JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

- Year & Sem – I^{st} Year , I^{st} Sem
- Subject – Engineering Physics
- Unit – Wave Optics-II (Diffraction of Light)
- Department- Applied Science (Physics)

Syllabus & Course outcomes

- **Syllabus:-**

  Newton's Rings, Michelson's Interferometer, Fraunhoffer Diffraction from a Single Slit. Diffraction grating: Construction, theory and spectrum, Resolving power and Rayleigh criterion for limit of resolution, Resolving power of diffraction grating, X-Ray diffraction and Bragg's Law.

- **Course outcomes :-**

CO1:- Students will be able to explain the basic concepts, theoretical principles and practical applications of interference, diffraction phenomena and their related optical devices in visible range and X-ray diffraction by crystals (i.e., Bragg's law).

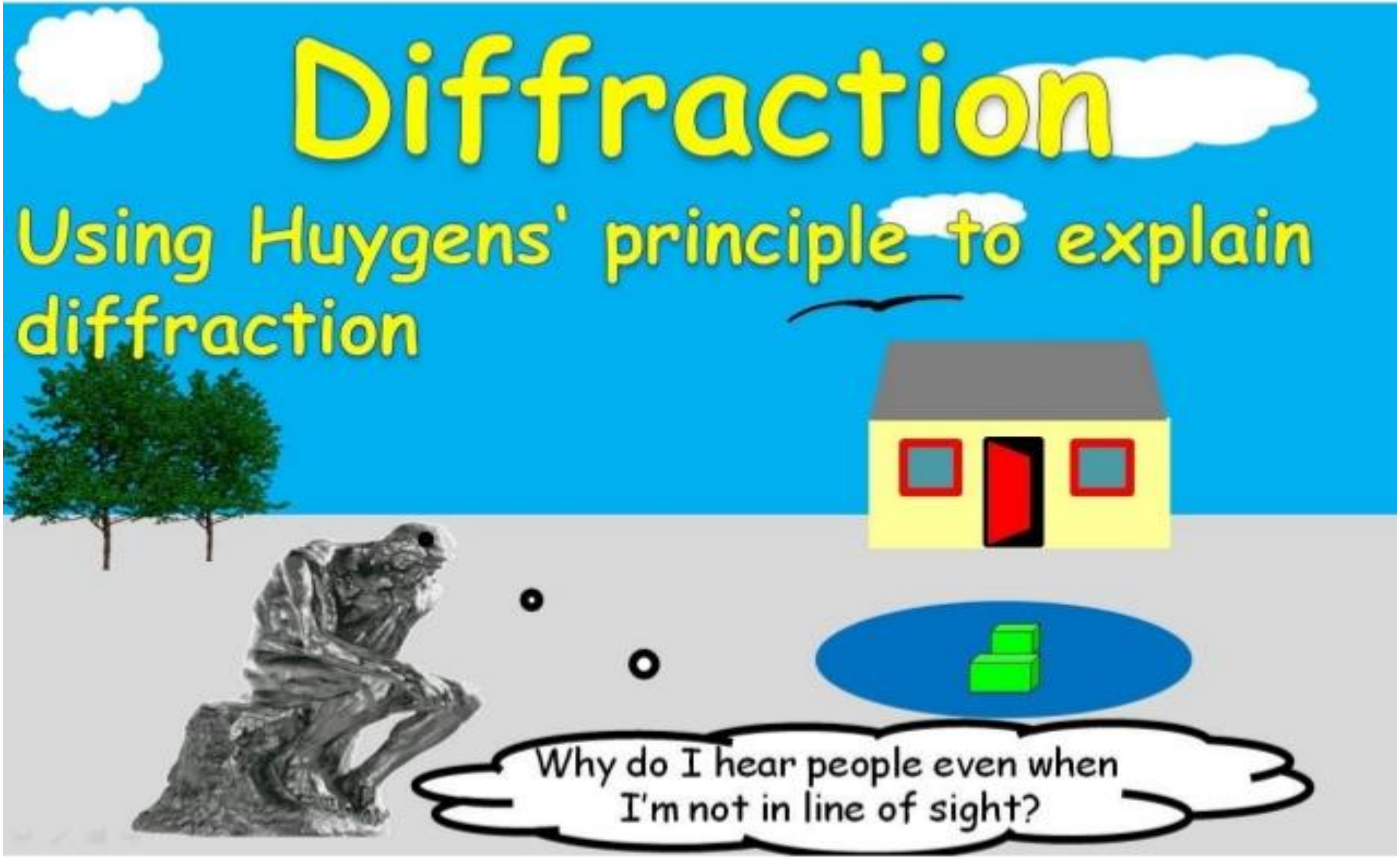
CONTENTS

Part :- 2

Lecture Plan

S. No	Topics	Lectures required	Lect. No.
1	Introduction of Diffraction of light	1	1
2	Fraunhoffer's diffraction, Single Slit:- formulation of resultant Intensity.	1	2
3	Diffraction Grating:- theory, construction and spectrum.	1	3
4	Resolving Power & Rayleigh criterion for limit of resolution.	1	4
5	Resolving power of diffraction grating	1	5
6	X-ray diffraction & Bragg's law.	1	6
7	Numericals	1	7

Diffraction

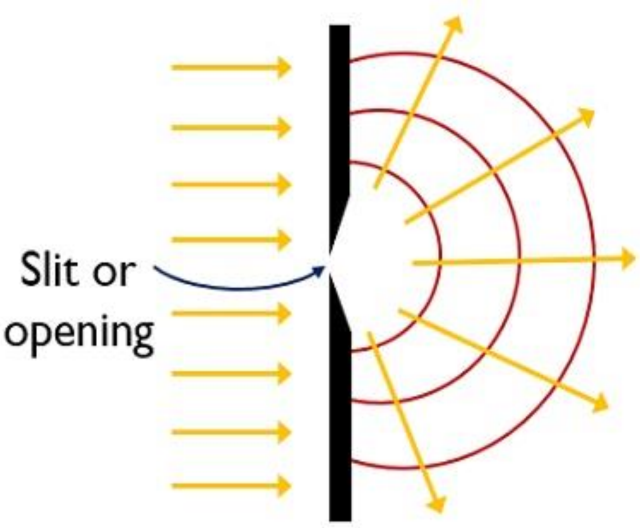


Definition of diffraction of light

- Diffraction of light :The phenomenon of bending of light waves around the corners of the obstacle and entering into the region of geometrical shadow of the obstacle is called diffraction**.** It was first observed by Scientist Grimaldi.
- Diffraction of light depends on following two factors:
- (1) Size of the slit or aperture (a)
- (2) Wavelength of the light wave incident (**λ**)

For maximum diffraction : **a= λ**

Note: first of all diffraction was observed in sound waves, since the wavelength of sound waves(**λ** =1m) and also size of the obstacle (a=1m)are nearly equal (**a= λ).**



