

## Unit-5

The errors, which occur during measurement are known as **measurement errors**.

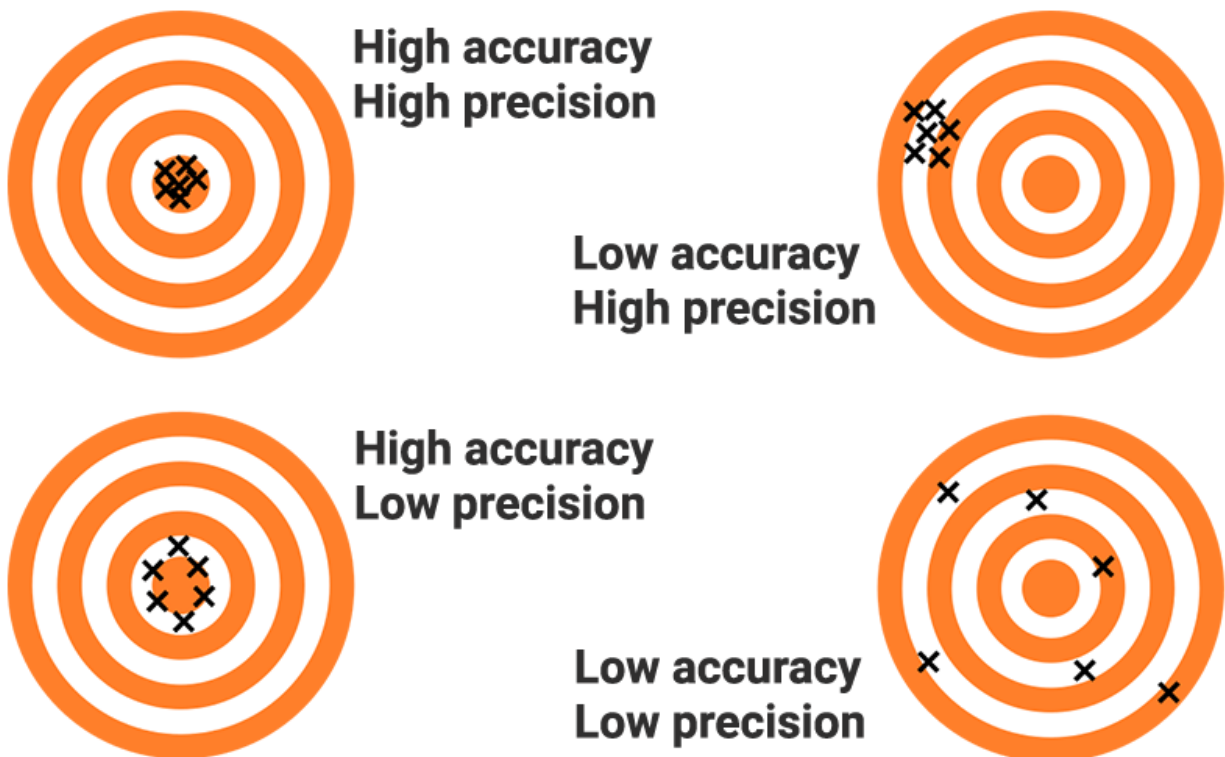
### Accuracy

It is the closeness with which an instrument reading approaches the true value of the quantity being measured.

Point accuracy- This is the accuracy of the instrument only at one point on its scale. It does not give any information about the general accuracy of the instrument.

Accuracy as “Percentage of Scale Range”.-When an instrument has uniform scale its accuracy may be expressed in terms of scale range. For eg. the accuracy of thermometer having scale of 500°C may be expressed as  $\pm 0.5\%$  scale range. This means that the accuracy of thermometer when the reading is 500°C is  $\pm 0.5\%$  which is negligible, but when the reading is 25°C, the error is very high and therefore specification of accuracy is highly misleading.

**3. Accuracy as Percentage of True Value** – Such type of accuracy of the instruments is determined by identifying the measured value regarding their true value. The accuracy of the instruments is neglected up to  $\pm 0.5$  percent from the true value.



# Precision

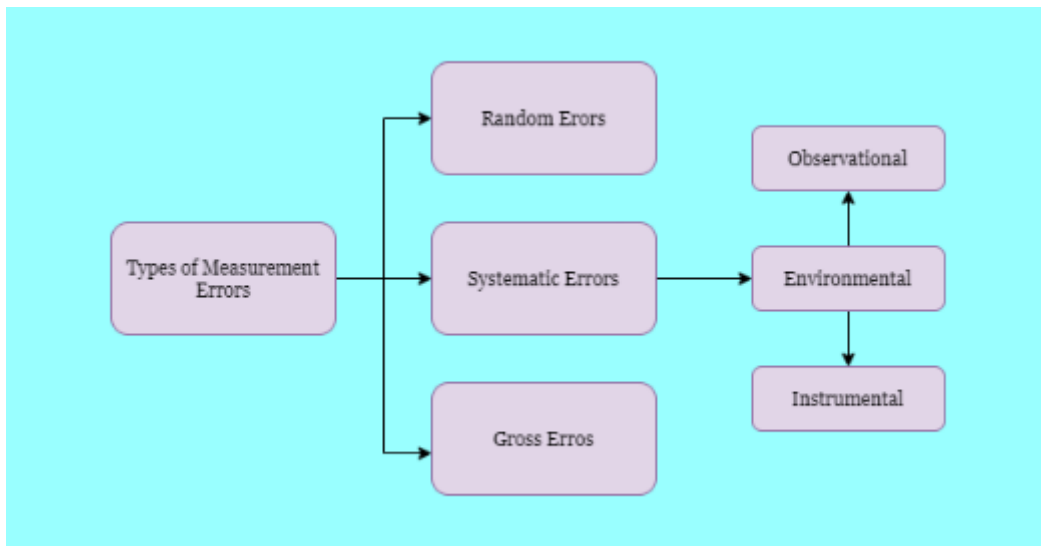
**Definition:** The term precision means two or more values of the measurements are closed to each other. The value of precision differs because of the observational error. The precision is used for finding the consistency or reproducibility of the measurement. The conformity and the number of significant figures are the characteristics of the precision.

