JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTER

Class – B.Tech Civil (IV SEM)
Subject –Hydraulics Engineeging
Unit – 2
Presented by – Ashish Boraida (Assistant Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional ,cultural and social enrichment of the lives of indivisuals and communities

MISSION OF INSTITUTE

Focus on evaluation of learining ,outcomes and motivate students to research apptitude by project based learning.

- Identify based on informed perception of indian ,regional and global needs ,the area of focus and provide plateform to gain knowledge and solutions.
-
- Offer oppurtunites for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

VISION AND MISSION OF DEPARTMENT

Vision

To become a role model in the field of Civil Engineering for the sustainable development of the society.

Mission

1)To provide outcome base education.

2)To create a learning environment conducive for achieving academic excellence.

3)To prepare civil engineers for the society with high ethical values.

Introduction, Objective and Outcome of Fluid Mechanics

Objective:

,
The primary purpose of the study of Fluid mechanics is to develop the capacity to understand important basic terms used in fluid mechanics, understand hydrostatics and buoyancy with practice of solving problems. Student could be able to understand Kinematics of flow and fluid dynamics, Bernoulli's equation and laminar flow with practice of solving problems in practical life for the benefit of society and mankind.

Outcomes

- Student will be able to understand Dimensional, Model Analysis and Turbulent Flow with problems.
- Student will be able to understand variable Flow in open channels , Gradually and Rapidly Varied Flow.
- Student will be able to understand Impact of Jets and hydraulic machines
- Student will be able to understand Hydrology, Ground water and Canal Hydraulics.

CONTENTS

- Boundry Layer Theory
- Displacement Thickness

BOUNDARY LAYER THEORY

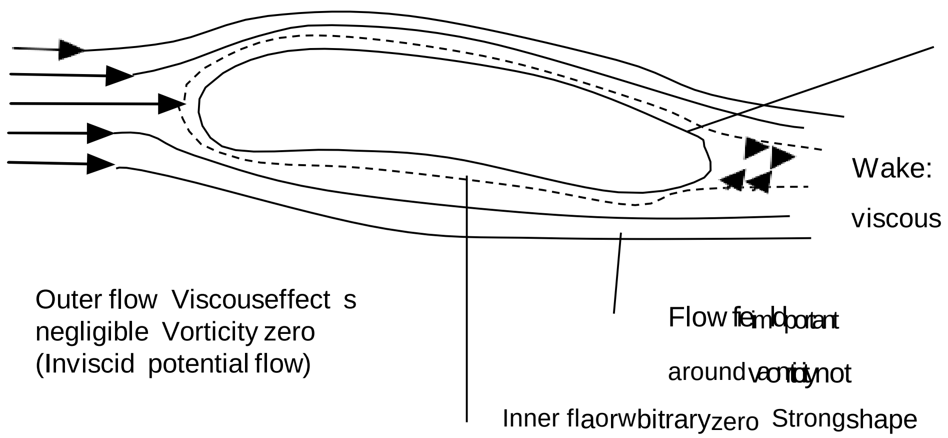
HIGH RENOLDS NUMBER FLOW → BOUNDARY LAYERS

$(Re \to \infty)$

BOUNDARY LAYER Thin region adjacent to surface of a body where viscous forces dominate over inertia forces

$$Re = \left(\frac{inertia\ forces}{viscous\ forces} \right)$$

$Re \gg 1$



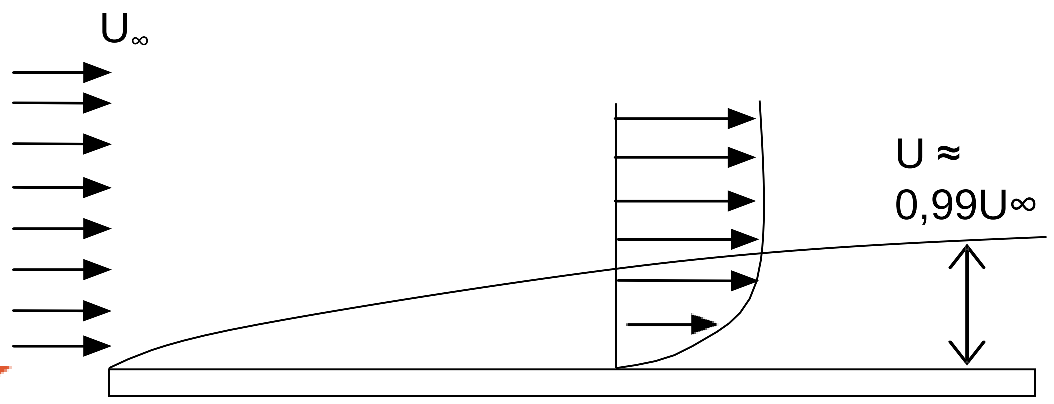
$$\frac{d\theta}{dx} + (\delta^* + 2\theta) \frac{1}{\theta}\frac{dU}{dx} = \frac{\tau_0}{\rho U^2}$$

