JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem

Subject –Engineering Mathematics-II

Unit – III

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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Particular Integral case -5

P.I when $X = xV$ or $x^2 V$

$$P.I = \frac{1}{f(D)} xV = x \cdot \frac{1}{f(D)} V - \frac{f'(D)}{\{f(D)\}^2} V$$

or $P.I = \frac{1}{f(D)} x^2 V$

$$= x^2 \frac{1}{f(D)} V + 2x \left\{ \frac{d}{dD} \cdot \frac{1}{f(D)} \right\} V + \left\{ \frac{d^2}{dD^2} \cdot \frac{1}{f(D)} \right\} \cdot V$$

Ex: $\dfrac{d^2y}{dx^2} + \dfrac{dy}{dx} = x \cos x$

Sol: $(D^2+D)y = x \cos x$

Auxiliary eqn is : $m^2+m=0$

$\Rightarrow m(m+1)=0 \Rightarrow m=0,-1$

$C.F = C_1 + C_2 e^{-x}$

$$P.I = \frac{1}{(D^2+D)} x\cos x$$

$$= x \cdot \frac{1}{D^2+D} \cos x - \frac{(2D+1)}{(D^2+D)^2} \cos x$$

$$= x \cdot \frac{1}{(-1+D)} \cos x - \frac{(2D+1)}{(-1+D)^2} \cos x$$

$$= x \cdot \frac{(D+1)}{D^2-1} \cos x - \frac{(2D+1)}{(D^2-2D+1)} \cos x$$

$$= x \cdot \frac{(D+1)}{(-1-1)} \cos x - \frac{(2D+1)}{(-1-2D+1)} \cos x$$

$$= -\frac{x}{2} (D+1) \cos x + \frac{1}{2D} (2D+1) \cos x$$

$$= -\frac{x}{2} (-\sin x + \cos x) + \frac{1}{2D} (-2\sin x + \cos x)$$

$$= +\frac{x}{2} (\sin x - \cos x) + \frac{1}{2} (2\cos x + \sin x)$$

hence the soln is

$$y = C.F + P.I$$

$$y = c_1 + c_2 e^{-x} + \frac{x}{2}(\sin x - \cos x) + \frac{1}{2}(2\cos x + \sin x).$$

Ex: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x\sin x$

Sol: $(D^2 - 2D + 1)y = x\sin x$,

The auxiliary eqn is

$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0$

$\Rightarrow m = 1, 1.$

$C.F = (c_1 + c_2 x)e^x$

$$P.I = \frac{1}{D^2 - 2D + 1} x \sin x = \frac{1}{(D-1)^2} x \sin x$$

$$= x \cdot \frac{1}{D^2 - 2D + 1} \sin x + \frac{d}{dD}\left[\frac{1}{(D-1)^2}\right] \sin x$$

$$= x \cdot \frac{1}{\cancel{-1} - 2D \cancel{+1}} \sin x - \frac{2}{(D-1)^3} \sin x$$

$$= -\frac{x}{2} \cdot \frac{1}{D} \sin x - \frac{2}{D^3 - 3D^2 + 3D - 1} \sin x$$

$$= \frac{x}{2}\cos x - \frac{2}{(-1)D - 3(-1) + 3D - 1}\sin x$$

$$= \frac{x}{2}\cos x - \frac{\cancel{2}}{\cancel{2}(D+1)}\sin x$$

$$= \frac{x}{2}\cos x - \frac{(D-1)}{D^2-1}\sin x = \frac{x}{2}\cos x - \frac{(D-1)}{-1-1}\sin x$$

$$= \frac{x}{2}\cos x + \frac{1}{2}(D-1)\sin x$$

$$= \frac{x}{2}\cos x + \frac{1}{2}(\cos x - \sin x)$$

hence the required sol is

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x)e^x + \frac{1}{2}(x\cos x - \sin x + \cos x)$$

Ex: $(D^2-1)y = x^2 \cos x$

Sol: Auxiliary eqn is:

