JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTER

Class – B.Tech Civil (III SEM)
Subject –Fluid Mechanics
Unit – 1
Presented by – Ashish Boraida (Assistant Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional ,cultural and social enrichment of the lives of indivisuals and communities

MISSION OF INSTITUTE

Focus on evaluation of learining ,outcomes and motivate students to research apptitude by project based learning.

- Identify based on informed perception of indian ,regional and global needs ,the area of focus and provide plateform to gain knowledge and solutions.
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- Offer oppurtunites for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

VISION AND MISSION OF DEPARTMENT

Vision

To become a role model in the field of Civil Engineering for the sustainable development of the society.

Mission

1)To provide outcome base education.

2)To create a learning environment conducive for achieving academic excellence.

3)To prepare civil engineers for the society with high ethical values.

Introduction, Objective and Outcome of Fluid Mechanics

Objective:

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The primary purpose of the study of Fluid mechanics is to develop the capacity to understand important basic terms used in fluid mechanics, understand hydrostatics and buoyancy with practice of solving problems. Student could be able to understand Kinematics of flow and fluid dynamics, Bernoulli’s equation and laminar flow with practice of solving problems in practical life for the benefit of society and mankind.

Outcomes

- Student will be able to understand basics of fluid mechanics, types of fluids.
- Student will be able to understand fluid statics and buoyancy.
- Student will be to understand Kinematics of flow and fluid dynamics and solving relevant problems.
- Student will be to understand Bernoulli’s equation and laminar flow with practice of solving problems.

CONTENTS

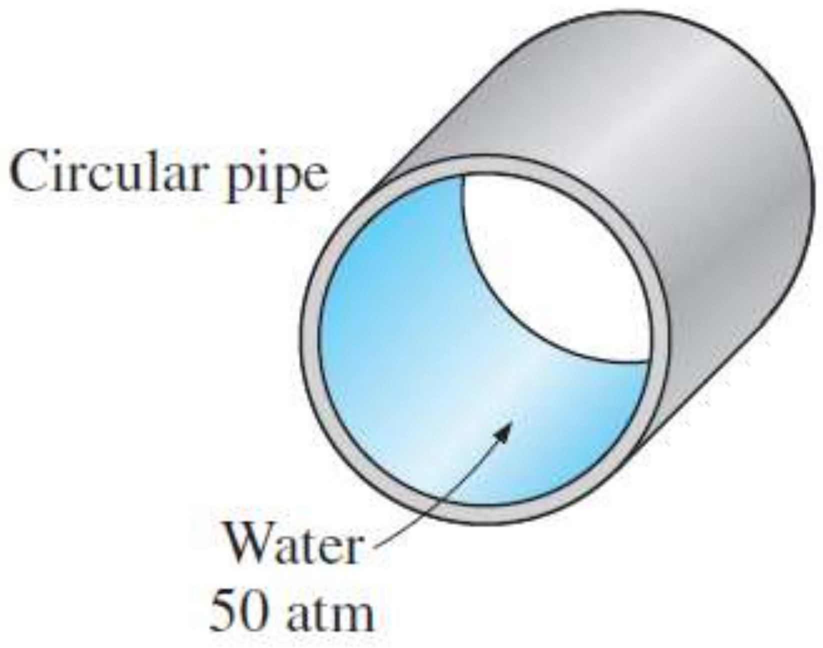
- Laminar Flow
- Laminar Flow in Pipes
- Pressure Drop
- Applications

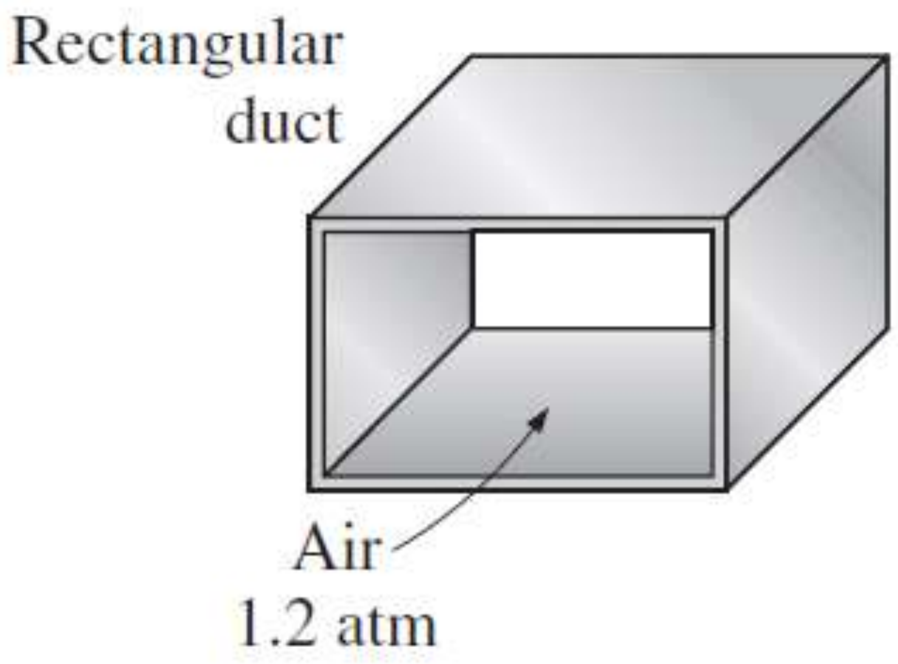
Objectives

- Have a deeper understanding of laminar and turbulent flow in pipes and the analysis of fully developed flow
- Calculate the major and minor losses associated with pipe flow in piping networks and determine the pumping power requirements
- Understand various velocity and flow rate measurement techniques and learn their advantages and disadvantages

INTRODUCTION

- Liquid or gas flow through *pipes* or *ducts* is commonly used in heating and cooling applications and fluid distribution networks.
- The fluid in such applications is usually forced to flow by a fan or pump through a flow section.
- We pay particular attention to *friction*, which is directly related to the *pressure drop* and *head loss* during flow through pipes and ducts.
- The pressure drop is then used to determine the *pumping power requirement*.





Circular pipes can withstand large pressure differences between the inside and the outside without undergoing any significant distortion, but noncircular pipes cannot.

Theoretical solutions are obtained only for a few simple cases such as fully developed laminar flow in a circular pipe.

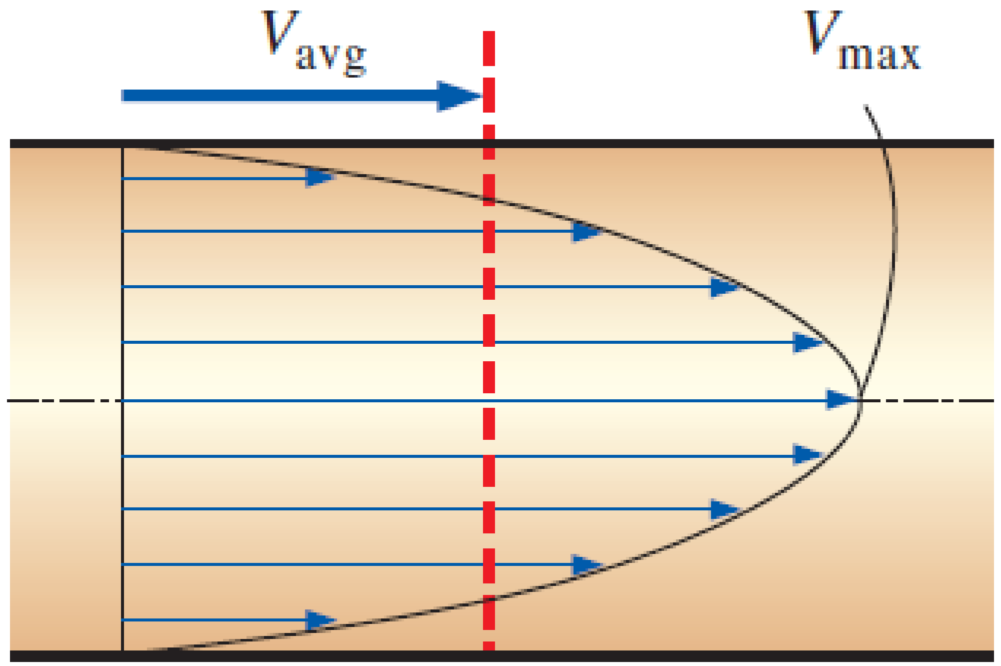
Therefore, we must rely on experimental results and empirical relations for most fluid flow problems rather than closed-form analytical solutions.

