JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTER

Year & Sem. – II & III Civil
Subject –Engineering Mechanics
Unit– 2
Presented by – Sumit Saini (Assistant Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional ,cultural and social enrichment of the lives of indivisuals and communities

MISSION OF INSTITUTE

Focus on evaluation of learining ,outcomes and motivate students to research apptitude by project based learning.

•Identify based on informed perception of indian ,regional and global needs ,the area of focus and provide plateform to gain knowledge and solutions.

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•Offer oppurtunites for interaction between academic and industry .

•Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

VISION AND MISSION OF DEPARTMENT

Vision

To become a role model in the field of Civil Engineering for the sustainable development of the s ociety.

Mission

1)To provide outcome base education.

2)To create a learning environment conducive for achieving academic excellence.

3)To prepare civil engineers for the society with high ethical values.

ANALYSIS OF PLANE TRUSSES

- Engineering Structures
- Rigid or perfect Truss
- Determination of Axial forces in the members of truss

  Method of Joints

  Method of Sections.

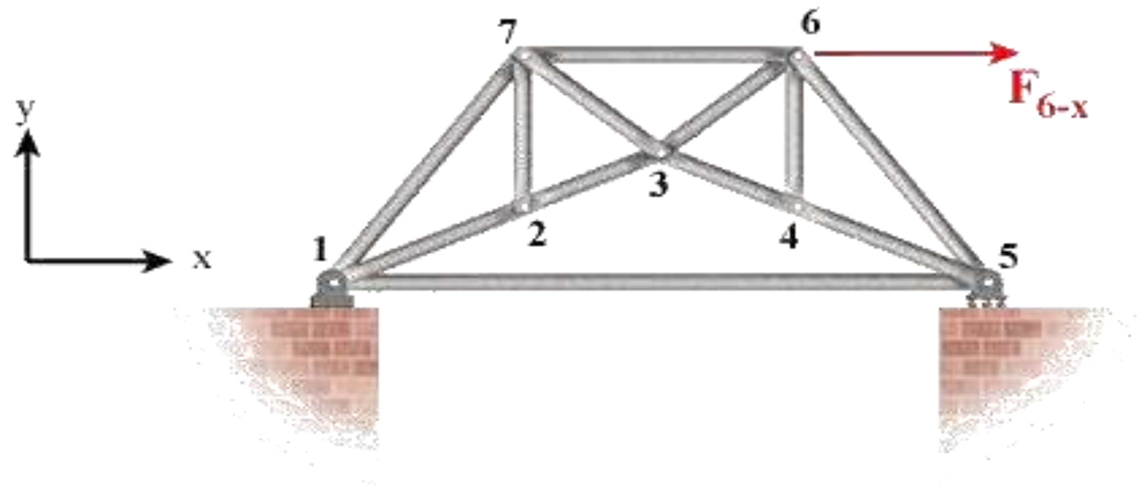


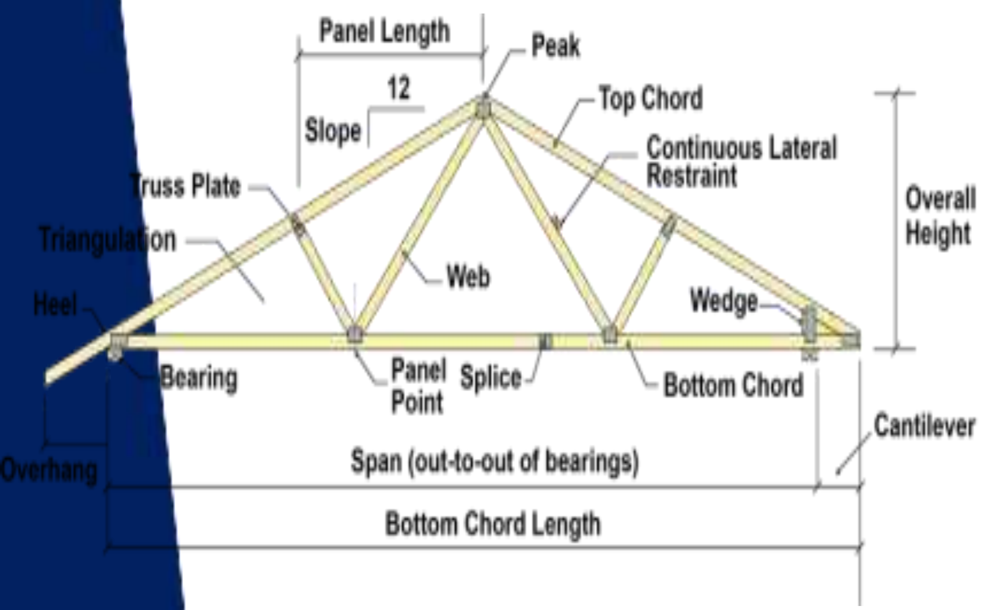
Fig. c Statically inderterminate truss with one redundant member

ENGINEERING STRUCTURES

ENGINEERING STRUCTURES:-- These may be defined as any system of connected member built to support of transfer forces acting on them and to safely withstand these forces.

The Engineering structures are broadly divided in to-

1.Trusses

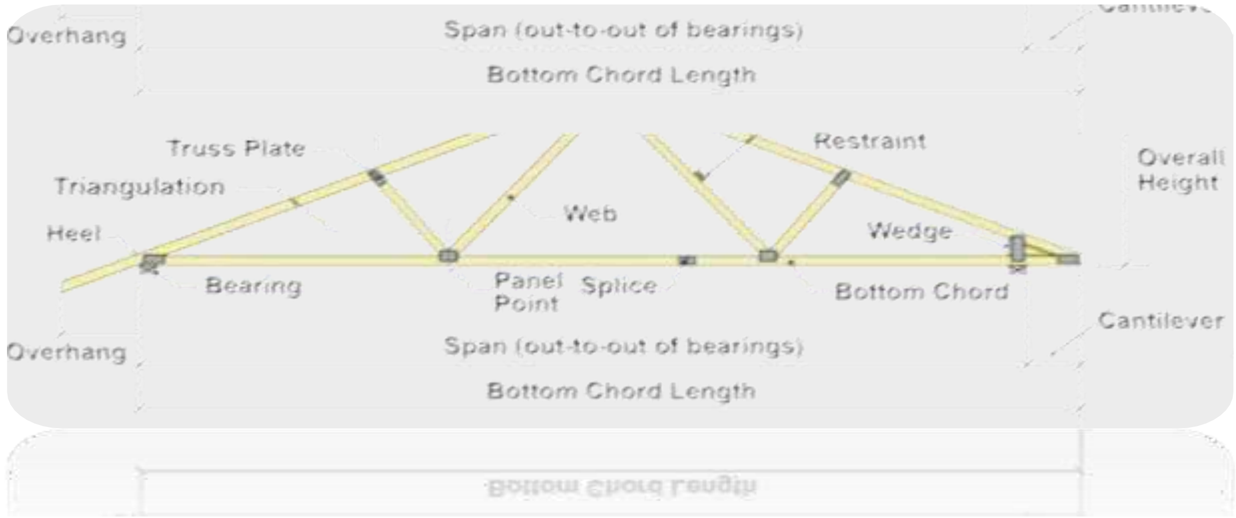


2. Frames

3. Machine

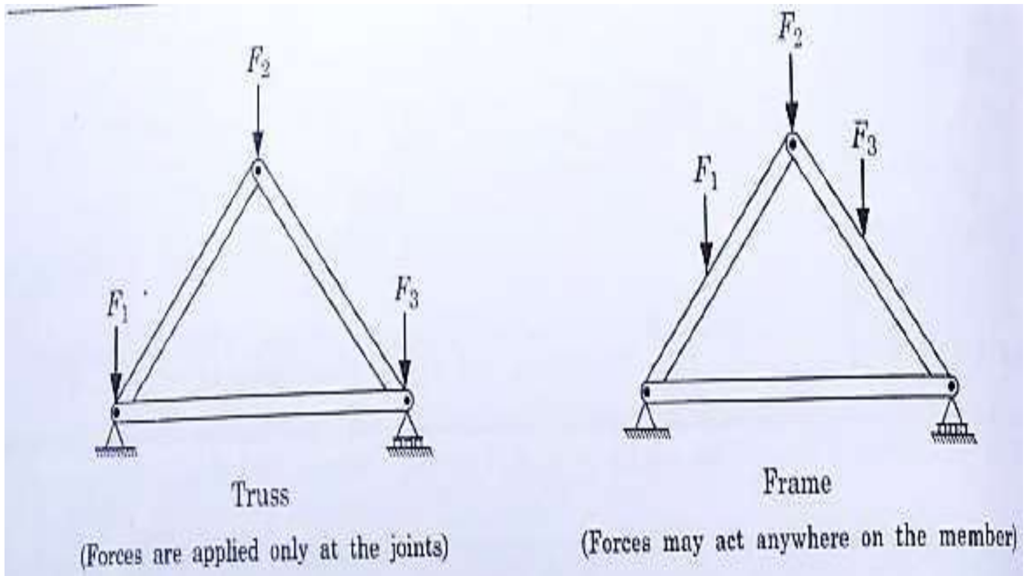
Trusses

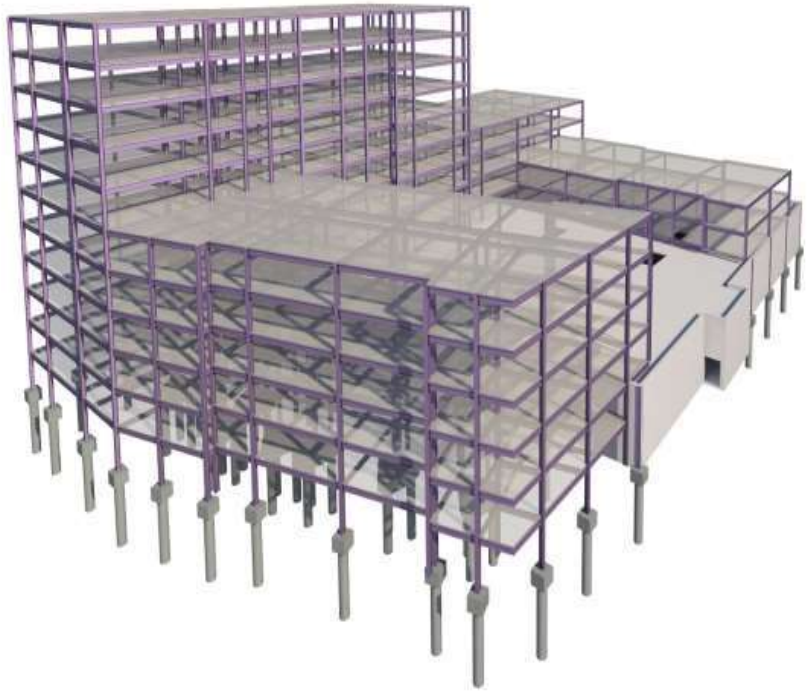
- A truss is a structure composed of slender members joined together at their end points.
- Each member only takes axial forces
- It is a system of uniform bars and members (of circular, channel and angle section etc.) joined together at their ends by riveting and welding.
- Trusses are constructed to support loads.
- The members of a truss are straight members and the loads are applied only on the joints.
- Every member of a truss is two force member.