Steady ,incompressible 2-D flow with no body forces. Valid for laminar flow

$$\tau_0 \sim \left(\frac{\partial u}{dy}\right)^n$$

O.D.E for $\theta(x)$

- **To solve eq. we first "assume" an approximate velocity profile inside the B.L Relate the wall shear stress to the velocity field**
- **Typically the velocity profile is taken to be a polynomial in y, and the degree of fluid this polynominal determines the number of boundary conditions which may be satisfied**
- **EXAMPLE:** $\frac{u}{U} = a + b\eta + c\eta^2 = f(\eta)$ **LAMINAR FLOW OVER A FLAT PLATE:**



Dimensionless gov. eqs.

$$\nabla . \vec{V} = 0$$

X ;

$$\frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} = -\frac{\partial P^*}{\partial x} + \underbrace{\frac{1}{Re}\left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2}\right)}_{\text{viscous terms}}$$

$$P^* = \frac{P}{\rho U_\infty^2}$$

Y;

$$\frac{\partial v}{\partial t} + u\frac{\partial v}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{\partial P}{\partial y} + \frac{1}{Re}\left(\frac{\partial^2 v}{\partial x^2} + \frac{\partial^2 v}{\partial y^2}\right)$$

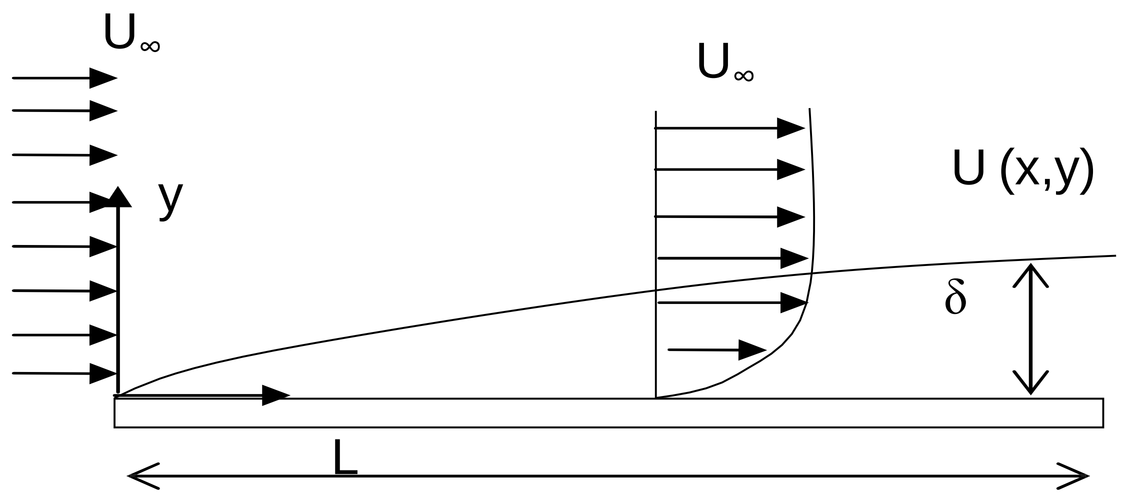
"Naïve" way of solving problem for

$$Re \rightarrow \infty \quad \Rightarrow \quad \frac{1}{Re} \rightarrow 0$$

If you drop the viscous term → Euler's eqs. (inviscid fluid)

- We can not satisfy all the boundary B.C.s because order of eqs. Reduces by 1

Inside B-L can not get rid of viscous terms



$$\delta^* = \frac{\delta}{L} \langle \frac{1}{100}$$

Derivation of B-L eqs. From the N-S eqs

- <u>Physically based argument</u> :determine the order of terms in N-S
- Limiting procedure as $Re \longrightarrow \infty$ eqs. and throw out small terms

Assumption 1

$$\delta^* = \frac{\delta}{L} \langle\langle 1$$

Term		**Order**
$\frac{\partial u^*}{\partial x^*}$	→	$\frac{(1)}{(1)} = 1$
$\frac{\partial v^*}{\partial y^*}$	→	$\frac{\delta^*}{\delta^*} = 1$
v^*	→	δ^*
$\frac{\partial v^*}{\partial x^*}$	→	$\frac{\delta^*}{1} = \delta^*$
$\frac{\partial^2 u^*}{\partial y^{*2}}$	→	$\frac{1}{\delta^{*2}}$
$\frac{du^*}{dt^*}$	→	$u^* \frac{\partial u^*}{\partial x^*} = 1$

$$\frac{\partial u^*}{\partial t} + u^* \frac{\partial u^*}{\partial x} + v^* \frac{\partial u^*}{\partial y} = -\frac{\partial P^*}{\partial x} + \frac{1}{\text{Re}}\left(\frac{\partial^2 u^*}{\partial x^2} + \frac{\partial^2 u^*}{\partial y^2}\right)$$

