JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem

Subject –Engineering Mathematics-II

Unit – III

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VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Particular Integral case -4

when $X = e^{ax} \cdot v$; V is any fn. of x

$$P.I = \frac{1}{f(D)}(e^{ax} \cdot v) = e^{ax} \cdot \frac{1}{f(D+a)} \cdot v$$

where $\frac{1}{f(D+a)} v$ can be evaluated by Previously discussed methods.

Ex: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = x^2 e^{3x}$

Sol: Auxilliary equ is

$m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0$

$\Rightarrow m = 1, 1.$

$C.F = (c_1 + c_2 x)e^x$

$$P.I = \frac{1}{(D^2-2D+1)} x^2 e^{3x} = \frac{1}{(D-1)^2} x^2 e^{3x}$$

$$= e^{3x} \cdot \frac{1}{(D+3-1)^2} x^2 \Rightarrow e^{3x} \cdot \frac{1}{(D+2)^2} x^2$$

$$= \frac{1}{4} e^{3x} \cdot \frac{1}{\left(1+\frac{D}{2}\right)^2} x^2 \Rightarrow \frac{1}{4} e^{3x} \left[1+\frac{D}{2}\right]^{-2} x^2$$

$$= \frac{1}{4} e^{3x} \left(1 - D + \frac{3D^2}{4} + \cdots\right) x^2$$

$$= \frac{1}{4} e^{3x} \left(x^2 - 2x + \frac{3}{2}\right) = \frac{1}{8} e^{3x} (2x^2 - 4x + 3)$$

hence the complete sol. is

$$y = C.F + P.I$$

$$y = (c_1 + c_2 x) e^{x} + \frac{1}{8} e^{3x} (2x^2 - 4x + 3).$$

Ex: $\frac{d^2y}{dx^2} + 3\frac{dy}{dx} + 2y = e^{2x}\sin x$

Sol: To find C.F, Auxiliary eqn is

$m^2 + 3m + 2 = 0 \Rightarrow (m+1)(m+2) = 0$

$m = -1, -2.$

$C.F = c_1 e^{-x} + c_2 e^{-2x}$

$$P.I = \frac{1}{D^2+3D+2} e^{2x} \sin x = e^{2x} \frac{1}{(D+2)^2+3(D+2)+2} \sin x$$

$$= e^{2x} \cdot \frac{1}{(D^2+4D+4+3D+8)} \sin x$$

$$= e^{2x} \cdot \frac{1}{(D^2+7D+12)} \sin x = e^{2x} \frac{1}{(-1+7D+12)} \sin x$$

$$= e^{2x} \cdot \frac{1}{(7D+11)} \sin x = e^{2x} \cdot \frac{(7D-11) \sin x}{(7D+11)(7D-11)}$$

$$= e^{2x} \cdot \frac{(7D-11)\sin x}{49D^2-121} = e^{2x} \cdot \frac{(7D-11)\sin x}{49(-1)-121}$$

$$= -\frac{e^{2x}}{170}(7D-11)\sin x$$

$$= \frac{-e^{2x}}{170}\left(7\frac{d}{dx}(\sin x) - 11\sin x\right)$$

$$= -\frac{e^{2x}}{170}(7\cos x - 11\sin x).$$

hence the Complete sol. is

$$y = C.F + P.I$$

$$y = c_1 e^{-x} + c_2 e^{-2x} - \frac{e^{2x}}{170}(7\cos x - 11\sin x).$$

Ex: $$\frac{d^4y}{dx^4}+\frac{d^2y}{dx^2}+y=e^{-\frac{x}{2}}\cos\left(\frac{\sqrt{3}}{2}x\right)$$