$$\dot{m} = \rho V_{avg} A_c = \int_{A_c} \rho u(r)\, dA_c$$

The value of the average velocity V_{avg} at some streamwise cross-section is determined from the requirement that the *conservation of mass* principle be satisfied

$$V_{avg} = \frac{\int_{A_c} \rho u(r)\, dA_c}{\rho A_c} = \frac{\int_0^R \rho u(r) 2\pi r\, dr}{\rho \pi R^2} = \frac{2}{R^2} \int_0^R u(r) r\, dr$$

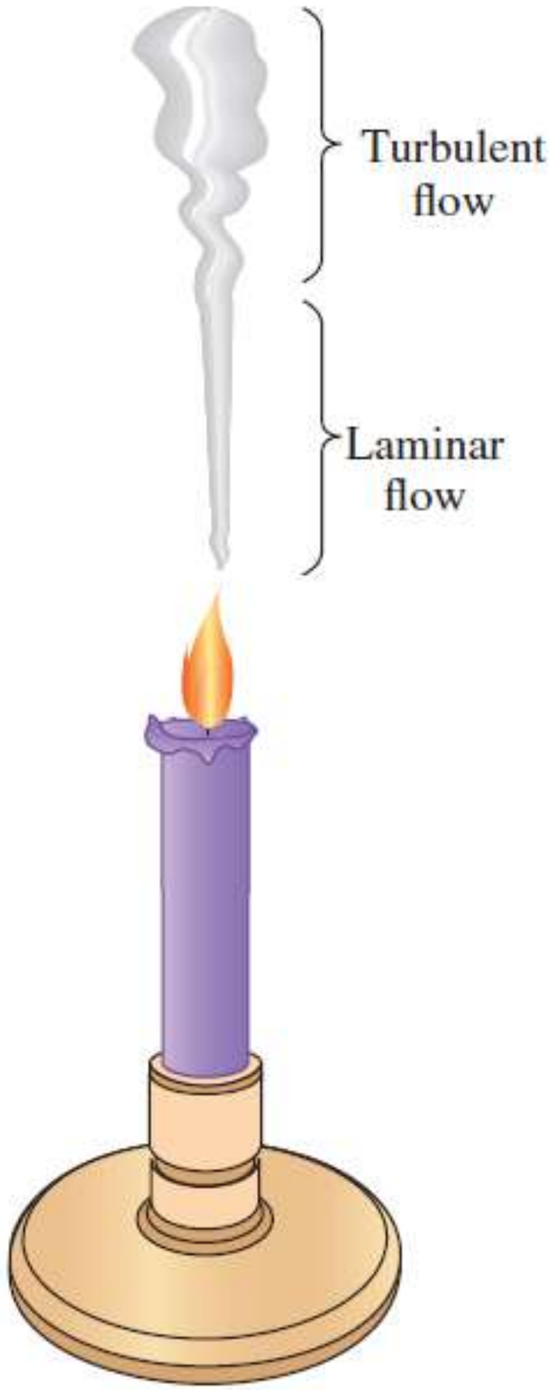
The average velocity for incompressible flow in a circular pipe of radius *R*



city V_{avg} is defined as the average
h a cross section. For fully
minar pipe flow, V_{avg} is half of the
locity.

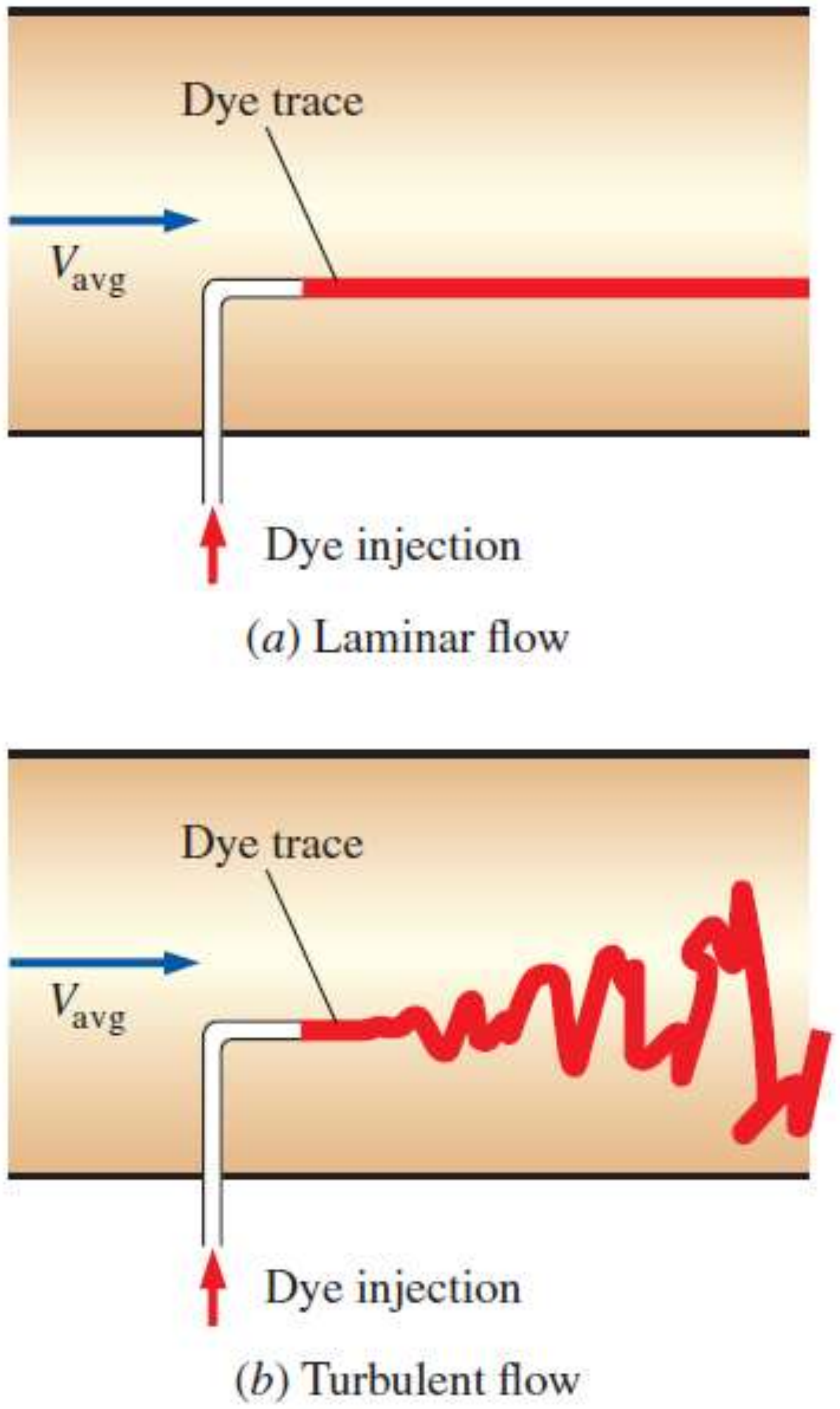
LAMINAR AND TURBULENT FLOWS

Laminar flow is encountered when highly viscous fluids such as oils flow in small pipes or narrow passages.