(1) (1) $\delta^* \frac{1}{\delta^*} = 1$ $\frac{(1)}{(1)} = 1$ δ^{*2} $\frac{(1)}{(1)^2}$ $\frac{(1)}{(\delta^*)^2}$

Neglect since of order $\frac{(1)}{(\delta^*)^2}$ >>>1

Also for y –direction

$$\frac{\partial v^*}{\partial t^*} + u^* \frac{\partial v^*}{\partial x^*} + v^* \frac{\partial v^*}{\partial y^*} = -\frac{\partial P^*}{\partial y^*} + \frac{1}{\text{Re}}\left(\frac{\partial^2 v^*}{\partial x^{*2}} + \frac{\partial^2 v^*}{\partial y^{*2}}\right)$$

* $(1)\frac{(\delta^*)}{(1)}$ $(\delta^*)\frac{(\delta^*)}{(\delta^*)}$ $(\delta^{*2})\left\{\frac{\delta^*}{(1)^2} + \frac{\delta^*}{(\delta^*)^2}\right\}$

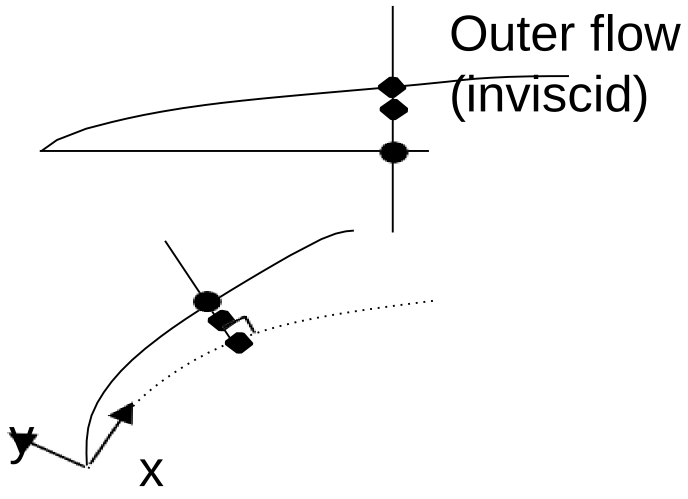
U(δ^*) U(δ^*) U(δ^*) U(δ^*)

$\frac{\partial P^*}{\partial y^*}$ $\Rightarrow$ U(δ^*) $\Rightarrow$ small relative to $\frac{\partial P^*}{\partial x^*}$ $\Rightarrow$ U(1)

To good approximation $P \cong P(x)$ $\Rightarrow$ pressure at the edge of B-L. is equal to pressure <u>on</u> boundary layer.

- Time – dependant $\Rightarrow$ $P \cong P(x, t)$ $\leftarrow$ known from the other flow
- Pressure at all points is the same
- Only need to consider x-direction B-L. eqs.

Prandtl (1904)



2-D planar

1) $$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0$$

2) $$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = -\frac{1}{\rho}\frac{\partial P}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2}$$

Governing eqs.for

B.L

B-L.eqs.
still non-linear but
parabolic type

unknows u,v (x,y,t)

$P \cong P(x,t)$ → known from the potential flow

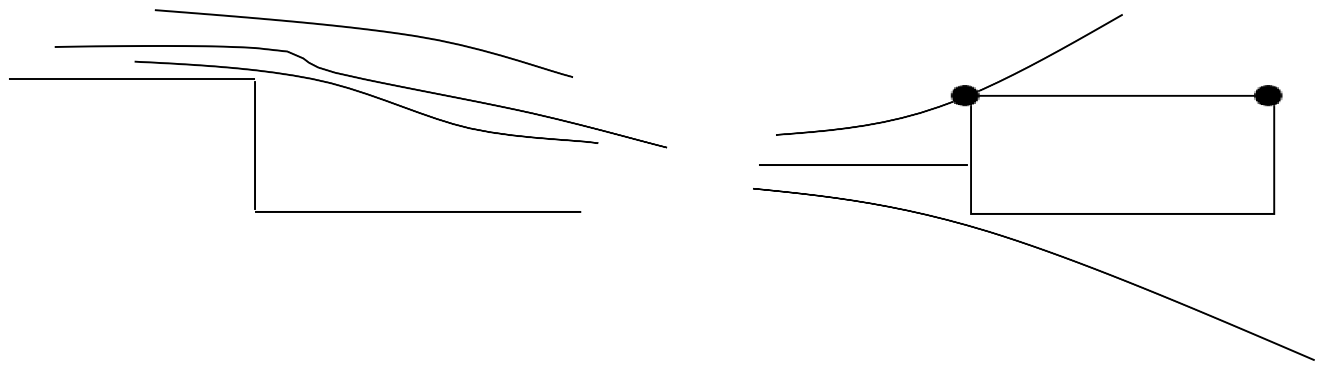
Need B.C.s & I.C.(time dependant)

•2-D, steady BCs

- u=ν=0 at y=0
- u=u(y) at x=0
- u= U_∞(x) y $\longrightarrow$ ∞ (y $\longrightarrow$ δ) $\longleftrightarrow$ marching condition

- B-L. eqs. can be solved exactly for several cases
- Can approximate solution for other cases Limitation of B.L eqs.: where they fail?