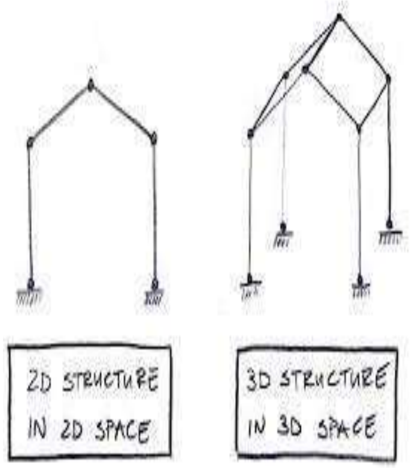


Frames

- It is a structure consisting of several bars or members pinned together and
- In which one and more than one of its members is subjected to more than two forces.
- They are designed to support loads and are stationary structures





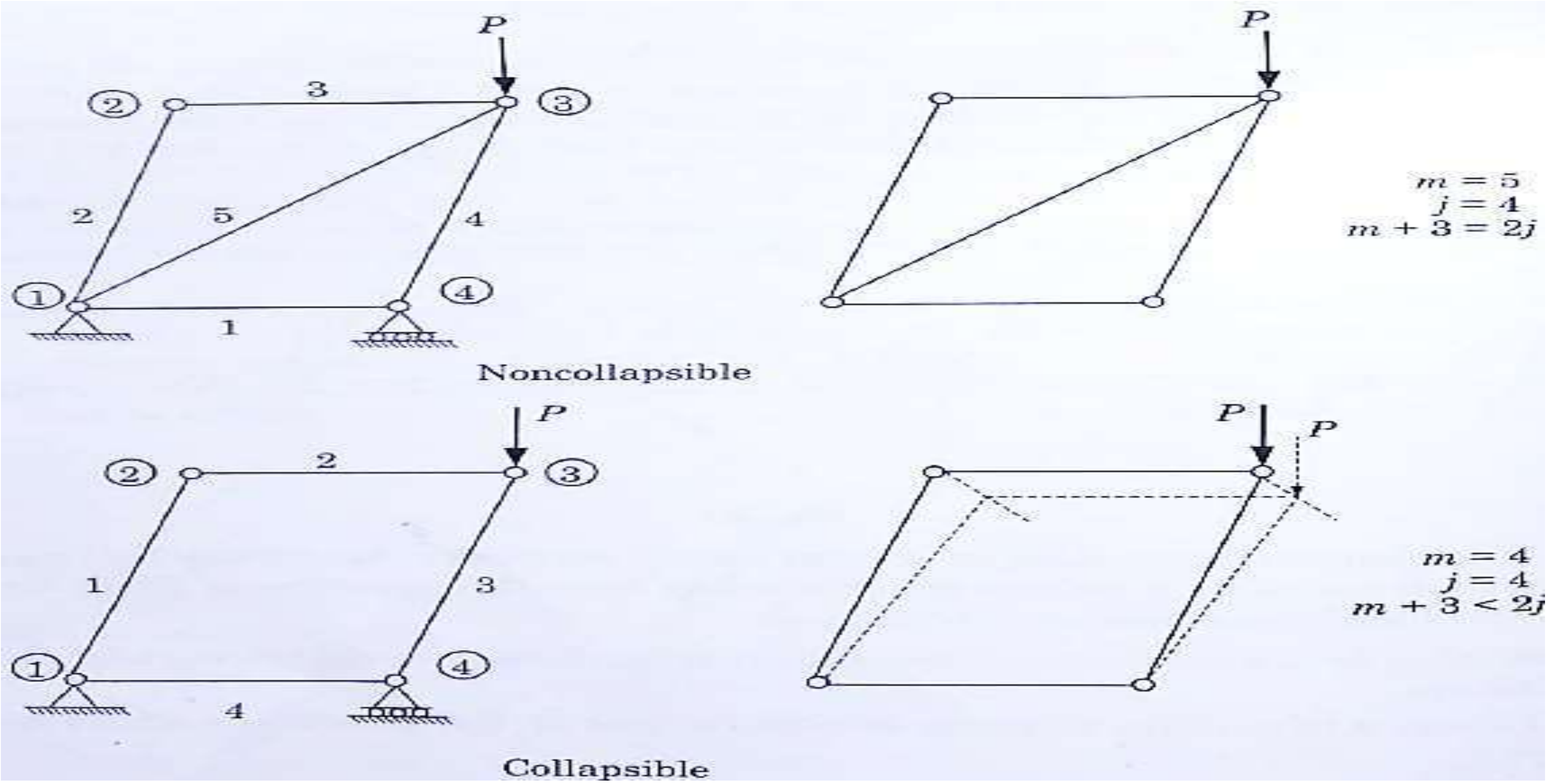


Frame structure

RIGID OR PERFECT TRUSS

- **The term rigid with reference to the truss, is used in the sense that the truss is non collapsible when external forces are removed**

- **Stable structure If (m+3 = 2 x j)**



Basic Assumption for Perfect Truss

1. The joints of simple truss are assumed to be pin connections and frictionless. The joints therefore can not resist moments.

2. The loads on the truss are applied in joints only.

3. The members of a truss are straight two force members with the forces acting collinear with the centerline of the members.

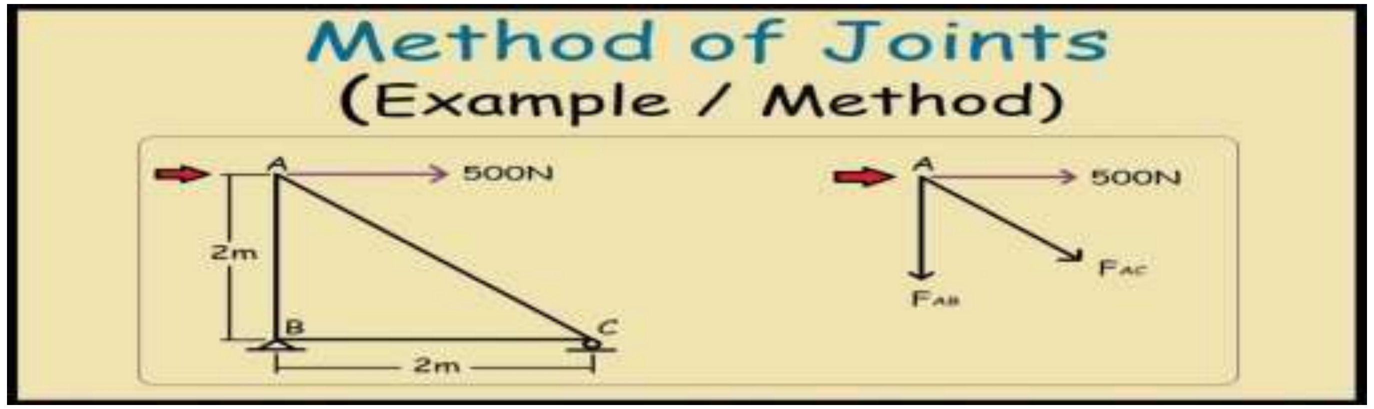
4. The weight of the members are negligibly small unless otherwise mentioned.

5. The truss is statically determinate

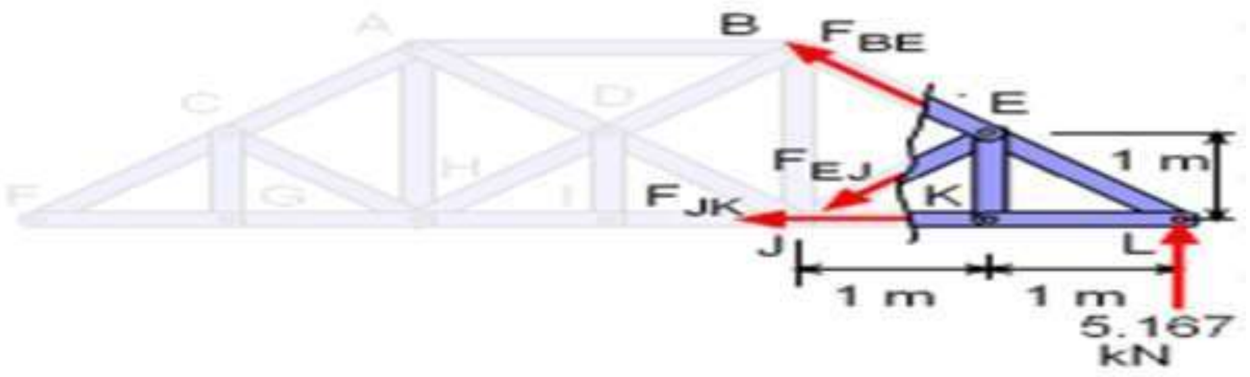
Determination of Axial forces in the members of truss

There are three methods :

1. METHOD OF JOINTS



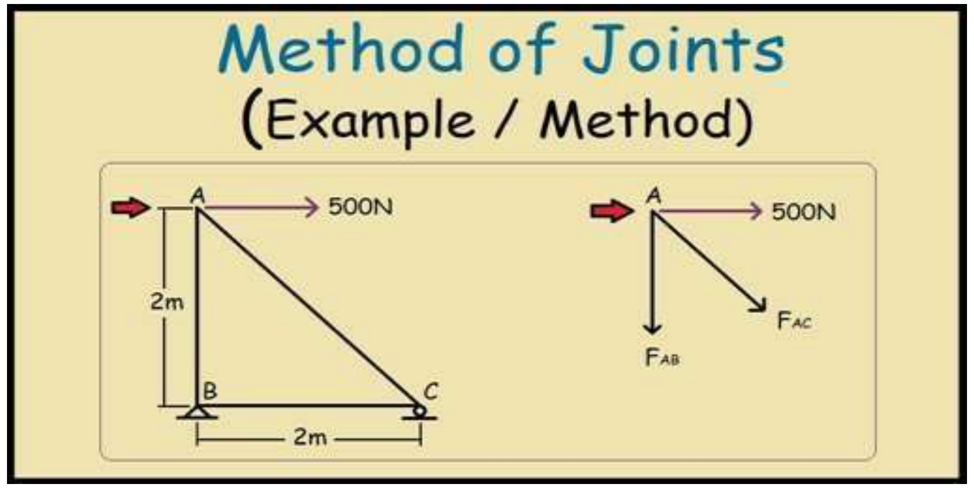
2 METHOD OF SECTION



Method of Joints

Principle;

If a truss is in equilibrium, then each of its joints must also be in equilibrium.



