JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

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Year & Sem – I Year & II Sem

Subject –Engineering Mathematics-II

Unit – I

Presented by – (Dr.Vishal Saxena, Associate Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

VECTORS

Vectors and their Dependences

Vectors: An ordered set of n real numbers is called an n-dimensional vector.

These n real numbers are called components of the vector

$$x = [x_1, x_2, ---, x_n]$$

Linear Combination of Vectors

Let there be r vectors $X_1, X_2, ----, X_r$ and vector X is expressed as

$$X = k_1X_1 + k_2X_2 + \cdots + k_rX_r$$

where $k_1, k_2, \ldots, k_r$ are scalars (positive, negative or zero)

Linearly Dependent

The vectors $X_1, X_2, \ldots, X_n$ are said to be linearly dependent if there exist scalars $k_1, k_2, \ldots k_n$ such that

$$k_1X_1 + k_2X_2 + \ldots + k_nX_n = 0$$

or any one of these vectors can be expressed as linear combination of others, then such vectors are linearly dependent.

$$X_m = -\frac{k_1}{k_m}X_1 - \frac{k_2}{k_m}X_2 - \ldots - \frac{k_n}{k_m}X_n$$

Linearly Independent

The vectors $X_1, X_2, \ldots X_n$ are said to be linearly independent if $\cancel{k_1}$ $k_1X_1 + k_2X_2 + \cdots + k_nX_n = 0$

is possible only for $k_1 = k_2 = \cdots = k_n = 0$

Linear dependence and independence can be checked by rank of matrix

(i) If the rank of matrix be equal to numbers of vectors then these are linearly independent.

(ii) If the rank of matrix be less than number of vectors then these are linearly dependent.

Q.1 Check for following vectors for linear dependent. If so, express one of these as linear combination of others

a) $x_1 = (1, 3, 4, 2)$; $x_2 = (3, -5, 2, 2)$; $x_3 = (2, -1, 3, 2)$

b) $x_1 = (1, 1, 1, 3)$; $x_2 = (1, 2, 3, 4)$; $x_3 = (2, 3, 4, 9)$

Sol^n

a) $$A = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix} = \begin{bmatrix} 1 & 3 & 4 & 2 \\ 3 & -5 & 2 & 2 \\ 2 & -1 & 3 & 2 \end{bmatrix}$$

$R_2 \to R_2 - 3R_1$, $R_3 \to R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & -14 & -10 & -4 \\ 0 & -7 & -5 & -2 \end{bmatrix}$$

$R_3 \to R_3 - \frac{1}{2} R_2$

$$\begin{bmatrix} 1 & 3 & 4 & 2 \\ 0 & -14 & -10 & -4 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

Hence rank of matrix $\rho(A) = 2$

So these three vectors are linearly dependent.

$$x_1 = \lambda x_2 + \mu x_3$$

$$(1, 3, 4, 2) = \lambda(3, -5, 2, 2) + \mu(2, -1, 3, 2)$$

$$\Rightarrow \left.\begin{aligned} 3\lambda + 2\mu &= 1 \\ -5\lambda - \mu &= 3 \\ 2\lambda + 3\mu &= 4 \\ 2\lambda + 2\mu &= 2 \end{aligned}\right\} \Rightarrow \mu = 2, \lambda = -1$$

$$x_1 = -x_2 + 2x_3$$

b)

$$A = \begin{bmatrix} 1 & 1 & 1 & 3 \\ 1 & 2 & 3 & 4 \\ 2 & 3 & 4 & 9 \end{bmatrix}$$

$R_2 \to R_2 - R_1$, $R_3 \to R_3 - 2R_1$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 1 & 2 & 3 \end{bmatrix}$$

$R_3 \to R_3 - R_2$

$$A \sim \begin{bmatrix} 1 & 1 & 1 & 3 \\ 0 & 1 & 2 & 1 \\ 0 & 0 & 0 & 2 \end{bmatrix}$$

$\rho(A) = 3$

Hence the three vectors are linearly independent.

The Rank - Nullity Theorem

Nullspace

Let $A = (a_{ij})_{m \times n}$, then the nullspace of matrix A is the set of all n-dimensional column vectors x such that

$$Ax = 0$$

nullity of A = no. of free variables in the system

Q. 2 Find the nullspace of the matrix

$$A = \begin{bmatrix} 0 & 1 & 2 & -1 & 3 \\ 1 & -1 & -3 & 1 & 1 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & 1 \end{bmatrix}$$

Sol^n First we find echelon form

$$A = \begin{bmatrix} 0 & 1 & 2 & -1 & 3 \\ 1 & -1 & -3 & 1 & 1 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & 1 \end{bmatrix}$$

$R_1 \leftrightarrow R_2$

$$\begin{bmatrix} 1 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 4 & 0 & 0 & 1 & -2 \\ 2 & 3 & 8 & -2 & 1 \end{bmatrix}$$