Diffraction of light wave

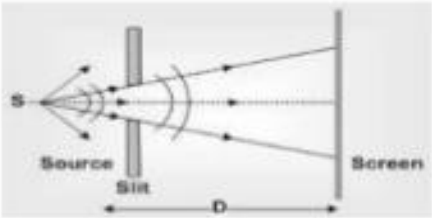
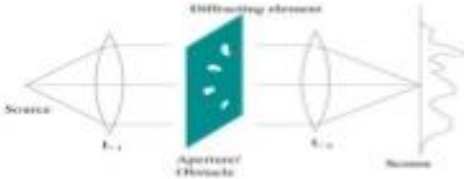
Circuit Globe

Difference between Diffraction and Interference

S. No.	Properties	Diffraction	Interference
1	Origin	Superposition between infinite coherent waves.	Superposition between two coherent waves.
2	Maxima	Never of equal intensity.	Always of equal intensity
3	Width of fringes	Never of equal width.	May or may not be of equal width.
4	Minima	Minima intensity may increase with order.	All minima have the same Intensity.
5	Number of fringes observed.	Always small	Generally large.

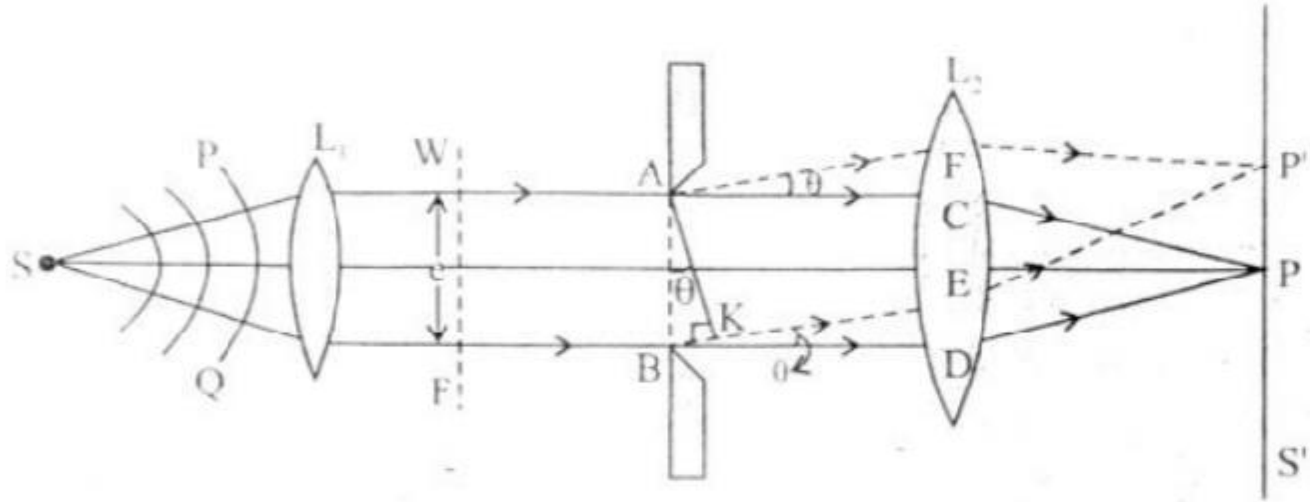
Classification of Diffraction

Diffraction phenomena of light can be divided into two different classes

Fresnel's Diffraction	Fraunhofer diffraction
	
Cylindrical wave fronts	Planar wave fronts
Source of screen at finite distance from the obstacle	Observation distance is infinite. In practice, often at focal point of lens.
Move in a way that directly corresponds with any shift in the object.	Fixed in position
Fresnel diffraction patterns on flat surfaces	Fraunhofer diffraction patterns on spherical surfaces.
Change as we propagate them further 'downstream' of the source of scattering	Shape and intensity of a Fraunhofer diffraction pattern stay constant.

Fraunhoffer's Diffraction Due to Single Slit

- Experimental Diagram:-



S = monochromatic source (λ wavelength)

S' = Screen

L_1, L_2 = converging (convex) lens

WF = plane wavefront

AB = Slit with width 'e' ($e \approx \lambda$)

θ = diffracting angle

AFP'+ BEP' = diffracted waves

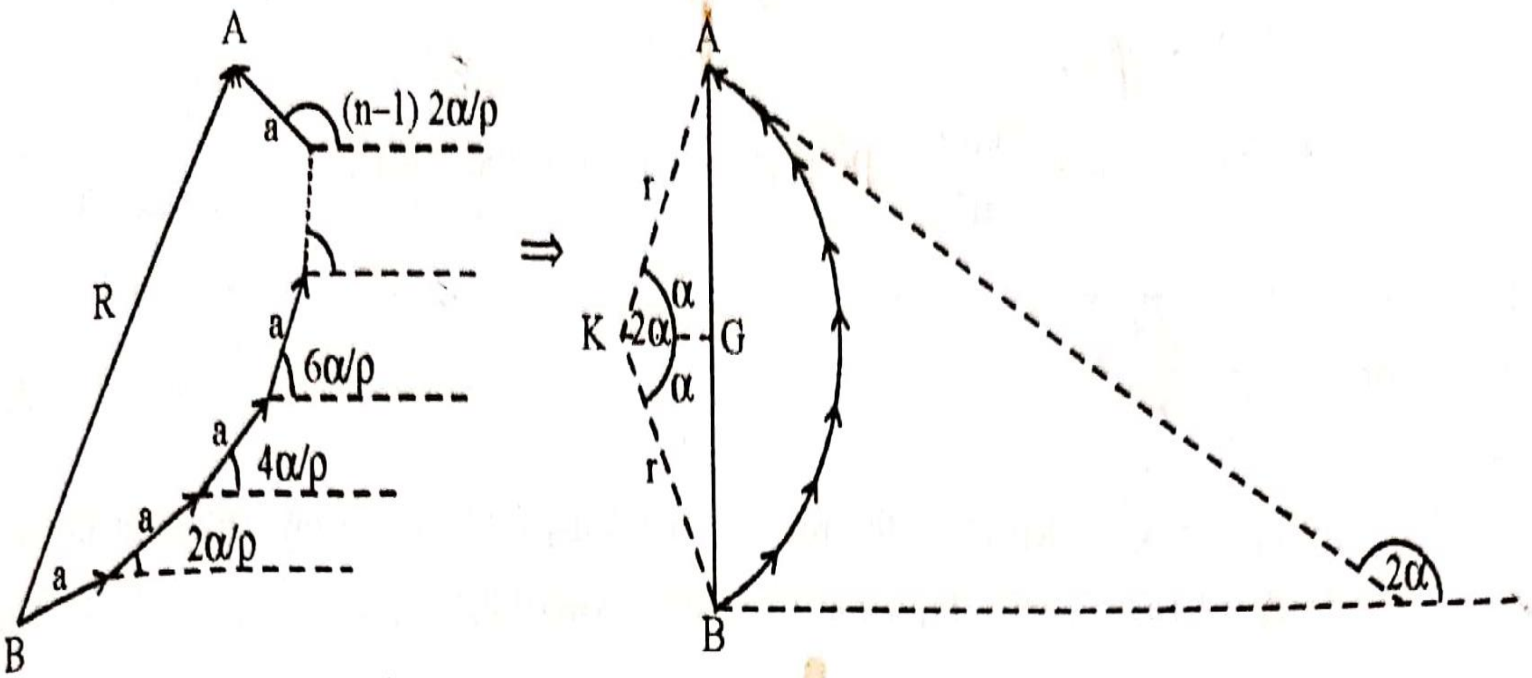
BK = path difference between the waves

P = Central (bright) maxima at Screen

P' = Point on screen (to calculate intensity)

