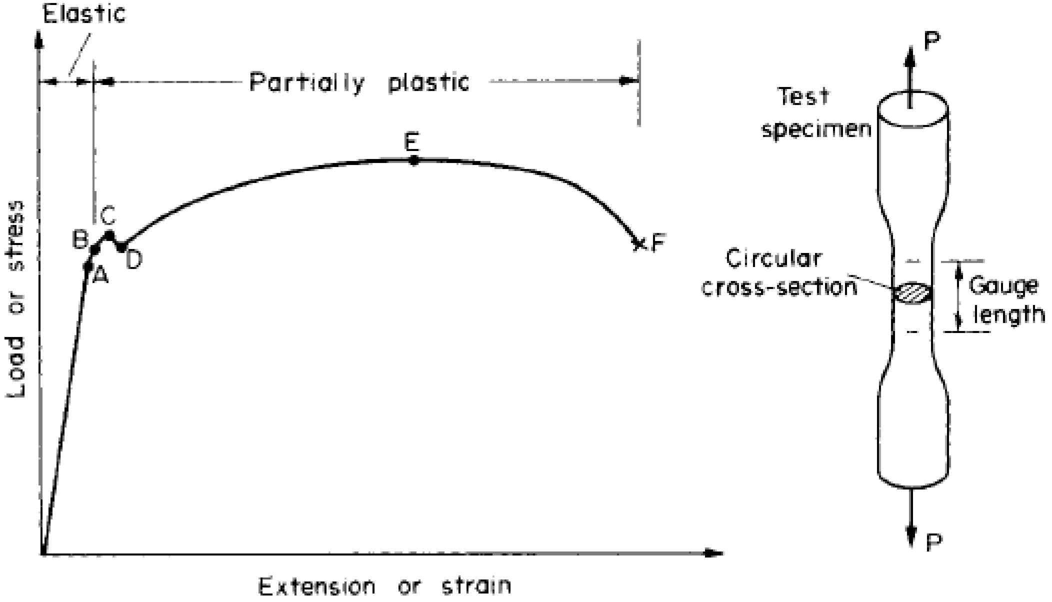


Fig. 1.3. Typical tensile test curve for mild steel.

1.8 STRESS-STRAIN CURVE (TENSILE TEST DIAGRAM)

- A→ Limit of proportionality; It is the point where the linear nature of the stress strain graph ceases
- B→ Elastic limit; It is the limiting point for the condition that material behaves elastically, but hooke's law does not apply . For most practical purposes it can be often assumed that limit of proportionality and elastic limits are the same
- Beyond the elastic limits, there will be some permanent deformation or permanent set when the load is removed
- C (Upper Yield point), D (Lower yield point)→ Points after which strain increases without correspondingly high increase in load or stress
- E→ Ultimate or maximum tensile stress; Point where the necking starts
- F→ Fracture point

RELATIONSHIPS BETWEEN STRESS & STRAIN

- **A) 1-Dimensional case** (due to pull or push or shear force)

  ς=Ee
- **B) 2-Dimensional case**
- Consider a body of length L, width B and height H. Let the body be subjected to an axial load. Due to this axial load, there is a deformation along the length of the body. This strain corresponding to this deformation is called longitudinal strain.
- Similarly there are deformations along directions perpendicular to line of application of fore. The strains corresponding to these deformations are called lateral strains

RELATIONSHIPS BETWEEN STRESS & STRAIN

δL = Increase in length,
δb = Decrease in breadth, and
δd = Decrease in depth.

Then longitudinal strain $= \dfrac{\delta L}{L}$

and lateral strain $= \dfrac{\delta b}{b}$ or $\dfrac{\delta d}{d}$

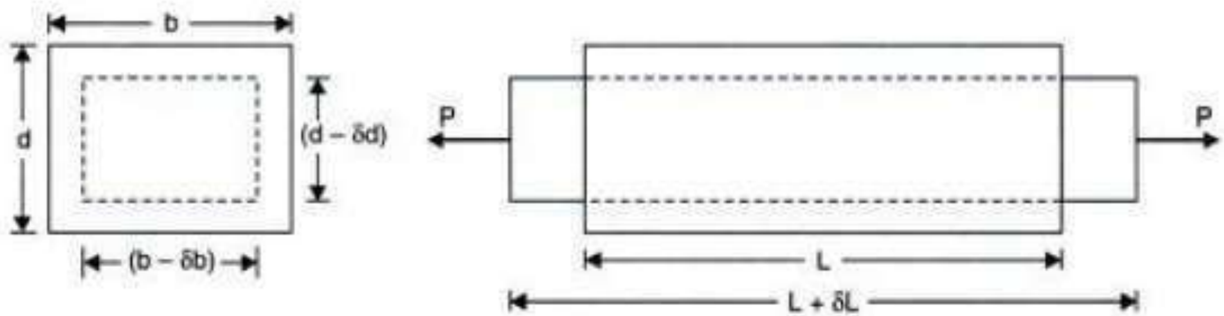


Fig. 1.3. Typical tensile test curve for mild steel.