The high precision means the result of the measurements are consistent or the repeated values of the reading are obtained. The low precision means the value of the measurement varies. But it is not necessary that the highly precise reading gives the accurate result.

Example – Consider the 100V, 101V, 102V, 103V and 105V are the different readings of the voltages taken by the voltmeter. The readings are nearly close to each other. They are not exactly same because of the error. But as the reading are close to each other then we say that the readings are precise.

## Measurement Error

**Definition:** The measurement error is defined as the difference between the true or actual value and the measured value. The true value is the average of the infinite number of measurements, and the measured value is the precise value.



### 1. Gross Errors

The gross error occurs because of the human mistakes. For examples consider the person using the instruments takes the wrong reading, or they can record the incorrect data. Such type of

error comes under the gross error. The gross error can only be avoided by taking the reading carefully.

For example – The experimenter reads the 31.5°C reading while the actual reading is 21.5°C. This happens because of the oversights. The experimenter takes the wrong reading and because of which the error occurs in the measurement.

Such type of error is very common in the measurement. The complete elimination of such type of error is not possible. Some of the gross error easily detected by the experimenter but some of them are difficult to find. Two methods can remove the gross error.

Two methods can remove the gross error. These methods are

- The reading should be taken very carefully.
- Two or more readings should be taken of the measurement quantity. The readings are taken by the different experimenter and at a different point for removing the error.

## 2. Systematic Errors

The systematic errors are mainly classified into three categories.

1. Instrumental Errors
2. Environmental Errors
3. Observational Errors

### 2 (i) Instrumental Errors

These errors mainly arise due to the three main reasons.

**(a) Inherent Shortcomings of Instruments** – Such types of errors are inbuilt in instruments because of their mechanical structure. They may be due to manufacturing, calibration or operation of the device. These errors may cause the error to read too low or too high.

For example – If the instrument uses the weak spring then it gives the high value of measuring quantity. The error occurs in the instrument because of the friction or hysteresis loss.

**(b) Misuse of Instrument** – The error occurs in the instrument because of the fault of the operator. A good instrument used in an unintelligent way may give an enormous result.

For example – the misuse of the instrument may cause the failure to adjust the zero of instruments, poor initial adjustment, using lead to too high resistance. These improper practices may not cause permanent damage to the instrument, but all the same, they cause errors.

**(c) Loading Effect** – It is the most common type of error which is caused by the instrument in measurement work. For example, when the voltmeter is connected to the high resistance circuit it gives a misleading reading, and when it is connected to the low resistance circuit, it gives the dependable reading. This means the voltmeter has a loading effect on the circuit.

The error caused by the loading effect can be overcome by using the meters intelligently. For example, when measuring a low resistance by the ammeter-voltmeter method, a voltmeter having a very high value of resistance should be used.

## 2 (ii) **Environmental Errors**

These errors are due to the external condition of the measuring devices. Such types of errors mainly occur due to the effect of temperature, pressure, humidity, dust, vibration or because of the magnetic or electrostatic field. The corrective measures employed to eliminate or to reduce these undesirable effects are

- The arrangement should be made to keep the conditions as constant as possible.
- Using the equipment which is free from these effects.
- By using the techniques which eliminate the effect of these disturbances.
- By applying the computed corrections.

## 2 (iii) **Observational Errors**

Such types of errors are due to the wrong observation of the reading. There are many sources of observational error. For example, the pointer of a voltmeter resets slightly above the surface of the scale. Thus an error **occurs** (because of parallax) unless the line of vision of the observer is exactly above the pointer. To minimise the parallax error highly accurate meters are provided with mirrored scales.

## 3. **Random Errors**

The error which is caused by the sudden change in the atmospheric condition, such type of error is called random error. These types of error remain even after the removal of the systematic error. Hence such type of error is also called residual error.

Following are the parameters that are used in statistical analysis.

- Mean
- Median
- Variance
- Deviation
- Standard Deviation

Now, let us discuss about these **statistical parameters**.

### Mean

Let  $x_1, x_2, x_3, \dots, x_n$  are the  $N$  readings of a particular measurement. The mean or **average value** of these readings can be calculated by using the following formula.