PQ = spherical wavefront

A slit, rectangular aperture of large length compared to its breadth is placed perpendicular to the plane of paper. Let the slit AB be illuminated by a parallel beam of monochromatic light of wavelength λ from a source S which is placed at the principal focus of the lens L_1. Let the width of the narrow slit be 'e', light is diffracted by the slit and is focussed by another convex lens L_2 on the screen, S' (placed at the principal focus of the lens L_2) Instead of a sharp image of the slit, a central bright band and alternate dark and bright bands of decreasing intensity symmetrical on both sides are obtained. This pattern is called diffraction pattern of a single slit.



The amplitude of wavelets due to each part of the slit is very small and the phase angle increases by infinitesimal small value from one part to another part in steps of $\frac{2\alpha}{\rho}$. The value of ρ can be assumed as it tends tend to infinity and the polygon would now take the shape of an arc AB as shown in fig. The phase difference between the first and the last amplitude vector is 2α. It will be equal to the tangents drawn at the points A and B.

$\therefore$ Angle $2\alpha = \dfrac{\text{arc AB}}{\text{radius}} = \dfrac{\rho a}{r}$

$$r = \frac{\rho a}{2\alpha}$$

Now a perpendicular *KG* is drawn from the centre K of an arc on the line AB, which will bisect the line AB in two equal parts. (BG and AG)

$\therefore$ $\angle AKG = \angle BKG = \dfrac{2\alpha}{2} = \alpha$ and $\sin\alpha = \dfrac{AG}{AK}$ [in Δ AKG]

so, $AB = 2AG = 2r\sin\alpha$ [$\because$ $AK = r$]

Substituting the value of r from equation we get

$$AB = \frac{2\rho a}{2\alpha}\sin\alpha = \frac{\rho a \sin\alpha}{\alpha} = A \text{ (Resultant amplitude)}$$

where α is given by equation.

$$\alpha = \frac{\pi}{\lambda} e \sin\theta$$

The resultant Intensity at point P' will be:-

$$I \propto A^2$$

$$I = KA^2 = K\rho^2 a^2 \frac{\sin^2\alpha}{\alpha^2}$$

or $$\boxed{I = I_0 \frac{\sin^2\alpha}{\alpha^2}}$$

Where $I_0 = KA_0^2 = K\rho^2 a^2$ is the maximum Intensity at the point where all the wave reach in the same phase.

Intensity distribution of diffraction Pattern

It is evident from the equation that the intensity at the point P depends on the path difference α and consequently on the angle of diffraction θ. So $\sin^2 \alpha / \alpha^2$ gives maximum and minimum intensity at different point for different value of θ.

1. Central Maximum

For the central point 'P' on the screen,

$$\theta = 0 \text{ and hence } \alpha = 0 \qquad \left[\because \alpha = \frac{\pi}{\lambda} e \sin\theta\right]$$

Hence intensity at P will be :

$$I = I_0\left(\frac{\sin^2\alpha}{\alpha^2}\right) = I_0 = \text{maximum intensity} \quad \left[\because \lim_{\alpha\to 0} \frac{\sin^2\alpha}{\alpha^2} = 1\right] \qquad \text{.....(}$$

The maximum intensity at θ = 0 is called central maximum

2. Principal Minima

For minimum intensity I should be zero.
So, $I = 0 \rightarrow \sin\alpha = 0$
For that $\alpha = 0, \pm \text{Л}, \pm 2\text{Л}, \pm 3\text{Л} \ldots$
But $\alpha \neq 0$, because at $\alpha = 0$, central maxima forms.
So for Principal minima the value of $\alpha = \pm n\text{Л}$
where $n = 1,2,3,\ldots$ and $n \neq 0$

Substituting the value of α, we get

$$\frac{\pi}{\lambda} e \sin\theta = \pm n\pi$$

or $\mathbf{e \sin \theta = \pm n\lambda}$**……. This the condition for minima**

Thus, the angular positions of first, second and third minima are given by $\boldsymbol{\theta} = \pm \lambda/e, \pm 2\lambda/e, \pm 3\lambda/e$ and so on.

3. Secondary Maxima

To find other direction of maximum intensity, differentiate the equation with respect to α and equate it to zero

$$\text{So, } \frac{dI}{d\alpha} = 0$$

$$\frac{dI}{d\alpha} = \frac{d}{d\alpha}\left(I_0 \frac{\sin^2\alpha}{\alpha^2}\right) = 0$$