Laminar: Smooth streamlines and highly ordered motion.
Turbulent: Velocity fluctuations and highly disordered motion.
Transition: The flow fluctuates between laminar and turbulent flows.
Most flows encountered in practice are turbulent.

Laminar and turbulent flow regimes of candle smoke.



The behavior of colored fluid injected into the flow in laminar and turbulent flows in a pipe.

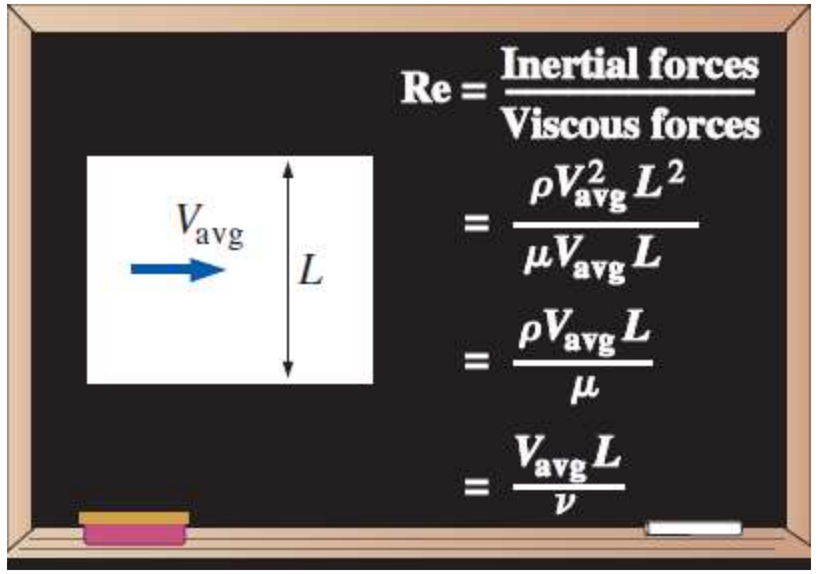
Reynolds Number

The transition from laminar to turbulent flow depends on the *geometry*, *surface roughness*, *flow velocity*, *surface temperature*, and *type of fluid*.

The flow regime depends mainly on the ratio of *inertial forces* to *viscous*

$$\text{Re} = \frac{\text{Inertial forces}}{\text{Viscous forces}} = \frac{V_{avg}D}{\nu} = \frac{\rho V_{avg}D}{\mu}$$

At large Reynolds numbers, the inertial forces, which are proportional to the fluid density and the square of the fluid velocity, are large relative to the viscous forces, and thus the viscous forces cannot prevent the random and rapid fluctuations of the fluid (turbulent).
At small or moderate Reynolds numbers, the viscous forces are large enough to suppress these fluctuations and to keep the fluid "in line" (laminar).



The Reynolds number can be viewed as the ratio of inertial forces to viscous forces acting on a fluid element.

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8–4 ■ LAMINAR FLOW IN PIPES

We consider steady, laminar, incompressible flow of a fluid with constant properties in the fully developed region of a straight circular pipe.

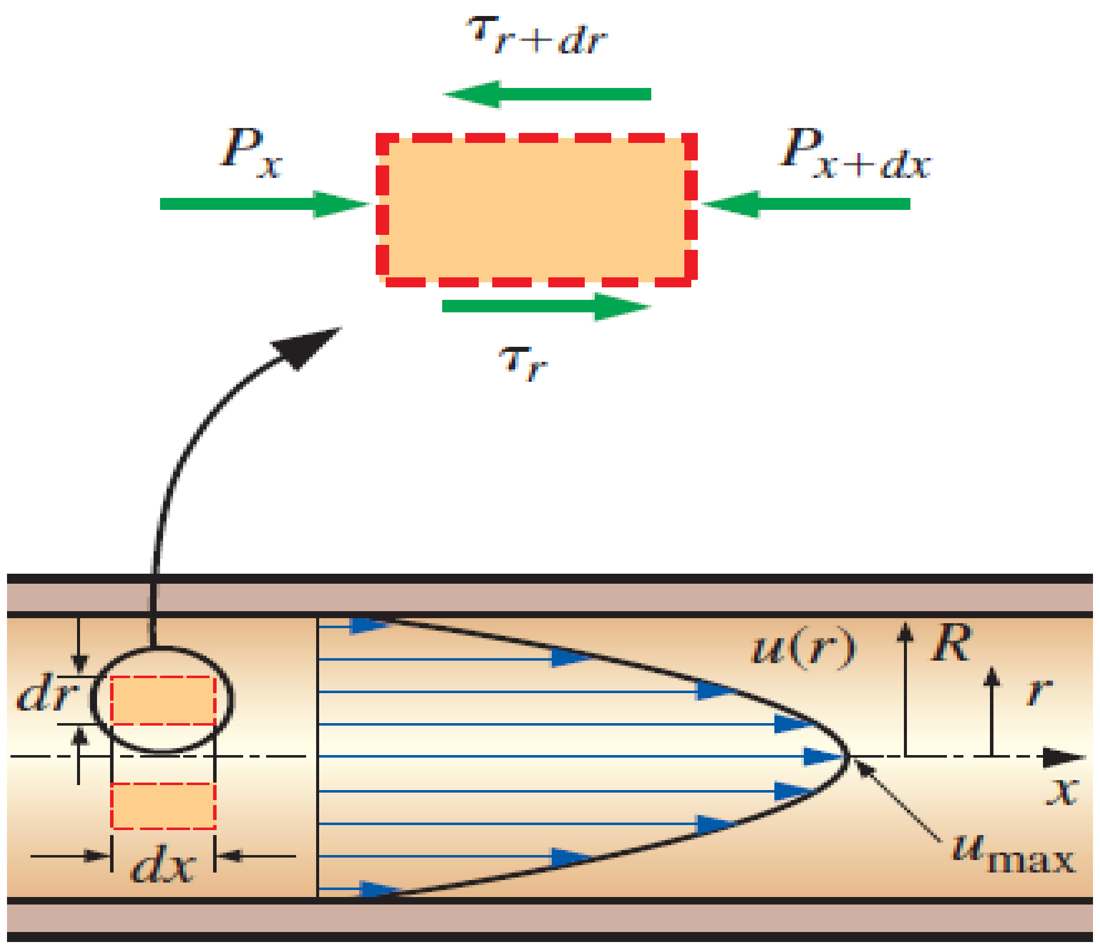
In fully developed laminar flow, each fluid particle moves at a constant axial velocity along a streamline and the velocity profile *u*(*r*) remains unchanged in the flow direction. There is no motion in the radial direction, and thus the velocity component in the direction normal to the pipe axis is everywhere zero. There is no acceleration since the flow is steady and fully developed.

