JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTER

Year & Sem. – II & III Civil
Subject –Engineering Mechanics
Unit No.– 4

Presented by – Sumit Saini (Assistant Professor)

SUMMARY

- This topic explains about the relationship between work, energy and power. Students will exposed to energy changes and mechanical efficiency.

LEARNING OUTCOME:

1. **Understand the concept of work, energy and power**
   1. Define work, energy and power
   2. calculate form of energy by using formulas:

      a) Kinetic Energy, $E_k = \frac{1}{2}mv^2$

      b) Potential Energy, $E_p = mgh$
   3. State Principle of Conservation of energy
   4. Describe conversion of energy from one form of energy to another.
   5. Apply the concept and formula of work, energy and power in solving the related problems'
   6. Calculate the efficiency of mechanical system.

$$\text{Efficiency} = \frac{P_o}{P_{in}} \times 100\% \quad and \; \text{Efficiency} = \frac{E_o}{E_{in}} \times 100\%$$

4.1.1 DEFINE WORK, ENERGY AND POWER

- Work done is the product of an applied force and the displacement of an object in the direction of the applied force.

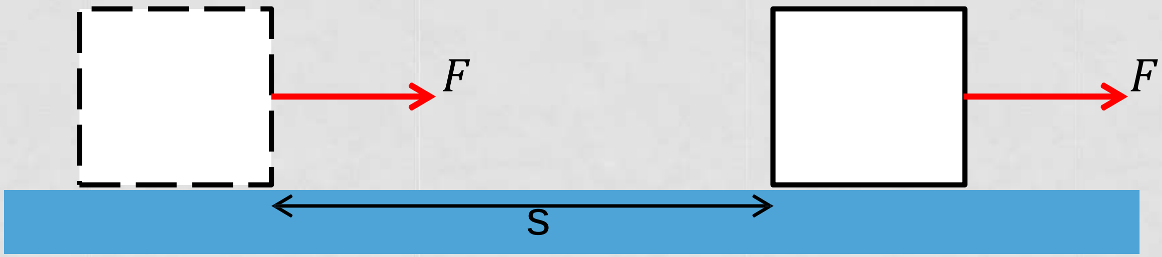


Figure 1 : force and the displacement in the same direction

$$W = F \times s$$

F = force (Newton)
s = displacement (m)
Unit work = J @ Nm

4.1.1 DEFINE WORK, ENERGY AND POWER

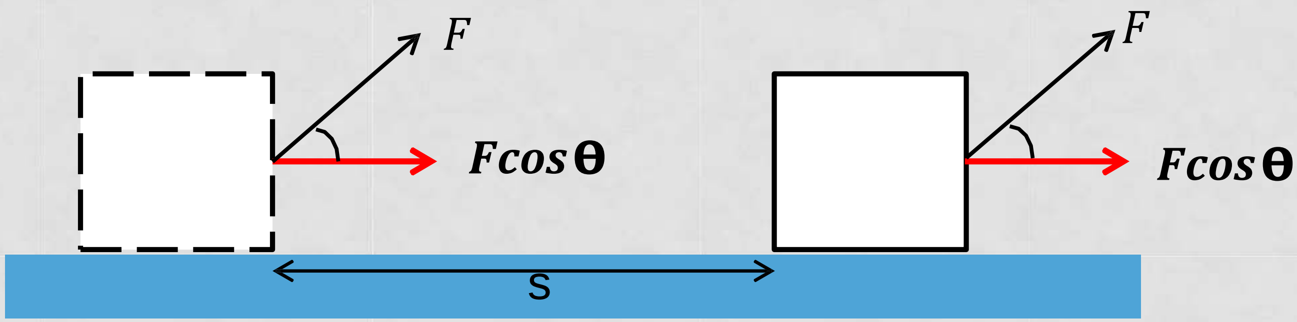


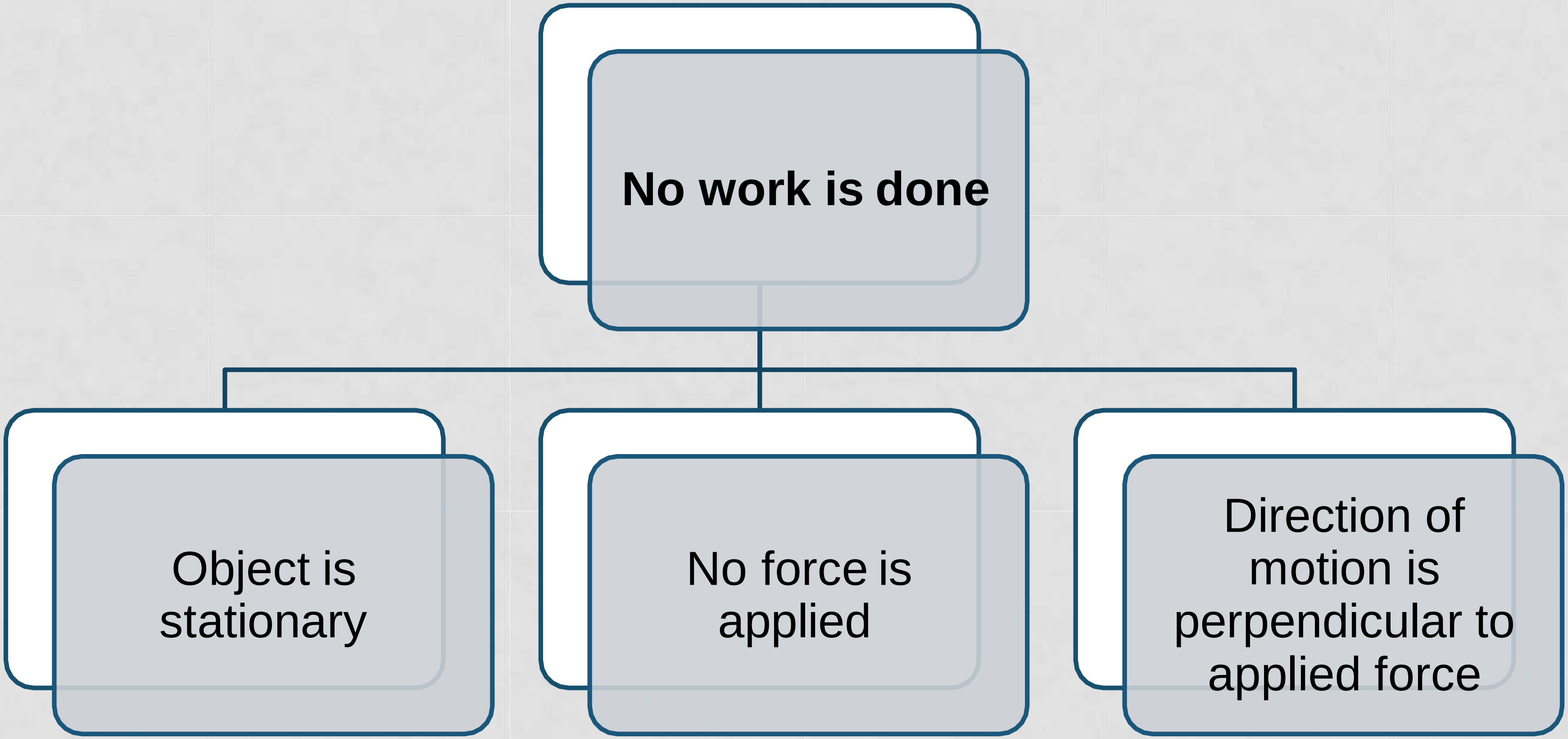
Figure 2 : force and the displacement in different direction

In this situation, we use:

$$W = Fs\cos\theta$$

F = force (Newton)
s = displacement (m)
Unit work = J @ Nm

4.1.1 DEFINE WORK, ENERGY AND POWER



NO WORK IS DONE

Figure 3(a) Pushing a wall

Figure 3(b) a satellite orbiting in space

Figure 3 : No work is done

DEFINITION ENERGY, E

- Energy is capacity to do work
- Unit : Joule (J)

Solar energy

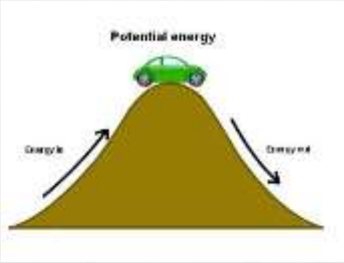
Kinetic energy

Electric energy

Form of energy

Gravitational potential energy

Chemical energy



Nuclear energy

GRAVITATIONAL POTENTIAL ENERGY, E_p

- Definition : energy of an object due to its **higher position** in the gravitational field.