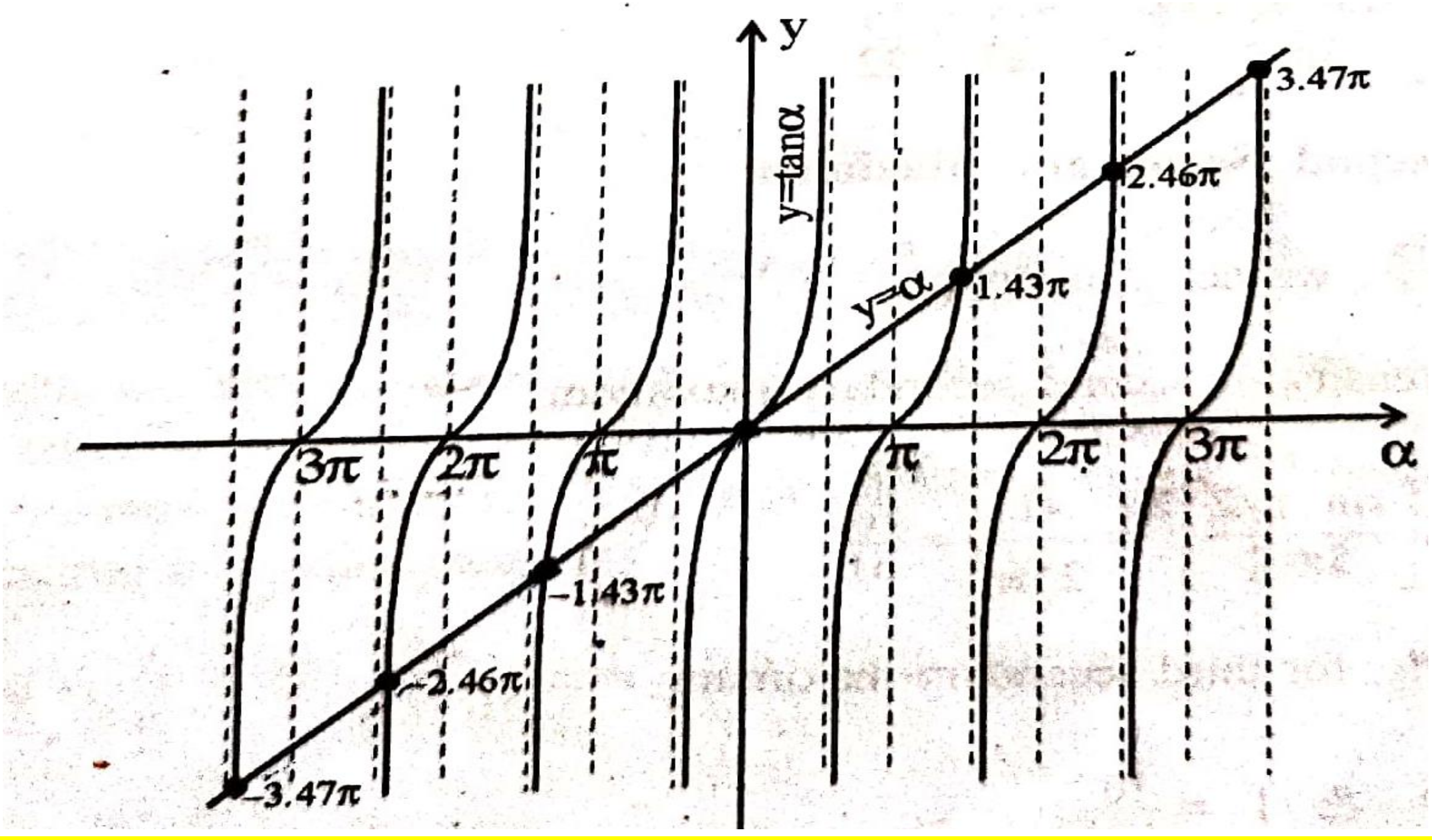
$$I_0\left[2\sin\alpha\cos\alpha\left(\frac{1}{\alpha^2}\right) + \sin^2\alpha\left(-\frac{2}{\alpha^3}\right)\right] = 0$$

$$\frac{I_0 2\sin\alpha}{\alpha^3}(\alpha\cos\alpha - \sin\alpha) = 0$$

Either Sin α = 0 or (α Cos α – Sin α) = 0, Since Sin α gives the position of minimum intensity, So for secondary maxima-

$$\alpha \cos\alpha - \sin\alpha) = 0$$

$$\alpha = \tan\alpha$$



The point of intersection of two curves gives the solution of equation. From the graph, we get the value of α given by α = 0, ± 1.43Л, ± 2.46Л, ± 3.47Л. But α = 0 shows the position of central maxima. Thus for other secondary maxima:-

α = ± 1.43Л, ± 2.46Л, ± 3.47Л…. etc.

From the above values, it is clear that the secondary maxima do not fall at midway between the two minima, but are slightly displaced towards the centre of the pattern. The value of α can be approximately given out for secondary maxima as-

α = ± 3Л/2, ± 5Л/2, ± 7Л/2 …….. etc.

$$\alpha = \pm\frac{3\pi}{2}, \pm\frac{5\pi}{2}, \pm\frac{7\pi}{2} \ldots \text{etc.}$$

$$\text{or} \quad \alpha = \frac{\pm(2n+1)\pi}{2} \quad \text{where } n = 1, 2, 3, \cdots$$

Or esinθ=(2n+1)λ/2 ………. This is the condition of maxima

Substituting the value of α in equation, the intensity of secondary maximas will be-

Ist Secondary Maxima :

Since we know that

$$I = I_0 \frac{\sin^2 \alpha}{\alpha^2}$$

Since $\alpha = \pm 3Л/2$, then

$I = 4I_O /9Л^2$

II^nd^ Secondary Maxima :

Since $\alpha = \pm 5Л/2$, then

$I = 4I_O /25Л^2$

III^rd^ Secondary Maxima :