## Formula for Finding the Mean of the Ungrouped Data

$$\text{Mean} = \frac{\text{Sum of the Variables Total}}{\text{Number of Variates}}$$

$$\text{Mean} = \frac{x_1 + x_2 + x_3 + x_4 + \dots + x_n}{n}$$

$$\text{Symbolically, } A = \frac{\sum x_i}{n}; i = 1, 2, 3, 4, \dots, n.$$

If the number of readings of a particular measurement are more, then the mean or average value will be approximately equal to **true value**

### Median

If the number of readings of a particular measurement are more, then it is difficult to calculate the mean or average value. Here, calculate the **median value** and it will be approximately equal to mean value.

For calculating median value, first we have to arrange the readings of a particular measurement in an **ascending order**. We can calculate the median value by using the following formula, when the number of readings is either an **odd number or even number**.

**Median**

**n is odd,**

$$\text{Median} = \left(\frac{n+1}{2}\right)^{th} \text{ observation}$$

**n is even,**

$$\text{Median} = \frac{\left(\frac{n}{2}\right)^{th} + \left(\frac{n}{2} + 1\right)^{th} \text{ observation}}{2}$$

## Deviation from Mean

The difference between the reading of a particular measurement and the mean value is known as *deviation from mean*. In short, it is called *deviation*. Mathematically, it can be represented as

$$d_i = x_i - m$$

Where,

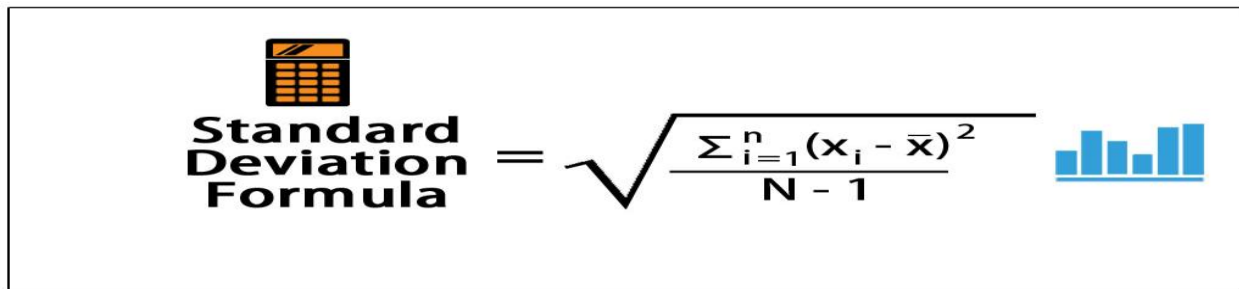
$d_i$  is the deviation of  $i^{\text{th}}$  reading from mean.

$x_i$  is the value of  $i^{\text{th}}$  reading.

$m$  is the mean or average value.

## Standard Deviation

The root mean square of deviation is called **standard deviation**. Mathematically, it can be represented as

A graphic showing the standard deviation formula. On the left is a calculator icon. In the center, the text "Standard Deviation Formula" is followed by an equals sign and a square root symbol. Inside the square root is the fraction  $\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{N - 1}$ . To the right of the formula is a bar chart icon with five bars of varying heights.

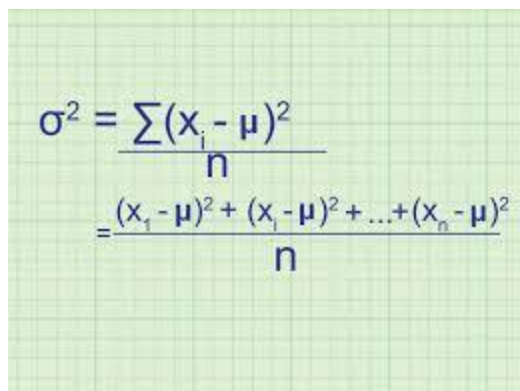
**Standard Deviation Formula** =  $\sqrt{\frac{\sum_{i=1}^n (x_i - \bar{x})^2}{N - 1}}$

We can use the above formula for standard deviation, when the number of readings,  $N$  is less than 20. When readings are greater than 20 we use  $N$  instead of  $N-1$ .

**Note** – If the value of standard deviation is small, then there will be more accuracy in the reading values of measurement.

## Variance

The square of standard deviation is called **variance**. Mathematically, it can be represented as

The variance formula is written on a green grid background. The formula is  $\sigma^2 = \frac{\sum (x_i - \mu)^2}{n}$ . Below this, the expanded form is shown:  $= \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}$ .
$$\sigma^2 = \frac{\sum (x_i - \mu)^2}{n}$$
$$= \frac{(x_1 - \mu)^2 + (x_2 - \mu)^2 + \dots + (x_n - \mu)^2}{n}$$

## Numericals on Combination of errors and limiting error

proof :-

$$X = x_1 + x_2 + x_3 \quad \text{--- (i)}$$

diff. partially on both sides.

$$\delta X = \delta x_1 + \delta x_2 + \delta x_3$$

$$\frac{\delta X}{X} = \left(\frac{x_1}{X}\right) \frac{\delta x_1}{x_1} + \left(\frac{x_2}{X}\right) \frac{\delta x_2}{x_2} + \left(\frac{x_3}{X}\right) \frac{\delta x_3}{x_3}$$

$$\frac{\delta X}{X} = \pm \left[ \frac{x_1}{X} \left(\frac{\delta x_1}{x_1}\right) + \frac{x_2}{X} \left(\frac{\delta x_2}{x_2}\right) + \frac{x_3}{X} \left(\frac{\delta x_3}{x_3}\right) \right]$$