Sol: Auxiliary equ is

$$m^4+m^2+1=0$$

$$\Rightarrow \quad m^4+2m^2+1-m^2=0 \quad \Rightarrow \quad (m^2+1)^2-m^2=0$$

$$(m^2-m+1)(m^2+m+1)=0$$

$$\Rightarrow \quad m=\frac{-1\pm i\sqrt{3}}{2}, \quad \frac{1\pm i\sqrt{3}}{2}$$

$$C.F = e^{-\frac{x}{2}}\left[c_1 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right] + e^{\frac{x}{2}}\left[c_3 \cos\left(\frac{\sqrt{3}}{2}x\right) + c_4 \sin\left(\frac{\sqrt{3}}{2}x\right)\right]$$

$$P.I = \frac{1}{(D^4+D^2+1)} e^{-x/2} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= \frac{1}{(D^2-D+1)(D^2+D+1)} e^{-x/2} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-\frac{x}{2}} \cdot \frac{1}{\left\{\left(D-\frac{1}{2}\right)^2 - \left(D-\frac{1}{2}\right) + 1\right\}\left\{\left(D-\frac{1}{2}\right)^2 - \left(D-\frac{1}{2}\right) + 1\right\}} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-\frac{x}{2}} \frac{1}{\left(D^2 - D + \frac{1}{4} + D + \frac{1}{2}\right)\left(D^2 - D + \frac{1}{4} - D + \frac{3}{2}\right)} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-x/2} \cdot \frac{1}{\left(D^2 + \frac{3}{4}\right)\left(-\frac{3}{4} - 2D + \frac{7}{4}\right)} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-\frac{x}{2}} \cdot \frac{1}{\left(D^2+\frac{3}{4}\right)(-2D+1)} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-\frac{x}{2}} \frac{1\,(1+2D)}{\left(D^2+\frac{3}{4}\right)(1-4D^2)} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= e^{-\frac{x}{2}} \frac{(1+2D)}{\left(D^2+\frac{3}{4}\right)\left\{1-4\left(-\frac{3}{4}\right)\right\}} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= \frac{1}{4} e^{-\frac{x}{2}} (1+2D) \cdot \frac{1}{\left(D^2+\frac{3}{4}\right)} \cos\left(\frac{\sqrt{3}}{2}x\right)$$

$$= \frac{1}{4} e^{-\frac{x}{2}} (1+2D) \frac{x}{2\left(\frac{\sqrt{3}}{2}\right)} \sin\left(\frac{\sqrt{3}x}{2}\right)$$

$$= \frac{1}{4\sqrt{3}} e^{-x/2} (1+2D) \left[x \sin\left(\frac{\sqrt{3}x}{2}\right) \right]$$

$$= \frac{1}{4\sqrt{3}} e^{-\frac{x}{2}} \left[x \sin\left(\frac{\sqrt{3}x}{2}\right) + 2\left\{ \sin\left(\frac{\sqrt{3}x}{2}\right) + \frac{x\sqrt{3}}{2} \cos\left(\frac{\sqrt{3}x}{2}\right) \right\} \right]$$

$$= \frac{1}{4\sqrt{3}} e^{-\frac{x}{2}} \left[(x+2) \sin\left(\frac{\sqrt{3}x}{2}\right) + x\sqrt{3} \cos\left(\frac{\sqrt{3}x}{2}\right) \right]$$

Hence the Complete sol. is

$$y = C \cdot F + P \cdot I$$

$$y = e^{-\frac{x}{2}}\left[C_1 \cos\left(\frac{\sqrt{3}x}{2}\right) + C_2 \sin\left(\frac{\sqrt{3}x}{2}\right)\right] + e^{x/2}\left[C_3 \cos\left(\frac{\sqrt{3}x}{2}\right)\right.$$

$$\left. + C_4 \sin\left(\frac{\sqrt{3}x}{2}\right)\right] + \frac{1}{4\sqrt{3}} e^{-\frac{x}{2}}\left[(x+2)\sin\left(\frac{\sqrt{3}x}{2}\right) + x\sqrt{3}\cos\left(\frac{\sqrt{3}x}{2}\right)\right].$$