Since $\alpha = \pm 7Л/2$, then

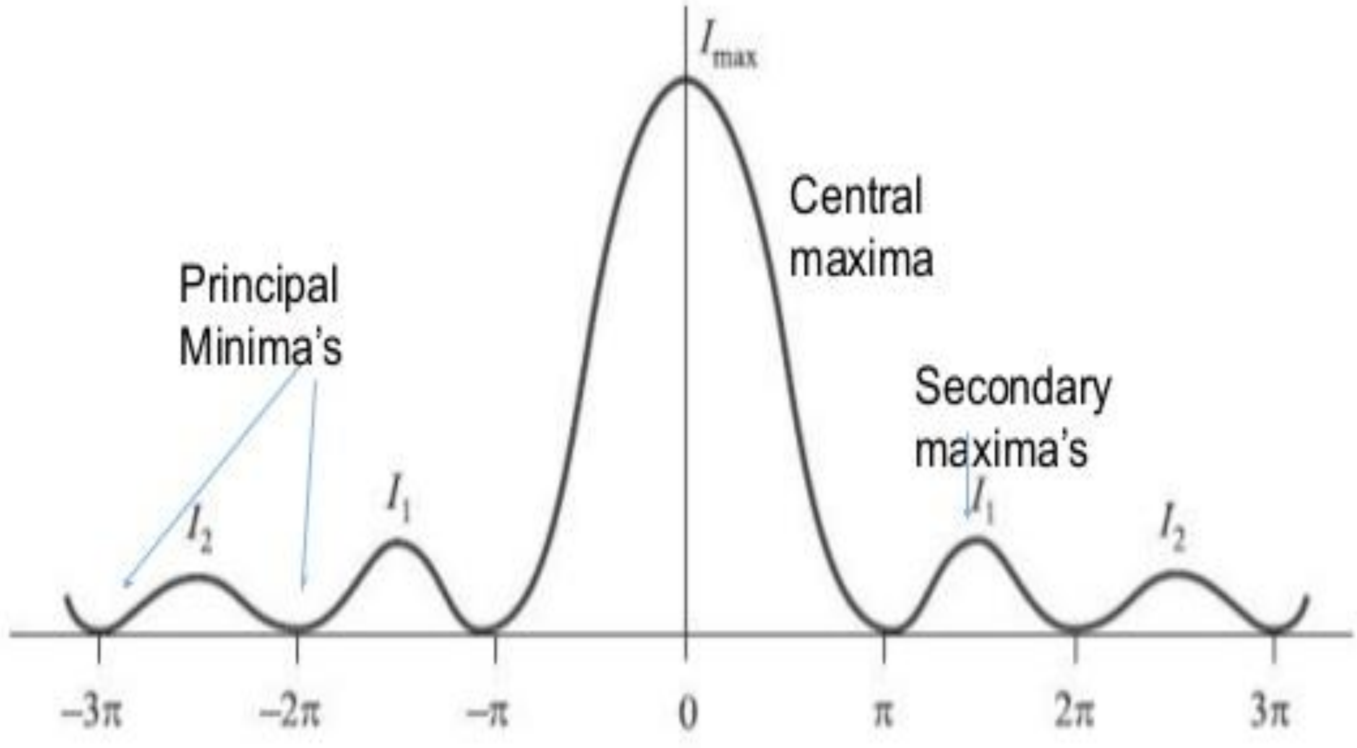
$I = 4I_O /49Л^2$

Ratio of relative intensities of successive maxima are in the ratio

$I_O : 4I_O /9Л^2 : 4I_O /25Л^2 : 4I_O /49Л^2$

Or **$1 : 4/9Л^2 : 4/25Л^2 : 4/49Л^2$**

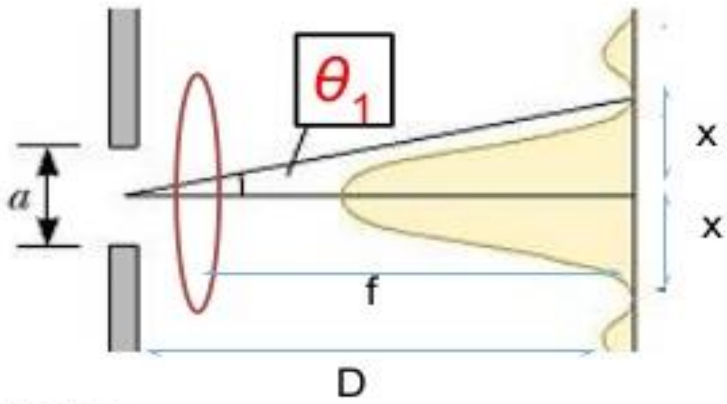
Intensity distribution by single slit diffraction



Width of central maxima

The width of he central maximum can be derived as the separation between the first minimum on either side of the central maximum.

If he first maximum is at distance x from the central maximum then



$$w = 2x$$ …………….. eq 1

We know that

$$\Delta = a\sin\theta = n\lambda$$ …..eq 2

for

$$n = 1, \theta = \theta_1$$

$$\sin\theta_1 = \pm\frac{\lambda}{a}$$ …..eq 3

From the diagram

$$\tan\theta_1 = \frac{x}{D} \approx \frac{x}{f}$$ …..eq 4

If θ is very small $\sin\theta_1 = \tan\theta_1 = \theta_1$

$$\frac{\lambda}{a} = \frac{x}{f}$$

$$x = \frac{\lambda f}{a}$$

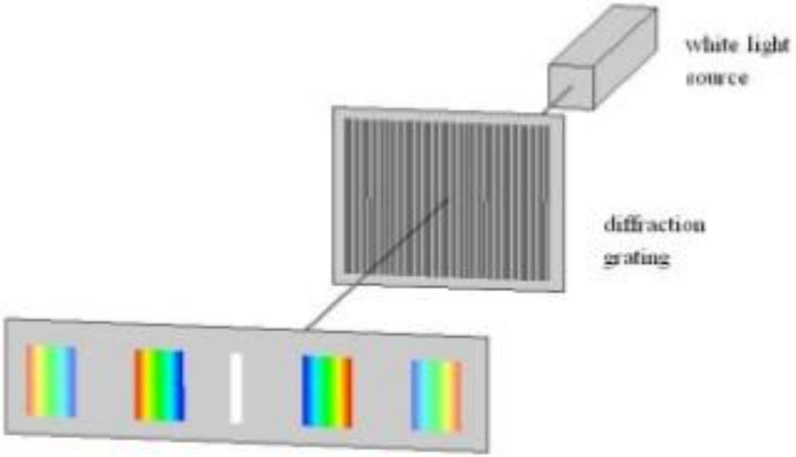
$$w = \frac{2\lambda f}{a}$$

Diffraction grating

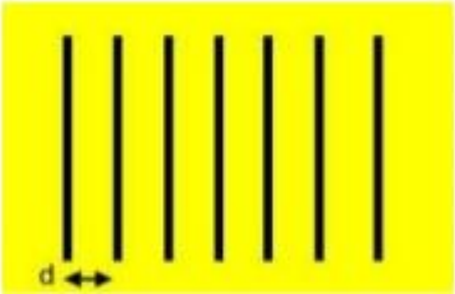
A diffraction grating is an arrangement equivalent to a large number of parallel slits of equal widths and separated from one another by equal opaque spaces.

Construction

Diffraction grating can be made by drawing a large number of equidistant and parallel lines on an optically plane glass plate with the help of a sharp diamond point. The rulings scatter the light and are effectively opaque, while the unruled parts transmit light and act as slits. The experimental arrangement of diffraction grating is shown



They are two type refection and transmission gratings



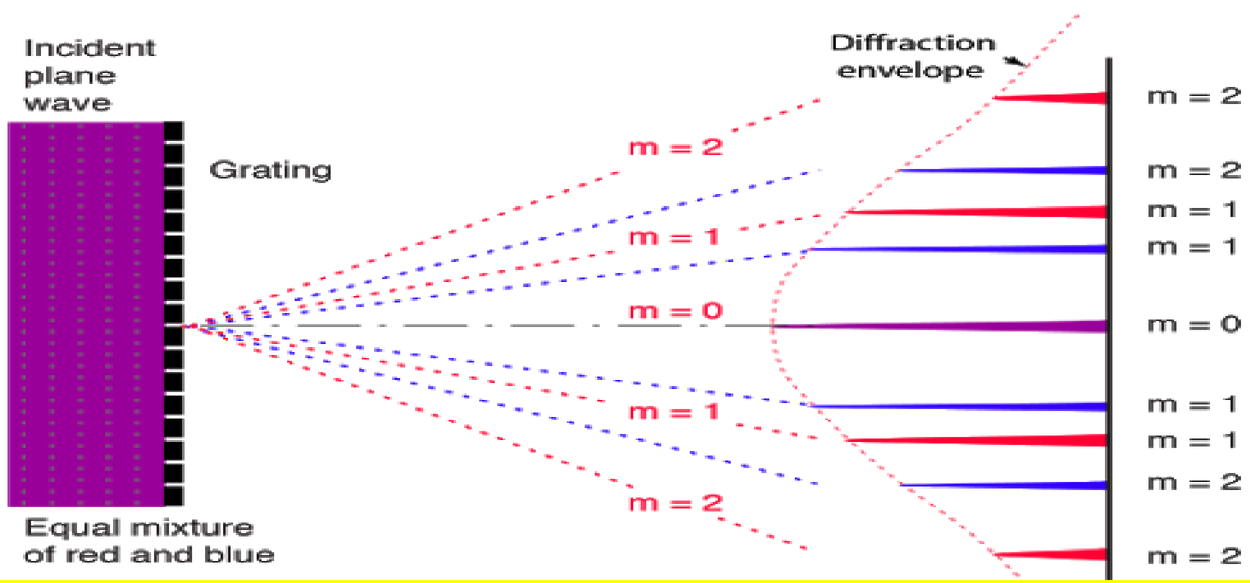
Diffraction due to N-parallels slits

- It consists of a large no. of shits arranged very close together intervals. This device is used to study the optical spectra of light source .
- Definition:- It is a plane glass plate on which a large number of equidistant, and fine lines are drawn (or ruled by means of fine diamond point worked with a ruling engine precession machine (put in constt . maintained room temp.}.