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Longitudinal strain is always of opposite sign of that of lateral strain. Ie if the longitudinal strain is tensile, lateral strains are compressive and vice versa
- Every longitudinal strain is accompanied by lateral strains in orthogonal directions
- Ratio of lateral strain to longitudinal strain is called Poisson's ratio (μ); Mathematically,
- μ=-Lateral strain/Longitudinal strain
- Consider a rectangular figure ABCD subjected a stress in σx direction and in σ y direction

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to $\varsigma x = \varsigma\ x/E$

  Strain along x direction due to $\varsigma\ y = -\mu\ x\ \varsigma y/E$

  Total strain in x direction $ex = \varsigma\ x/E - \mu x\ \varsigma y/E$

  Similarly total strain in y direction, $ey = \varsigma\ y/E - \mu\ x\ \varsigma x/E$
- In the above equation tensile stresses are considered as positive and compressive stresses as negative
- **C) 3 Dimensional case:-**

  Consider a 3 D body subjected to 3 orthogonal normal stresses in x,y and z directions respectively

RELATIONSHIPS BETWEEN STRESS & STRAIN

- Strain along x direction due to ςx= ς x/E

  Strain along x direction due to ς y=-μ x ςy/E

  Strain along x direction due to ς z=-μ x ςz/E

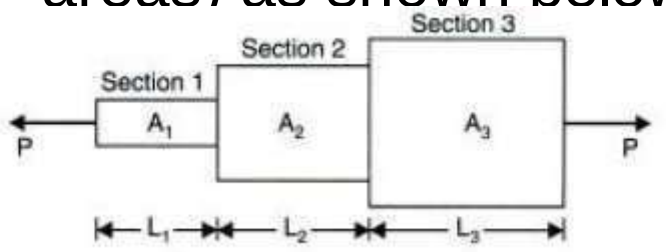
  Total strain in x direction ex= ς x/E - μ x (ςy/E + ςz/E)

  Similarly total strain in y direction, ey= ς y/E - μ x (ςxE + ςz/E)

  Similarly total strain in z direction, ez= ς z/E - μ x (ςxE + ςy/E)

1.10 ANALYSIS OF BARS OF VARYING CROSS SECTION

- Consider a bar of different lengths and of different diameters (and hence of different cross sectional areas) as shown below. Let this bar be subjected to



- The total change in length will be obtained by adding the changes in length of individual sections
- Total stress in section 1: $\varsigma 1 = E1 \times \Delta L1/L1$

  $\varsigma 1 \times L1/E1 = \Delta L1$

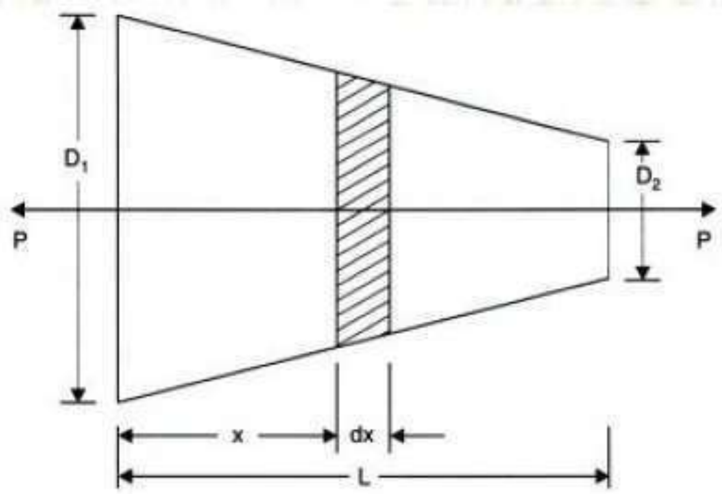
  $\varsigma 1 = P/A1$; Hence $\Delta L1 = PL1/A1E1$
- Similarly, $\Delta L2 = PL2/A2E2$; $\Delta L3 = PL3/A3E3$

1.10 ANALYSIS OF BARS OF VARYING CROSS SECTION

- Hence total elongation $\Delta L = P x (L1/A1E1 + L2/A2E2 + L3/A3E3)$
- If the Young's modulus of different sections are the same, E1=E2=E3=E; Hence $\Delta L = P/E x (L1/A1 + L2/A2 + L3/A3)$
- When a number of loads are acting on a body, the resulting strain, according to principle of superposition, will be the algebraic sum of strains caused by individual loads
- While using this principle for an elastic body which is subjected to a number of direct forces (tensile or compressive) at different sections along the length of the body, first the free body diagram of individual section is drawn. Then the deformation of each section is calculated and the total deformation is equal to the algebraic sum of deformations of individual sections

1.11 ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider a bar uniformly tapering from a diameter D1 at one end to a diameter D2 at the other end
- Let
- P→ Axial load acting on the bar
- L→ Length of bar
- E→ Young's modulus of the material



1. 11 ANALYSIS OF UNIFORMLY TAPERING CIRCULAR ROD

- Consider an infinitesimal element of thickness dx, diameter Dx at a distance x from face with diameter D1.