4. Multiplication/division of variables.

$$X = x_1 x_2 x_3 \quad \text{--- (i)} \quad \frac{1}{X} = \frac{1}{x_1 x_2 x_3} \quad \text{--- (ii)}$$

$$\frac{\delta X}{X} = \pm \left[ \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} + \frac{\delta x_3}{x_3} \right] \quad ; \quad \text{(proof: Apply log on both sides, diff... you will get...)}$$

5.  $y = \frac{x_1^m \cdot x_2^n}{x_3^p}$

$$\frac{\delta y}{y} = \pm \left[ m \frac{\delta x_1}{x_1} + n \frac{\delta x_2}{x_2} + p \frac{\delta x_3}{x_3} \right]$$

If any error is lying within the limits that means min. and max. value, it is known as unknown error. It is denoted by " $\pm$ " symbol.  $\text{E.g.} - (100 \pm 5) \Omega \Rightarrow (95 \Omega \text{ to } 100 \Omega)$

Question

Two resistors are given as  $R_1 = 100 \pm 4\%$  ( $100 \pm 4 \Omega$ )

$$R_2 = 50 \pm 2\%$$
 ( $50 \pm 4 \Omega$ )

when they are connected in series, Find the equivalent resistance.

i) Find  $(R_1 - R_2)$  ; (ii) Find  $R_1 R_2$  (iii)  $\frac{R_1}{R_2}$

In case of multiplication & division the % limiting errors are simply added but don't add the error in value form.

Q2 :- Two resistors are given as  $R_1 = 100 \pm 6\Omega = (100 \pm 6\%)$   
 $R_2 = (50 \pm 2\Omega) = (50 \pm 4\%)$

(i)  $R_1 + R_2$  (ii)  $R_1 - R_2$  (iii)  $R_1 R_2$  (iv)  $\frac{R_1}{R_2}$

Sol :- (i)  $R_{eq} = R_1 + R_2 = 100 + 50 = 150\Omega$

$$\therefore \frac{\delta x}{x} = \pm \left( \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right) = \pm \left( \frac{100}{150} \times (6) + \frac{50}{150} (4) \right)$$

$$= \cancel{11.33\%} \quad 5.33\%$$

$$\therefore R_{eq} = 150 \pm \cancel{11.33\%} = 150 \pm 8\Omega = (100 \pm 6\Omega) + (50 \pm 2\Omega)$$

$$= (150 \pm 8\Omega)$$

$$\boxed{\text{Nominal value} \left( \frac{x}{100} \right) = \text{Error in value}}$$

(ii)  $R_{eq} = R_1 - R_2 = 100 - 50 = 50\Omega = (100 \pm 6) - (50 \pm 2)$   
 $= 50 \pm 8\Omega$

$$\therefore \frac{\delta x}{x} = \pm \left( \frac{100}{50} (6) + \frac{50}{50} (2) \right) = \pm 16\%$$

$$\therefore R_{eq} = 50 \pm 16\% = (50 \pm 8)\Omega$$

(iii)  $R_{eq} = R_1 R_2 = (100 \pm 6\%)(50 \pm 4\%) = 5000 \pm (10\%)$   
 $= (5000 \pm 500)\Omega$

(iv)  $R_{eq} = \frac{R_1}{R_2} = \frac{100}{50} \pm (6 \pm 4) = 2 \pm 10\% = (2 \pm 0.2)\Omega$

(v)  $R_{eq} = \frac{R_1 R_2}{(R_1 + R_2)}$



Solution:-

$$\begin{aligned} \text{(i)} \quad (R_1 + R_2) &= (100 \pm 4\%) + (50 \pm 2\%) \\ &= (100 \pm 4\Omega) + (50 \pm 2\Omega) \\ &= 150 \pm 6\Omega \\ &= 150 \pm \frac{6}{150} \times 100 \cong 150 \pm 3.33\% \end{aligned}$$

$$\begin{aligned} \frac{\delta x}{x} &= \pm \left[ \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right] \\ &= \pm \left[ \frac{100}{150} \times 4 + \frac{50}{150} \times 2 \right] \\ &= \pm 3.33\% \end{aligned}$$

$$\therefore R_{eq} = 150 \pm 3.33\% = 150 \pm 5\Omega \quad (145\Omega \text{ to } 155\Omega)$$

$\downarrow$  nominal value       $\downarrow$  Limiting error       $\downarrow$  error in value form.