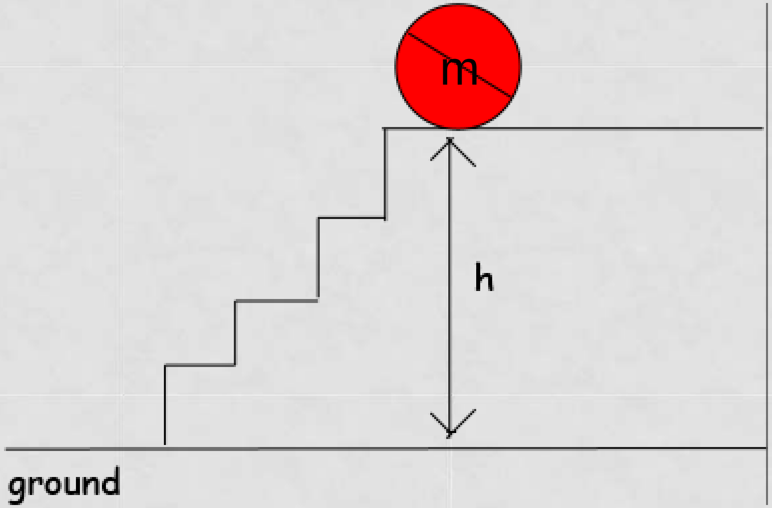


Figure 4: Gravitational Potential Energy

$$E_p = m\,g\,h$$

m = mass (kg)
g = gravity acceleration =9.81m/s^2
h= height

KINETIC ENERGY, E_k

- Definition : energy of an object due to its **motion**
- Unit : Joule (J)

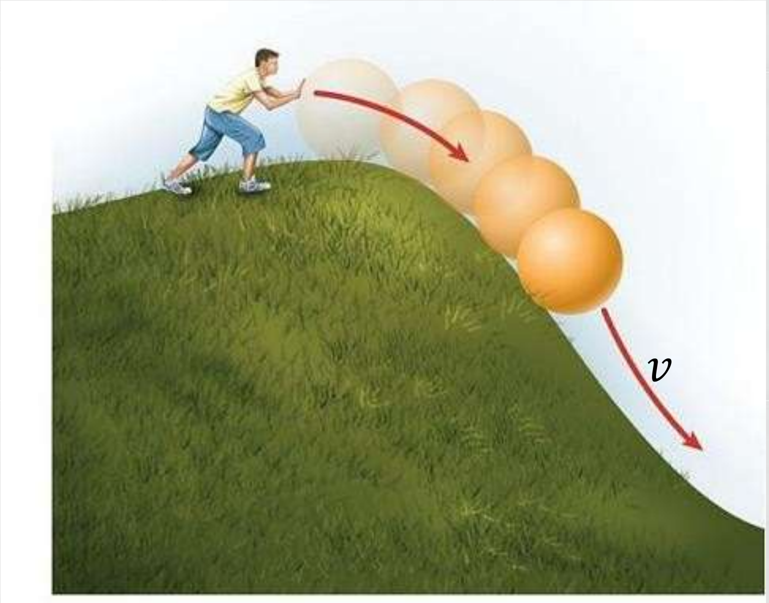


Figure 5: kinetic energy

$$E_k = \frac{1}{2} m v^2$$

m = mass (kg)

v = velocity $\left(\frac{m}{s}\right)$

DEFINITION POWER, P

- Definition : the rate at which work is done or the amount of work done per second
- Unit : J/s @ Watt

$$P = \frac{W}{t} = \frac{mgh}{t} = F \times v$$

$m =$ mass (kg)

$v = velocity \quad \left(- \right)$

$W = work \ (J) \quad s$

$t = time \quad (s)$

$F = force \quad (N)$

$g = gravity\ acceleration = 9.81\ m/s^2$

4.1.2 CALCULATE FORM OF ENERGY USING FORMULA E_p AND E_k

EXAMPLE

A car is moving with the velocity of 10 m/s and is having mass of 250 Kg. Calculate its Kinetic energy?

$v = 10\, m/s$ →

Solution :

The car posses kinetic energy so use formula $E_k = \frac{1}{2} m v^2$

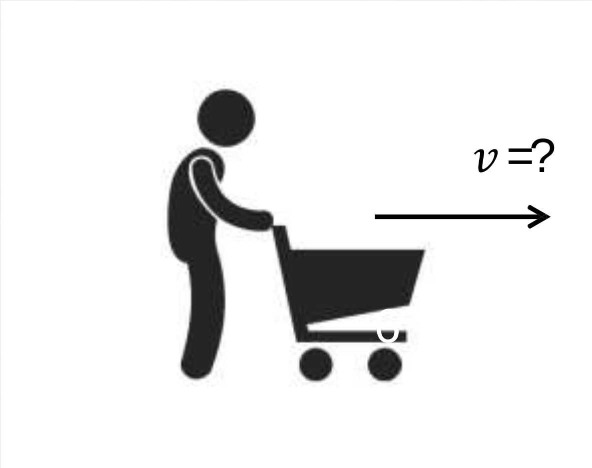
Then,

$E_k = \frac{1}{2} m v^2 \qquad = \frac{1}{2} (250)(10^2) = 12500\, J$

4.1.2 CALCULATE FORM OF ENERGY USING FORMULA E_p AND E_k

EXAMPLE

A man is carrying a trolley of mass 6 kg and having Kinetic energy of 40 J. Calculate its velocity with which he is running?