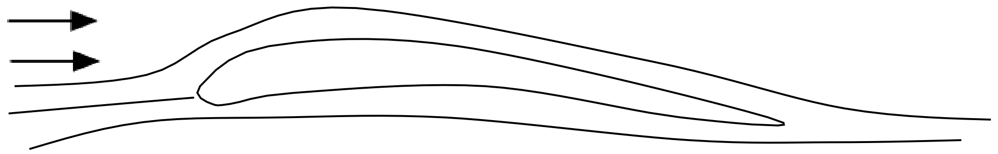
(1) Abrupt chances

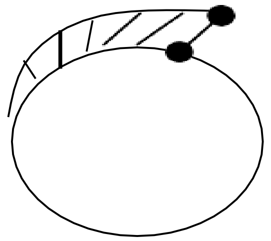


(2) Eqs. are not applicable near the leading edge

L is small ⟶ $\delta^* = \frac{\delta}{L} \langle\langle 1$ invalid

(3) Where the flow separates not valid beyond the separation point





Bernouilli eqs. ρ =constant

$$\frac{P}{\rho} + \frac{V^2}{2} = \text{constant} \quad \Longrightarrow \quad \frac{1}{\rho}\frac{dP}{dx} + \frac{1}{2}2U\frac{dV}{dx} = 0$$

Valid along the streamlines

$$-\frac{1}{\rho}\frac{dP}{dx} = U\frac{dU}{dx}$$

substitute the B.L eqs u,v can be found

known

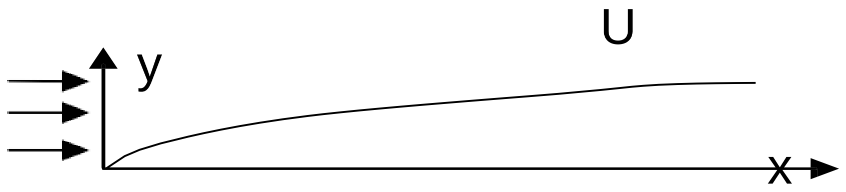
SIMILARITY SOLUTION TO B.L. EQS

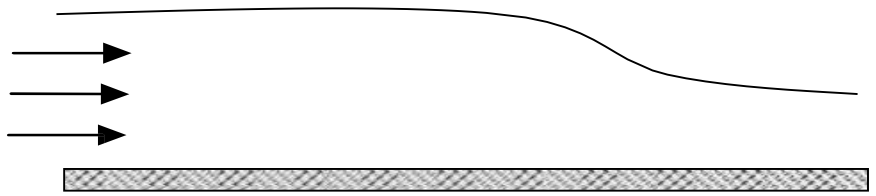
Example 1

Zero pressure gradient

Flow over a semi-infinite flat plate $\frac{dp}{dx} = 0$ p = constant

Steady ,laminar & U=constant $\left(\frac{dp}{dx} = 0 \right)$





- Bernouilli eqs. outsideB.L $\quad p + \frac{1}{2}\rho U^2 = cons.$

U=constant , $\quad \frac{dp}{dx} = 0$

Governing (B.L. eqs.) become

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \qquad (1)$$

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \nu\frac{\partial^2 u}{\partial y^2} \qquad (2)$$

B.C.

- $y=0$ $u=v=0$ (no-slip) & $y \longrightarrow \infty$, $u \longrightarrow U$
- $x=0$ $u=U$

Blasuis(1908) :

1.Introduce the stream function $\psi(x,y)$

- Recall ; $u=\dfrac{\partial \psi}{\partial y}$ $v=-\dfrac{\partial \psi}{\partial x}$