$$\text{(ii)} \quad (R_1 - R_2) = (100 \pm 4\%) + (50 \pm 2\%)$$

$$x = x_1 - x_2 = 100 - 50 = 50\Omega$$

$$\frac{\delta x}{x} = \pm \left[ \frac{x_1}{x} \frac{\delta x_1}{x_1} + \frac{x_2}{x} \frac{\delta x_2}{x_2} \right] = \pm \left[ \frac{100}{50} \times 4 + \frac{50}{50} \times 2 \right]$$

$$\frac{\delta x}{x} = \pm 10\%$$

$$R_{eq} = 50 \pm 10\% = 50 \pm 5\Omega \Rightarrow (45 \text{ to } 55)\Omega$$

$\downarrow$  nominal value       $\downarrow$  error in value form

$$\begin{aligned} \text{(iii)} \quad R_1 R_2 &\Rightarrow x = x_1 x_2 \Rightarrow \frac{\delta x}{x} = \frac{\delta x_1}{x_1} + \frac{\delta x_2}{x_2} \\ \therefore x &= 100 \times 50 = 5000\Omega; \quad \frac{\delta x}{x} = 4 + 2 = \pm 6\% \\ \therefore R_{eq} &= R_1 R_2 = 5000 \pm 6\% = (5000 \pm 300)\Omega \end{aligned}$$

$$\gamma = 0.8453 \bar{D}$$

$$\therefore \gamma = 0.6745 \sigma = \frac{0.4765 \bar{D}}{h} = 0.8453 \bar{D}$$

$$h = \frac{0.4765}{(0.6745) \sigma}$$

$$h = \frac{0.906}{\sigma}$$

$$h = \frac{1}{0.906 \sigma}$$

$$\boxed{h \sigma = \frac{1}{0.906}}$$

$$\sigma_{\bar{D}} = \frac{\sigma}{\sqrt{n}} \times \frac{1}{\sqrt{2}} = \frac{\sigma_m}{\sqrt{2}}$$

$$\therefore \boxed{\sigma_{\bar{D}} = \frac{\sigma_m}{\sqrt{2}}}$$

mean probable error =  $\gamma_m$

$$\gamma_m = \frac{0.6745 \sigma}{\sqrt{(n-1)}} = \frac{\gamma}{\sqrt{(n-1)}} \text{ for } (n \leq 20)$$

$$\boxed{\gamma_m = \frac{\gamma}{\sqrt{n}}}; n > 20.$$

standard deviation of mean  $\boxed{\sigma_{\bar{D}} = \frac{\sigma}{\sqrt{n}}}$

standard deviation of standard deviation

$$\Rightarrow \boxed{\sigma_{\sigma} = \frac{\sigma}{\sqrt{2n}}}$$

minimum range of error =  $x_{avg} - x_{min}$

maximum range of error =  $x_{max} - x_{avg}$ .

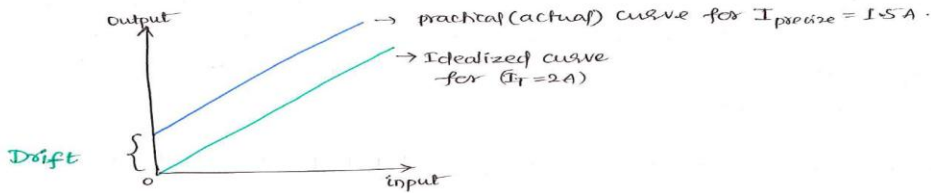
Average range of error =  $\frac{(x_{avg} - x_{min}) + (x_{max} - x_{avg})}{2}$

$$\boxed{\text{Avg. Range of error} = \frac{x_{max} - x_{min}}{2}}$$

Highly precise instrument doesn't mean that highly accurate.  
 Because the most precise instruments will give the wrong reading  
 so that precision never confirms accuracy.

→ $I_{true} = 2A$ ;	(1.5A) reading is	Reproducibility
out of 10 readings	⇒ repeated 5 times	⇒ 50%
	⇒ " 8 "	⇒ 8%
	⇒ " 10 "	⇒ 100%

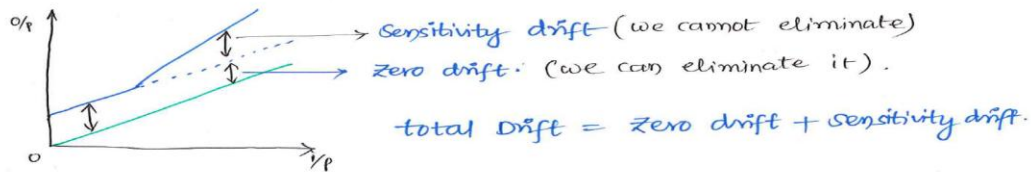
Reproducibility refers to the degree of repeatability.



\* Reproducibility :- Refers to the degree of repeatability.

A perfectly reproducible instrument is having zero drift.

Zero Drift can be eliminated by recalibrating the instrument.

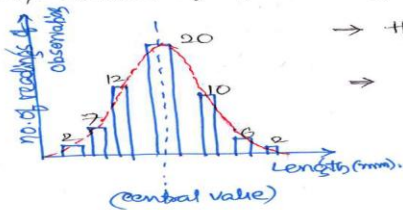


(Ans) also

**Central value :-** If we make a large no. of measurements and if the plus effects are equal to the minus effects, they would cancel each other and we would obtain the scatter around a central value.

This condition is frequently met in practice.

**Histogram :-** When a no. of multisample observations are taken experimentally there is a scatter of data about some central value. One method of presenting test results in the form of a histogram.



→ Histogram is also called as a **frequency distribution curve**.

→ With more and more data taken at smaller and smaller increments the histogram would finally change into a **smooth curve**.

The most probable value of measured variable (variate) is the arithmetic mean of the no. of readings taken. Theoretically, an infinite no. of readings would give the best result, although in practice only finite no. of measurements can be made.