Solution :

The man posses kinetic energy so use formula $\boldsymbol{E_k = \frac{1}{2} m v^2}$

Then,

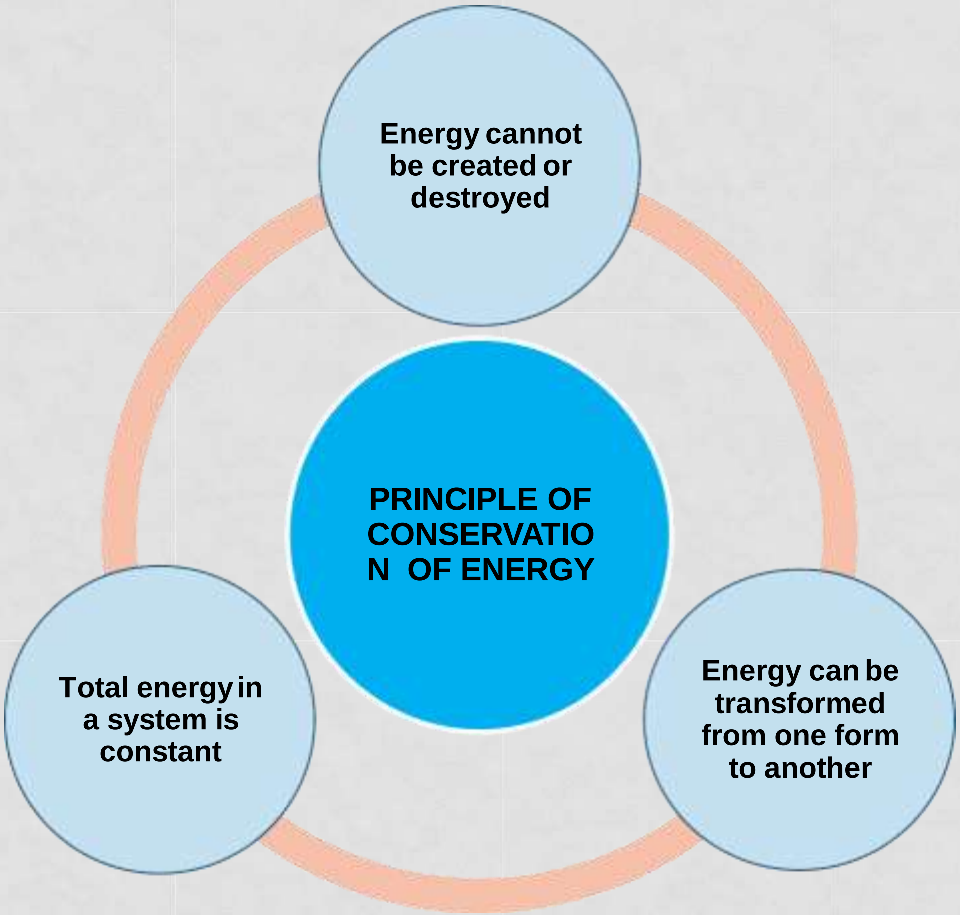
$$\boldsymbol{E_k = \frac{1}{2} m v^2}$$