It is a plane glass plate on which a large number of equidistant ; parallel , and fine lines are ruled by means of a fine diamond point worked with a ruling engine precision machine The number of lines ruled on grating varies from 10000 to 30000 per inch and width of ruled space varies from 2 to 4 inches. This device is used To study the optical spectra of light sources.

Principle of Grating

Light diffracted from large number of slits add in phases to give rise maxima different colours at different positions



Grating is example of Fraunhoffer diffraction.

Grating surface is kept for normal incidence of light beam for source.

Suppose (a+b)-Grating element, a=width of each shit

b- width of opaque space.

There are N slits- suppose S_1, S_2-------------S_n, slits act as N- Coherent sources, Hence N waves when interfere, produce maxima and minima.

Calculation of path diff. and phase diff. between waves.

$$\sin\theta = \frac{BC}{(a+b)} \quad = BC = (a+b)sin\theta$$

$(a+b)sin\theta$ = path diff. bet^n waves S_1 & S_2

$\frac{2\lambda}{\lambda}[(a+b)sin\theta]$ = phase diff bet^n waves S_1 & S_2

$\frac{2\lambda}{\lambda}[(a+b)sin\theta]$ = **2β**

It shows that the phase difference between successive wave is constant and equal to 2β and phase increase in arithmetic progression.

Amplitude of N vibrations in a direction θ, each of amplitude

$A_o = A \; \sin\alpha \; / \; \alpha$ having a common phase difference

$2\beta = 2\Pi \, (a + b) \sin\theta \; / \; \lambda$

We shall do so by the graphical method. For it let us draw equal lengths $M P_1$, $P_1 P_2$, P_3 ...... $P_{N-1} P_N$ representing equal amplitudes A_0 As we pass on from one

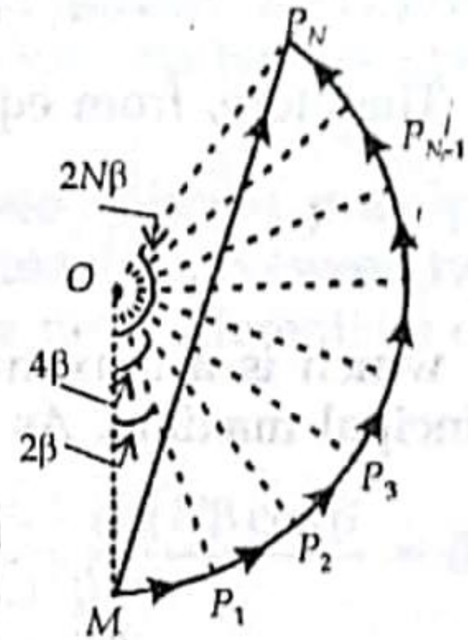
length to another, the inclination goes on changing by the same amount 2β representing equal phase difference between them. Thus polygon of N amplitudes is constructed. The closing side MP_N of the polygon represents the resultant amplitude R. If O is the centre of the polygon, then from geometry, we have

$$MP_1 = 2.OM \sin\beta$$

and $$MP_N = 2.OM \sin\beta$$

Eliminating OM, we get

$$MP_N = 2.\frac{MP_1}{2\sin\beta} \cdot \sin N\beta = A_0 \frac{\sin N\beta}{\sin\beta}$$



Hence the resultant intensity is given by

$$I = R_0^2 \frac{\sin^2\alpha}{\alpha^2} \frac{\sin^2 N\beta}{\sin^2\beta}$$

The first factor $R_0^2 \dfrac{\sin^2\alpha}{\alpha^2}$ expresses intensity distribution due to diffraction at a single slit while the second factor $\dfrac{\sin^2 N\beta}{\sin^2\beta}$ may be said to arise from interference between the N diffracted waves from the N slits.

Principal Maxima

If $\sin\beta = 0$

i.e., $\beta = \pm n\pi$

then $\sin N\beta = 0$

and $\dfrac{\sin NB}{\sin\beta} = \dfrac{0}{0}$ becomes indeterminate. But it may be evaluated by finding its limit as $\beta \rightarrow \pm n\pi$ by the usual method of differentiating the numerator and the denominator.

Thus

$$\lim_{\beta\to\pm n\pi} \frac{\sin N\beta}{\sin\beta} = \lim_{\beta\to\pm n\pi} \frac{\dfrac{d}{d\beta}\sin N\beta}{\dfrac{d}{d\beta}\sin\beta}$$

$$= \lim_{\beta\to\pm n\pi} \frac{N\cos N\beta}{\cos\beta} = \frac{N\cos N(\pm n\pi)}{\cos(\pm n\pi)} = \pm N$$

Therefore the intensity is given by

$$I = R_0^2 \frac{\sin^2 \alpha}{\alpha^2} . N^2$$

Which is maximum. These maxima is are most intense and hence they are called Principal Maxima.

$$\beta = \pm n\pi$$

or $$\frac{\pi}{\lambda}(a+b)\sin\theta = \pm n\pi$$

or $$(a+b)\sin\ \theta = \pm\ n\lambda$$

where $n = 0, 1, 2, 3$ etc. represents the order of the interference maximum. For $n = 0$, we get the *zero order maximum*. For $n = 1, 2, 3$ etc., we obtain the first, second, third etc., principal maxima. The $\pm$ sign indicates that there are two principal maxima for each order lying on either side of the zero order maximum. The distance $(a + b)$ is called the *grating element* and is generally denoted by e. This equation simply implies that maxima occur when waves from corresponding points on successive slits reach with a path difference of a whole number of λ, For first order this path difference is λ while for second order it is 2λ.

Minima : A series of minima occur when

$$\sin N\beta = 0 \qquad \text{but } \sin\beta \neq 0,$$

$$\frac{\sin N\beta}{\sin\beta} = 0$$

and from equation (3-12), we have $I = 0$

which is a minimum. Thus for minima, we have

$$\sin N\beta = 0$$

or

$$N\beta = \pm m\pi$$

or $$N.\frac{\pi}{\lambda}(a+b)\sin\theta = \pm m\pi$$

or $$N(a+b)\sin\theta = \pm m\lambda$$

Where m has all integers value except 0, 2N,….nN because these value of m makes Sin β = 0 which correspond to the position of principle maxima. Thus m=0 gives the principle maximum, m=1,2,3,…N-1 gives the minima and then m=N gives again a principle maxima. Hence there are N-1 equally spaced minima between two adjacent maxima.