  Deformation of the element d(Δx)= P x dx/ (AxE)

  Ax=π/4 x Dx²; Dx= D1 - (D1 – D2)/L x x

  Let (D1-D2)/L=k; Then Dx= D1-kx

  d(ΔLx)= 4 x P x dx/(π x (D1-kx)² x E)

  Integrating from x=0 to x=L 4PL/(πED1D2)

$$\int_0^L d(\Delta x) = \int_0^L 4 \times P \times dx / (\pi \times (D1-kx)^2 \times E)$$

  Let D1-kx=λ; then dx= -(d λ/k)

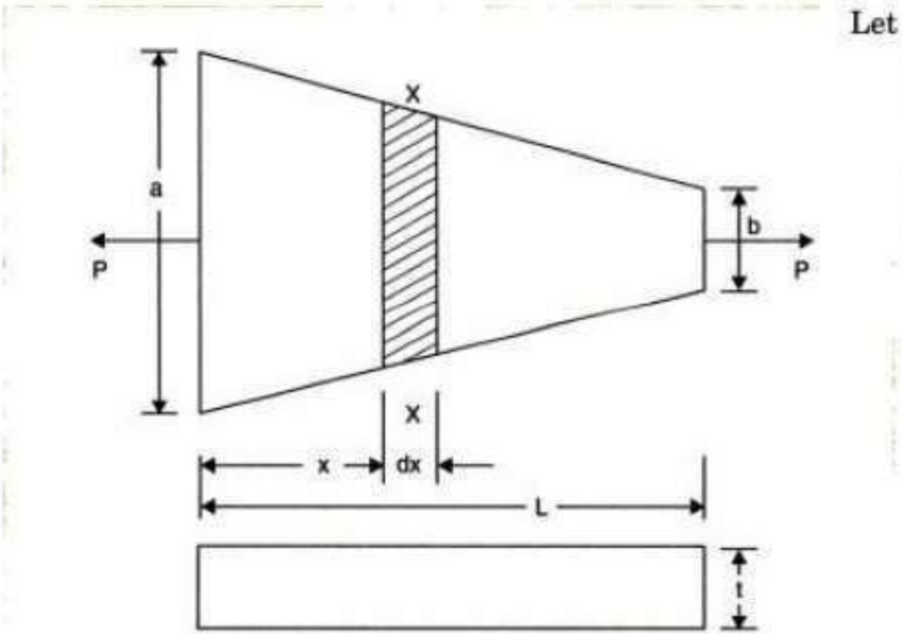
  When x=0, λ=D1; When x=L, λ=D2

$$\int_0^L d(\Delta L_x) = \int_{D1}^{D2} -4 \times P \times dx / (\pi \times \lambda^2\, k \times E)$$

$$\Delta L_x = 4PL/(\pi ED1D2)$$

1.12 ANALYSIS OF UNIFORMLY TAPERING RECTANGULAR BAR

A bar of constant thickness and uniformly tapering in width from one end to the other end is shown in Fig. 1.14.

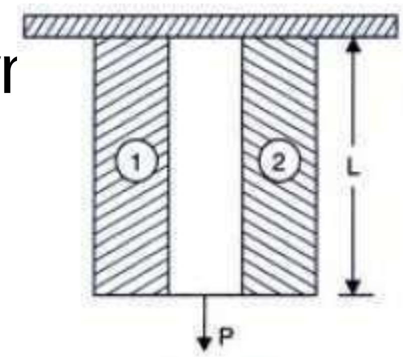


Let P = Axial load on the bar
L = Length of bar
a = Width at bigger end
b = Width at smaller end
E = Young's modulus
t = Thickness of bar

$$dL = \frac{PL}{Et(a-b)} \log_e \frac{a}{b} .$$

1.13 ANALYSIS OF BARS OF COMPOSITE SECTIONS

- A bar, made up of two or more bars of equal lengths but of different materials rigidly fixed with each other and behaving as one unit for elongation and shortening when subjected to axial loads is called composite bar.
- Consider a composite bar as shown



- Let

P→ Applied load

L→ Length of bar

A1→ Area of cross section of Inner member

A2→ Cross sectional area of Outer member

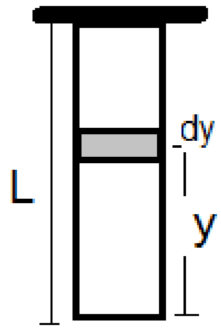
1.13 ANALYSIS OF BARS OF COMPOSITE SECTIONS

- Strain developed in the outer member= Strain developed in the inner member

  ς1/E1 = ς2/E2
- Total load (P)= Load in the inner member (P1) + Load in the outer member (P2)
- ς1 x A1 + ς2 x A2= P
- Solving above two equations, we get the values of ς1, ς2 & e1 and e2

PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

- Consider a bar of length L, area of cross section A rigidly fixed at one end. Let ρ be the density of the material. Consider an infinitesimal element of thickness [illegible]ce y from the bottom of the bar.