Ex: $(D^2-1)y = \cosh x \cos x$

Sol: Auxiliary eqn is : $m^2-1=0$

$(m+1)(m-1)=0 \Rightarrow m = 1, -1$

$C.F = C_1 e^{x} + C_2 e^{-x}$

$$R.I = \frac{1}{(D+1)(D-1)} \left\{ \frac{1}{2}(e^{x}+e^{-x}) \cos x \right\}$$

$$= \frac{1}{2} \frac{1}{(D-1)(D+1)} e^{x} \cos x + \frac{1}{2} \cdot \frac{1}{(D-1)(D+1)} e^{-x} \cos x$$

$$= \frac{1}{2} e^{x} \cdot \frac{1}{(D+1-1)(D+1+1)} \cos x + \frac{1}{2} e^{-x} \frac{1}{(D-1-1)(D+1-1)} \cos x$$

$$= \frac{1}{2} e^{x} \frac{1}{D^2+2D} \cos x + \frac{1}{2} e^{-x} \frac{1}{D^2-2D} \cos x$$

$$= \frac{1}{2} e^{x} \frac{1}{-1+2D} \cos x + \frac{1}{2} e^{-x} \frac{1}{-1-2D} \cos x$$

$$= \frac{1}{2} e^{x} \frac{(2D+1)}{4D^2-1} \cos x - \frac{1}{2} e^{-x} \frac{(2D-1)}{4D^2-1} \cos x$$

$$= \frac{1}{2} e^{x} \frac{(2D+1)}{-4-1} \cos x - \frac{1}{2} e^{-x} \frac{(2D-1)}{-4-1} \cos x$$

$$= -\frac{1}{10}e^{x}(-2\sin x + \cos x) + \frac{1}{10}e^{-x}(-2\sin x - \cos x)$$

$$= \frac{2}{5}\sin x\left(\frac{e^{x}-e^{-x}}{2}\right) - \frac{\cos x}{5}\left(\frac{e^{x}+e^{-x}}{2}\right)$$

$$= \frac{2}{5}\sin x \sinh x - \frac{1}{5}\cos x \cosh x$$

hence the required sol is

$$y = C.F + P.I$$

$$y = c_1 e^{x} + c_2 e^{-x} + \frac{2}{5}\sin x \sinh x - \frac{1}{5}\cos x \cosh x$$

Ex: $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = xe^x \cos x$

Sol: we have $(D^2 - 2D + 1)y = xe^x \cos x$

A.E is $m^2 - 2m + 1 = 0 \Rightarrow (m-1)^2 = 0 \Rightarrow m = 1, 1$

$C.F = (C_1 + C_2 x)e^x$

$$P.I = \frac{1}{D^2 - 2D + 1} xe^x \cos x = \frac{1}{(D-1)^2} xe^x \cos x$$

$$= e^x \frac{1}{(D-1+1)^2} x\cos x = e^x \cdot \frac{1}{D^2} x\cos x$$

$$= e^x \cdot \frac{1}{D}\left[\int x\cos x \, dx\right] = e^x \cdot \frac{1}{D}\left[x\sin x - (1)(-\cos x)\right]$$

$$= e^x \cdot \frac{1}{D}(x\sin x + \cos x)$$

$$= e^x \cdot \int (x\sin x + \cos x)\, dx$$

$$= e^x \left[x(-\cos x) - (1)(-\sin x) + \sin x \right]$$

$$= e^x \left[-x\cos x + \sin x + \sin x \right]$$

$$= e^x \left[-x\cos x + 2\sin x \right]$$

hence the complete sol is

$$y = C.F + P.I$$

$$y = (C_1 + C_2 x)e^x + e^x \left[2\sin x - x\cos x \right].$$

Ex: $(D^2-4D+4)y = 8x^2 e^{2x} \sin 2x$

Sol: A.E is $m^2-4m+4=0 \Rightarrow (m-2)^2=0$

$\Rightarrow m = 2, 2$

$C.F = (C_1+C_2x)e^{2x}$

$$P.I = \frac{1}{D^2-4D+4}\, 8x^2 e^{2x} \sin 2x$$

$$= 8 \cdot \frac{1}{(D-2)^2}\, x^2 e^{2x} \sin 2x$$

$$= 8 e^{2x} \frac{1}{(D-2+2)^2}\, x^2 \sin 2x = 8 \cdot e^{2x} \frac{1}{D^2}\, x^2 \sin 2x$$