Procedure:

1. Start with a joint that has no more than two unknown forces
2. Establish the x and y axis;
3. At this joint, $\sum F_x = 0$ and $\sum F_y = 0$;
4. After finding the unknown forces applied on this joint, these forces become the given values in the analysis of the n ext joints.
5. At this joint, $\sum F_x = 0$ and $\sum F_y = 0$;

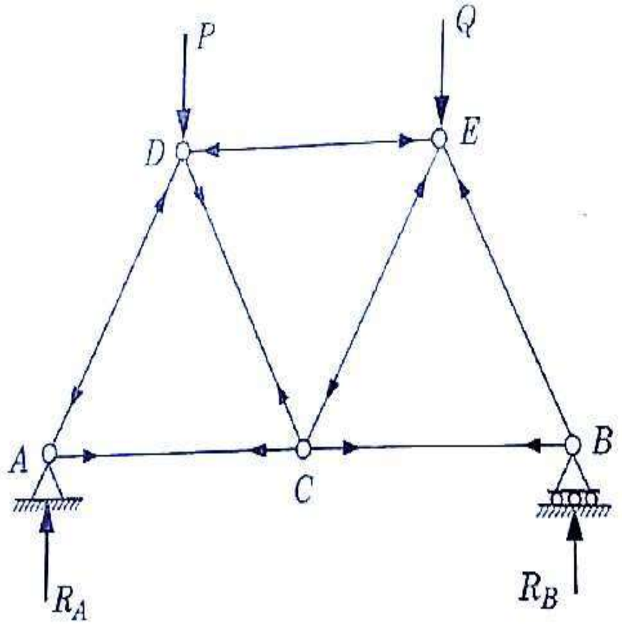
Method of Joints

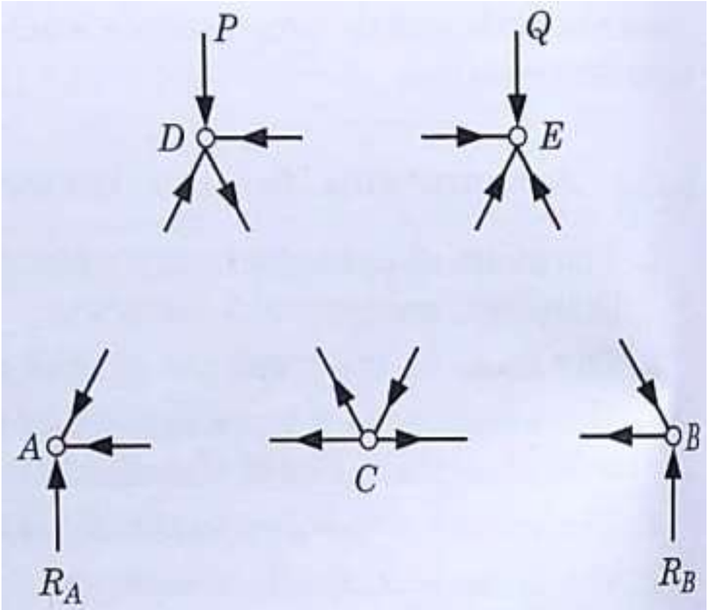
Tips:

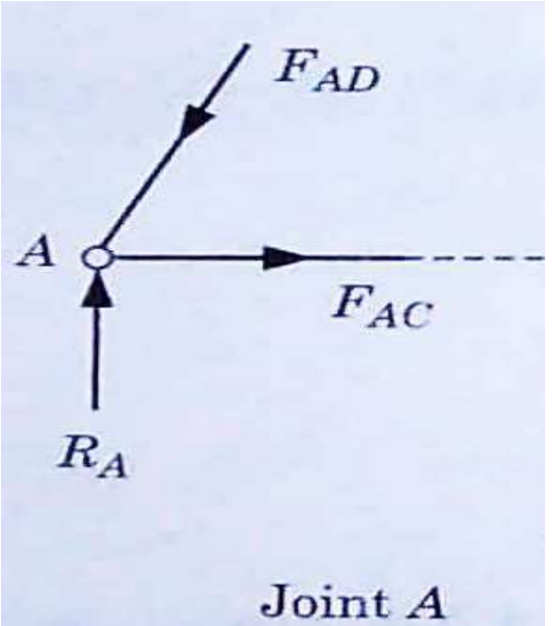
1. The joints with external supports always connect with two truss members. Thus many ti mes, the analysis starts from analyzing the supports. Therefore very often the analysis be gins with finding the reaction forces applied at the supports.

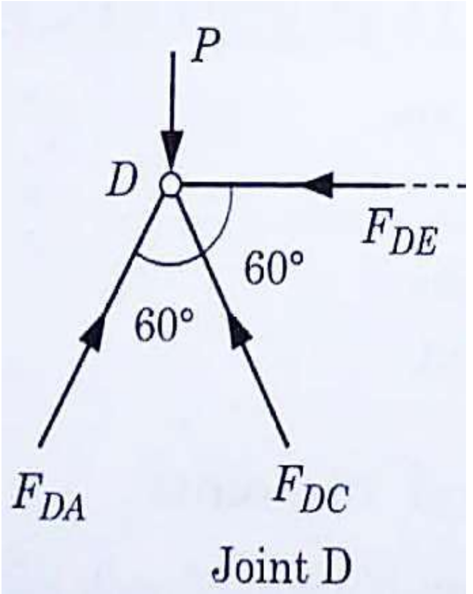
2. Pay attention to symmetric systems and zero-force members. Identification of these special case s sometimes will make the whole analysis WAY EASIER!!

Method of Joint





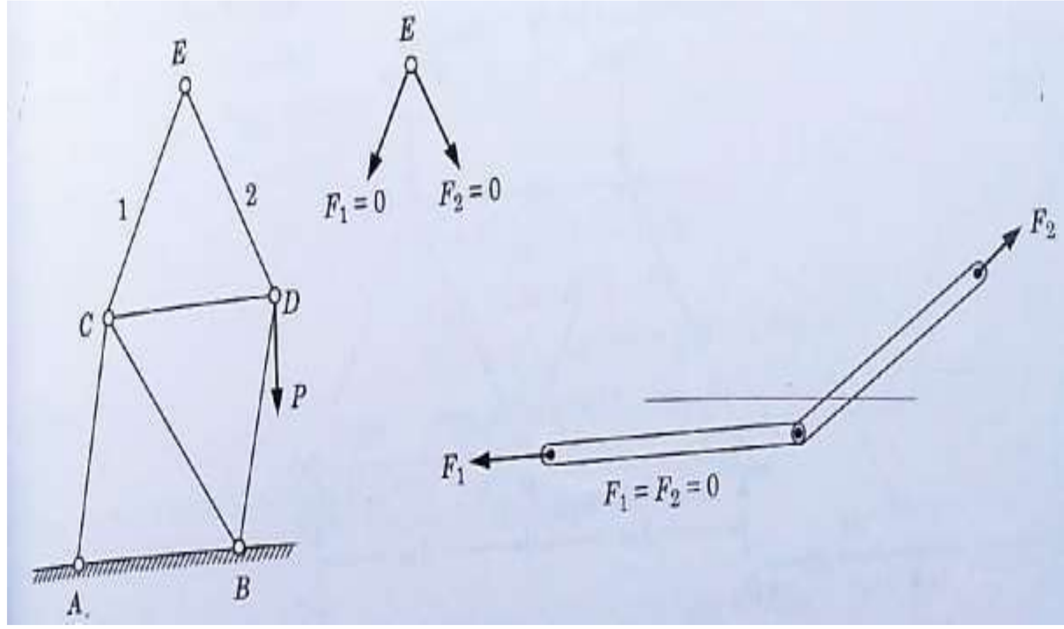


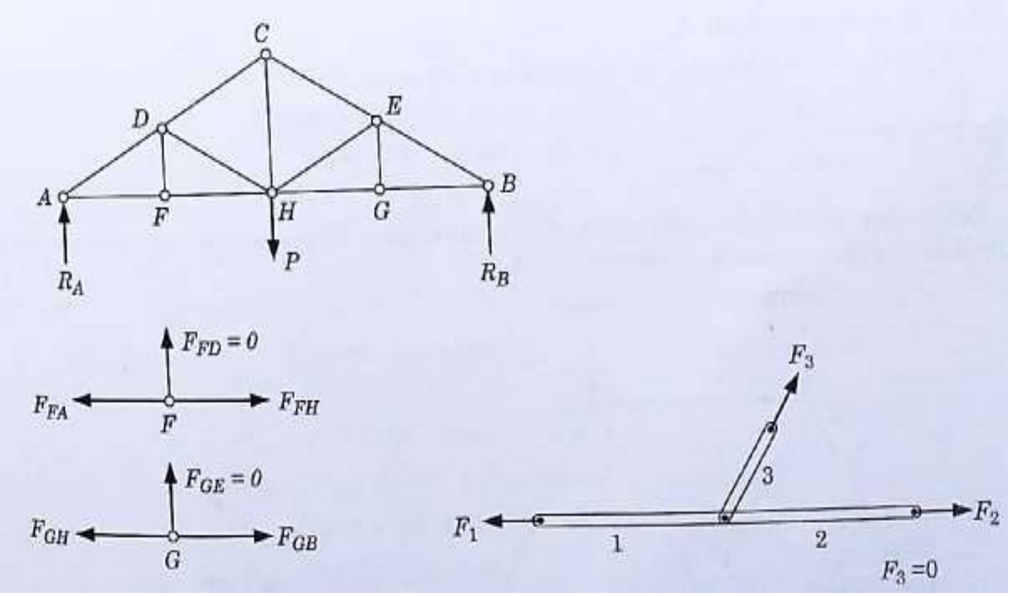


At any joint having two unknown forces
$\sum F_x = 0$ and $\sum F_y = 0$

SPECIAL CONDITIONS

1. When two members meet at a joint are not collinear and there is no external force acting at the joint, then the forces in both the members are zero.
2. When there are three members meeting at a joint, of which two are collinear and third member be at an an gle and if there is no load at the joint, the force in third member is zero.





Method of Joint

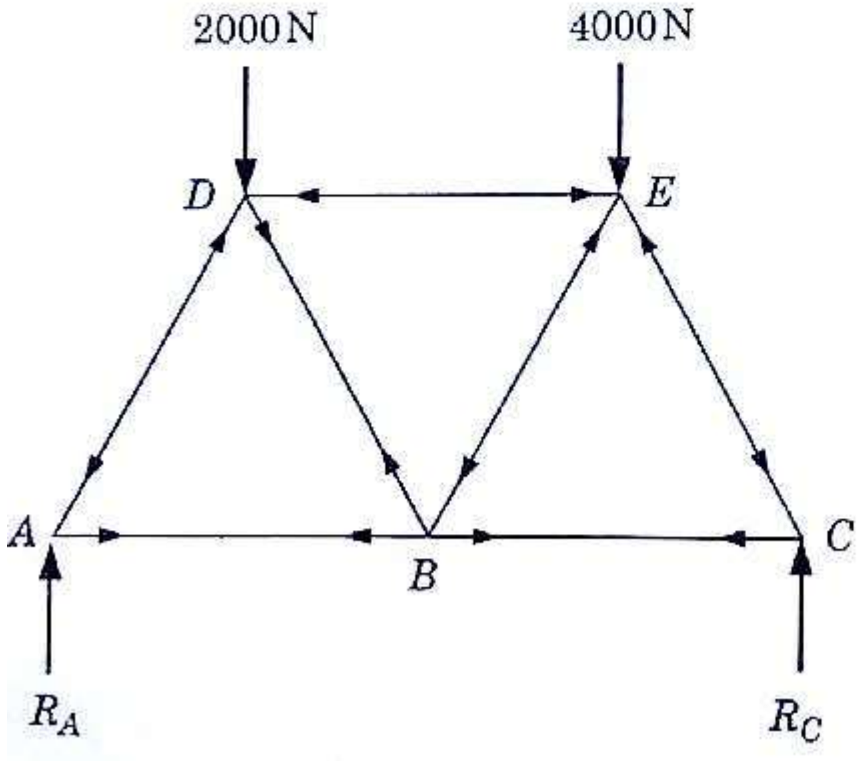
- The method of joints is one of the simplest methods for determining the force acting on the individual members of a truss because it only involves two force equilibrium equations.

