JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

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Year & Sem – I Year & II Sem
Subject –Engineering Mathematics-II
Unit – I
Presented by – (Dr.Vishal Saxena, Associate Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

Solution of system of simultaneous linear equation(examples)

Q. 4 Solve the following system of equations

$5x+3y+7z=4,\ 3x+26y+2z=9,\ 7x+2y+10z=5$

Solve The augmented matrix is given as

$$K \sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 3 & 26 & 2 & 9 \\ 7 & 2 & 10 & 5 \end{array}\right]$$

$R_2 \to 5R_2 - 3R_1$, $R_3 \to 5R_3 - 7R_1$

$$K \sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 121 & -11 & 33 \\ 0 & -11 & 1 & -3 \end{array}\right]$$

$R_2 \to \frac{R_2}{11}$

$$\sim \begin{bmatrix} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & -11 & 1 & -3 \end{bmatrix}$$

$R_3 \to R_3 + R_2$

$$\sim \left[\begin{array}{ccc|c} 5 & 3 & 7 & 4 \\ 0 & 11 & -1 & 3 \\ 0 & 0 & 0 & 0 \end{array}\right]$$

$$\rho(K) = 2 < 3$$

$$5x + 3y + 7z = 4$$

$$11y - z = 3$$

$$y = k, \quad z = 11k - 3, \quad x = 5 - 16k$$

Q 5 For what values of λ and μ, does the system of equations

$2x+3y+5z=9$, $7x+3y-2z=8$ and $2x+3y+\lambda z=\mu$

has (i) no solution (ii) a unique solution

(iii) infinite number of solutions

Sol^n The given system is

$$2x+3y+5z=9$$
$$7x+3y-2z=8$$
$$2x+3y+\lambda z=\mu$$

$$\begin{bmatrix} 2 & 3 & 5 \\ 7 & 3 & -2 \\ 2 & 3 & 1 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 9 \\ 8 \\ \mu \end{bmatrix}$$

$$K \sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 7 & 3 & -2 & 8 \\ 2 & 3 & \lambda & \mu \end{array}\right]$$

$R_3 \to R_3 - R_1$, $R_2 \to R_2 - \frac{7}{2} R_1$

$$K \sim \left[\begin{array}{ccc|c} 2 & 3 & 5 & 9 \\ 0 & -15/2 & -39/2 & 9/2 \\ 0 & 0 & \lambda - 5 & \mu - 9 \end{array}\right]$$

Case I If $\lambda - 5 \neq 0$, $\mu \neq 0$ i.e. $\lambda \neq 5$, $\mu \neq 9$ then

$\rho(A) = \rho(K) = 3$ = no. of variables

System has a unique solution

Case II If $\lambda - 5 = 0$ and $\mu - 9 \neq 0$ i.e. $\lambda = 5$ but $\mu \neq 9$

then $\rho(A) = 2$ and $\rho(K) = 3$

$\rho(A) \neq \rho(K) \Rightarrow$ system is inconsistent

$\Rightarrow$ no solution

Case III If $\lambda - 5 = 0 = \mu - 9$

$\Rightarrow \lambda = 5$, $\mu = 9$ then

$\rho(A) = \rho(K) = 2$

System is consistent and has infinite no. of solutions

Q. 6 For what values of η the equations $x+y+z=1$, $2x+y+4z=\eta$ and $4x+y+10z=\eta^2$ have no solution and solve them completely in each case.

Sol^n $x+y+z=1$

$2x+y+4z=\eta$

$4x+y+10z=\eta^2$

$$\begin{bmatrix} 1 & 1 & 1 \\ 2 & 1 & 4 \\ 4 & 1 & 10 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 1 \\ 2 \\ 2^2 \end{bmatrix}$$

$$K \sim \left[\begin{array}{ccc|c} 1 & 1 & 1 & 1 \\ 2 & 1 & 4 & 2 \\ 4 & 1 & 10 & 2^2 \end{array}\right]$$

$R_2 \to R_2 - 2R_1$, $R_3 \to R_3 - 4R_1$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & n-2 \\ 0 & -3 & 6 & n^2-4 \end{bmatrix}$$

$R_3 \to R_3 - 3R_2$

$$\sim \begin{bmatrix} 1 & 1 & 1 & 1 \\ 0 & -1 & 2 & n-2 \\ 0 & 0 & 0 & n^2-3n+2 \end{bmatrix}$$