$R_3 \to R_3 - 4R_1$, $R_4 \to R_4 - 2R_1$

$$\begin{bmatrix} 1 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 4 & 12 & -3 & -6 \\ 0 & 5 & 14 & -4 & 3 \end{bmatrix}$$

$R_3 \to R_3 - 4R_2,\ R_4 \to R_4 - 5R_2$

$$\begin{bmatrix} 1 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 4 & 1 & -18 \\ 0 & 0 & 4 & 1 & -18 \end{bmatrix}$$

$R_4 \to R_4 - R_3$

$$\begin{bmatrix} 1 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 4 & 1 & -18 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

The solution set of $Ax = 0$ is $A'x = 0$

$$\begin{bmatrix} 1 & -1 & -3 & 1 & 1 \\ 0 & 1 & 2 & -1 & 3 \\ 0 & 0 & 4 & 1 & -18 \\ 0 & 0 & 0 & 0 & 0 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \\ x_4 \\ x_5 \end{bmatrix} = 0$$

no. of nonzero rows $= 3$

no. of free variables $= 2$

Let x_4 & x_5 be free variables then we have

$4x_3 + x_4 - 18x_5 = 0 \Rightarrow x_3 = -\frac{1}{4}x_4 + \frac{9}{2}x_5$ (from 3 row)

$x_2 + 2x_3 - x_4 + 3x_5 = 0$

$$x_2 + 2\left(-\frac{1}{4}x_4 + \frac{9}{2}x_5\right) - x_4 + 3x_5 = 0$$

(from 2 row)

$$2x_2 - x_4 + 18x_5 - 2x_4 + 6x_5 = 0$$

$$2x_2 - 3x_4 + 24x_5 = 0 \Rightarrow x_2 = \frac{3}{2}x_4 - 12x_5$$

$$x_1 - x_2 - 3x_3 + x_4 + x_5 = 0$$

(from 1 row)

$$x_1 - \left(\frac{3}{2}x_4 - 12x_5\right) - 3\left(-\frac{1}{4}x_4 + \frac{9}{2}x_5\right) + x_4 + x_5 = 0$$

$$2x_1 + 8x_5 = 0 \Rightarrow x_1 = -4x_5$$

If we want to remove fractions, Let

$t_1 = \frac{x_4}{4}$, $t_2 = \frac{x_5}{2}$

then

$x = (-8t_2, 6t_1 - 24t_2, -t_1 + 9t_2, 4t_1, 2t_2)^T$

$= t_1(0, 6, -1, 4, 0)^T + t_2(-8, -24, 9, 0, 2)^T$

Here $\rho(A) = 3$, $N(A) = 2$

Sum of nullity & rank $= 2 + 3 = 5$

Q. 2 Find the basis of nullspace of the matrix

$$A = \begin{bmatrix} 1 & -2 & 0 & 4 \\ 3 & 1 & 1 & 0 \\ -1 & -5 & -1 & 8 \end{bmatrix}$$

Sol^n Echelon form of the matrix

$$\begin{bmatrix} 1 & -2 & 0 & 4 \\ 0 & 7 & 1 & -12 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

On expressing in terms of free variables ($\therefore$ i.e. x_3 & x_4)

$$x = \frac{1}{7}\begin{bmatrix} -2 \\ -1 \\ 7 \\ 0 \end{bmatrix} x_3 + \frac{1}{7} x_4 \begin{bmatrix} -4 \\ 12 \\ 0 \\ 7 \end{bmatrix}$$

Let $x = t_1(-2,-1,7,0)^T + t_2(-4,12,0,7)^T$

so basis of A is

$$N(A) = \begin{bmatrix} -2 & -4 \\ -1 & 12 \\ 7 & 0 \\ 0 & 7 \end{bmatrix}$$

Q: Are the vectors are linearly dependent. If so express one of these as linear combination of others.

a. $x_1 = (1, 2, 1)$, $x_2 = (2, 1, 4)$, $x_3 = (4, 5, 6)$, $x_4 = (1, 8, -3)$

b. $x_1 = (1, 2, 4)$, $x_2 = (2, -1, 3)$, $x_3 = (0, 1, 2)$, $x_4 = (-3, 7, 2)$

c. $x_1 = (1, 2, 3)$, $x_2 = (3, -2, -1)$, $x_3 = (1, -6, -5)$.

Ans: a. Linearly dependent,

Linear Combination: $2x_1 + x_2 - x_3 + 0x_4 = 0$

b. Linearly dependent,

Linear Combination: $-9x_1 + 12x_2 - 5x_3 + 5x_4 = 0$

c. Vectors are linearly independent.

Refrences

1.Advanced Engineering Mathematics by Prof.ERWIN KREYSZIG (Ch.10,page no.557-580)
2. Advanced Engineering Mathematics by Prof.H.K Dass (Ch.14,page no.851-875)
3.Advanced Engineering Mathematics by B.V RAMANA (Ch.20,pageno.20.1.20.5)
4.NPTEL Lectures available on

http://www.infocobuild.com/education/audio-video-courses/mathematics/TransformTechniquesForEngineers-IIT-Madras/lecture-47.html

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Thank you!