  Since only two equations are involved, only two unknowns can be solved for at a time. Ther efore, you need to solve the joints in a certain order. That is, you need to work from the sides towards the center of the truss.

- Since you need to work in a certain order, the Method of Sections (which will be covered lat er) can be more useful if you just want to know the forces acting on a particular member clo se to the center of the truss.

Method of Joint

- Using method of joints find the axial forces in all the members of a truss with the loading shown infigure



Sol : As there is no horizontal external force acting on the truss, so horizontal component of a reaction at A is zero
therefore
Lets take moment at A

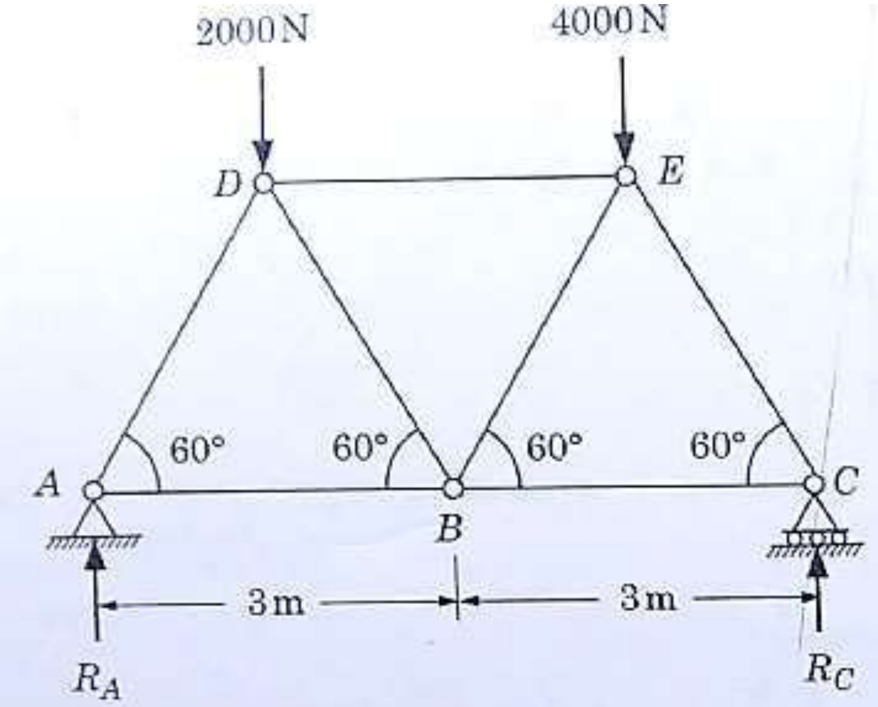
$\sum$ MA= 0
-2000 x 1.5-4000 x 4.5+ Rc x 6 = 0
Rc= 3500 N

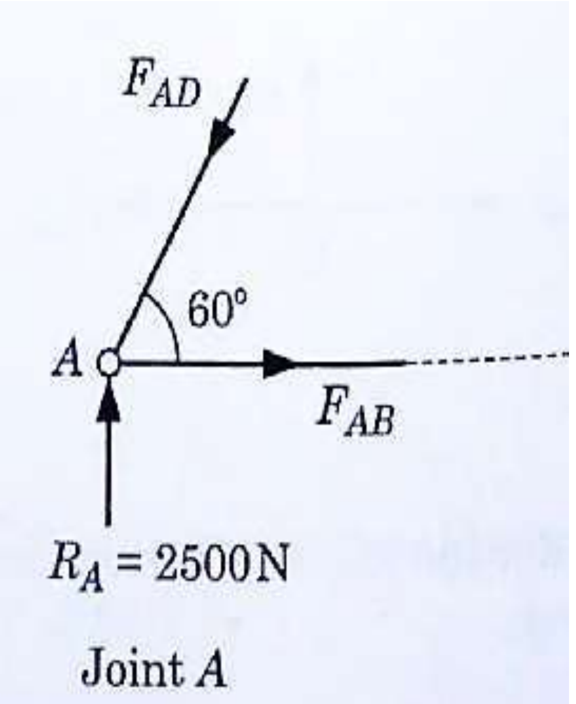
$\sum$ Fy = 0
Ra + Rc-2000-4000 = 0 Ra = 2500 N

Two unknown reactions are known now

Method of joint

- Using method of joints find the axial forces in all the members of a truss with the loading shown infigure





Joint-A

Lets begin with joint A, at which there are two unknown forces. We can not begin with joint D because there are three unknown forces acting at joint D therefore ,
Consider a free body diagram at joint A. Equations of equilibrium can be written as

$\sum F_x = 0$, $F_{ab} - F_{ad} \cos 60^0 = 0$

$\sum F_y = 0$, $R_a - F_{ad} \sin 60^0 = 0$

F_{ad} = Ra/ sin 60 = 2500/ 0.866 = 2887 N (Comp)
C = F_{ad} cos 60^0 = 2887 x 0.5 = 1443 N (Tensile)

The forces F_{ad} & F_{ab} are both positive therefore the assumed direction of forces are correct

Method of joint

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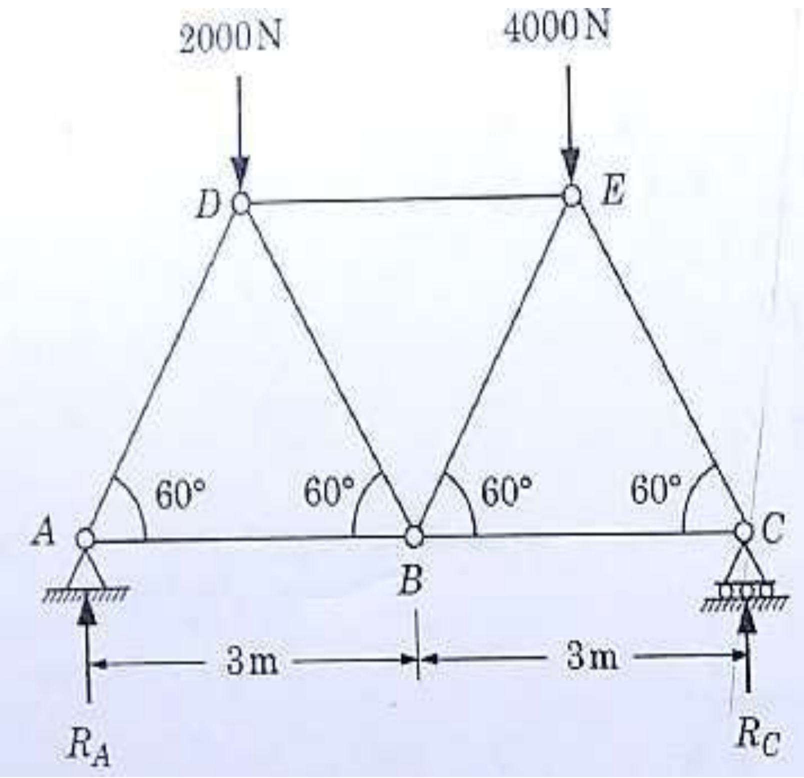
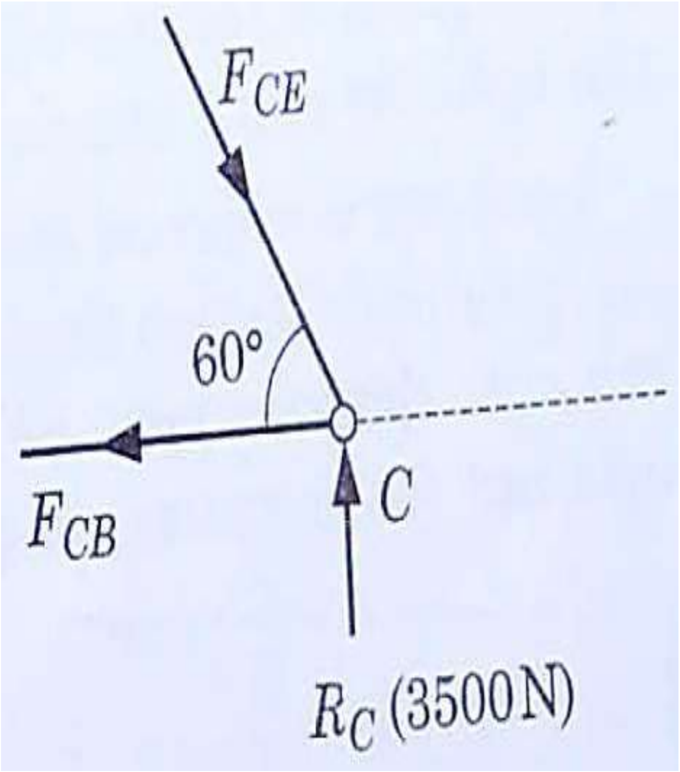




Joint-C
Therefore ,
Consider a free body diagram at joint C. Equations of equili brium can be written as

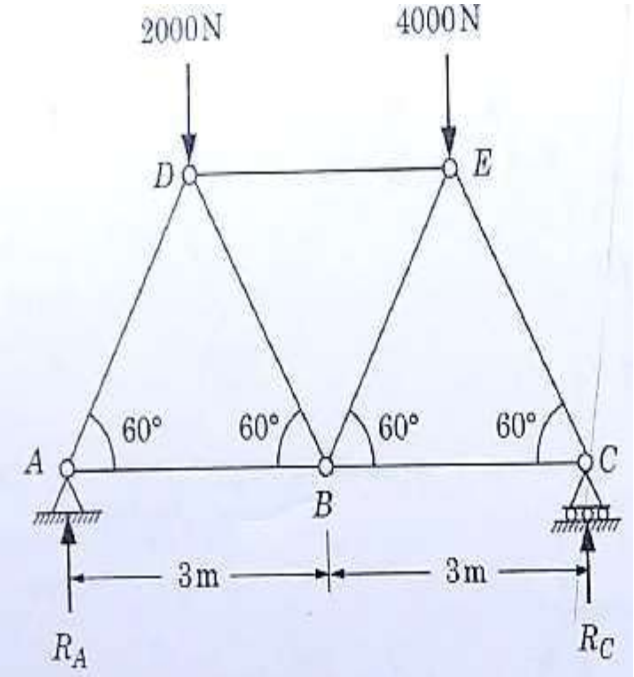
$\sum Fx = 0$, $F_{ce} \cos 60^0 - F_{cb} = 0$

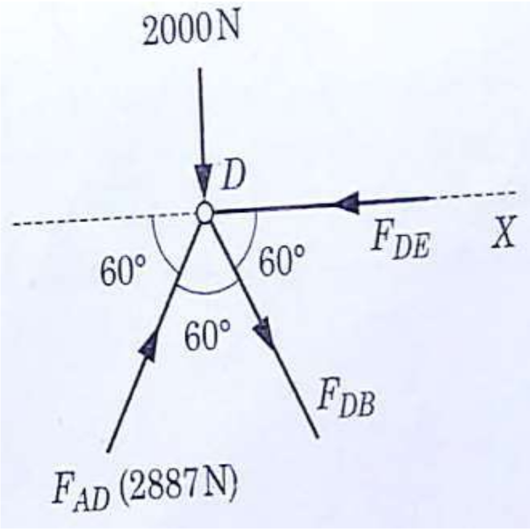
$\sum Fy = 0$, $R_c - F_{ce} \sin 60^0 = 0$