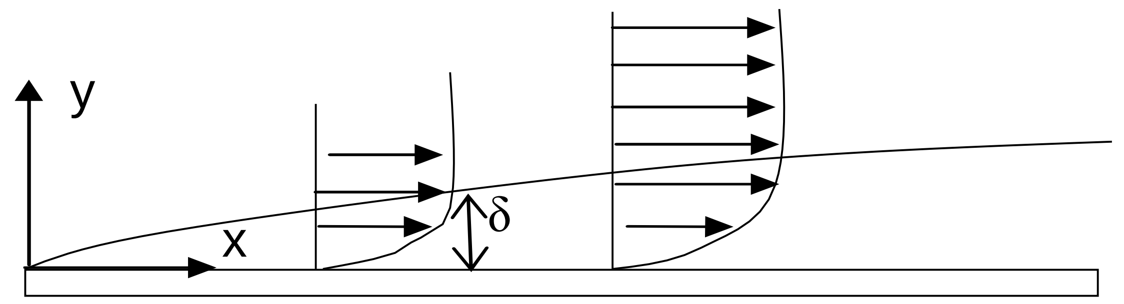
note that ψ satisfies cont. eqs. substitute intoB.L. mom. Eqs

$$\frac{\partial \psi}{\partial y} \cdot \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial \psi}{\partial x} \cdot \frac{\partial^2 \psi}{\partial y^2} = \nu \frac{\partial^3 \psi}{\partial y^3} \qquad (2')$$

- Now, assume that we have a similarity "stretching" variable, which has all velocity profiles on plate scaling on δ .

i.e

$$\frac{u}{U_\infty} = f\left(\frac{y}{\delta}\right)$$



$$\delta = g(U_\infty, x, \nu)$$ → dimensional analysis

$$\frac{\delta}{x} = g\left(\frac{U_\infty x}{\nu}\right) = g(\mathrm{Re})$$ ⇒ $\frac{1}{\mathrm{Re}} \sim \mathrm{U}(\delta^2)$ $\delta \sim \sqrt{\nu}$

⇒ $\delta \sim \sqrt{\frac{\nu x}{U_\infty}}$ $\frac{m^2}{s} \cdot \frac{m}{m} . s = m$ [ ] $\frac{\delta}{x} \sim \frac{1}{\sqrt{\mathrm{Re}_x}}$ both U(δ)

Viscous dif. Depth

$$\mathrm{Re} = \frac{U_\infty x}{\nu} \qquad \delta \approx 5\sqrt{\frac{\nu x}{U_\infty}}$$

Let $\eta = \frac{y}{\delta}$ [-] similarity variable

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}} \quad \Rightarrow \quad \frac{u}{U} = f(\eta)$$

Use similarity profile assumption to turn 2

P.D.E $\Rightarrow$ 1 O.D.E

$$u = \frac{\partial \psi}{\partial y}\bigg|_{x = fixed} \qquad \psi = \int_0^y u\,dy = \int_0^y Uf(\eta)dy = \int_0^\eta Uf(\eta) \sqrt{\frac{\nu x}{U}} d\eta$$

$$\psi = \sqrt{U\nu x}\int_0^{\eta} f(\eta)d\eta = \sqrt{U\nu x}F(\eta)$$

$$\underbrace{\int_0^{\eta} f(\eta)d\eta}_{F(\eta)}$$

$$\psi = \sqrt{U\nu x}F(\eta)$$

$$\psi = \sqrt{U\nu x}F(\eta)$$

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$$

$$\psi - \psi_0 = \int_0^y u\,dy \qquad d\psi = \frac{\partial \psi}{\partial y}dy + \frac{\partial \psi}{\partial x}dx$$

- Now, substitute ψ into P.D.E for ψ(x,y) to get O.D.E for F(η)

$$\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}F + \sqrt{U_\infty \nu x}F'\frac{\partial \eta}{\partial x}$$

$$F' = \frac{dF}{d\eta} \qquad F'' = \frac{d^2F}{d\eta^2}$$

$$\frac{\partial \eta}{\partial x} = -\frac{1}{2} y \sqrt{\frac{U_\infty}{\nu x}} \frac{1}{x} = -\frac{1}{2x}\eta \qquad \frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(F - \eta F') \qquad \frac{\partial \psi}{\partial y} = \sqrt{U_\infty \nu x} F' \sqrt{\frac{U_\infty}{\nu x}} = U_\infty F'$$

$$\frac{\partial^2 \psi}{\partial x \partial y} = -\frac{U_\infty}{2x}\eta F'' \qquad \frac{\partial^2 \psi}{\partial y^2} = U_\infty \sqrt{\frac{U_\infty}{\nu x}} F'' \qquad \frac{\partial^3 \psi}{\partial y^3} = \frac{U_\infty^2}{\nu x} F'''$$

Substituting into eq. (2’)

$$U_\infty F'\left(-\frac{U_\infty}{2x}\eta F''\right) - \frac{1}{2}\left(\frac{U_\infty \nu}{x}\right)^{1/2}(F - \eta F') U_\infty \left(\frac{U_\infty}{\nu x}\right)^{1/2} F'' = \nu \frac{U_\infty^2}{\nu x} F'''$$

or

$$-\frac{U_\infty^2}{2x}\eta F'' F' - \frac{1}{2}\frac{U_\infty^2}{x} F'' F + \frac{1}{2}\frac{U_\infty^2}{x}\eta F'' F' = \frac{U_\infty^2}{x} F'''$$

$$F''' + \frac{1}{2} F F'' = 0$$

blasius eq. 3rd order , non linear ODE

Note: $F''' + FF'' = 0$ for $\eta = y\sqrt{\frac{U_\infty}{2\nu x}}$ BVP

BC's are

At y=0 u=v=0 $\Rightarrow$ $\eta = 0$

BC 1) $u\big|_{y=0} = \frac{\partial \psi}{\partial y}\Big|_{y=0} = 0$ $\Rightarrow$ $U_\infty F'\big|_{\eta=0} = 0$ **F'(0)=0**