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} = 0$$

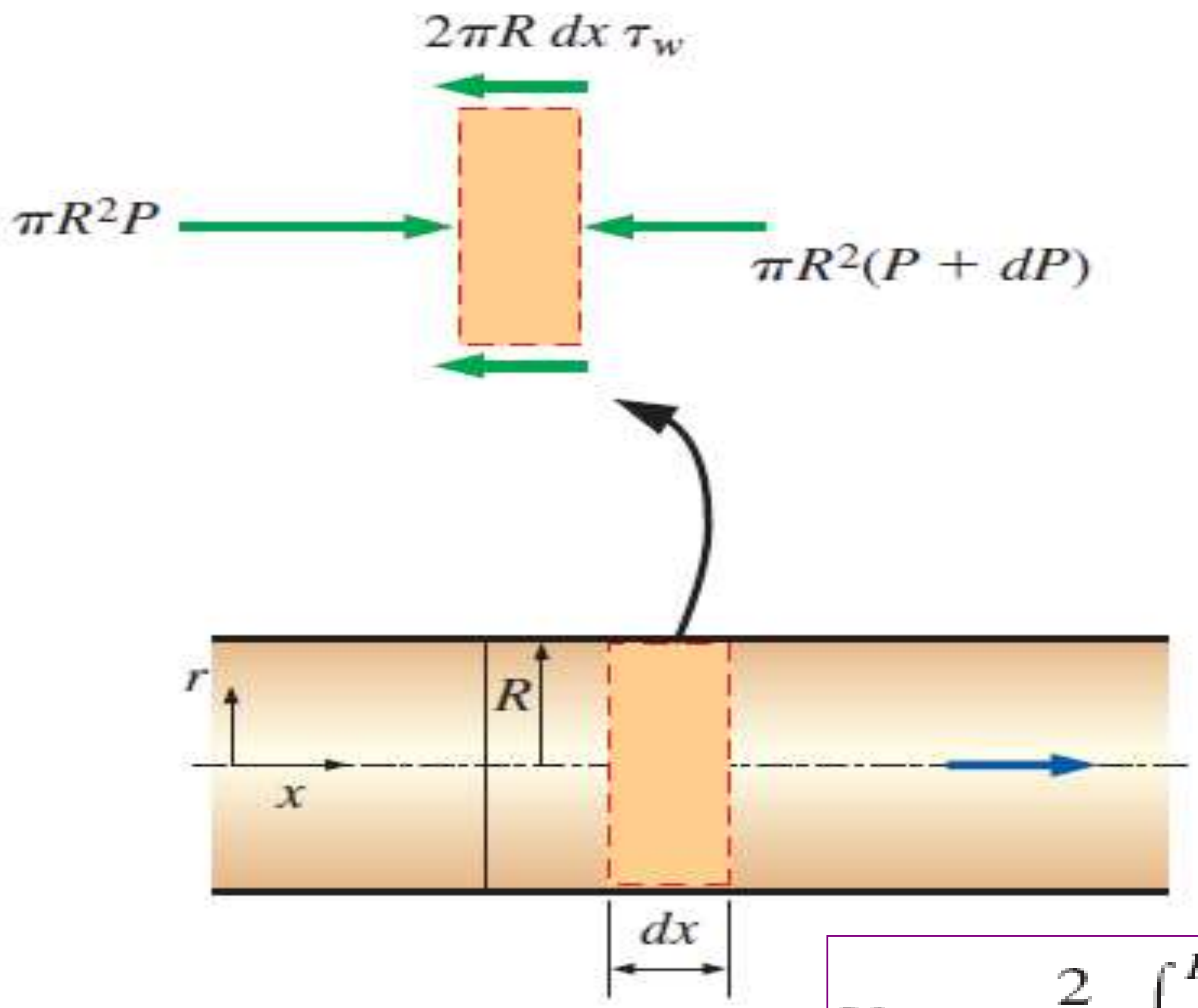
$$r\frac{P_{x+dx} - P_x}{dx} + \frac{(r\tau)_{r+dr} - (r\tau)_r}{dr} = 0$$

$$r\frac{dP}{dx} + \frac{d(r\tau)}{dr} = 0 \qquad \tau = -\mu(du/dr)$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx}$$



Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with a horizontal pipe in fully developed laminar flow.



$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = \frac{r^2}{4\mu}\left(\frac{dP}{dx}\right) + C_1 \ln r + C_2$$

$\partial u/\partial r = 0$ at $r = 0$ Boundary conditions

$u = 0$ at $r = R$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right)$$

Average velocity

$$V_{avg} = \frac{2}{R^2}\int_0^R u(r) r\,dr = \frac{-2}{R^2}\int_0^R \frac{R^2}{4\mu}\left(\frac{dP}{dx}\right)\left(1 - \frac{r^2}{R^2}\right) r\,dr = -\frac{R^2}{8\mu}\left(\frac{dP}{dx}\right)$$

Force balance:

$$\pi R^2 P - \pi R^2(P + dP) - 2\pi R\,dx\,\tau_w = 0$$

Simplifying:

$$\frac{dP}{dx} = -\frac{2\tau_w}{R}$$

$$u(r) = 2V_{avg}\left(1 - \frac{r^2}{R^2}\right)$$ Velocity profile

$$u_{max} = 2V_{avg}$$ Maximim velocity at centerline

Free-body diagram of a fluid disk element of radius *R* and length *dx* in fully developed laminar flow in a horizontal pipe.

Pressure Drop and Head Loss

A quantity of interest in the analysis of pipe flow is the *pressure drop* ΔP since it is directly related to the power requirements of the fan or pump to maintain flow. We note that dP/dx = constant, and integrating from $x = x_1$ where the pressure is P_1 to $x = x_1 + L$ where the pressure is P_2 gives

$$\frac{dP}{dx} = \frac{P_2 - P_1}{L}$$

Laminar flow:

$$\Delta P = P_1 - P_2 = \frac{8\mu L V_{avg}}{R^2} = \frac{32\mu L V_{avg}}{D^2}$$

A pressure drop due to viscous effects represents an irreversible pressure loss, and it is called pressure loss ΔP_L.