$$\boldsymbol{40\ J = \frac{1}{2}(6)(v^2)}$$

$$\boldsymbol{v^2 = \frac{40 \times 2}{6} = 13.33}$$

$$v = \sqrt{13.33} = 3.65\ \frac{}{s}$$

4.1.3 STATE PRINCIPLE OF CONSERVATION OF ENERGY



4.1.4 DESCRIBE CONVERSION FROM ONE FORM OF ENERGY TO ANOTHER

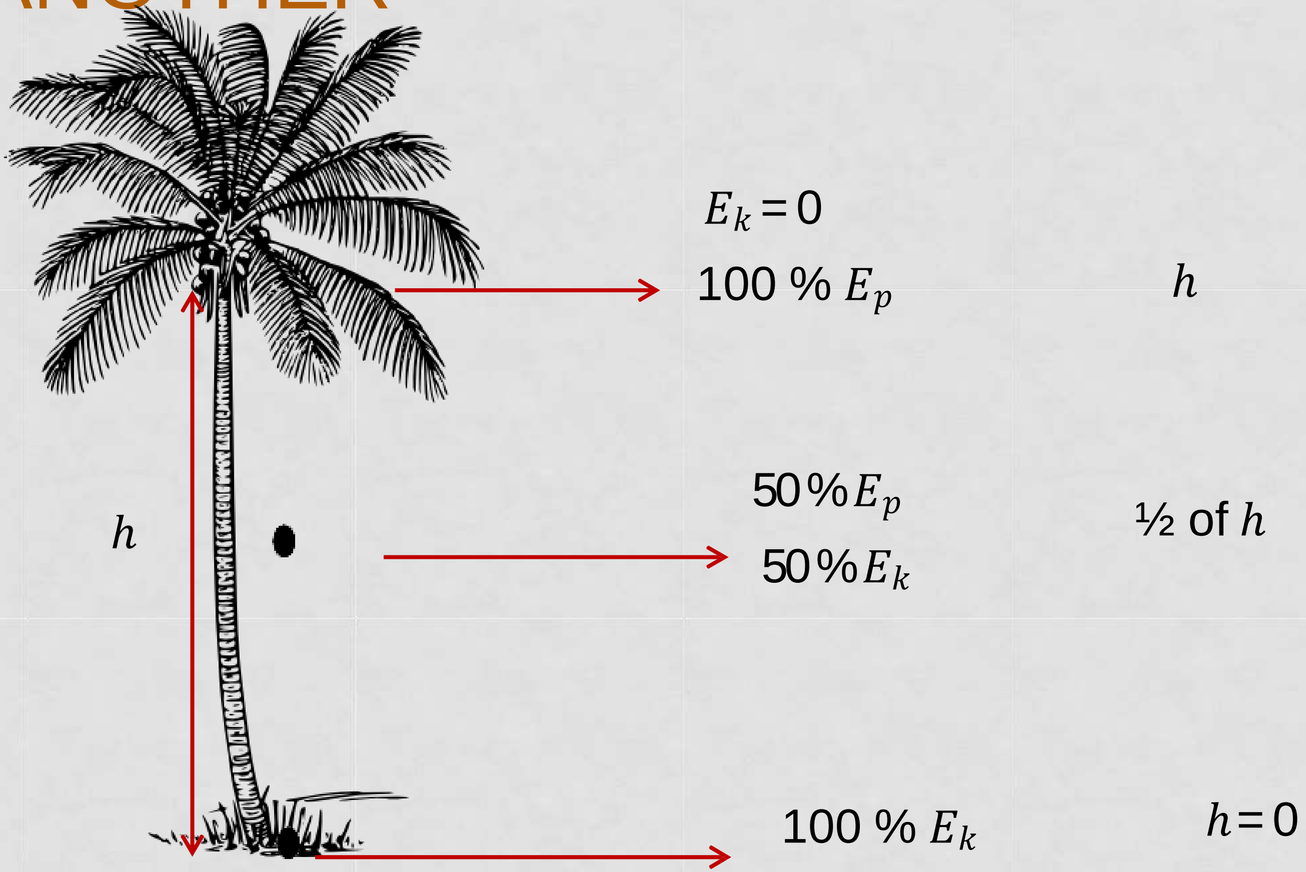


Figure 6(a): Conversion of Energy

4.1.4 DESCRIBE CONVERSION FROM ONE FORM OF ENERGY TO ANOTHER

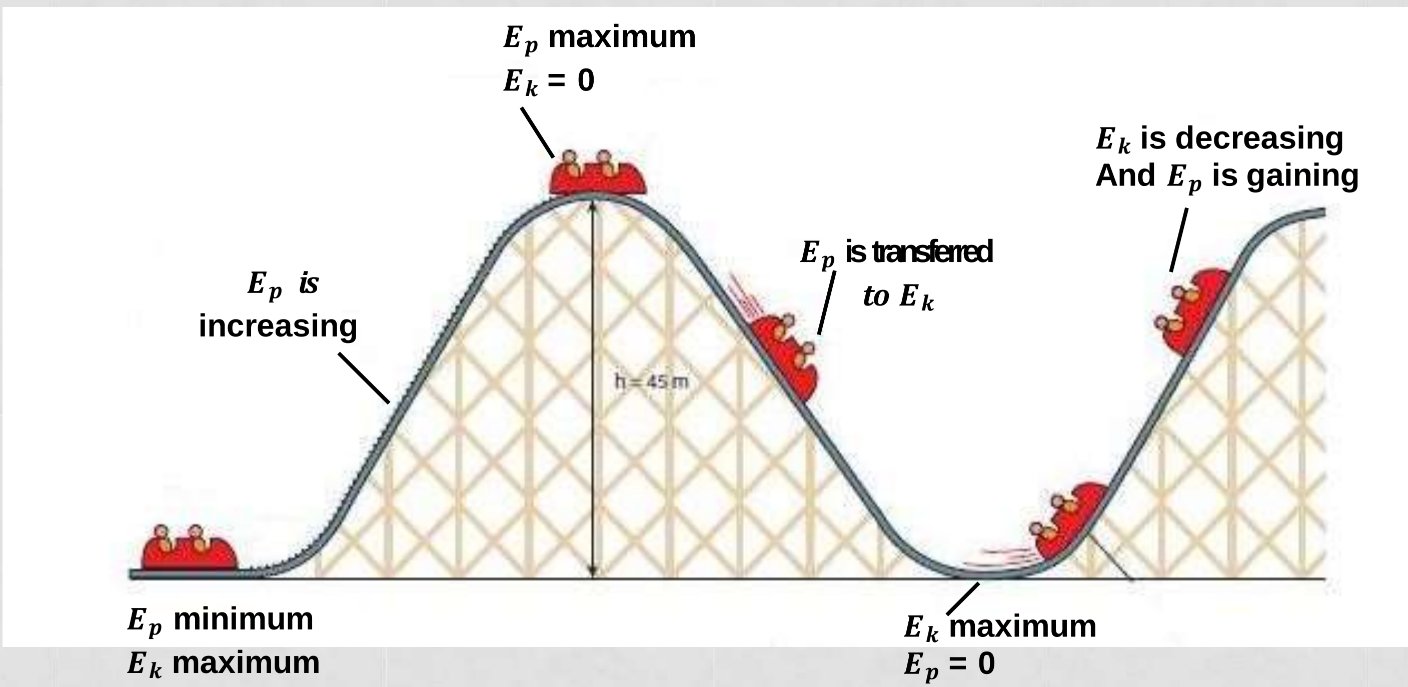


Figure 6(b): Conversion of Energy

L

4.1.4 DESCRIBE CONVERSION FROM ONE FORM OF ENERGY TO ANOTHER

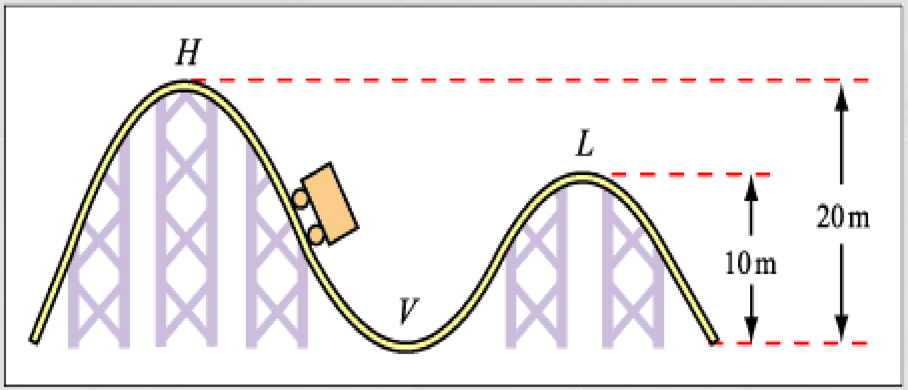
EXAMPLE

Figure below shows a simplified route of a roller coaster. Initially, the cart runs down from the starting point . Neglecting the effects of friction, find its speed at

(a) the "valley" *V*

(b) at the top of second hill *L*

(c) If friction is taken in consideration, will the answers be smaller or larger than that found in part (a) and (b)?



4.1.4 DESCRIBE CONVERSION FROM ONE FORM OF ENERGY TO ANOTHER

Solution :

a) $E_{p\,(at\,H)} = E_{k\,(at\,V)}$

$$mgh = \frac{1}{2}mv^2$$

$$v^2 = 2gh$$

$$v = \sqrt{2gh} = \sqrt{2\left(\frac{9.81}{s}\right)(20)}$$

$$= 19.81 \frac{}{s}$$

b) $E_{p\,(at\,H)} + E_{K(at\,H)} = E_{p(at\,L)} + E_{K(at\,L)}$

$$m(9.81)(20) + 0 = m(9.81)(10) + \frac{1}{2}mv^2$$

$$196.2m = 98.1m + \frac{1}{2}mv^2$$

$$\frac{1}{2}mv^2 = 196.2\text{m} - 98.1\text{m} = 98.1\text{m}$$

$$mv^2 = 196.2m$$

$$v = \sqrt{196.2} = 14.0\frac{m}{s}$$

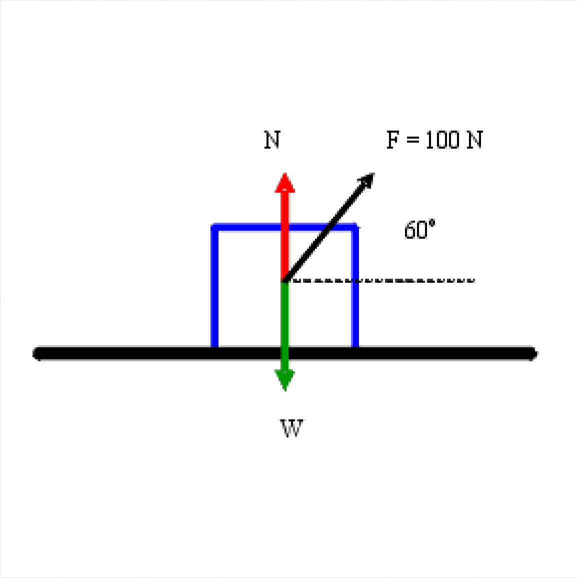
4.1.4 DESCRIBE CONVERSION FROM ONE FORM OF ENERGY TO ANOTHER

c) The results would be smaller than found in part (a) and (b), because some energy is lost by friction.

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

A box is dragged across a floor by a 100N force directed 60^{o} above the horizontal. How much work does the force do in pulling the object 8m?



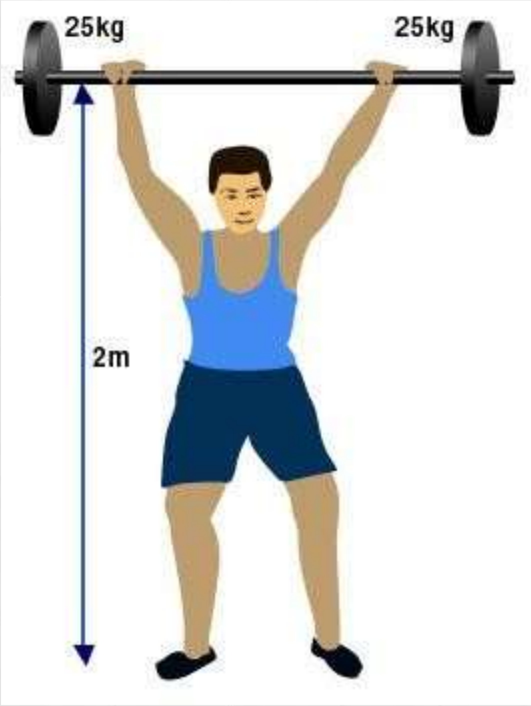
Solution:

$$W \quad = Fs\cos\theta = 100 \times 8\cos 60^{\circ} = 400\,Nm = 400J$$

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

Calculate the work done by the weight lifter in lifting the weights?