BC 2) $v\big|_{y=0} = 0$ $\Rightarrow$ $-\frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(F - \eta F') = 0$ **F(0)=0**

BC 3) (x,y $\longrightarrow \infty$) $\longrightarrow$ U_∞

$\frac{\partial \psi}{\partial y}\Big|_{y\to\infty} \to U_\infty$ $U_\infty F'\big|_{\eta\to\infty} = U_\infty$ $F'(\eta \to \infty) \longrightarrow 1$ $F'(\infty) = 1$

F(η) dimensionless function

Or At x=0 $u = U_\infty$ ⇒ $U_\infty F'\big|_{\substack{x=0 \\ \eta\to\infty}} = U_\infty$

F'(∞)=1 same with BC 3) Matching B.C

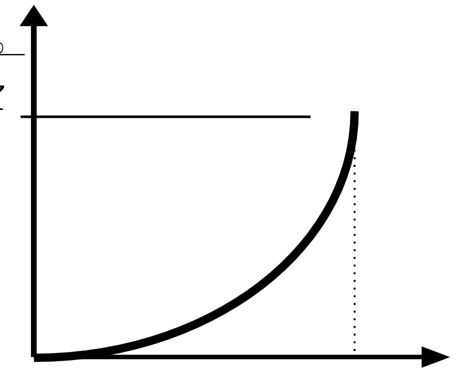
- Solution to blasius eg a)power series
  b)runge-kutta
- results tabulated form for F,F',F'',etc

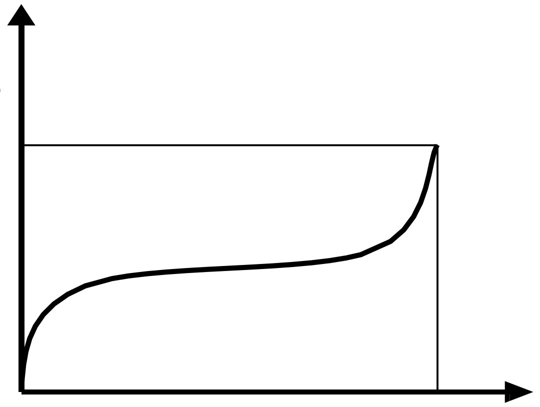
p.g 121

$\eta = y\sqrt{\frac{U_\infty}{\nu x}}$	F	$F' = \frac{u}{U_\infty}$	F''
0	0	0	0.33206
#	#	#	#
5.0	3.28329	0.99155	0.01591

F”= 0.33206 ← From the solution

- Velocity profile





$$v = -\frac{\partial \psi}{\partial x} = \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(\eta F' - F)$$

$$\frac{v}{U_\infty} = \frac{1}{2}\mathrm{Re}_x^{-1/2}\left[\eta F' - F\right]$$

$$\eta \to \infty \qquad v_\infty = \frac{1}{2}\sqrt{\frac{U_\infty \nu}{x}}(5x1 - 3.28)$$

$$\frac{v_\infty}{U_\infty} = 0.86\frac{1}{\sqrt{\mathrm{Re}_x}}$$

Shear stress distribution along the flat plate

$$\tau = \mu\left(\frac{\partial u}{\partial y} + \frac{\partial v}{\partial x}\right) \qquad \tau(x,y)$$

$$\frac{\partial u^*}{\partial y^*} \gg \frac{\partial v^*}{\partial x^*} \qquad \tau \cong \mu \frac{\partial u}{\partial y}$$

$$For \quad Re_x = 10^4 \Rightarrow \frac{\nu_\infty}{U_\infty} = 0.00865 \approx \frac{1}{100}$$

$$\text{For} \quad Re_x = 10^6 \Rightarrow \frac{\nu_\infty}{U_\infty} = 0.000865 \approx \frac{1}{1000}$$