$$\Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2}$$

pressure loss for all types of fully developed internal flows

$$f = \frac{64\mu}{\rho D V_{avg}} = \frac{64}{\text{Re}}$$

Circular pipe, laminar

$\rho V_{avg}^2/2$ dynamic pressure

$$f = \frac{8\tau_w}{\rho V_{avg}^2}$$

Darcy friction factor

$$h_L = \frac{\Delta P_L}{\rho g} = f\frac{L}{D}\frac{V_{avg}^2}{2g}$$

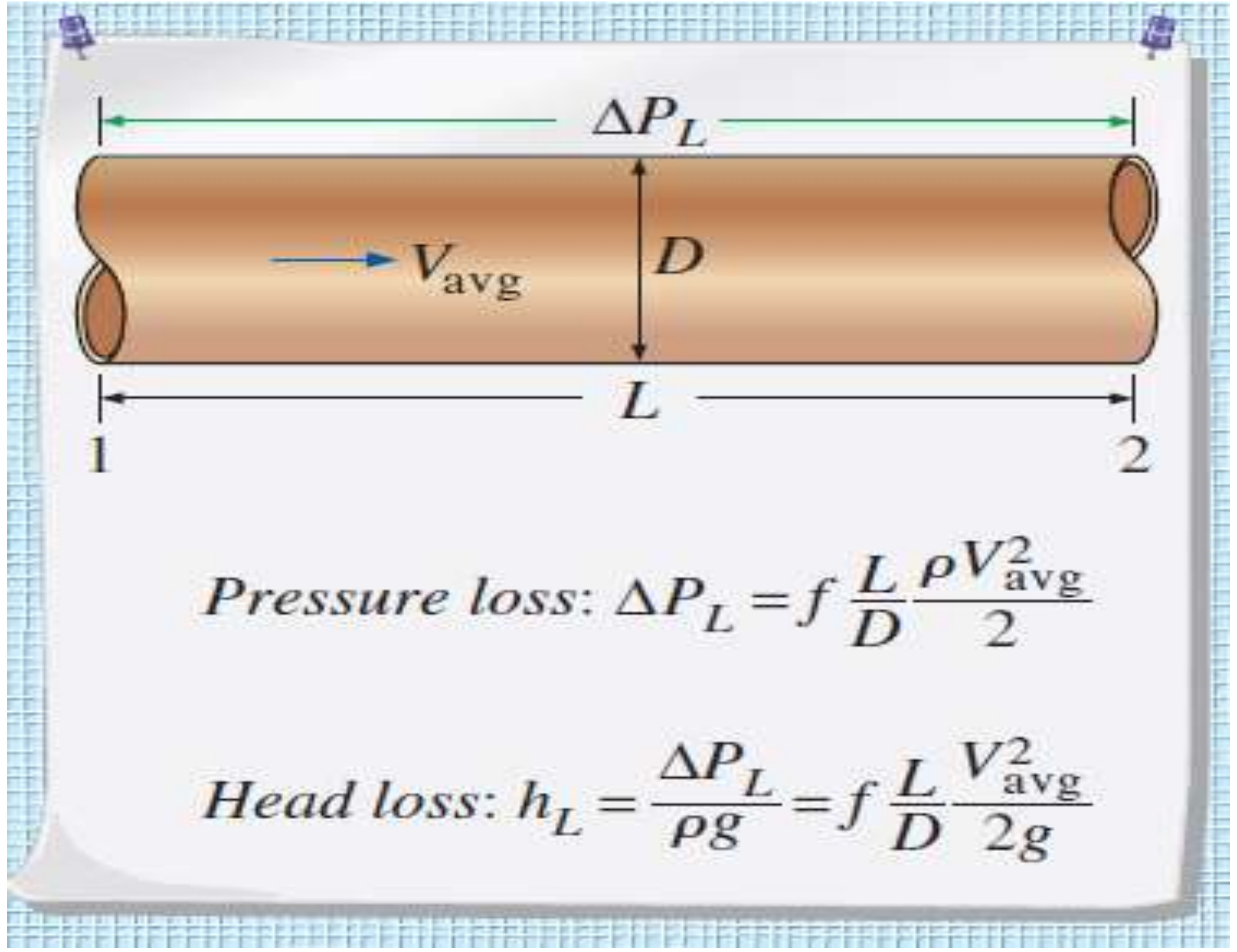
Head loss

In laminar flow, the friction factor is a function of the Reynolds number only and is independent of the roughness of the pipe surface.

The head loss represents the additional height that the fluid needs to be raised by a pump in order to overcome the frictional losses in the pipe.

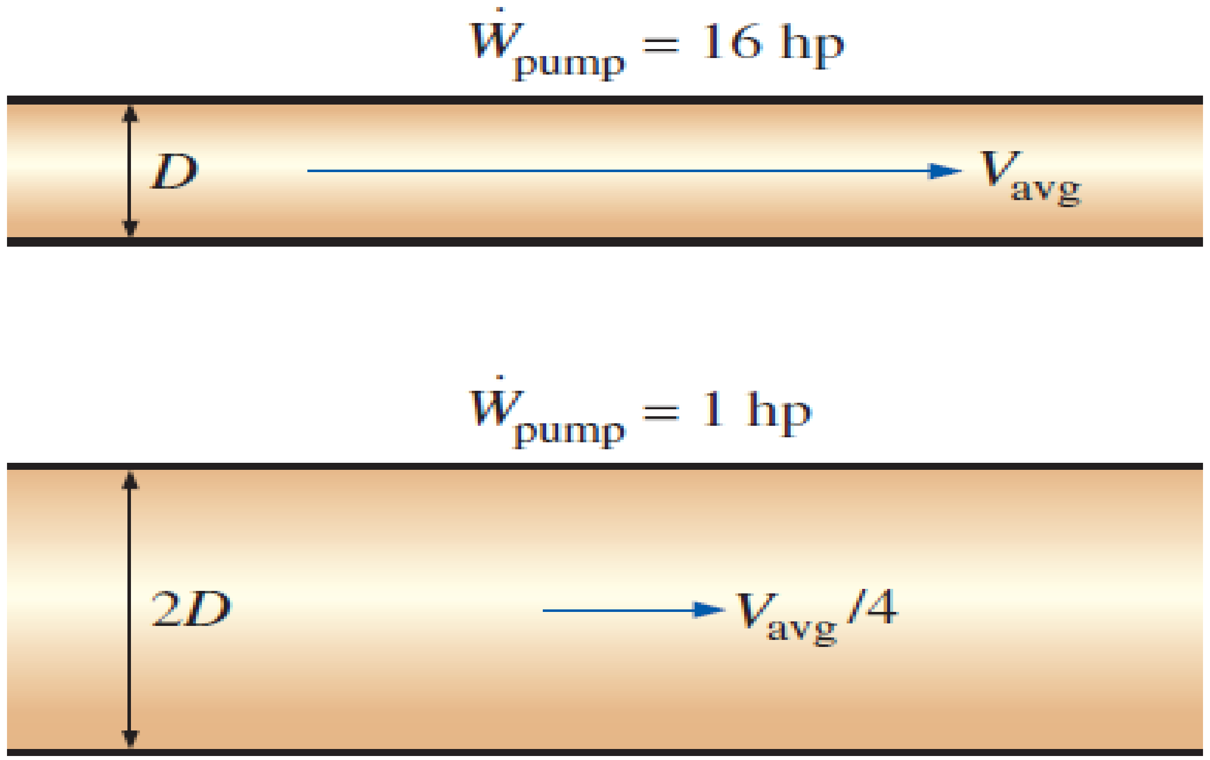
$$\dot{W}_{pump,\,L} = \dot{V}\,\Delta P_L = \dot{V}\rho g h_L = \dot{m} g h_L \qquad V_{avg} = \frac{(P_1 - P_2)R^2}{8\mu L} = \frac{(P_1 - P_2)D^2}{32\mu L} = \frac{\Delta P\, D^2}{32\mu L}$$ Horizontal pipe

$$\dot{V} = V_{avg} A_c = \frac{(P_1 - P_2)R^2}{8\mu L}\pi R^2 = \frac{(P_1 - P_2)\pi D^4}{128\mu L} = \frac{\Delta P\, \pi D^4}{128\mu L}$$ Poiseuille's law



For a specified flow rate, the pressure drop and thus the required pumping power is proportional to the length of the pipe and the viscosity of the fluid, but it is inversely proportional to the fourth power of the diameter of the pipe.

The relation for pressure loss (and head loss) is one of the most general relations in fluid mechanics, and it is valid for laminar or turbulent flows, circular or noncircular pipes, and pipes with smooth or rough surfaces.



The pumping power requirement for a laminar flow piping system can be reduced by a factor of 16 by doubling the pipe diameter.

The pressure drop ΔP equals the pressure loss ΔP_L in the case of a horizontal pipe, but this is not the case for inclined pipes or pipes with variable cross-sectional area.