$m^2-1=0 \Rightarrow m = 1, -1$

$C.F = c_1 e^{x} + c_2 e^{-x}$

$$P.I = \frac{1}{D^2-1} x^2 \cos x$$

$$= x^2 \cdot \frac{1}{D^2-1}\cos x + 2x \cdot \frac{d}{dD}\left(\frac{1}{D^2-1}\right)\cos x + \frac{d^2}{dD^2}\left(\frac{1}{D^2-1}\right)\cos x$$

$$= x^2 \cdot \frac{1}{-1-1}\cos x + 2x\left\{\frac{-2D}{(D^2-1)^2}\right\}\cos x + \frac{(6D^2+2)}{(D^2-1)^3}\cos x$$

$$= -\frac{x^2}{2}\cos x + 2x \cdot \left\{\frac{-2D\cos x}{(-1-1)^2}\right\} + \frac{2(3D^2\cos x + \cos x)}{(-1-1)^3}$$

$$= -\frac{1}{2}x^2\cos x + x\sin x - \frac{1}{4}(-3\cos x + \cos x)$$

$$= -\frac{x^2}{2}\cos x + x\sin x + \frac{1}{2}\cos x$$

$$= x\sin x + \frac{1}{2}(1 - x^2)\cos x$$

Thus the general sol is

$$y = C.F + P.I$$

$$y = c_1 e^{x} + c_2 e^{-x} + x\sin x + \frac{1}{2}(1-x^2)\cos x$$

Ex: $(D^4 + 2D^2 + 1)y = x^2 \cos x$

Sol: The auxiliary eqn is

$m^4 + 2m^2 + 1 = 0 \Rightarrow (m^2+1)^2 = 0$

$m = i, i, -i, -i$

$C.F = (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x$

$$P.I. = \frac{1}{D^4+2D^2+1} x^2 \cos x$$

$$= \text{Real Part of } \frac{1}{(D^2+1)^2} x^2 e^{ix} \quad \cdots (1)$$

Now $\frac{1}{(D^2+1)^2} x^2 e^{ix} = e^{ix} \cdot \frac{1}{[(D+i)^2+1]^2} x^2$

$$= e^{ix} \cdot \frac{1}{[D^2+2iD+i^2+1]^2} x^2 = e^{ix} \frac{1}{(D^2+2iD)^2} x^2$$

$$= e^{ix}\frac{1}{D^2(D+2i)^2}x^2 = e^{ix}\times -\frac{1}{4D}\left(1+\frac{D}{2i}\right)^{-2}x^2$$

$$= -\frac{1}{4}e^{ix}\frac{1}{D^2}\left[1-\frac{D}{i}-\frac{3}{4}D^2\right]x^2$$

$$= -\frac{1}{4}e^{ix}\frac{1}{D^2}\left[x^2+2ix-\frac{3}{2}\right]$$

$$= -\frac{1}{4}e^{ix}\left[\frac{x^4}{12}+\frac{ix^3}{3}-\frac{3}{4}x^2\right]$$

$$= -\frac{1}{4}(\cos x + i\sin x)\left[\frac{x^4}{12} + \frac{ix^3}{3} - \frac{3}{4}x^2\right]$$

Now from (1), Its real Part is

$$P.I = -\frac{1}{4}\left[\left(\frac{x^4}{12} - \frac{3}{4}x^2\right)\cos x - \frac{x^3}{3}\sin x\right]$$

$$= \left(-\frac{1}{48}x^4 + \frac{3}{16}x^2\right)\cos x + \frac{x^3}{12}\sin x$$

hence the general sol is

$$y = C.F + P.I.$$

$$y = (c_1 + c_2 x)\cos x + (c_3 + c_4 x)\sin x - \frac{1}{48}x^4 \cos x + \frac{3}{16}x^2 \cos x + \frac{1}{12}x^3 \sin x.$$