$$= 8 e^{2x} \frac{1}{D} \left[\frac{x^2}{2}(-\cos 2x) - 2x\left(-\frac{\sin 2x}{4}\right) + \frac{2\cos 2x}{8} \right]$$

$$= 8e^{2x} \cdot \frac{1}{D}\left[-\frac{x^2}{2}\cos 2x + \frac{x\sin 2x}{2} + \frac{\cos 2x}{4}\right]$$

$$= 8e^{2x}\left[-\frac{x^2}{2}\left(\frac{\sin 2x}{2}\right) - \left(-\frac{2x}{2}\right)\left(-\frac{\cos 2x}{4}\right) + (-1)\left(-\frac{\sin 2x}{8}\right)\right.$$

$$\left. + \frac{x}{2}\left(-\frac{\cos 2x}{2}\right) - \left(\frac{1}{2}\right)\left(-\frac{\sin 2x}{4}\right) + \frac{\sin 2x}{8}\right]$$

$$= e^{2x}\left[-2x^2 \sin 2x - 2x\cos 2x + \sin 2x - 2x\cos 2x + \sin 2x + \sin 2x\right]$$

$$= e^{2x}\left[-2x^2 \sin 2x - 4x\cos 2x + 3\sin 2x\right]$$

$$= -e^{2x}\left[4x\cos 2x + (2x^2-3)\sin 2x\right]$$

hence the complete sol is

$$y = C.F + P.I$$

$$y = (C_1 + C_2 x)e^{2x} - e^{2x}\left[4x\cos 2x + (2x^2-3)\sin 2x\right].$$

Practice Problems

(1). $\frac{d^2y}{dx^2} - 4\frac{dy}{dx} + 4y = e^x \cos 2x$

Ans: $y = (c_1 + c_2 x)e^{2x} - \frac{1}{25} e^x (4\sin 2x + 3\cos 2x)$

(2) $\frac{d^2y}{dx^2} + y = xe^{2x}$

Ans: $y = c_1 \cos x + c_2 \sin x + \frac{e^{2x}}{25}(5x-4)$

(3) $$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 5y = e^{2x}\sin x$$

Ans: $$y = e^{x}(C_1 \sin 2x + C_2 \cos 2x) - \frac{1}{10}e^{2x}(\cos x - 2\sin x)$$

(4) $$\frac{d^3y}{dx^3} - 3\frac{d^2y}{dx^2} + 3\frac{dy}{dx} - y = xe^{x} + e^{x}$$

Ans! $$y = e^{x}\left\{(C_1 + C_2 x + C_3 x^2) + \frac{1}{16}x^3 + \frac{1}{24}x^4\right\}.$$

(5) $(D^3-7D-6)y = x^2e^{2x}$

Ans: $y = c_1e^{-x} + c_2e^{-2x} + c_3e^{3x} - \frac{e^{2x}}{12}\left\{x^2 + \frac{5}{6}x + \frac{97}{72}\right\}$

(6) $(D^2-4D+13)y = e^{2x}\cos 3x$

Ans: $y = e^{2x}(c_1\cos 3x + c_2\sin 3x) + \frac{1}{16}xe^{2x}\sin 3x$

(7) $\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + 4y = e^x\cos x$

Ans: $y = e^x(c_1\cos\sqrt{3}x + c_2\sin\sqrt{3}x) + \frac{1}{2}e^x\cos x$

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Thank you!