**Measure of Dispersion from the mean :-**

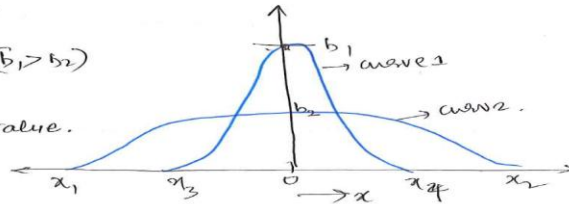
Dispersion :- The property which denotes the extent to which the values are dispersed about the central value called dispersion.

dispersion = spread = scatter

curve 1 :- Greater precision ( $b_1 > b_2$ )

curve 2 :- Lower precision

$x$  ⇒ deviation from the central value.



Dispersion is more for curve 2. That means ....

(12)

A large dispersion indicates that some factors involved in the measurement process are not under close control and  $\therefore$  it becomes more difficult to estimate the measured quantity with confidence and definiteness.

$$\text{Range} \Rightarrow (x_2 - x_1); (x_4 - x_3);$$

**Deviation** :- Deviation is the departure of the observed reading from the arithmetic mean of the group of readings.

$$d_1 = x_1 - \bar{x}$$

$$d_2 = x_2 - \bar{x}$$

$$\dots \dots \dots$$

$$d_n = x_n - \bar{x}$$

$$\Sigma d_i = d_1 + d_2 + \dots + d_n$$

$$= (x_1 - \bar{x}) + (x_2 - \bar{x}) + \dots + (x_n - \bar{x})$$

$$= (x_1 + x_2 + x_3 + \dots + x_n) - n\bar{x}$$

$$= n\bar{x} - n\bar{x}$$

$$\therefore \bar{x} = \frac{\Sigma x_i}{n}$$

$$\Sigma d_i = 0.$$

$\therefore$  Algebraic sum of deviations is always zero.

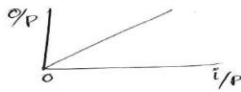
Highly precise instruments yield a low value of average deviation between the readings. The average deviation is an indication of the precision of the instruments used in making the measurements.

$$\text{i.e. } \bar{D} = \frac{|d_1| + |d_2| + \dots + |d_n|}{n} = \text{avg. deviation.}$$

$$(\text{precision}) \propto \frac{1}{(\text{avg. deviation})}$$

Linearity :- slope of curve (o/p vs i/p) = constant. (12)

i.e. (Output)  $\propto$  (Input).  
 proportional output ; slopes = constant  
 i.e. Uniform scale reading.



non-Linearity :-



slopes  $\neq$  constant  
 (i.e. variable slope).  
 (Eg:) slope (Output)  $\propto$  (Input)<sup>2</sup>.  
 cramped scale at lower end.



Eg:

$$\theta \propto I^2$$

$$I = 1A \Rightarrow \theta = 1^2$$

$$I = 2A \Rightarrow \theta = 2^2$$

$$I = 5A \Rightarrow \theta = 5^2$$

⋮

$$I = 10A \Rightarrow \theta = 10^2$$

$$I = 20A \Rightarrow \theta = 20^2$$

$$I = 30A \Rightarrow \theta = 30^2$$

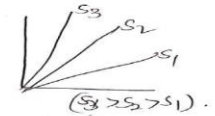
If the output follows the input with a proportional relationship then the instrument is said to be linear. Otherwise, if the output follows the input with a square law relationship

then the instrument is said to be non-linear. For a non-linear instrument's scale is cramped at lower end.

Sensitivity :- Slope of curve  $\equiv$  sensitivity =  $\frac{d(o/p)}{d(i/p)} = \frac{dy}{dx}$  ;

It is defined as the ratio of infinitesimal change in output to the change in input is known as sensitivity.

For linear instruments - constant sensitivity  
non-linear instrument - variable sensitivity.



High sensitive (elements) instruments will respond for high small inputs.

Q:- There are 3 sets of data having average deviations of values 0.6, 0.3, and 0.5 for A, B, C set of data respectively. Then what is the correct order of precision of measurements of A, B, C set of data.

(a)  $A > B > C$

(b)  $A > C > B$

(c)  $B > C > A$

(d) We cannot say until we get information about standard deviation.

Sol:- precision  $\propto \frac{1}{(\text{avg. deviation})}$   $\propto \frac{1}{(\text{dispersion} = \text{spread} = \text{scatter})}$

### Normal (or) Gaussian Distribution Curve of Errors :-

This is the basis for the major part of study of random errors. This type of distribution is most frequently met in normal practices.

The law of probability states the normal occurrence of deviations from average value of an infinite no. of measurements (or) observations can... be expressed as..

$$y = \frac{b}{\sqrt{\pi}} \exp(-b^2 x^2) ; y = \frac{1}{\sigma\sqrt{2\pi}} \exp(-x^2/2\sigma^2).$$

$x$  = magnitude of deviation from mean.

$y$  = no. of readings at any deviation  $x$ .