At the wall (y=0)

$$\tau_0(x) = \mu \frac{\partial u}{\partial y}\bigg|_{y=0}$$

$$\downarrow$$

$$\tau_w(x)$$

$$\tau_0(x) = \mu \frac{\partial^2 \psi}{\partial y^2}\bigg|_{y=0} = \mu U_\infty \sqrt{\frac{U_\infty}{\nu x}} F''\big|_{\eta=0}$$

$$\downarrow$$

Distribution along the wall

$$\tau_0(x) = \mu \sqrt{\frac{U_\infty^3}{\nu x}} F''(0)$$

$$\downarrow$$

0.332

Non dimensionalize :

$$C_f = \frac{\tau_0}{\frac{1}{2}\rho U_\infty^2} = \frac{2F''(0)}{\sqrt{\mathrm{Re}_x}} = \frac{0.664}{\sqrt{\mathrm{Re}_x}} \qquad \mathrm{Re}_x = \frac{U.x}{\nu}$$

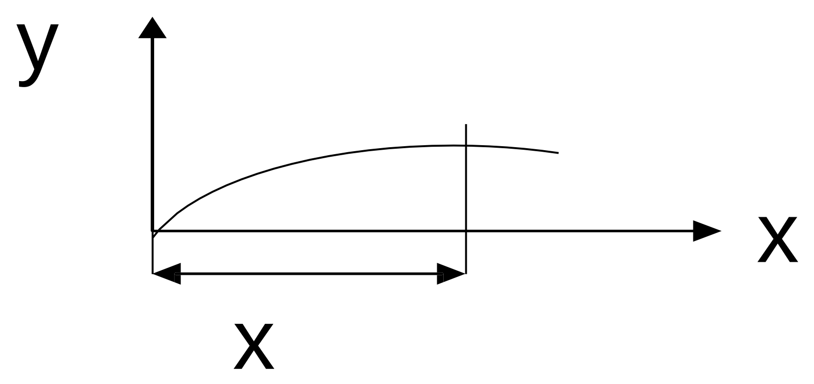
$$C_f = 0.664\sqrt{\frac{\nu}{Ux}}$$

Friction coef.

$$\text{Note :} \quad x \to 0 \quad \Rightarrow \quad \tau_0 \to \infty$$

$$\nu \qquad \to \infty$$

B.L eqs.are not valid near the leading edge



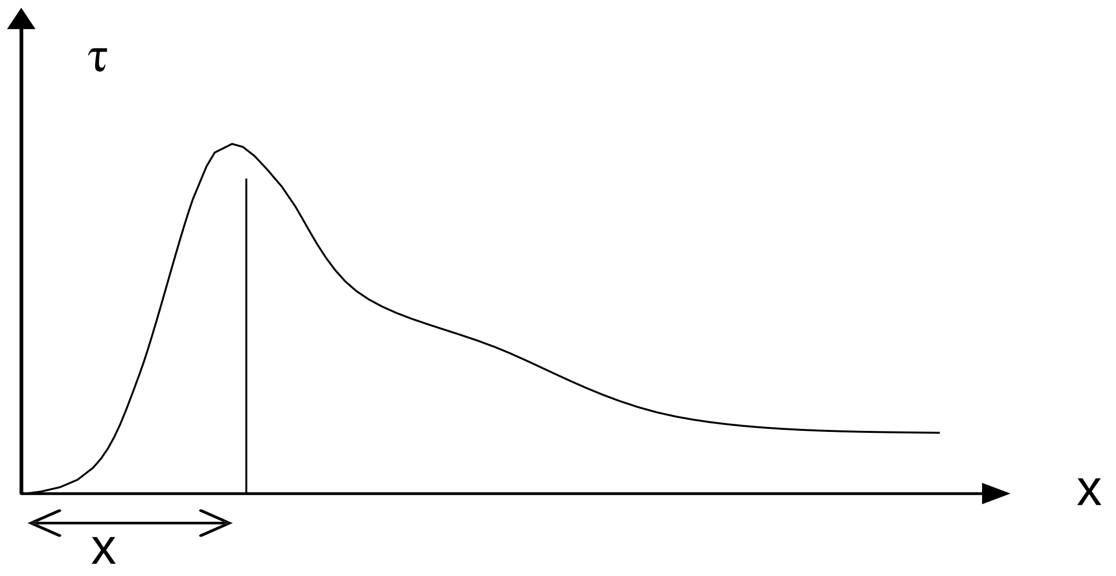
Up to the point we are considering

Drag force acting on the flat plate We have to integrate shear stress

$$F_D = \int_0^x \tau_0(\zeta)\,d\zeta$$

↓

per unit width



$$2F_D = 1.328(b)\sqrt{U_\infty^3 \mu \rho x}$$

dimensionless drag coef.(C_D)

we have 2 wetted sides

$$C_D = \frac{2F_D}{\frac{1}{2}\rho U_\infty^2 A}$$

A=2bx

Width normal to the blackboard

$$C_D = \frac{1.328}{\sqrt{Re_x}}$$

valid for laminar flow i.e for $Re_x < 5.10^5$ to 10^6

for $Re_x > 10^6$ $\rightarrow$ turbulent drag becomes considerably greater

Boundary Layer Thickness : δ

$$\eta = y\sqrt{\frac{U_\infty}{\nu x}} \quad \text{at } \eta = 5 \quad \Rightarrow \quad \frac{u}{U} = 0.99 \quad \rightarrow y = \delta \qquad \text{(Table)}$$

$$5 \cong \delta\sqrt{\frac{U_\infty}{\nu x}} \qquad \delta \cong \frac{5x}{\sqrt{Re_x}} \qquad Re_x = \frac{U_\infty x}{\nu}$$