$F_{ce} = Rc/ \sin 60 = 3500/ 0.866 =$ 4041 N
(Comp)
$F_{cb} = F_{ce} \times \cos 60^0 = 4041 \times 0.5 =$ 2020.5 N
(Tensile)

The forces Fce & Fcb are both positive therefore the assumed direction of forces are correct

Method of Joint





Joint-D

Therefore ,
Consider a free body diagram at joint C. Equations of equilibrium can be written as

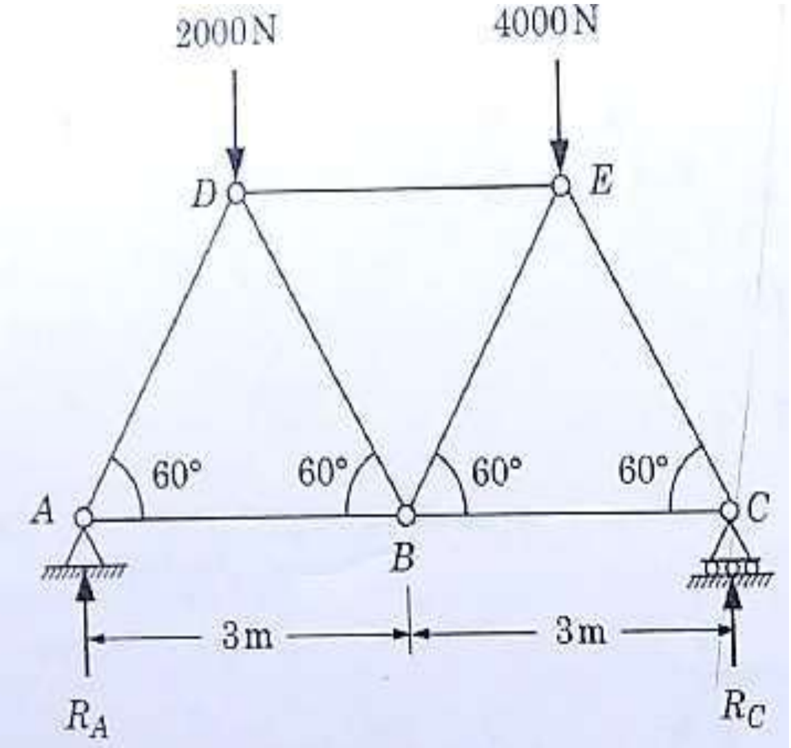
$\sum Fx = 0$, $F_{db} \cos 60^0 + F_{ad} \cos 60^0 - F_{de} = 0$

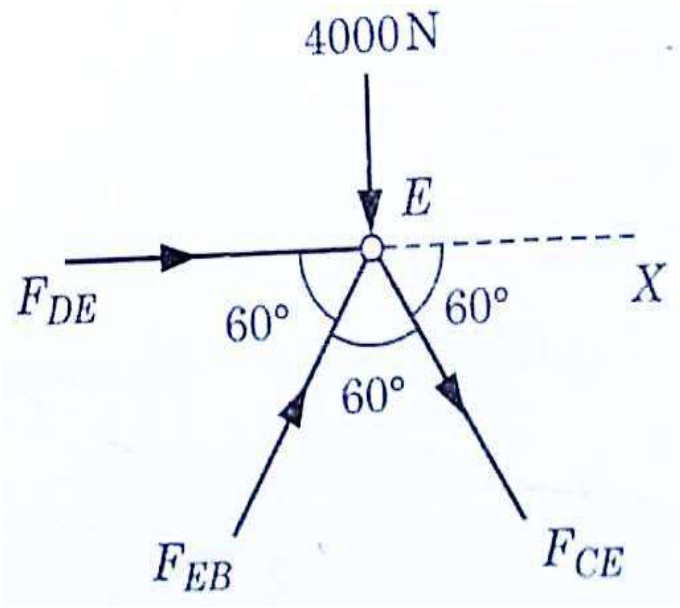
$\sum Fy = 0$, $R_c - F_{ce} \sin 60^0 = 0$

$F_{ce} = Rc / \sin 60 = 3500 / 0.866 = 4041$ N
(Comp)
$F_{cb} = F_{ce} \times \cos 60^0 = 4041 \times 0.5 = 2020.5$ N
(Tensile)

The forces F_{ce} & F_{cb} are both positive therefore the assumed direction of forces are correct

Method of joint





Joint-E

Therefore ,

Consider a free body diagram at joint C. Equations of equilibrium can be written as

$\sum F_x = 0$, $F_{ce} \cos 60^0 - F_{cb} = 0$

$\sum F_y = 0$, $R_c - F_{ce} \sin 60^0 = 0$

$F_{de} = F_{ce} \cos 60^0 + F_{ad} \cos 60^0$

$= 577 \times 0.5 + 2887 \times 0.5 = 1732$ N (Comp)

$F_{ce} = 4041$ N (Comp) known

The forces F_{ce} & F_{de} are both positive therefore the assumed direction of forces are correct

Method of Joint

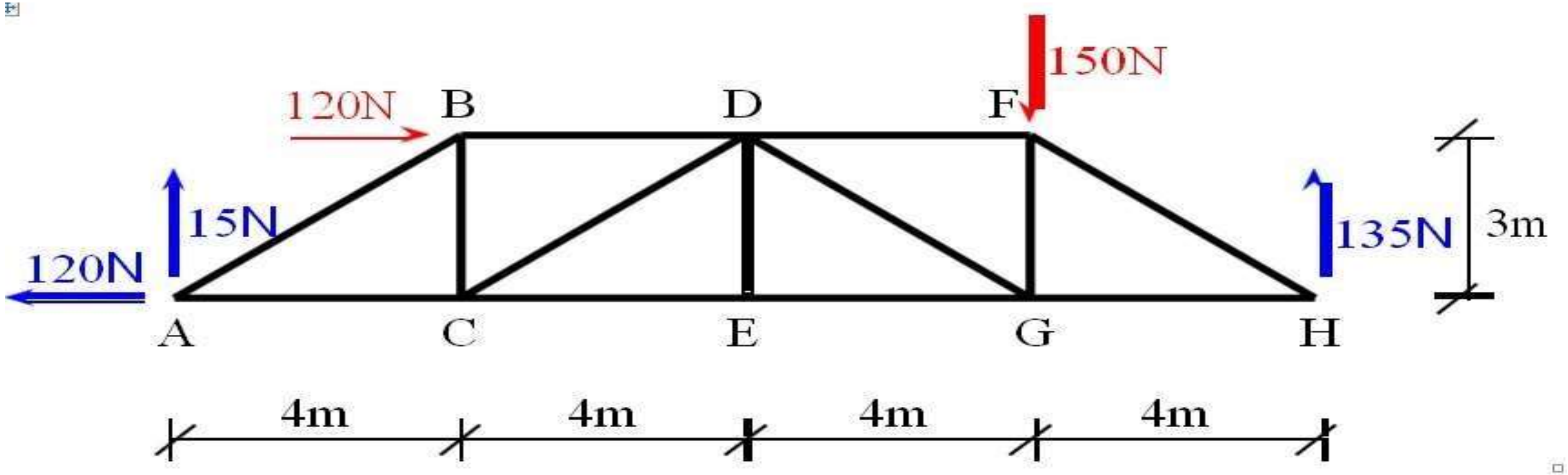
- $\sum Fx = 0$, $F_{eb} \cos 60^0 + F_{de} - F_{de} \cos 60^0 = 0$

  $$F_{eb} = 4041 \times 0.5 - 1732 / 0.5 = 577 N \text{ (comp)}$$

- There is no need to consider the equilibrium of the joint B as all the forces
  have been determined

Method of Section

1.The Method of Sections involves analytically cutting the truss into sections and solving for static equilibrium for each section.



Method of Section

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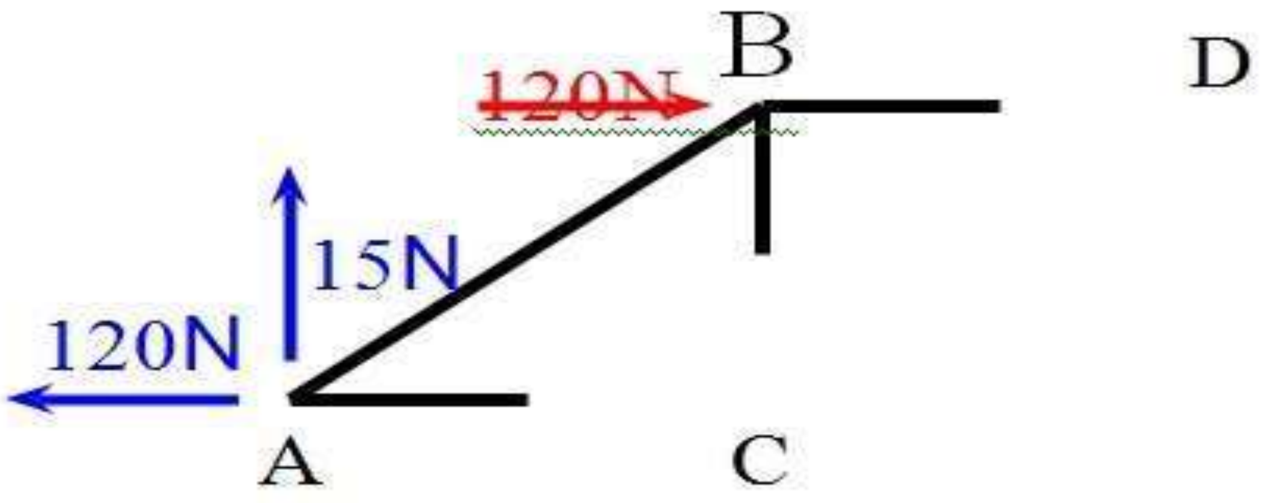
Method of Sections - Cutting through AC, BC and BD

Let's create a section by cutting through members AC, BC and BD. Recall that we want to cut through at most three members.



Let's redraw this section enlarged.

Method of Section

Method of Sections - Cutting through AC, BC and BD

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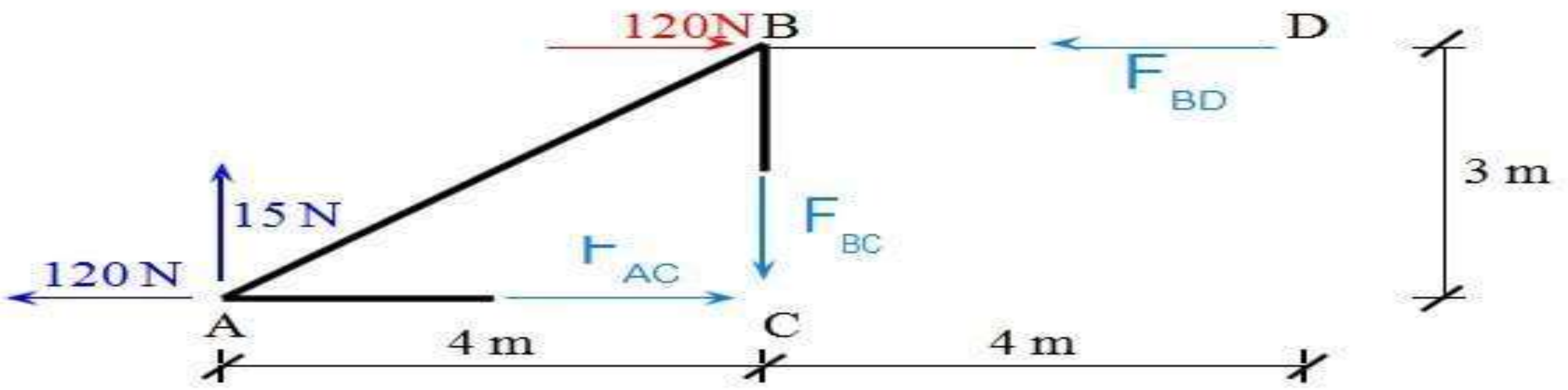

Since F_{BC} is the only force that has a vertical component, it must point down to balance the 15 N force (A_Y).