(The probability of occurrence of deviation  $x$ )

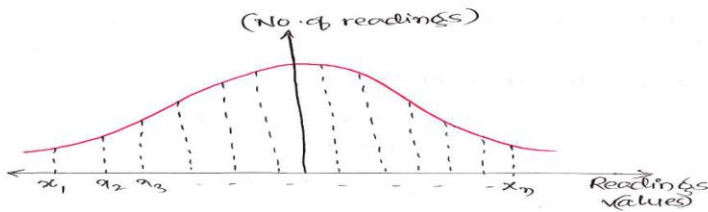
$b$  = a constant called precision index

$\sigma$  = standard deviation.

$\sigma$  = usually known as quantity of interest.

## Gaussian statistical Analysis :-

(2)



$$\text{Arithmetic mean (AM)} = \bar{x} = \frac{x_1 + x_2 + x_3 + \dots + x_n}{n} = \boxed{\frac{\sum_{i=1}^n x_i}{n} = \bar{x}}$$

$$\text{Deviations (d}_i\text{)} ; d_1 = x_1 - \bar{x} ; d_2 = x_2 - \bar{x} \dots , d_n = x_n - \bar{x}$$

$$\boxed{d_i = x_i - \bar{x}}$$

$$\text{mean deviation} = \bar{D} = \frac{|d_1| + |d_2| + \dots + |d_n|}{n}$$

$$\text{standard deviation } (\sigma) = \sqrt{\frac{d_1^2 + d_2^2 + \dots + d_n^2}{n}}$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n |d_i|^2}{n}} \quad \text{for infinite no. of observation. i.e. } n > 20.$$

$$\sigma = \sqrt{\frac{\sum_{i=1}^n |d_i|^2}{(n-1)}} \quad \text{for finite no. of observations (ii) } n \leq 20.$$

$$\text{Variance (V)} = \sigma^2 = (\text{standard deviation})^2$$

$$\text{probable error } (\gamma) = 0.6745 \sigma ; \sigma \propto \gamma$$

$$\boxed{\gamma = 0.6745 \sigma}$$

$$\boxed{\gamma = \frac{0.4765}{h}}$$

$h \rightarrow$  precision index



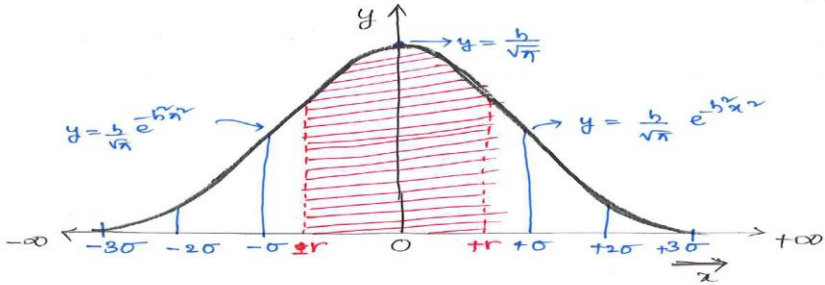


Fig:- Normal probability curve (or) Gaussian curve.

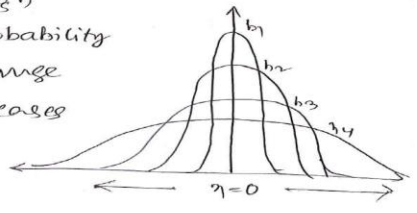
→ The value of  $b$  determines the sharpness of the curve since the curve drops sharply owing to the term  $(-b^2)$  being in the exponent. The sharp curve evidently indicates that the deviations are more closely grouped together around deviation  $x=0$ .

⇒ It is clear that the probability that a variate lies in a given range becomes less as the deviation of the range becomes greater.

If greater the value of  $b$ ... for a more probability less.

∴ Thus the name... **precision index** for  $b$  is reasonable.

→ A large value of  $b$  represents high precision of the data because the probability of occurrence of variates in a given range falls off rapidly as the deviation increases because the variates tends to cluster (becomes closer) into a narrow range.



### Probable Error:-

The confidence in the best value (most probable value) is connected with the sharpness of the distribution curve.

Total area of the Gaussian curve = 1.

$$\frac{1}{\sigma\sqrt{2\pi}} \int_{-\infty}^{+\infty} \exp(-x^2/2\sigma^2) dx = 1.$$

A convenient measure of precision is the quantity ( $r$ ), called probable error.

$$\frac{b}{\sqrt{\pi}} \int_{-r}^{+r} \exp(-b^2 x^2) dx = \frac{1}{2}$$

$$\therefore r = 0.6745\sigma = \frac{0.4769}{b}$$

### average deviation for normal curve:

$$\bar{D} = \int_{-\infty}^{+\infty} |x| y dx$$

$$b = \frac{1}{\sqrt{\pi} \bar{D}} \Rightarrow \bar{D} = \frac{r}{0.4769\sqrt{\pi}} = \frac{r}{0.8453}$$

$$r = 0.8453 \bar{D}$$

$$PE = r = 0.6745\sigma = \frac{0.4769}{b} = 0.8453 \bar{D}.$$

for finite readings ( $n$ ); - probable error ( $r_m$ ) =  $0.6745 \frac{\sigma}{\sqrt{n}}$

standard deviation of mean  $\sigma_m = \frac{\sigma}{\sqrt{n}}$

standard deviation of standard deviation  $\sigma_{\sigma} = \frac{\sigma}{\sqrt{2n}} = \frac{\sigma_m}{\sqrt{2}}$

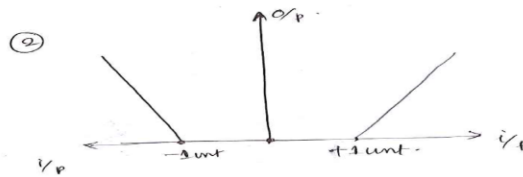
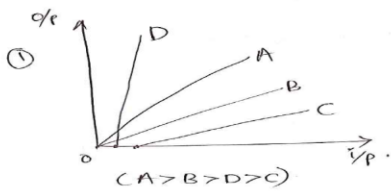
$\therefore (\text{Time-const})_{\text{mech.}} > (\text{Time-constant})_{\text{electrical}}$ .