Solution:

$$E_p = F \times s = mg$$

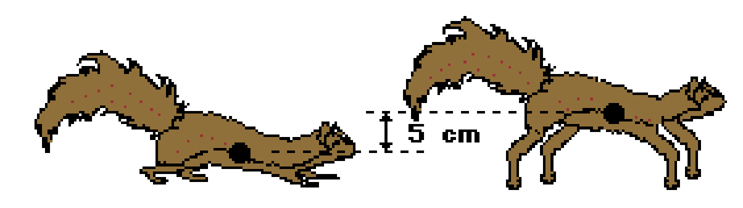
$$= (25kg + 25kg)\left(\frac{9.81m}{s^2}\right)(2m)$$

$$= 981\,J$$

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

A tired squirrel (mass of approximately 1 kg) does push-ups by applying a force to elevate its center-of-mass by 5 cm in order to do a mere 0.50 Joule of work. If the tired squirrel does all this work in 2 seconds, then determine its power.



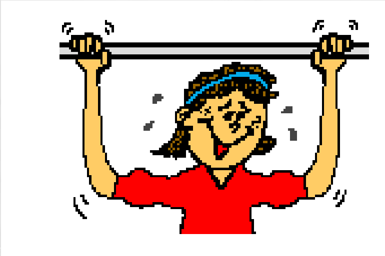
Solution :

$$P = \frac{W}{t} = \frac{0.50 J}{2 s} = 0.25 W$$

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

When doing a *chin-up*, a physics student lifts her 42 kg body a distance of 0.25 meters in 2 seconds. What is the power delivered by the student's biceps ?



Solution :

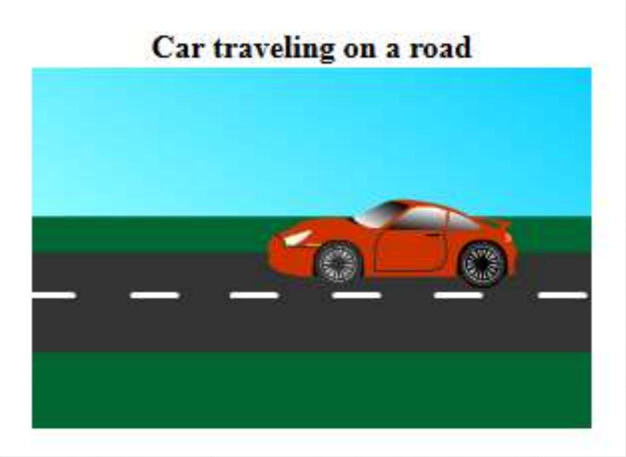
$$P = \frac{W}{t} = \frac{F \times s}{t} = \frac{(m\,g)\,s}{t} = \frac{(42)(\frac{9.81m}{s^2})(0.25m)}{2}$$

$$= 51.5W$$

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

A 300 kg car has a kinetic energy of 500 J. Find its speed.



Solution :

$$E_k = \frac{1}{2} m v^2$$

$$500\,J = \frac{1}{2}(300\,kg)\,v^2$$

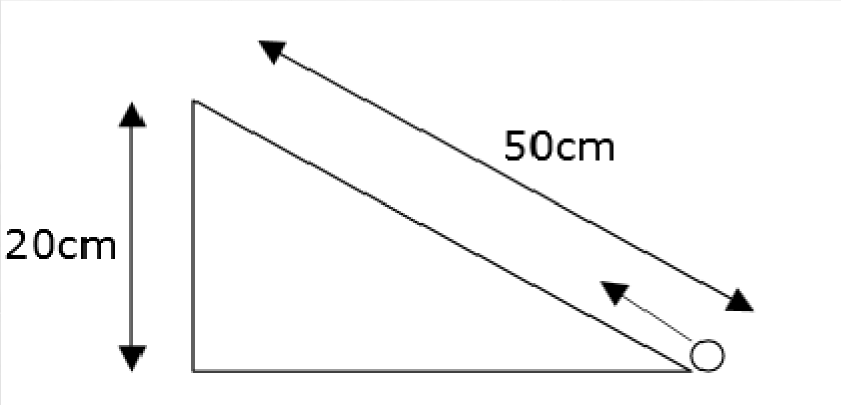
$$v^2 = \frac{500 \times 2}{300} = 3.33$$

$$v = \sqrt{3.33} = 1.83\ m/s$$

4.1.5 APPLY THE CONCEPT AND FORMULA OF WORK, ENERGY AND POWER IN SOLVING THE RELATED PROBLEMS

EXAMPLE

A 800g ball is pulled up a slope as shown in the diagram. Calculate the potential energy it gains



Solution :

$$E_p = mgh$$

$$= \left(\frac{800}{1000}\right)\left(\frac{9.81m}{s^2}\right)\left(\frac{20}{100}\right) = 1.5696\text{ J}$$

Converts gram to kg

Converts cm to meter

4.1.6 CALCULATE THE EFFICIENCY OF MECHANICAL SYSTEM

- The efficiency of an engine or machine is defined as:

$$\textbf{Efficiency},\eta = \frac{Useful\ energy\ output\ (work\ done)}{energy\ input} \times 100\%$$

$$= \frac{E_0}{E_{in}} \times 100\%$$

@

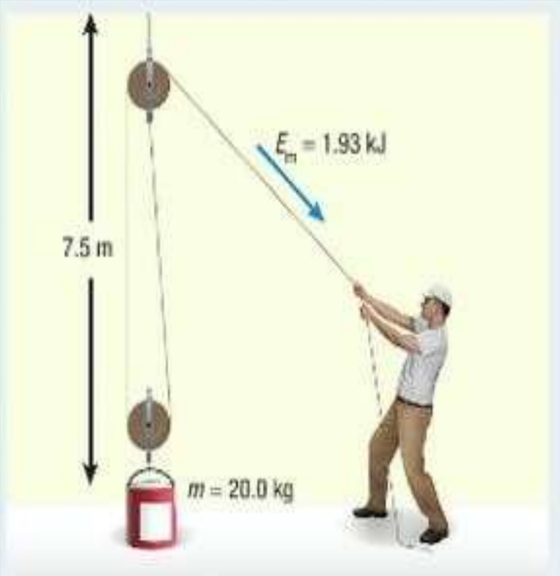
$$\textbf{Efficiency},\eta = \frac{Useful\ power}{power\ input} \times 100\%$$

$$= \frac{P_0}{P_{in}} \times 100\%$$

4.1.6 CALCULATE THE EFFICIENCY OF MECHANICAL SYSTEM

EXAMPLE

Find the efficiency of a rope and pulley system if a painter uses 1.93kJ of mechanical energy to pull on the rope and lift a 20kg paint barrel at constant speed to a height of 7.5m above the ground?



Solution :

$$E_{in} = 1.93\,kJ = 1.93 \times 1000 = 1930\,J$$

$$E_o = mgh = 20 \times 9.81 \times 7.5 = 1471.5\,J$$

$$\text{Efficiency} = \frac{E_o}{E_{in}} \times 100 = \frac{1471.5}{1930} \times 100 = 76.24\%$$

4.1.6 CALCULATE THE EFFICIENCY OF MECHANICAL SYSTEM

EXAMPLE

A heat engine gives out 400 J of heat energy as the useful work. Calculate the energy given to it as input if its efficiency is 40%?