Secondary Maxima:-

As there are N-1 minima between two adjacent principle maxima, there must be N-2 other maxima, known as secondary maxima between two principle maxima.

To find the position of secondary maxima, we first differentiate equation w.r.t. β and then equate to zero thus

$$\frac{dI}{d\beta} = \frac{R_o^2 \sin^2 \alpha}{\alpha^2} 2 \left[\frac{\sin NB}{\sin \beta}\right] \frac{N \sin N\beta \sin \beta - \sin N\beta \cos \beta}{\sin^2 \beta} = 0$$

$$N \cos N\beta \sin \beta - \sin N\beta \cos \beta = 0$$

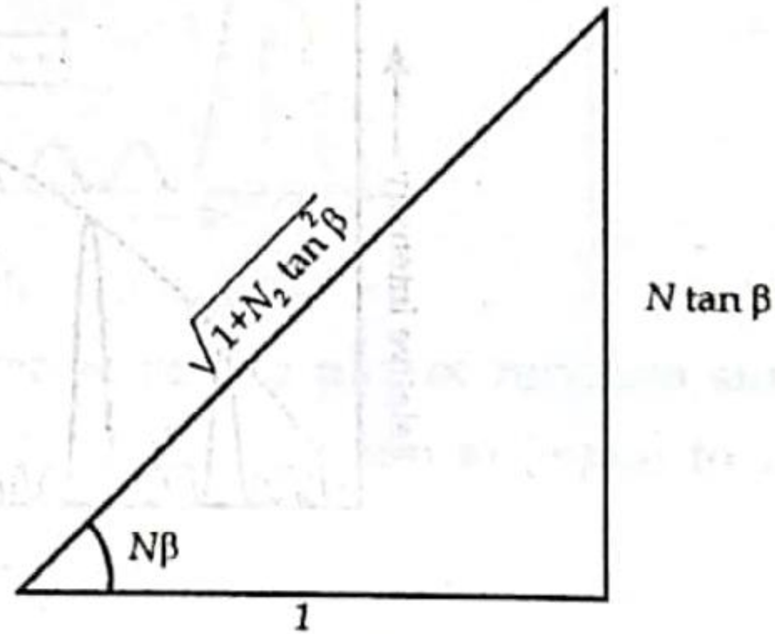
$$N \tan \beta = \tan N\beta .$$

The roots of this equation other than those for which $\beta = \pm n\pi$ (which corresponds to principal maxima) give the positions of secondary maxima.

To find the value of $\frac{\sin^2 N\beta}{\sin^2 \beta}$ from the equation $N \tan \beta = \tan N\beta$, we make use of triangle

$$\sin N\beta = \frac{N \tan \beta}{\sqrt{(1 + N^2 \tan^2 \beta}}$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2 \tan^2 \beta}{(1 + N^2 \tan^2 \beta) \sin^2 \beta}$$



$$= \frac{N^2}{(1 + N^2 \tan^2 \beta) \cos^2 \beta}$$

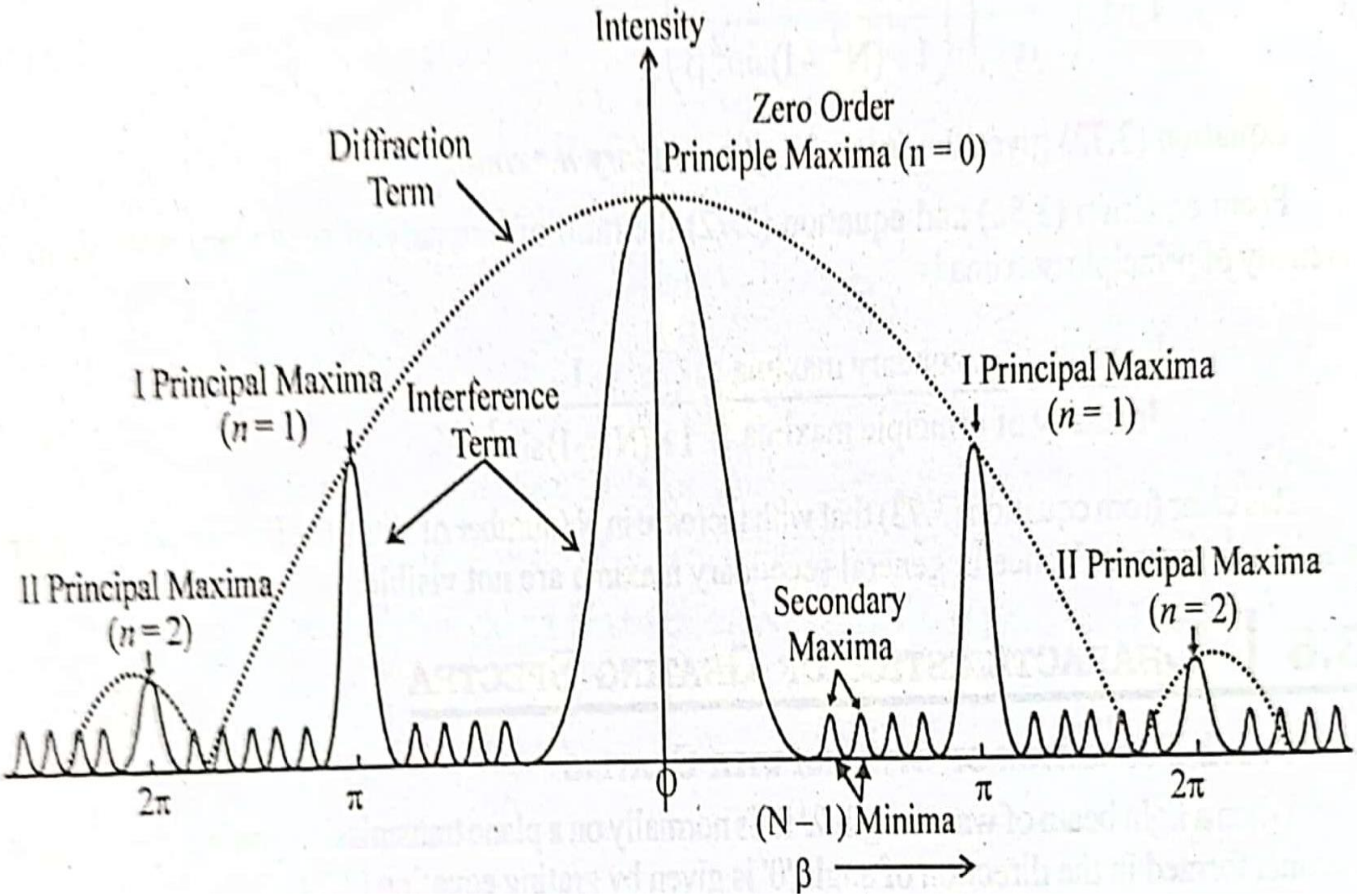
$$= \frac{N^2}{\cos^2 \beta + N^2 \sin^2 \beta}$$