The time taken by the instrument in order to give the response is known as dead-time.

There is no electrical inertia b/c the mass of electron is very very small. Where as every mechanical body will offer some inertia so that it takes considerable amount of time in order to give the response. B/c mechanical time constants are always greater than that of electrical time constants.

**Dead zone** :- For the largest value of input, the response of the instrument is zero. Beyond this input value the instrument gives the response. The corresponding portion of input where the output is known as dead zone.

**Threshold** :- At what particular input value, the instrument will give the response is known as threshold (or) pick-up

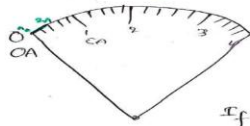
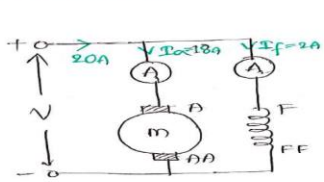


① Identify the from the above plot which instrument is the best inst is A

② Find the dead zone, threshold =  $\pm 1$  unit  
 dead zone = 2 units

**Resolution** :- refers to clarity (or) certainty.

(16)



FSD (40A)  $\Rightarrow$  1 unit =  $\frac{20}{4 \times 5} = 1A$ .

$I_f$   $A_1 = A_2$  (0-20A)  $\Rightarrow$   $A_1$  0-20,  $A_2$  0-5

more no. of divisions  $\Rightarrow$  more clarity, more resolutions.

The smallest value of change in input that we can detect with more clarity (or) more certainty is known as resolution.

**Fidelity** :- The readings obtained had much faith, fully represented in the second book is known as fidelity.

**Significant figures** :-

When we are representing the significant figures in a resultant variable always we will select the min. significant figure of the given variable.

**Types of standards.**

1. International stds  $\Rightarrow$  IEEE, ISO

eg:- std. voltage  $\Rightarrow$  Western company.

Western std. cell  $\Rightarrow$   $E = 1.0183 = 1.0183$  volt

(Highest accuracy).  $\Rightarrow$  not available for common man (patents required)

2. National standards (primary standards). ISIRI.

(much more accurate), not available

3. secondary standards (Industrial standards).

Every industry has its own standards..., not available

**Q:-** Five students given the current readings by using ammeters and recorded as 10.03 A, 10.11 A, 10.12 A, 10.08 A. (16)

Find the (i) AM

(ii) deviations ( $d_i$ ); (iii) mean deviations ( $\bar{D}$ )

(iv) std. deviation ( $\sigma$ ); (v) variance =  $\sigma^2$ ; (vi) probable error

(vii) mean probable error; (viii)  $\sigma_m$  (std. deviation of mean)

(ix)  $\sigma_{\sigma}$  (std. deviation of std. deviation).

(x) min. range of error, max. range of error.

**Sol:-**

AM =  $\bar{x} = 10 + 0.085 \approx 10.08$

$d_1 = 10.03 - 10.08 = -0.05$

$d_2 = 10.11 - 10.08 = +0.03$

$d_3 = 10.12 - 10.08 = +0.04$

$d_4 = 10.08 - 10.08 = 0.00$

$\bar{D} = \frac{10.05 + (0.03) + 0.04 + 0}{4}$

$\bar{D} = 0.03$

variance =  $\sigma^2 = 0.001684$

$\sigma = \sqrt{\frac{d_1^2 + d_2^2 + d_3^2 + d_4^2}{(n-1)}}$

$= \sqrt{\frac{0.05^2 + 0.03^2 + 0.04^2 + 0}{(4-1)}}$

$= \sqrt{\frac{25 + 9 + 16}{3 \times 10^4}}$

$\sigma = 0.0408$ .

mean probable error ( $r$ ) =  $0.6745 \sigma = 0.6745 \times 0.0408 = 0.0275$

std. deviation of mean ( $\sigma_m$ ) =  $\frac{\sigma}{\sqrt{n}} = \frac{0.0408}{\sqrt{4}} = 0.0204$

st. de. of st. de. ( $\sigma_{\sigma}$ ) =  $\frac{1}{\sqrt{2}} \sigma_m = 0.0144$

mean probable error ( $r_m$ ) =  $\frac{r}{\sqrt{n-1}} = \frac{0.0275}{\sqrt{3}} = 0.015$

min. range of error =  $x_{avg} - x_{min} = 10.08 - 10.03 = 0.05$

max. " " =  $x_{max} - x_{avg} = 10.12 - 10.08 = 0.04$

AVG " " =  $\frac{0.05 + 0.04}{2} = \pm 0.045$ .