Solution :

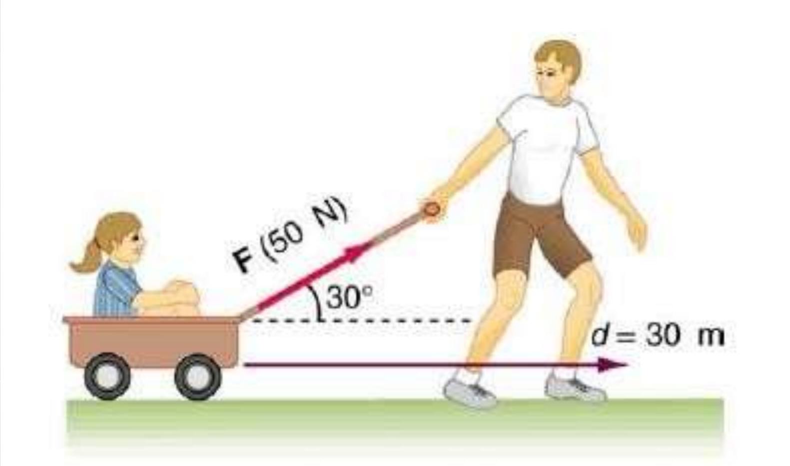
$$\text{Efficiency} = \frac{E_0}{E_{in}} \times 100\%$$

$$40\% = \frac{400J}{E_{in}} \times 100\%$$

$$E_{in} = \frac{400J}{40\%} \times 100\% = 1000\,J$$

EXERCISE

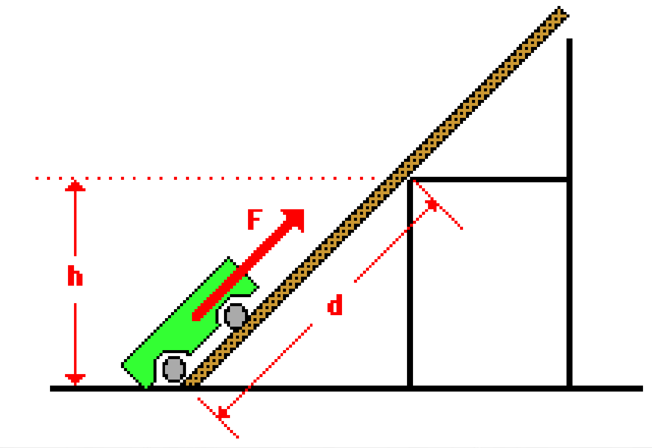
1) How much work is done by the boy pulling his sister 30.0 m in a wagon as shown in the above figure? Assume no friction acts on the wagon.



Answer = 1299.038 J

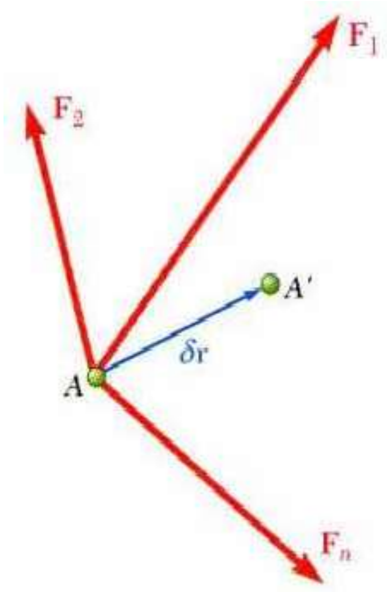
EXERCISE

2) A cart is loaded with a brick and pulled at constant speed along an inclined plane to the height of a seat-top. If the mass of the loaded cart is 3.0 kg and the height of the seat top is 0.45 meters, then what is the potential energy of the loaded cart at the height of the seat-top?



Answer : E_p = 13.2 J

Principle of Virtual Work



- *Imagine* the small *virtual displacement* of particle which is acted upon by several forces.
- The corresponding *virtual* work,

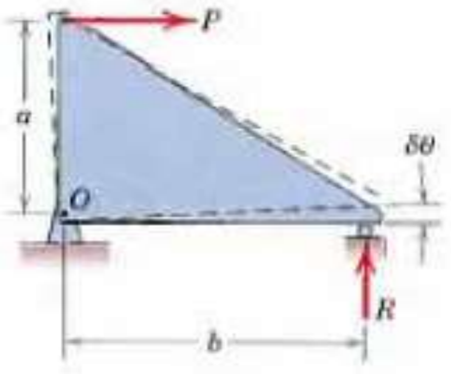
$$\delta U = \vec{F}_1 \cdot \delta\vec{r} + \vec{F}_2 \cdot \delta\vec{r} + \vec{F}_3 \cdot \delta\vec{r} = \left(\vec{F}_1 + \vec{F}_2 + \vec{F}_3\right)\cdot\delta\vec{r}$$

$$= \vec{R}\cdot\delta\vec{r}$$

Principle of Virtual Work:

- If a particle is in equilibrium, the total virtual work of forces acting on the particle is zero for any virtual displacement.
- If a rigid body is in equilibrium, the total virtual work of external forces acting on the body is zero for any virtual displacement of the body.
- If a system of connected rigid bodies remains connected during the virtual displacement, only the work of the external forces need be considered.

Virtual Work for a rigid body



$P a \delta\theta + R b \delta\theta = 0$

This is equivalent to taking moment about 0.

Similarly by *virtual translations in the* x *and* y *directions we can obtain* $\Sigma F_x = 0$ *and* $\Sigma F_y = 0$.

Note that internal forces do *not perform work due to cancellation from equal and opposite forces*

To summarize

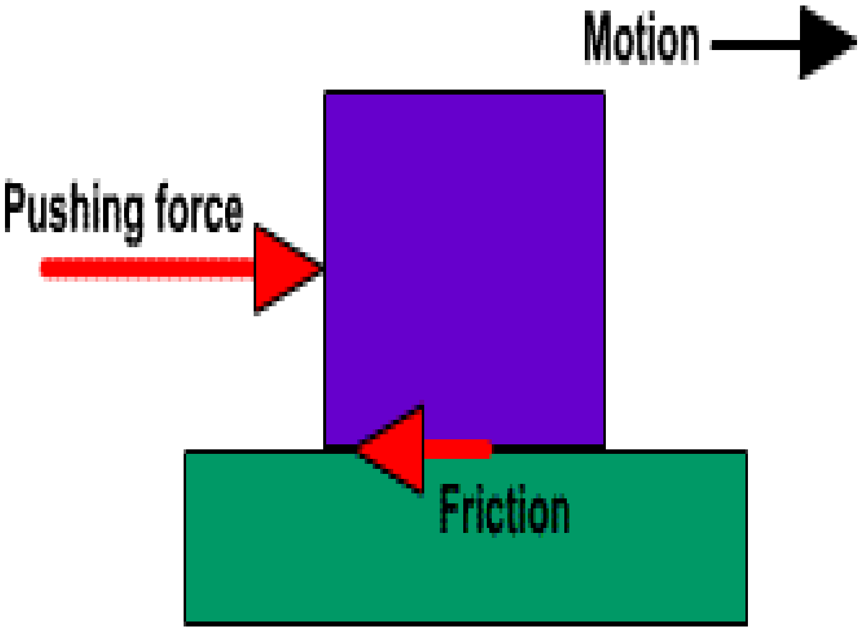
- ***Principle of virtual work***
  - **The virtual work done by external active forces on an ideal mechanical system in equilibrium is zero for any all virtual displacements consistent with the constraints.**
- Ideal system:
  - All surfaces, joints etc. are frictionless.
  - We will deal with ideal system in this course.
- Consistent with constraints:
  - The *virtual displacement* should be such that they should not do allow the *non-active* forces to do any work.