Taking moments about point B has both forces at A giving clockwise moments. Therefore, F_{AC} must point to the right to provide a counter-clockwise moment.

Taking moments about point C has the 15 N force acting at A and the 120 N acting at B giving clockwise moments. Therefore, F_{BD} must point to the left to provide a counter-clockwise moment.

Method of Section

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- Solving in the order of the previous page:
- $F_Y = +15N - F_{BC} = 0$

  $F_{BC} = 15N$ (tension)
- $M_B = -(120N)(3m) - (15N)(4m) + F_{AC}(3m) = 0$

  $F_{AC} = 140N$ (tension)
- $M_C = -(15N)(4m) - (120N)(3m) + F_{BD}(3m) = 0$

  $F_{BD} = 140N$ (compression)

Method of Section

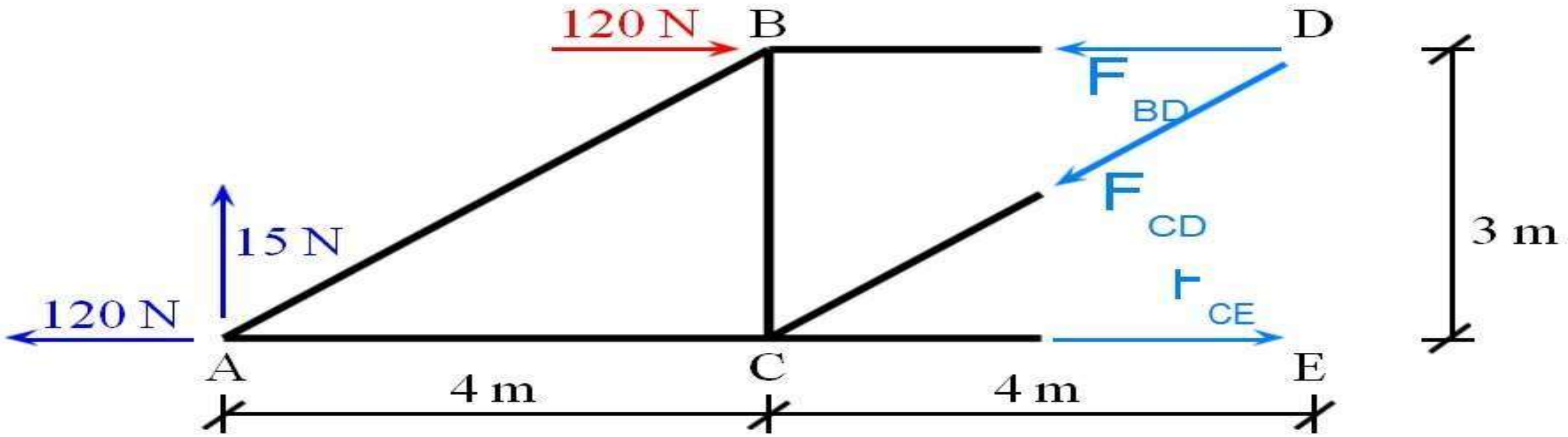
Method of Sections - Important Points

- When drawing your sections, include the points that the cut members would have connecte d to if not cut. In the section just looked at, this would be points C and D.

  Each member that is cut represents an unknown force. Look to see if there is a direction (h orizontal or vertical) that has only one unknown. If this true, you should balance forces in t hat direction. In the section just looked at, this would be the forces in the vertical direction since only F_{BC} has a vertical component.

  If possible, take moments about points that two of the three unknown forces have lines of forces that pass through that point. This will result in just one unknown in that moment e quation. In the section just looked at, taking moments about point B eliminates the unknow ns F_{BC} and F_{BD}. Similarly, taking moments about point C eliminates the unknowns F_{BC} and F_{AC} from the equation.

Method of Sections - Cutting through BD, CD and CE



- Since we know (from the previous section) the direction of F_{BD} we draw that in first. W e could also reason this direction by taking moments about point C.
- Since F_{CD} is the only force that has a vertical component, it must point down to balance t he 15 N force (A_Y).
- Taking moments about point D has the 120 N force and 15 N force acting at A giving cl ockwise moments. Therefore F_{CE} must point to the right to give a counter-clockwise m oment to balance this out.

Method of Section

Solving in the order of the previous page:

- $F_Y = +15N - 3/5\, F_{CD} = 0$

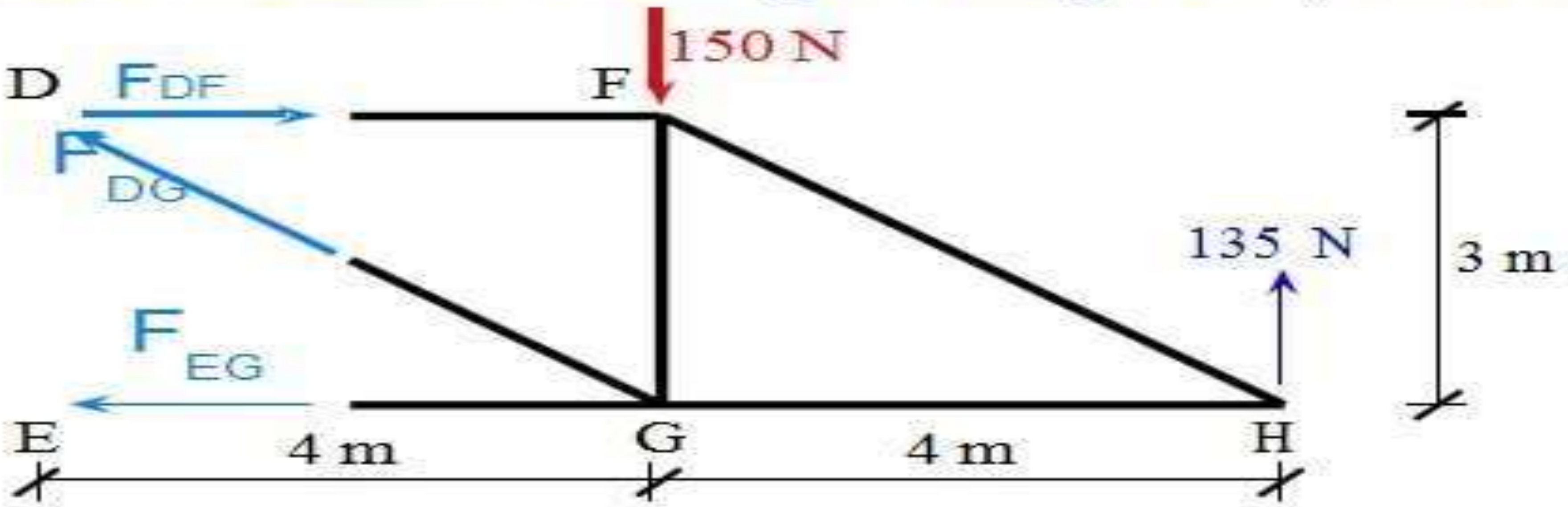
$$F_{CD} = 5/3(15N) = 25N \text{ (compression)}$$

- $M_D = -(120N)(3m) - (15N)(8m) + F_{CE}(3m) = 0$

$$F_{CE} = \quad = 160N \text{ (tension)}$$

Method of Section

Method of Sections - Cutting through DF, DG and EG



Since F_{DG} is the only unknown with a vertical component, it must point up since the 150 N force at F is bigger the 135 N force at H. Taking moments about point G has the 135 N force at H giving a counter-clockwise moment. Therefore F_{DF} must point to the right to give a clockwise moment about point G to balance this out.

Taking moments about point D has the 150 N force acting clockwise and the 135 N force acting counter-clockwise. The 135 N force has twice the moment arm so F_{EG} must point left to give a clockwise moment to balance this out.

Method of Section

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- Solving in the order of the previous page:

- $F_Y = -150N + 135N + F_{FG} = 0$

$F_{FG} = 150N - 135N = 15N \text{ (compression)}$

- $M_F = +(135N)(4m) - F_{GH}(3m) = 0$

$F_{GH} = 180N \text{ (tension)}$

Method of Section

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- Method of Sections - Remaining members
- For the rest of the members, AB, DE and FH, the only sections that would cut through them amount t o applying the Method of Joints.
- To solve for the force in member AB, you would cut through AB and AC. This is equivalent to applying the method of joints at joint A.
- To solve for the force in member FH, you would cut through FH and GH. This is equivalent to applying the method of joints at joint H.
- To solve for the force in member DE, you would cut through CE, DE and EG. This is equivalent to ap plying the method of joints at joint E.

Solved Example

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$$F_{DE} = \frac{1500}{0.866}$$

$$F_{DE} = 1732\ \text{N(T)} \quad \textit{Ans.}$$

It may be noted that the moment centre B chosen above, does not lie on the section of the truss under consideration.

Example 9.3 Determine the axial forces in the bars of a plane truss loaded as shown in Fig. 9.13.

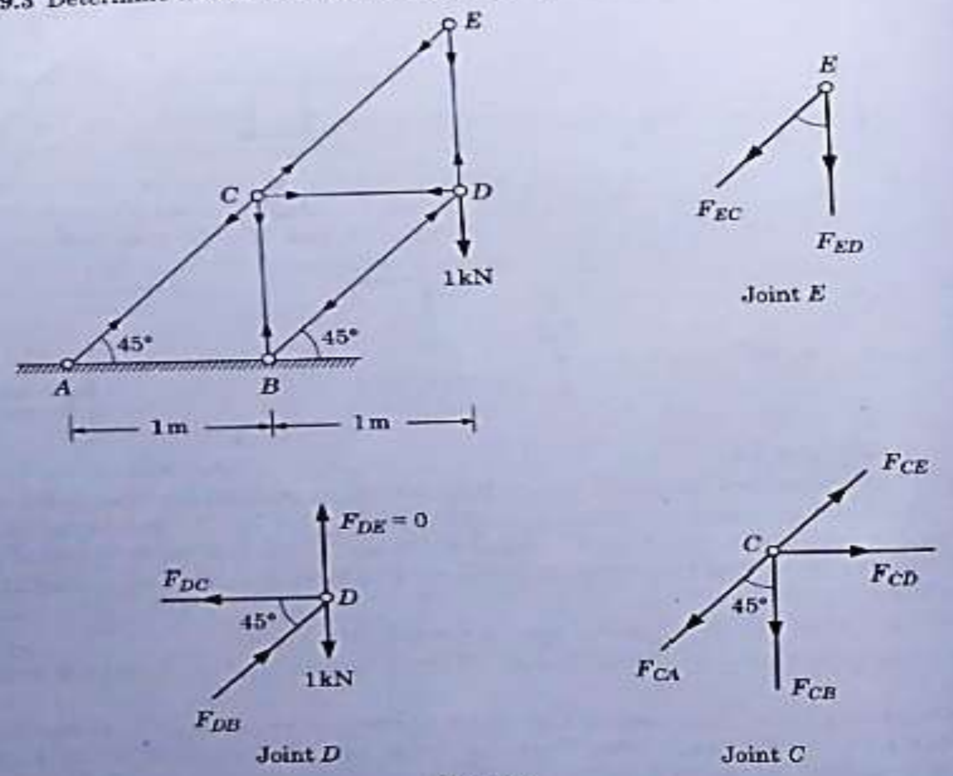


Fig. 9.13

Solution: As the truss is hinged to the foundation, it is not necessary to find the support reactions.