δ:defined as the distance from the wall for which $u = 0.99U_\infty$

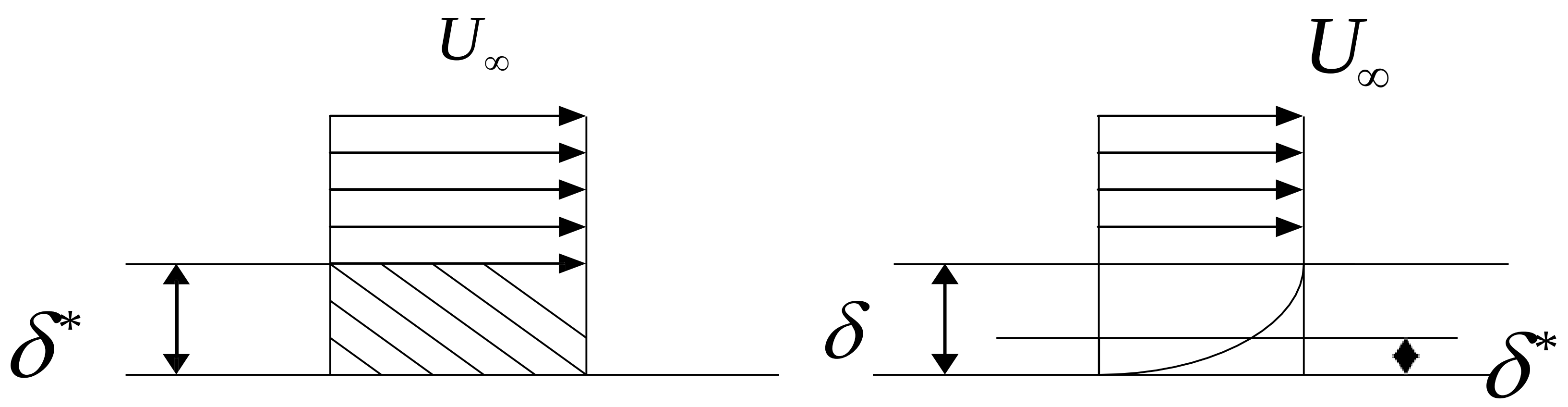
Boundary Layer Parameter (thicknesses)

Most widely used is δ but is rather arbitrary $y=\delta$ when $u=0.99\,U_\infty$

→ hard to establish

→ more physical parameters are needed

Displacement thickness: δ



an imaginary displacement of fluid from the surface to account for “lost” mass flow in boundary layer

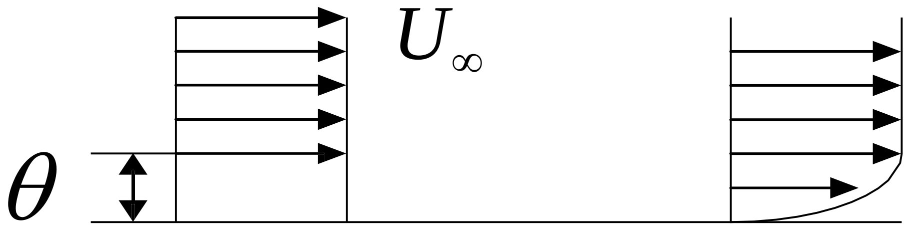
$$\dot{m}_{tot} = \int_0^\infty \rho u\,dy = \int_{y=\delta^*}^\infty \rho U_\infty dy = \int_0^\infty \rho U_\infty dy - \underbrace{\int_0^{\delta^*} \rho U_\infty dy}_{-\rho U_\infty \delta^*} \quad \text{or}$$

$$\rho U_\infty \delta^* = \int_0^\infty (\rho U_\infty - \rho u)\,dy$$

$$\delta^* = \int_0^\infty \left(1 - \frac{u}{U_\infty}\right) dy$$

if $\rho = cons.$ $\delta > \delta^*$ always by definition

Momentum thickness: θ



an imaginary displacement of fluid of velocity U_∞ to account for “lost” momentum due to the formation of a boundary layer velocity profile

$$\rho U_\infty^2 \theta = \int_0^\infty (\rho u dy) U_\infty - \int_0^\infty (\rho u dy) u$$

Mass flow in B.L

Possible momentum

actual momentum

"lost" momentum

JECRC Foundation

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Thank you!

STAY HOME, STAY SAFE