Why principle of Virtual Work

- For complex mechanisms (we will solve some problems) we do not need to dismember the system.
- We obtain the *active unknown force* in one shot without bothering about the *reactive* forces.
- Such type of analysis will be a stepping stone to VW analysis using *deformations* when you teach *Solid Mechanics, Structural Mechanics etc*. not to mention powerful

Definition of Friction

"The opposing force, which acts in the opposite direction of the movement of the upper Body ,is called "FRICTION FORCE", or simply " FRICTION ".



Types of Friction

- Generally we classify the friction in two categories –

(1) Static friction

(2) Dynamic friction

(1) Static friction : - when the body is in rest condition then the friction force exerted by the body is called "static friction".

(2) Dynamic friction

- when the body is in moving condition then the friction force exerted by the body is called is called "Dynamic friction“.
- Dynamic friction is also classified into three categories - (i) sliding friction

  (ii) Rolling friction

  (iii) Pivot friction

(i) sliding friction : -

when a body slide over another body then the friction force exerted by the body is called "Sliding friction".

(ii) Rolling friction:-

A body which has balls or rollers roles on the another body then the friction force exerted by the second body on the first body is called "Rolling friction".

(iii) Pivot friction:-

The friction force experienced by a body , due to the motion of rotation as in case of foot step bearings.

SOME OTHER TYPES OF FRICTION

(i) Dry friction - Friction between two un -lubricated or dry surface.

(ii) Lubricated friction - Friction between two lubricated surface.

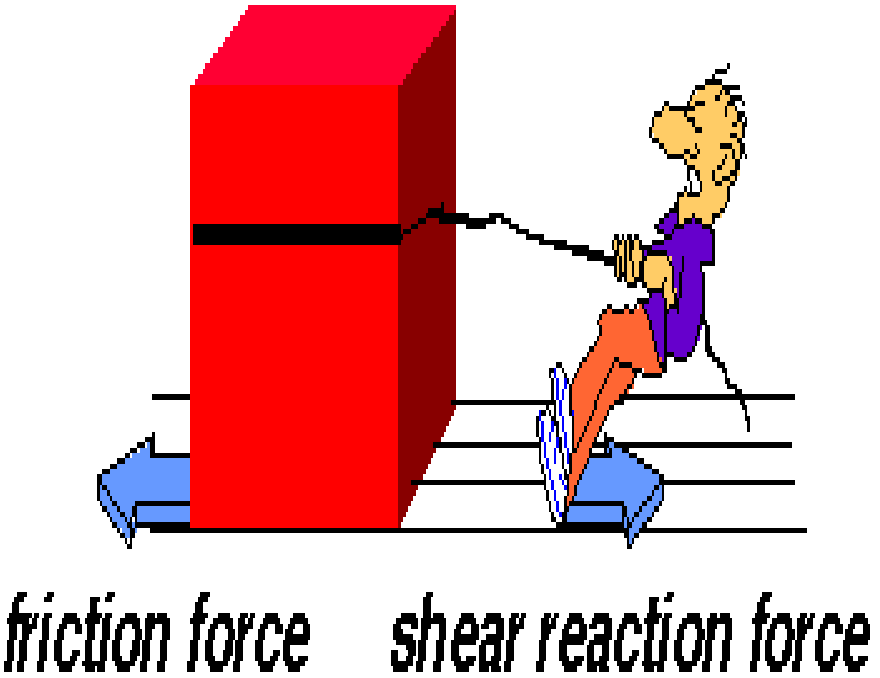
Lubricated friction can also be classified into two categories –

(a) Greasy friction - when there is a very thin layer of lubricant between the surfaces.

(b) Fluid friction : - when we introduce a thick layer of lubricant between two moving surfaces

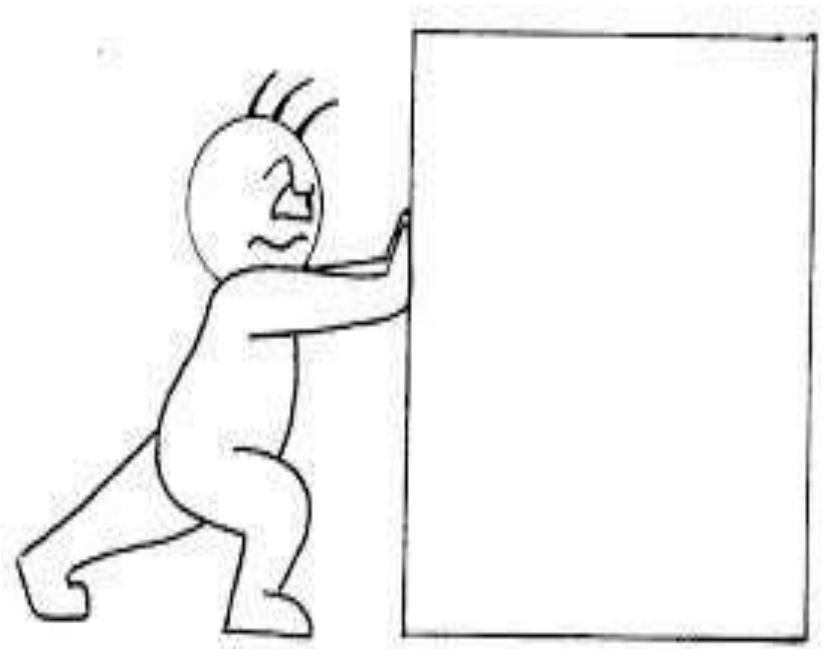
WHERE FRICTION IS DESIRABLE

- During walking on the road.
- In writing on the paper.
- To hold or grabbing something.



WHERE FRICTION IS NOT DESIRABLE

- During walking we have to put extra effort.
- Due to friction the life of the machine part reduced.
- it is not required in mechanical parts where two parts of machine meet.



A large effort required to move this block than on a smoother one.

LIMITING FRICTION FORCE

- The maximum value of the friction force at which the body resting on the another body start to slide.

If Limiting friction force is “F” and applied force is “P”, then

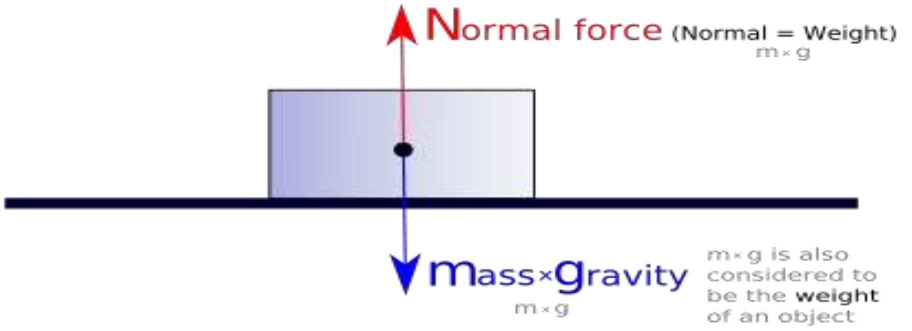
(i) If $F > P$ (no motion between two bodies)