- The force acting on the element considered= weight of the portion below it=ρAgy

PRODUCED IN A BAR DUE TO ITS SELF WEIGHT

- Tensile stress developed= Force acting on the element/Area of cross section= ρgy.

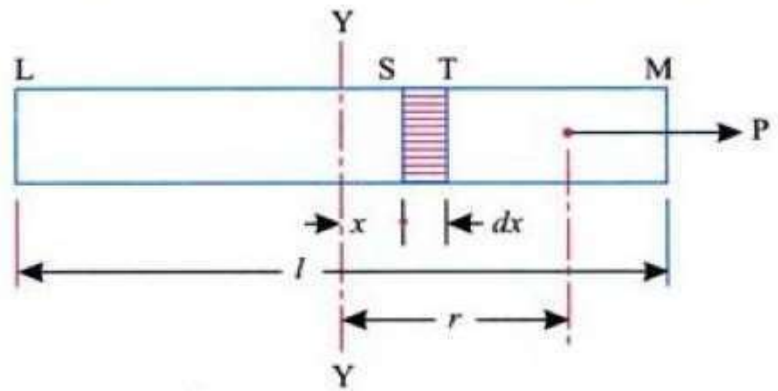
- From the above equation, it is clear that the maximum stress at the section where y=L, ie at the fixed end (ρgL) and minimum stress is at the free end(=0)

$$\Delta L_y = \int_0^L \rho g y\, dy / AE = \rho g L^2 / 2AE$$

- Elongation due to self weight

1.15 STRESS IN BAR DUE TO ROTATION

Consider a bar of length l rotating about the axis y at a constant angular velocity ω. Consider an infinitesimal element of thickness dx at a distance x from the axis of rotation.



Tensile force on element ST= Centrifugal force on element TM

Centrifugal force on element TM= Mass of element TM x r x ω^2= {l/2 – (x+dx)} x A x ρ x r x ω^2

r= x + ½ x (l/2- (x+dx)

As dx is numerically very small, x + dx≈x

Hence tensile force on element ST=(l/2 – x) x A x{x + ½ x (l/2-x)} x ρ x ω^2

= A x ρ x ω^2 x (l^2/4 - x^2)/2

1.15 STRESS IN BAR DUE TO ROTATION

Tensile stress developed= Tensile force/cross sectional area= $A \times \rho \times \omega^2 \times (l^2/4 - x^2)/2A$

$\sigma rod = \rho \times \omega^2 \times (l^2/4 - x^2)/2$

$\sigma rod = 0$, when $x = l/2$

σrod =Maximum when $d(\sigma rod)/dx = 0$; ie when $x=0$

$\sigma rodmax == \rho \times \omega^2 \times l^2/8$

Extension of element= $\sigma rod \times dx/E$

Extension of entire bar= $\int_0^l \rho \times \omega^2 \times (l^2/4 - x^2)dx/2 = \rho \times \omega^2 \times l^3/12E$

Extension of entire bar= $\rho \times \omega^2 \times l^3/12E$

1.16 THERMAL STRESS

- Thermal stresses are the stresses induced in a body due to change in temperature. Thermal stresses are set up in a body, when the temperature of the body is raised or lowered and the body is restricted from expanding or contracting
- Consider a body which is heated to a certain temperature

  Let

  L= Original length of the body

  Δ T=Rise in temp

  E=Young's modulus

  α=Coefficient of linear expansion

  dL= Extension of rod due to rise of temp
- If the rod is free to expand, Thermal strain developed

  $e_t = \Delta L / L = \alpha \times \Delta T$

1.16 THERMAL STRESS

- The extension of the rod, $\Delta L = L \times \alpha \times \Delta T$
- If the body is restricted from expanding freely, Thermal stress developed is $\varsigma t/et=E$
- $\varsigma t = E \times \alpha \times \Delta T$
- Stress and strain when the support yields:-

If the supports yield by an amount equal to δ, then the actual expansion is given by the difference between the thermal strain and δ

Actual strain, $e = (L \times \alpha \times \Delta T - \delta)/L$

Actual stress = Actual strain x E = $(L \times \alpha \times \Delta T - \delta)/L \times E$