Let us now mark by inspection the axial forces in all the members as shown in Fig. 9.13.

Now consider the equilibrium of the various joints.

Joint E

The joint E is in equilibrium under two forces F_{EC} and F_{ED} which are non-collinear. Hence they must be zero.

$$F_{EC} = F_{ED} = 0$$

Joint D

Consider the equilibrium of the joint D.

$\Sigma F_x = 0$: $\quad F_{DB} \cos 45° - F_{DC} = 0$

$$F_{DC} = \frac{F_{DB}}{\sqrt{2}}$$

$\Sigma F_y = 0$: $\quad F_{DB} \sin 45° - 1 = 0$

$$F_{DB} = \frac{1}{\sin 45°} = \sqrt{2}$$

$$F_{DB} = \sqrt{2}\ \text{kN(C)}$$

$$F_{DC} = \frac{F_{BD}}{\sqrt{2}} = \frac{\sqrt{2}}{\sqrt{2}}$$ Joint E

$$F_{DC} = 1\ \text{kN(T)}$$

Joint C

$\Sigma F_x = 0$: $\quad -F_{CA} \sin 45° + F_{CD} = 0$

$$F_{CA} = \frac{F_{CD}}{\sin 45°} = \frac{1}{1/\sqrt{2}}$$

$$F_{CA} = \sqrt{2}\ \text{kN(T)}$$

$\Sigma F_y = 0$: $\quad -F_{CA} \sin 45° - F_{CB} = 0$

$$F_{CB} = -F_{CA} \sin 45°$$

$$F_{CB} = -\sqrt{2} \times \frac{1}{\sqrt{2}} = -1\ \text{kN}$$

$$F_{CB} = 1\ \text{kN(C)}$$

(As the force F_{CB} in the member CB is negative, reverse the assumed direction of force).

$$\left.\begin{aligned} F_{DC} &= 1\ \text{kN(T)} \\ F_{DB} &= \sqrt{2}\ \text{kN(C)} \\ F_{CA} &= \sqrt{2}\ \text{kN(T)} \\ F_{CB} &= 1\ \text{kN(C)} \\ F_{EC} &= 0 \\ F_{ED} &= 0 \end{aligned}\right\} \textit{Ans.}$$

Example 9.4 Find the axial forces in the members BC, BG, BF, GC, GF and GE of the truss supported and loaded as shown. Use the method of joints.

Solution: Let us first evaluate the angle θ which may be needed while resolving the forces (Fig. 9.14).

From triangle AFG,

$$\frac{FG}{AF} = \tan 30^\circ$$

$$FG = AF \tan 30^\circ$$

$$FG = \frac{2l}{\sqrt{3}}$$

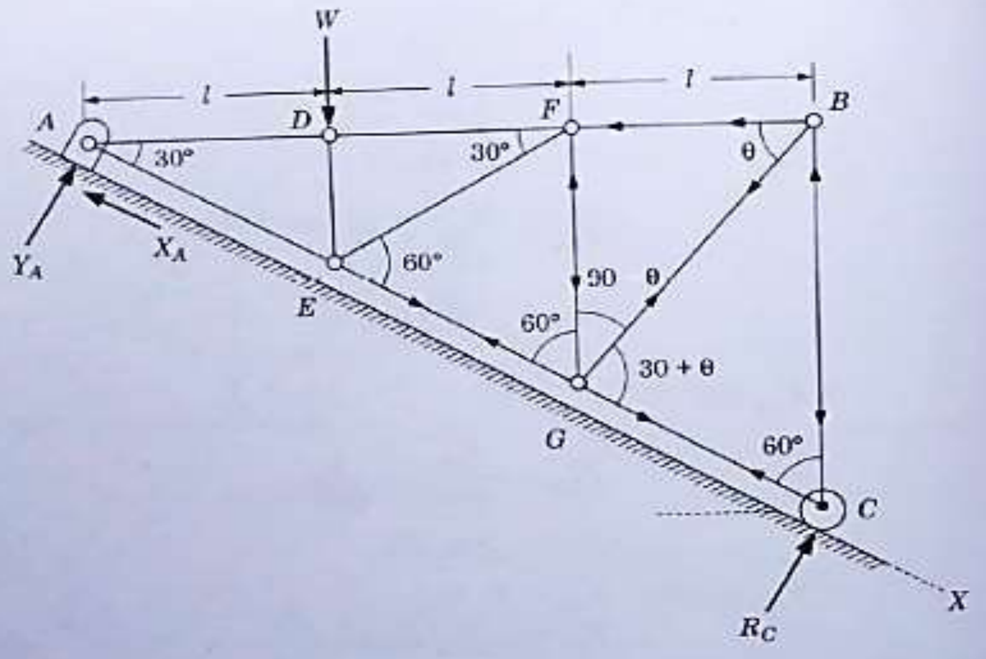


Fig. 9.14

From triangle FBG,

$$\frac{FG}{FB} = \tan\theta$$

$$\theta = \tan^{-1}\frac{FG}{RB}$$

$$\theta = \tan^{-1}\frac{2l}{\sqrt{3}l} = \frac{2}{\sqrt{3}}$$

$$\theta = 49.10^\circ$$

Support Reactions : Consider the equilibrium of the entire truss as a free-body. Taking moments about A.

$\Sigma M_A = 0$:

$$-W(AD) + R_C(AC) = 0$$

$$-Wl + R_C = \frac{3l}{\cos 30^\circ} = 0$$

$$R_C = \frac{W}{2\sqrt{3}}$$

$\Sigma F_x = 0$: (Along AC)

$$W \sin 30^\circ - X_A = 0$$

$$X_A = \frac{W}{2}$$

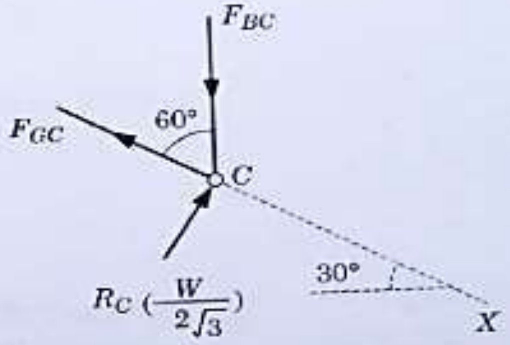
$\Sigma F_y = 0$: (Normal to AC)

$$Y_A + R_C - W\cos 30^\circ = 0$$

$$Y_A = W\cos 30^\circ - R_C = W\frac{\sqrt{3}}{2} - \frac{W}{2\sqrt{3}}$$

$$Y_A = \frac{W}{\sqrt{3}}$$

Joint C



$\Sigma F_x = 0$: $F_{BC}\cos 60^\circ - F_{GC} = 0$ ...(i)

$\Sigma F_y = 0$: $F_{BC}\sin 60^\circ - R_C = 0$

$$F_{BC} = \frac{R_C}{\sin 60^\circ} = \frac{W}{2\sqrt{3}}\left(\frac{2}{\sqrt{3}}\right)$$

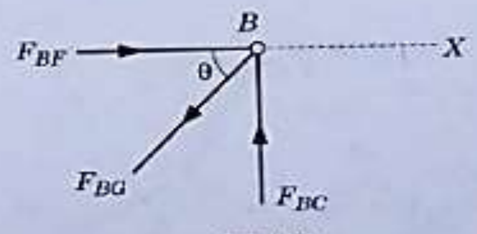
$$F_{BC} = \frac{W}{3}\text{(C)} \quad \textit{Ans.}$$

Substituting for F_{BC} in (i)

$$F_{GC} = F_{BC}\cos 60^\circ = \frac{W}{3}\left(\frac{1}{2}\right)$$

$$F_{GC} = \frac{W}{6}\text{(T)} \quad \textit{Ans.}$$

Joint B



Joint B

$\Sigma F_x = 0$: $F_{BF} - F_{BG}\cos\theta = 0$ ...(ii)

$\Sigma F_y = 0$: $-F_{BG}\sin\theta + F_{BC} = 0$

$\theta = 49.10^\circ$

$\sin 49.1^\circ = 0.756$

$\cos 49.1^\circ = 0.655$

$$F_{BG} = \frac{F_{BC}}{\sin\theta}$$

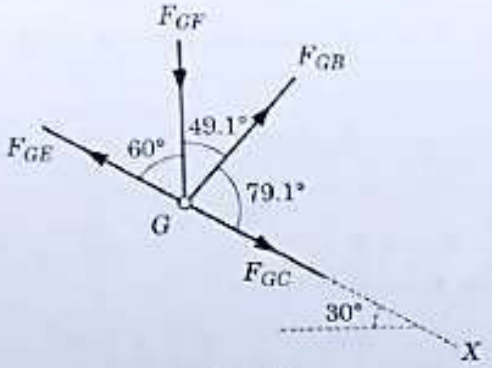
$$= \frac{W}{3}\left(\frac{1}{0.756}\right)$$

$$F_{BG} = 0.441,\ W\text{(T)} \quad \textit{Ans.}$$

Substituting for F_{BG} in (ii)

$$F_{BF} = F_{BG} \cos 49.1° = 0.441\,W \times 0.655$$
$$F_{BF} = 0.289\,W\,(C) \quad Ans.$$

Joint G



Joint G

$$\Sigma F_x = 0: \quad -F_{GE} + F_{GC} + F_{GB} \cos 79.1° + F_{GF} \cos 60° = 0 \quad \text{...(iii)}$$
$$\Sigma F_y = 0: \quad -F_{GF} \sin 60° + F_{GB} \sin 79.1° = 0$$

Or
$$F_{GF} = \frac{F_{GB} \sin 79.1}{\sin 60°} = \frac{0.441\,W}{0.887}$$
$$F_{GF} = 0.5\,W(C) \quad Ans.$$

Solving (iii)
$$F_{GE} = 0.5\,W(T) \quad Ans.$$

Example 9.5 A truss is loaded and supported as shown. Determine the axial forces in the members CE, CG and FG.