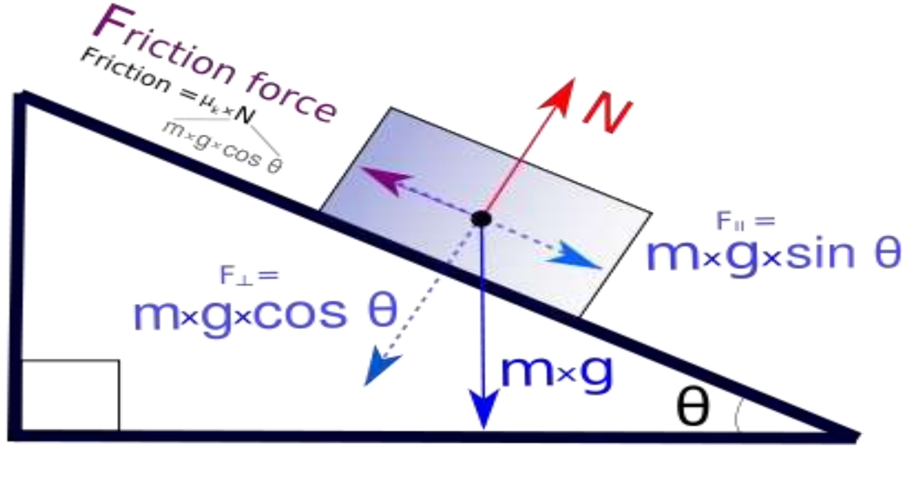
(ii) If $F = P$ (no motion between bodies)

(iii) If $F = P$ (motion will be there)

NORMALREACTION/FORCE

- It is the net force compressing two parallel surface together and the direction of the reaction force will be perpendicular to the surface on which its act.





COEFFICIENT OF FRICTION

- "It is defined as the ratio of the ratio of the limiting friction force to the normal reaction force".

- It is represented by the "μ".

- it is given by -> $\mu = F / Fn$

(limiting friction/normal reaction)

STATIC FRICTION

- "When friction occurs between two bodies which are not in motion then the friction is known as STATIC FRICTION“, and it is represented by "µs".
- It is given by

  $F\{max\} = \mu s * Fn$

DYNAMIC FRICTION

" When the friction occurs between two bodies which have relative motion to each other, then the friction is known as the DYNAMIC FRICTION", and it is represented by "µ"or "µd".

F {max} = µ * Fn.

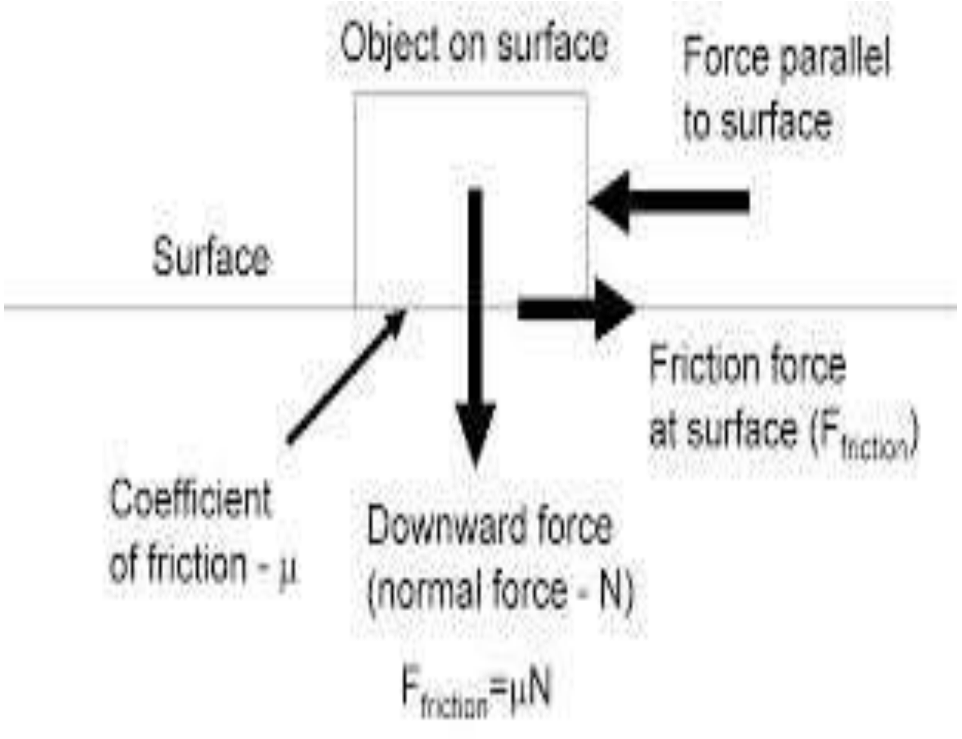
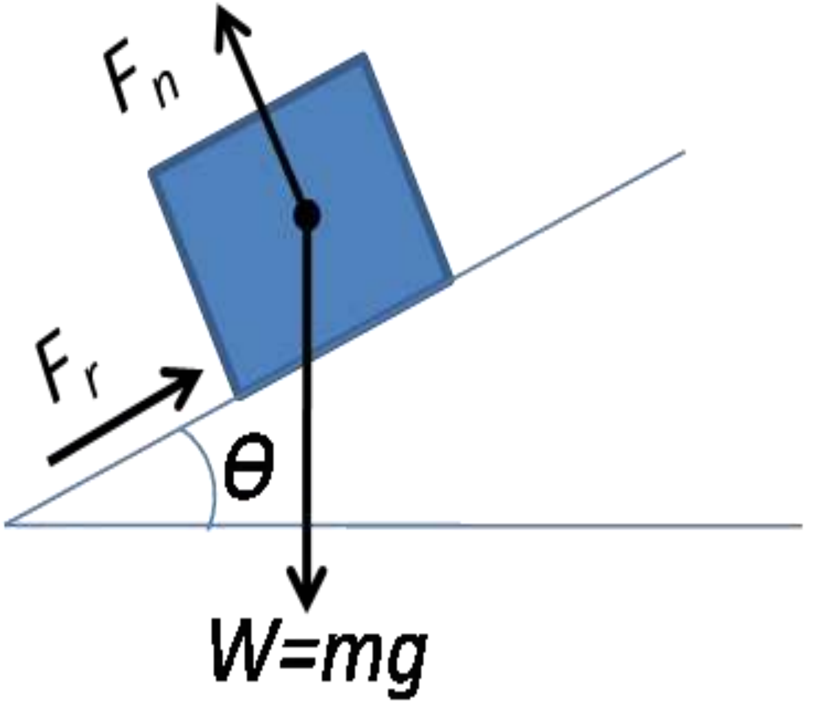


Figure 1 – Basic Definitions of the Coefficient of Friction

ANGLE OF REPOSE

"The minimum inclination of any plane to the horizontal is such that the body resting on the inclined plane tend to move down the plane, then this minimum inclination of the plane is known as ***ANGLE OF REPOSE".***



LAWS OF STATIC FRICTION

i. The applied force will be equal in magnitude with friction force.

ii. The ratio of the limiting friction force to the normal reaction force is called coefficient of static friction.

iii. The force of friction is dependent on the roughness of the surface but independent on the area of contact.

LAWS OF DYNAMIC FRICTION

i. The direction of the friction force will be opposite to the direction of motion of the body.

ii. The ratio of the dynamic friction to the normal reaction is a constant.

iii. As we increase the speed of the moving body then the friction force decrease.

LAWS OF SOLID FRICTION

i. The force of friction is dependent on the material of the surface but independent on the relative velocity of the sliding body and area of contact surface.

ii. The friction force is directly proportional to the normal reaction force.

LAWS OF FLUID FRICTION

i. The friction force is different for different lubricants.

ii. The friction force is independent on the load as well as substances of the bearing surfaces.

iii. The friction force reduce with increasing in the temperature of the lubricant.