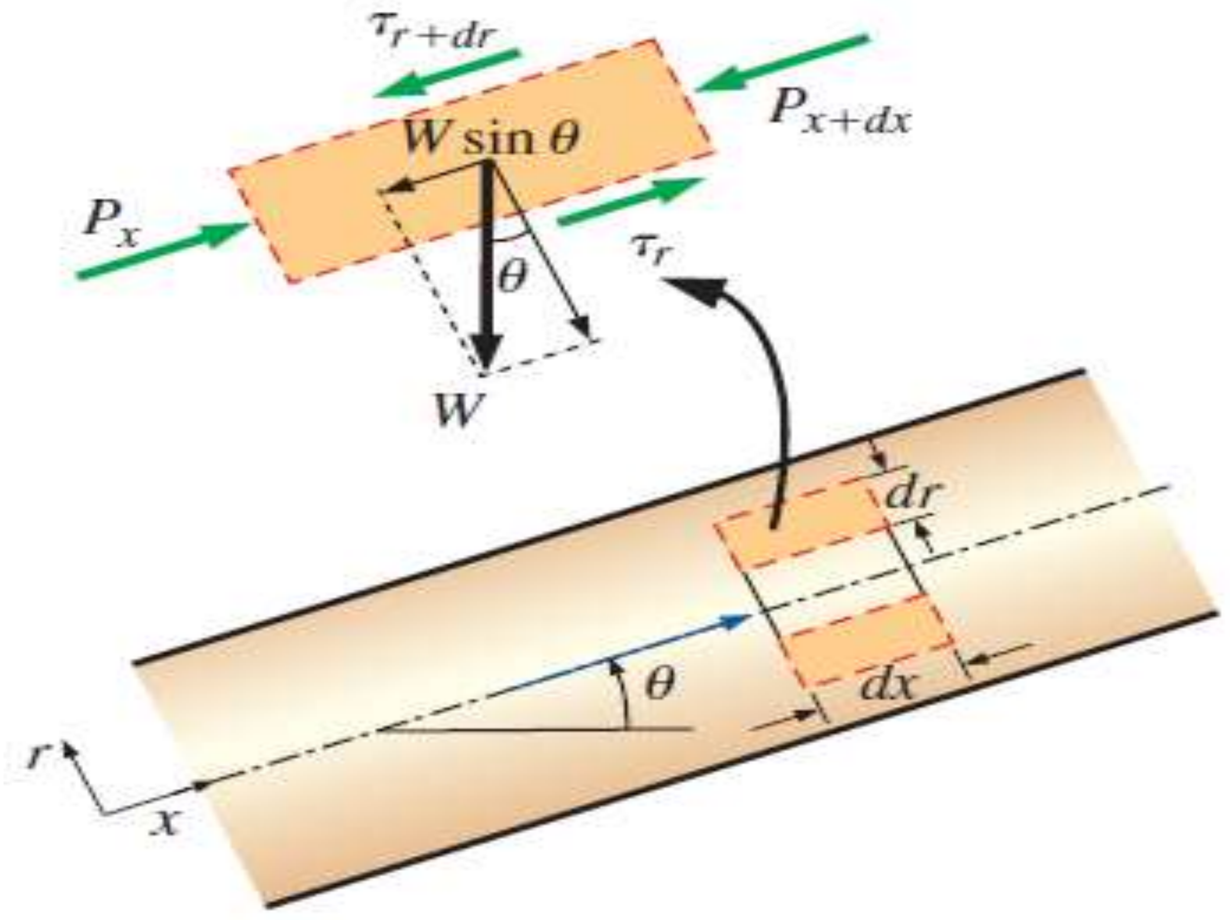
This can be demonstrated by writing the energy equation for steady, incompressible one-dimensional flow in terms of heads as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^2}{2g} + z_1 + h_{\text{pump, } u} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^2}{2g} + z_2 + h_{\text{turbine, } e} + h_L$$

$$P_1 - P_2 = \rho(\alpha_2 V_2^2 - \alpha_1 V_1^2)/2 + \rho g[(z_2 - z_1) + h_{\text{turbine, } e} - h_{\text{pump, } u} + h_L]$$

Therefore, the pressure drop $\Delta P = P_1 - P_2$ and pressure loss $\Delta P_L = \rho g h_L$ for a given flow section are equivalent if (1) the flow section is horizontal so that there are no hydrostatic or gravity effects ($z_1 = z_2$), (2) the flow section does not involve any work devices such as a pump or a turbine since they change the fluid pressure ($h_{\text{pump, } u} = h_{\text{turbine, } e} = 0$), (3) the cross-sectional area of the flow section is constant and thus the average flow velocity is constant ($V_1 = V_2$), and (4) the velocity profiles at sections 1 and 2 are the same shape ($\alpha_1 = \alpha_2$).

Effect of Gravity on Velocity and Flow Rate in Laminar Flow



Free-body diagram of a ring-shaped differential fluid element of radius *r*, thickness *dr*, and length *dx* oriented coaxially with an inclined pipe in fully developed laminar flow.

$$W_x = W \sin\theta = \rho g V_{\text{element}} \sin\theta = \rho g(2\pi r\, dr\, dx) \sin\theta$$

$$(2\pi r\, dr\, P)_x - (2\pi r\, dr\, P)_{x+dx} + (2\pi r\, dx\, \tau)_r - (2\pi r\, dx\, \tau)_{r+dr} - \rho g(2\pi r\, dr\, dx)\sin\theta = 0$$

$$\frac{\mu}{r}\frac{d}{dr}\left(r\frac{du}{dr}\right) = \frac{dP}{dx} + \rho g \sin\theta$$

$$u(r) = -\frac{R^2}{4\mu}\left(\frac{dP}{dx} + \rho g \sin\theta\right)\left(1 - \frac{r^2}{R^2}\right)$$

$$V_{\text{avg}} = \frac{(\Delta P - \rho g L \sin\theta)D^2}{32\mu L}$$

$$\dot{V} = \frac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$$

Laminar Flow in Circular Pipes

(Fully developed flow with no pump or turbine in the flow section, and $\Delta P = P_1 - P_2$)

Horizontal pipe: $\dot{V} = \dfrac{\Delta P \, \pi D^4}{128\mu L}$

Inclined pipe: $\dot{V} = \dfrac{(\Delta P - \rho g L \sin\theta)\pi D^4}{128\mu L}$

Uphill flow: $\theta > 0$ and $\sin\theta > 0$

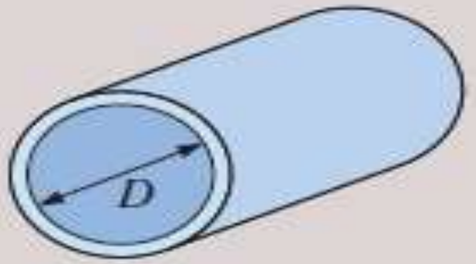
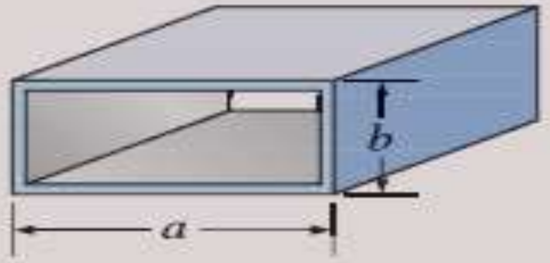
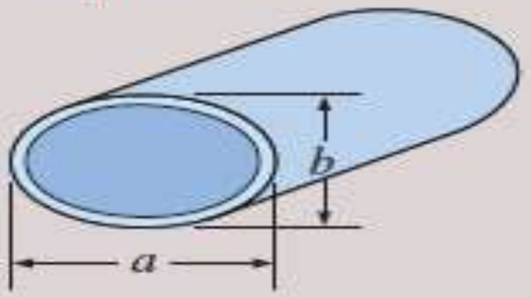
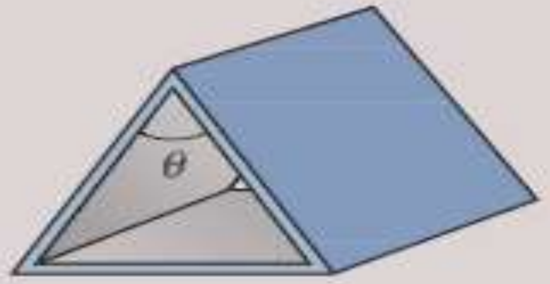
Downhill flow: $\theta < 0$ and $\sin\theta < 0$

The relations developed for fully developed laminar flow through horizontal pipes can also be used for inclined pipes by replacing ΔP with $\Delta P - \rho g L \sin\theta$.