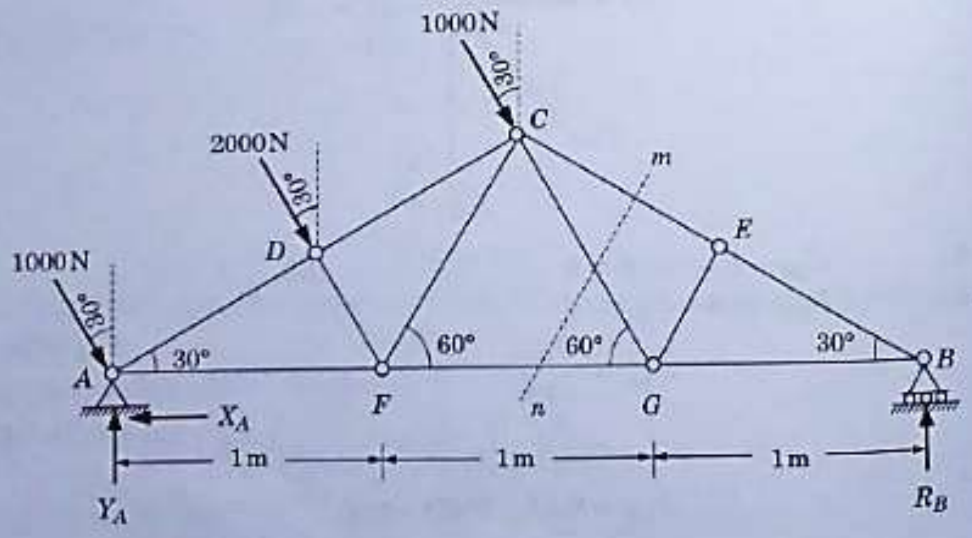


Fig. 9.15

Solution:

Support Reactions : Consider the equilibrium of the entire truss as a free-body.

Taking moments about A,

$$\Sigma M_A = 0: \quad R_B(3) - 2000 \times AD - 1000 \times AC = 0$$
$$3R_B - 2000\,AF \cos 30° - 1000\,AG \cos 30° = 0$$
$$3R_B = 2000 \times \frac{\sqrt{3}}{2} + 1000 \times 2 \times \frac{\sqrt{3}}{2}$$
$$R_B = \frac{2000}{\sqrt{3}}\,N$$

$$\Sigma F_y = 0: \quad R_B + Y_A - 1000 \cos 30° - 2000 \cos 30° - 1000 \cos 30° = 0$$
$$Y_A = \frac{\sqrt{3}}{2}(4000) - \frac{2000}{\sqrt{3}} = \frac{4000}{\sqrt{3}}$$
$$Y_A = \frac{4000}{\sqrt{3}}\,N$$

$$\Sigma F_x = 0: \quad -X_A + 1000 \sin 30° + 2000 \sin 30° + 1000 \sin 30° = 0$$
$$X_A = 1000 \times 0.5 + 2000 \times 0.5 + 1000 \times 0.5$$
$$X_A = 2000\,N$$

Pass a section mn through the truss cutting the members CE, CG and FG. Consider the equilibrium of the right hand portion of the truss as shown in Fig. 9.16.

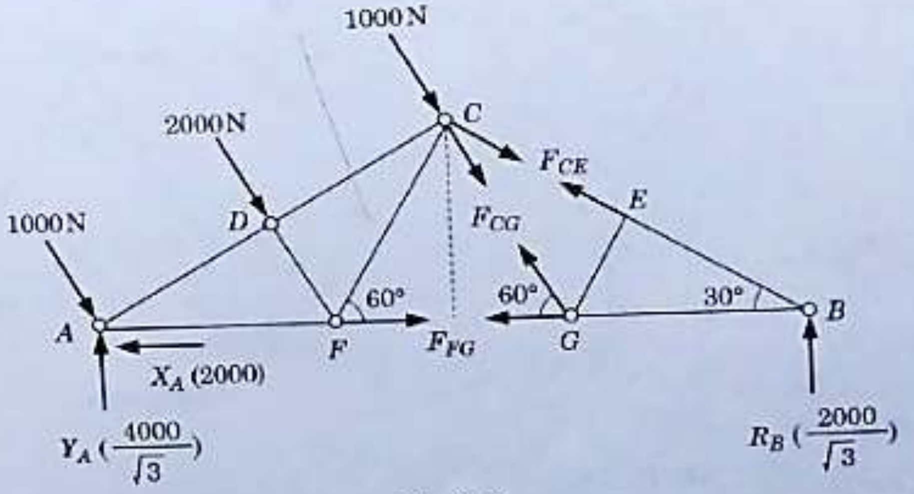


Fig. 9.16

Taking moments about C,

$$\Sigma M_C = 0: \quad -F_{FG} \times 0.5 \tan 60° + R_B(1.5)$$
$$-F_{FG} \times 0.5 \times \sqrt{3} + \frac{2000}{\sqrt{3}} \times 1.5 = 0$$
$$F_{FG} = 2000\,N(T) \quad Ans.$$

Taking moments about G,

$\Sigma M_G = 0: \quad F_{CE}(1 \times \sin 30°) + R_B(1) = 0$

$$F_{CE} = -\frac{2000}{\sqrt{3}} \times \frac{1}{0.5} = -2309 \text{ N}$$

Reverse the sign of the force F_{CE}

$$F_{CE} = 2309 \text{ N(C)} \quad \textit{Ans.}$$

Taking moments about B,

$\Sigma M_B = 0: \quad F_{CG}(1 \times \sin 60°) = 0$

$$F_{CG} = 0 \quad \textit{Ans.}$$

Example 9.6 A hexagonal truss formed of 11 bars of 2 m length each. It is hinged at one end and roller supported at the other end. Find the axial forces in the members CD and GB.

Solution: Support Reactions: Consider the equilibrium of the entire truss as a free-body Fig. 9.17.

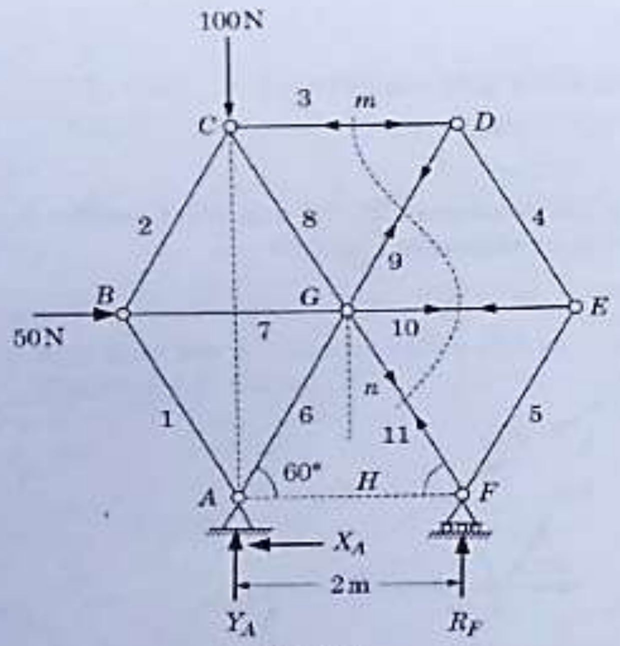


Fig. 9.17

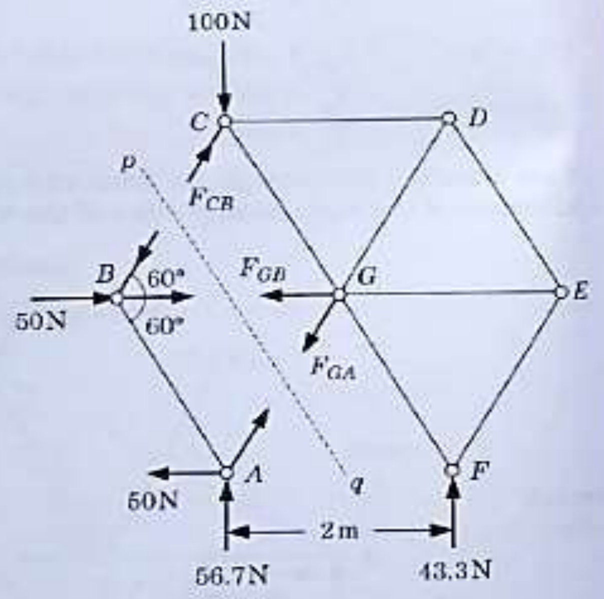


Fig. 9.18

Taking moments about A,

$\Sigma M_A = 0: \quad R_F(AF) - 50\,(HG) = 0$

$$HG = 2 \sin 60° = 2 \times \frac{\sqrt{3}}{2} = \sqrt{3}$$

$$R_F = \frac{50 \times \sqrt{3}}{2}$$

$$R_F = 43.3 \text{ N}$$

$\Sigma F_x = 0: \quad 50 - X_A = 0$

$$X_A = 50 \text{ N}$$

$\Sigma F_y = 0: \quad Y_A + R_F - 100 = 0$

$$Y_A = 100 - 43.3$$

$$Y_A = 56.7 \text{ N}$$

Pass a section mn through the truss cutting the members CD, GD, GE and GF and consider the equilibrium of the right hand portion of the truss. Note that the section mn cuts four members.

Taking moments about G,

$\Sigma M_G = 0: \quad R_F(1) - F_{CD}(2 \sin 60°) = 0$

$$43.3 \times 1 - F_{CD} \times 2 \times 0.866 = 0$$

$$F_{CD} = 25.0 \text{ N(C)} \quad \textit{Ans.}$$

Next pass a section pq cutting the members CB, GB and GA.

Consider the equilibrium of the right hand portion of the truss (Fig. 9.18).

Take moments about B

$\Sigma M_B = 0: \quad -F_{GA}(2 \sin 60°) + 43.3(3) - 100(1) = 0$

$$-F_{GA}(2 \times 0.866) + 129.9 - 100 = 0$$

$$F_{GA} = 17.26 \text{ N(T)}$$

Take moments about G,

$\Sigma M_G = 0: \quad -F_{CB}(2 \sin 60°) + 100\,(1) + 43.3\,(1) = 0$

$$F_{CB} = \frac{143.3}{2 \times 0.866}$$

$$F_{CB} = 82.74 \text{ N(C)}$$

$\Sigma F_x = 0: \quad F_{CB} \cos 60° - F_{GA} \cos 60° - F_{GB} = 0$

$$F_{GB} = \cos 60°(F_{CB} - F_{GA})$$

$$F_{GB} = 0.5(82.74 - 17.26)$$

$$F_{GB} = 32.74 \text{ N(T)} \quad \textit{Ans.}$$

Example 9.7 Determine the forces in the bars DC, DH and FH of the truss loaded and supported as shown in Fig. 9.19.

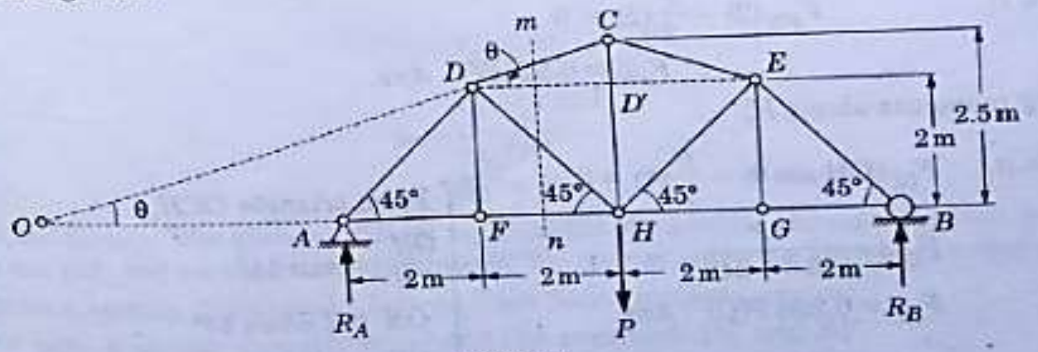


Fig. 9.19

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Thank you!

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