Case I If $\eta^2 - 3\eta + 2 = 0$

$(\eta - 1)(\eta - 2) = 0$

$\eta = 1, 2$

$\Rightarrow \rho(A) = 2 = \rho(A)$

Infinite many solution

If $\eta = 1$ $x + y + z = 1$, $-y + 2z = -1$

$\Rightarrow z = -3a$, $y = 2a + 1$ $z = a$

If $\eta = 2$ $\quad x+y+z=1,\ -y+2z=0$

$z=b,\ y=2b,\ x=-3b+1$

Case II If $\eta^2-3\eta+2\neq 0$ i.e. $\eta\neq 2,\ \eta\neq 1$

$\rho(k)=3$

$\rho(A)=2,\ \rho(k)=3$

$\rho(A)\neq\rho(k)$

$\Rightarrow$ System is inconsistent $\Rightarrow$ no solution

Q. Show that the equations $-2x+y+z=a$, $x-2y+z=b$, $x+y-2z=c$ have no solution unless $a+b+c=0$, in which case they have infinitely many solutions.

Sol: The given eqn can be written as

$$AX=B$$

$$\begin{bmatrix} -2 & 1 & 1 \\ 1 & -2 & 1 \\ 1 & 1 & -2 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} a \\ b \\ c \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 1 & -2 & 1 & b \\ 1 & 1 & -2 & c \end{array}\right]$$

$$R_2 \to R_2 + \frac{R_1}{2}, \qquad R_3 \to R_3 + \frac{R_1}{2}$$

$$= \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 0 & -3/2 & 3/2 & b + \frac{a}{2} \\ 0 & 3/2 & -3/2 & c + a/2 \end{array}\right]$$

$$R_3 \to R_3 + R_2$$

$$= \left[\begin{array}{ccc|c} -2 & 1 & 1 & a \\ 0 & -3/2 & 3/2 & b+\frac{a}{2} \\ 0 & 0 & 0 & a+b+c \end{array}\right]$$

Now if $a+b+c=0$ then Rank of $[A|B] = 2 = \rho[A]$ and in that case the System of eqns have infinite many Sol.

Ex: Test for consistency & solve them

$2x-3y+7z=5,\quad 3x+y-3z=13,\quad 2x+19y-47z=32$

Sol: The given eqn can be written as

$$AX=B$$

$$\begin{bmatrix} 2 & -3 & 7 \\ 3 & 1 & -3 \\ 2 & 19 & -47 \end{bmatrix} \begin{bmatrix} x \\ y \\ z \end{bmatrix} = \begin{bmatrix} 5 \\ 13 \\ 32 \end{bmatrix}$$

$$[A|B] = \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 3 & 1 & -3 & 13 \\ 2 & 19 & -47 & 32 \end{array}\right]$$

$$R_2 \to R_2 - \frac{3}{2}R_1, \quad R_3 \to R_3 - R_1$$

$$= \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -\frac{27}{2} & 11/2 \\ 0 & 22 & -54 & 27 \end{array}\right]$$

$$R_3 \to R_3 - 4R_2$$

$$= \left[\begin{array}{ccc|c} 2 & -3 & 7 & 5 \\ 0 & 11/2 & -27/2 & 11/2 \\ 0 & 0 & 0 & 5 \end{array}\right]$$

here $\rho(A) = 2$ & $\rho(A|B) = 3$

& $\rho(A) \neq \rho(A|B)$. (No sol. exists).

8 Examine whether the following eq^{n}s are consistent and solve them if they are consistent

a)

$$x + 2y - z = 3$$

$$3x - y + 2z = 1$$

$$2x - 2y + 3z = 2$$

$$x - y + z = -1$$

b)

$$4x - 2y + 6z = 8$$
$$x + y - 3z = -1$$
$$15x - 3y + 9z = 21$$

c)

$$x + y + z = 6$$
$$x - y + 2z = 5$$
$$3x + y + z = 8$$
$$2x - 2y + 3z = 7$$

d: $x+y+z=6,$

$x+2y+3z=14,$

$x+4y+7z=30$

Ans:

a: $x = -1, \quad y = 4, \quad z = 4$

b: $x = 1, \quad y = 3k - 2, \quad z = k$

c: $x = 1, \quad y = 2, \quad z = 3$

d: $x = k - 2, \quad y = 8 - 2k, \quad z = k$

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Thank you!