Laminar Flow in Noncircular Pipes

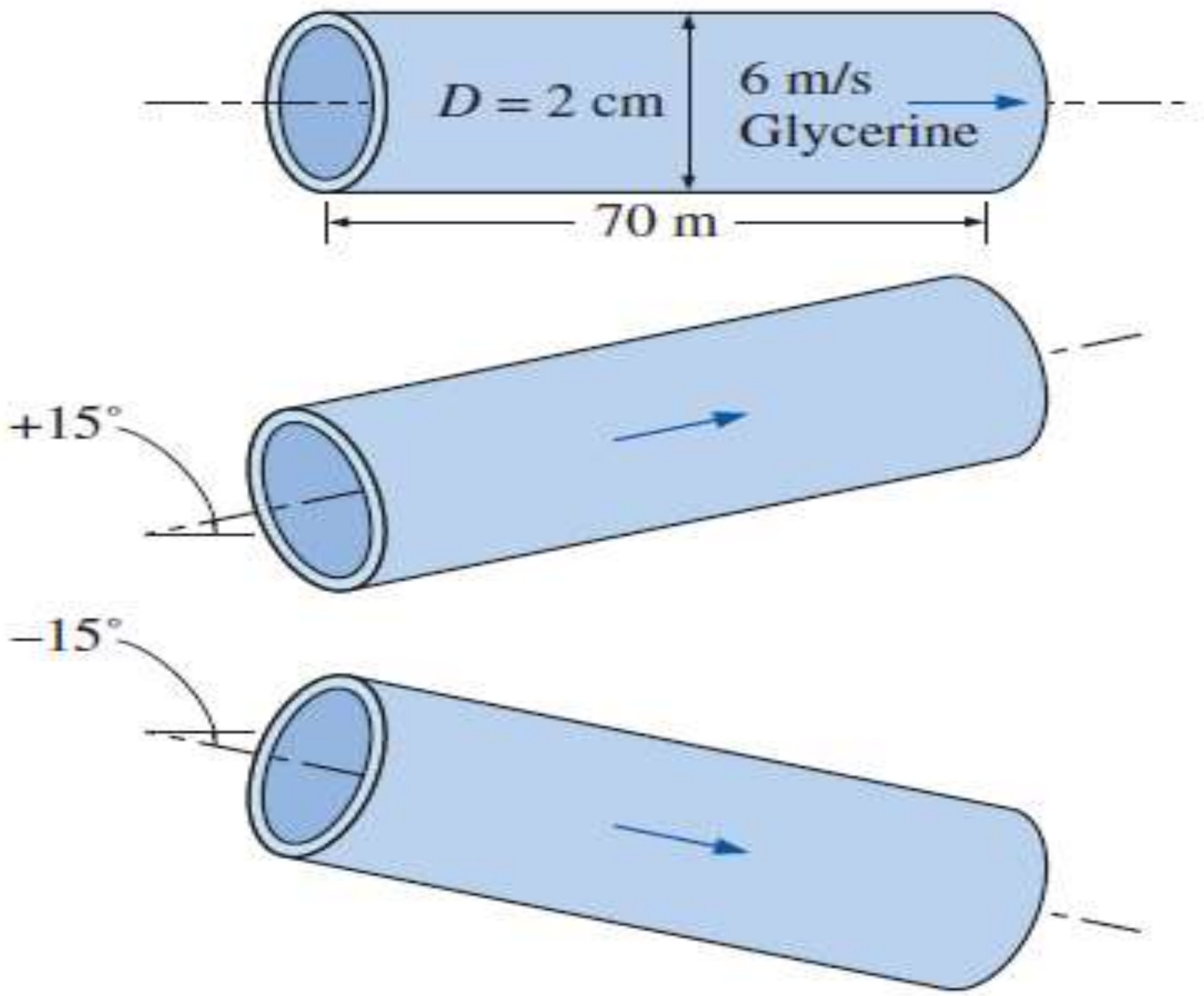
The friction factor *f* relations are given in Table 8–1 for *fully developed laminar flow* in pipes of various cross sections. The Reynolds number for flow in these pipes is based on the hydraulic diameter $D_h = 4A_c/p$, where A_c is the cross-sectional area of the pipe and p is its wetted perimeter

Friction factor for fully developed *laminar flow* in pipes of various cross sections ($D_h = 4A_c/p$ and $\text{Re} = V_{avg} D_h/\nu$)

Tube Geometry	a/b or θ°	Friction Factor f
Circle	—	64.00/Re
		
Rectangle	a/b	
	1	56.92/Re
	2	62.20/Re
	3	68.36/Re
	4	72.92/Re
	6	78.80/Re
	8	82.32/Re
	∞	96.00/Re
Ellipse	a/b	
	1	64.00/Re
	2	67.28/Re
	4	72.96/Re
	8	76.60/Re
	16	78.16/Re
Isosceles triangle	θ	
	10°	50.80/Re
	30°	52.28/Re
	60°	53.32/Re
	90°	52.60/Re
	120°	50.96/Re

EXAMPLE 8–1 Laminar Flow in Horizontal and Inclined Pipes

Consider the fully developed flow of glycerin at 40°C through a 70-m-long, 4-cm-diameter, horizontal, circular pipe. If the flow velocity at the centerline is measured to be 6 m/s, determine the velocity profile and the pressure difference across this 70-m-long section of the pipe, and the useful pumping power required to maintain this flow. For the same useful pumping power input, determine the percent increase of the flow rate if the pipe is inclined 15° downward and the percent decrease if it is inclined 15° upward. The pump is located outside this pipe section.



Properties The density and dynamic viscosity of glycerin at 40°C are $\rho = 1252 \text{ kg/m}^3$ and $\mu = 0.3073 \text{ kg/m·s}$, respectively.

Analysis The velocity profile in fully developed laminar flow in a circular pipe is expressed as

$$u(r) = u_{\max}\left(1 - \frac{r^2}{R^2}\right)$$

Substituting, the velocity profile is determined to be

$$u(r) = (6 \text{ m/s})\left(1 - \frac{r^2}{(0.02 \text{ m})^2}\right) = \mathbf{6(1 - 2500}r^2\mathbf{)}$$

where u is in m/s and r is in m. The average velocity, the flow rate, and the Reynolds number are

$$V = V_{\text{avg}} = \frac{u_{\max}}{2} = \frac{6 \text{ m/s}}{2} = 3 \text{ m/s}$$

$$\dot{V} = V_{\text{avg}}A_c = V(\pi D^2/4) = (3 \text{ m/s})[\pi(0.04 \text{ m})^2/4] = 3.77 \times 10^{-3} \text{ m}^3\text{/s}$$

$$\text{Re} = \frac{\rho V D}{\mu} = \frac{(1252 \text{ kg/m}^3)(3 \text{ m/s})(0.04 \text{ m})}{0.3073 \text{ kg/m·s}} = 488.9$$

which is less than 2300. Therefore, the flow is indeed laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{488.9} = 0.1309$$

$$h_L = f\frac{LV^2}{D\,2g} = 0.1309\,\frac{(70 \text{ m})}{(0.04 \text{ m})}\,\frac{(3 \text{ m/s})^2}{2(9.81 \text{ m/s}^2)} = 105.1\text{m}$$