Ex: $(D^2+1)y = x^2 \sin 2x$

Sol: The auxiliary eqn is

$$m^2+1=0 \Rightarrow m^2=-1 \Rightarrow m=\pm i$$

$$C.F = C_1 \cos x + C_2 \sin x$$

$$P.I = \frac{1}{D^2+1} x^2 \sin 2x$$

$$= \text{Imaginary Part of} \left\{ \frac{1}{D^2+1} x^2 e^{2ix} \right\} \quad \ldots (1)$$

Now $\dfrac{1}{(D^2+1)} x^2 e^{2ix}$

$$= e^{2ix} \cdot \frac{1}{(D+2i)^2+1} x^2$$

$$= e^{2ix} \cdot \frac{1}{(D^2+4iD-3)} x^2 = \frac{e^{2ix}}{-3} \left(1 - \frac{(D^2+4iD)}{3} \right)^{-1} x^2$$

$$= -\frac{1}{3} e^{2ix}\left[1 + \frac{D^2+4iD}{3} + \left(\frac{D^2+4iD}{3}\right)^2 + \cdots\right] x^2$$

$$= -\frac{1}{3} e^{2ix}\left[1 + \frac{4iD}{3} + \frac{D^2}{3} - \frac{16}{9} D^2 + \cdots\right] x^2$$

$$= -\frac{1}{6} e^{2ix}\left[x^2 + \frac{8ix}{3} + \frac{2}{3} - \frac{32}{9}\right]$$

$$= -\frac{1}{3}\left(\cos 2x + i\sin 2x\right)\left(x^2 + \frac{8}{3} ix - \frac{26}{9}\right)$$

$$= \left(-\frac{1}{3} x^2 \cos 2x + \frac{26}{27}\cos 2x + \frac{8}{27} x\sin 2x\right) + i\left(-\frac{8}{9} x\cos 2x - \frac{x^2}{3}\sin 2x + \frac{26}{27}\sin 2x\right) \quad \ldots (2)$$

from ① & ②, we get

$$P.I = -\frac{8}{9}x\cos 2x + \frac{1}{27}(26 - 9x^2)\sin 2x$$

Complete soln is : $y = C.F + P.I$

$$y = c_1\cos x + c_2\sin x - \frac{8}{9}x\cos 2x + \frac{1}{27}(26 - 9x^2)\sin 2x.$$

Practice Problems

1. $$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = x\cos x$$

Ans: $$y = (c_1 + c_2 x)e^{-x} + \frac{1}{2}(x\sin x + \cos x - \sin x).$$

2. $$\frac{d^2y}{dx^2} + 4y = x\sin x$$

Ans: $$y = c_1\cos 2x + c_2\sin 2x + \frac{1}{3}x\sin x - \frac{2}{9}\cos x$$

3. $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \sin x$$

Ans: $y = (C_1 + C_2 x)e^x - e^x(2\cos x + x\sin x)$

4. $$(D^2+4)y = x\sin 2x$$

Ans: $y = C_1 \cos 2x + C_2 \sin 2x - \frac{x^2}{8}\cos 2x + \frac{1}{16} x \sin 2x$

5. $(D^2+4)y = x\sin^2 x$

Ans: $y = C_1\cos 2x + C_2\sin 2x + \frac{1}{8}x - \frac{1}{16}x^2\sin 2x - \frac{1}{32}x\cos 2x$

6. $(D^2-1)y = x\sin x + (1+x^2)e^x$

Ans: $y = C_1 e^x + C_2 e^{-x} - \frac{1}{2}(x\sin x + \cos x) + \frac{1}{12}xe^x(2x^2-3x+9)$

7. $\dfrac{d^2y}{dx^2} + y = x^2 \sin x$

Ans: $y = C_1 \cos x + C_2 \sin x - \dfrac{1}{12}\left\{(2x^3 - 3x)\cos x - 3x^2 \sin x\right\}$

8. $(D^4 - 1)y = x^2 \sin x$

Ans: $y = C_1 e^{x} + C_2 e^{-x} + C_3 \cos x + C_4 \sin x + \dfrac{1}{12} x^3 \cos x$

$- \dfrac{5}{8} x \cos x - \dfrac{3}{8} x^2 \sin x.$

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Thank you!