The energy balance for steady, incompressible one-dimensional flow is given by Eq. 8–28 as

$$\frac{P_1}{\rho g} + \alpha_1 \frac{V_1^{\ 2}}{2g} + z_1 + h_{\text{pump, u}} = \frac{P_2}{\rho g} + \alpha_2 \frac{V_2^{\ 2}}{2g} + z_2 + h_{\text{turbine, e}} + h_L$$

For fully developed flow in a constant diameter pipe with no pumps or turbines, it reduces to

$$\Delta P = P_1 - P_2 = \rho g(z_2 - z_1 + h_L)$$

Then the pressure difference and the required useful pumping power for the horizontal case become

$$\Delta P = \rho g(z_2 - z_1 + h_L)$$
$$= (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(0 + 105.1 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ kg/m·s}^2}\right)$$
$$= \mathbf{1291 \text{ kPa}}$$

$$\dot{W}_{\text{pump, u}} = \dot{V}\Delta P = (3.77 \times 10^3 \text{ m}^3\text{/s})(1291 \text{ kPa})\left(\frac{1 \text{ kW}}{\text{kPa·m}^3\text{/s}}\right) = \mathbf{4.87 \text{ kW}}$$

The elevation difference and the pressure difference for a pipe inclined upwards 15° is

$$\Delta z = z_2 - z_1 = L\sin 15° = (70 \text{ m})\sin 15° = 18.1 \text{ m}$$

$$\Delta P_{\text{upward}} = (1252 \text{ kg/m}^3)(9.81 \text{ m/s}^2)(18.1 \text{ m} + 105.1 \text{ m})\left(\frac{1 \text{ kPa}}{1000 \text{ kg/m·s}^2}\right)$$
$$= 1366 \text{ kPa}$$

Then the flow rate through the upward inclined pipe becomes

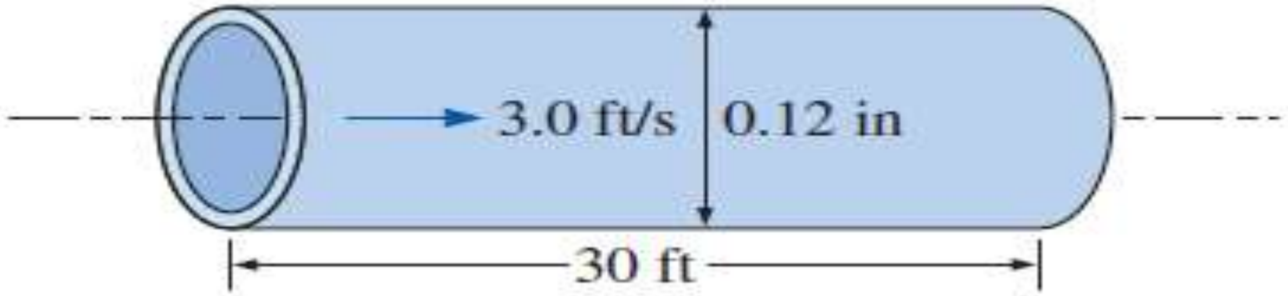
$$\dot{V}_{\text{upward}} = \frac{\dot{W}_{\text{pump}, u}}{\Delta P_{\text{upward}}} = \frac{4.87\ \text{kW}}{1366\ \text{kPa}}\left(\frac{1\ \text{kPa}\cdot\text{m}^3/\text{s}}{1\ \text{kW}}\right) = 3.57 \times 10^{-3}\ \text{m}^3/\text{s}$$

which is a decrease of **5.6 percent** in flow rate. It can be shown similarly that when the pipe is inclined 15° downward from the horizontal, the flow rate will increase by **5.6 percent.**

Discussion Note that the flow is driven by the combined effect of pumping power and gravity. As expected, gravity opposes uphill flow, enhances downhill flow, and has no effect on horizontal flow. Downhill flow can occur even in the absence of a pressure difference applied by a pump. For the case of $P_1 = P_2$ (i.e., no applied pressure difference), the pressure throughout the entire pipe would remain constant, and the fluid would flow through the pipe under the influence of gravity at a rate that depends on the angle of inclination, reaching its maximum value when the pipe is vertical. When solving pipe flow problems, it is always a good idea to calculate the Reynolds number to verify the flow regime—laminar or turbulent.

EXAMPLE 8–2 Pressure Drop and Head Loss in a Pipe

Water at 40°F (ρ = 62.42 lbm/ft^3 and μ = 1.038 × 10^{-3} lbm/ft·s) is flowing steadily through a 0.12-in- (= 0.010 ft) diameter 30-ft-long horizontal pipe at an average velocity of 3.0 ft/s (Fig. 8–18). Determine (*a*) the head loss, (*b*) the pressure drop, and (*c*) the pumping power requirement to overcome this pressure drop.



Properties The density and dynamic viscosity of water are given to be ρ = 62.42 lbm/ft^3 and μ = 1.038 × 10^{-3} lbm/ft·s, respectively.

Analysis (*a*) First we need to determine the flow regime. The Reynolds number is

$$\text{Re} = \frac{\rho V_{\text{avg}} D}{\mu} = \frac{(62.42\ \text{lbm/ft}^3)(3\ \text{ft/s})(0.01\ \text{ft})}{1.038 \times 10^{-3}\ \text{lbm/ft·s}} = 1803$$

which is less than 2300. Therefore, the flow is laminar. Then the friction factor and the head loss become

$$f = \frac{64}{\text{Re}} = \frac{64}{1803} = 0.0355$$

$$h_L = f\frac{L}{D}\frac{V_{\text{avg}}^2}{2g} = 0.0355\,\frac{30\ \text{ft}}{0.01\ \text{ft}}\,\frac{(3\ \text{ft/s})^2}{2(32.2\ \text{ft/s}^2)} = \mathbf{14.9\ ft}$$

(*b*) Noting that the pipe is horizontal and its diameter is constant, the pressure drop in the pipe is due entirely to the frictional losses and is equivalent to the pressure loss,

$$\Delta P = \Delta P_L = f\frac{L}{D}\frac{\rho V_{avg}^2}{2} = 0.0355\,\frac{30\text{ ft}}{0.01\text{ ft}}\frac{(62.42\text{ lbm/ft}^3)(3\text{ ft/s})^2}{2}\left(\frac{1\text{ lbf}}{32.2\text{ lbm}\cdot\text{ft/s}^2}\right)$$

$$= \mathbf{929\text{ lbf/ft}^2 = 6.45\text{ psi}}$$

(*c*) The volume flow rate and the pumping power requirements are

$$\dot{V} = V_{avg}A_c = V_{avg}(\pi D^2/4) = (3\text{ ft/s})[\pi(0.01\text{ ft})^2/4] = 0.000236\text{ ft}^3\text{/s}$$

$$\dot{W}_{pump} = \dot{V}\,\Delta P = (0.000236\text{ ft}^3\text{/s})(929\text{ lbf/ft}^2)\left(\frac{1\text{ W}}{0.737\text{ lbf}\cdot\text{ft/s}}\right) = \mathbf{0.30\text{ W}}$$

Therefore, power input in the amount of 0.30 W is needed to overcome the frictional losses in the flow due to viscosity.

Discussion The pressure rise provided by a pump is often listed by a pump manufacturer in units of head (Chap. 14). Thus, the pump in this flow needs to provide 14.9 ft of water head in order to overcome the irreversible head loss.

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Thank you!

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