$$\frac{\sin^2 N\beta}{\sin^2 \beta} = \frac{N^2}{1 + (N^2 - 1) \sin^2 \beta}$$

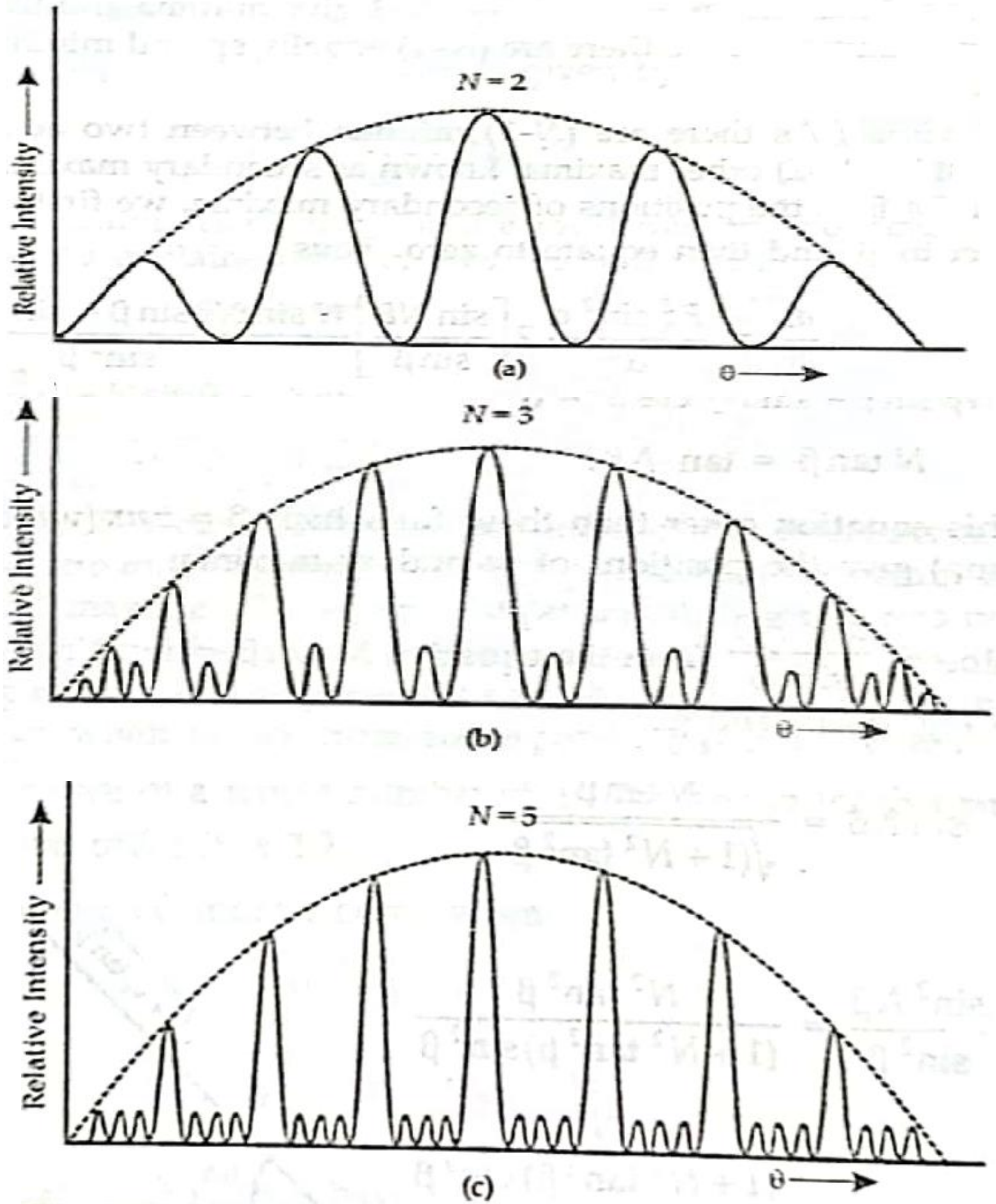
which shows that the intensity of secondary maxima varies as $\frac{N^2}{1 \div (N^2 - 1) \sin^2 \beta}$ while that of principal maxima varies as N^2. Therefore, the ratio of the intensity of these secondary maxima to the intensity of principal maxima is $\frac{1}{1 \div (N^2 - 1) \sin^2 \beta}$

Diffraction Pattern

Diffraction pattern of plane transmission grating is product of two components so the resultant pattern is overlapped pattern due to the pattern of both component (Diffraction due to single slit and Interference component).

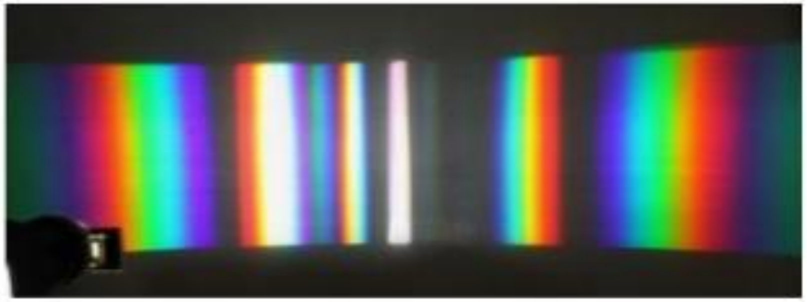


Intensity
Zero Order
Principle Maxima (n = 0)
Diffraction
Term
I Principal Maxima
(n = 1)
Interference
Term
I Principal Maxima
(n = 1)
II Principal Maxima
(n = 2)
II Principal Maxima
(n = 2)
Secondary
Maxima
2π
π
0
π
2π
(N – 1) Minima
β

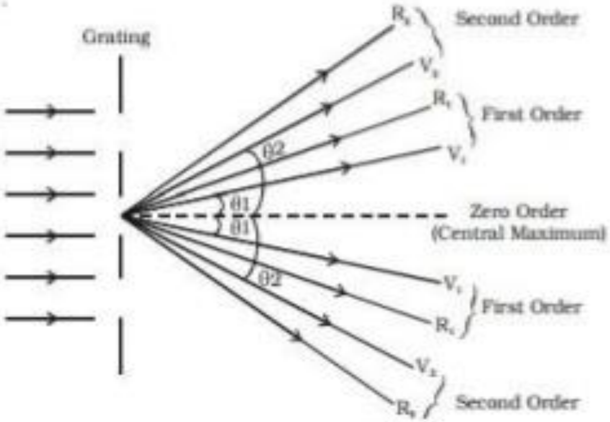

N = 2
Relative Intensity
(a)
θ
N = 3
Relative Intensity
(b)
θ
N = 5
Relative Intensity
(c)
θ

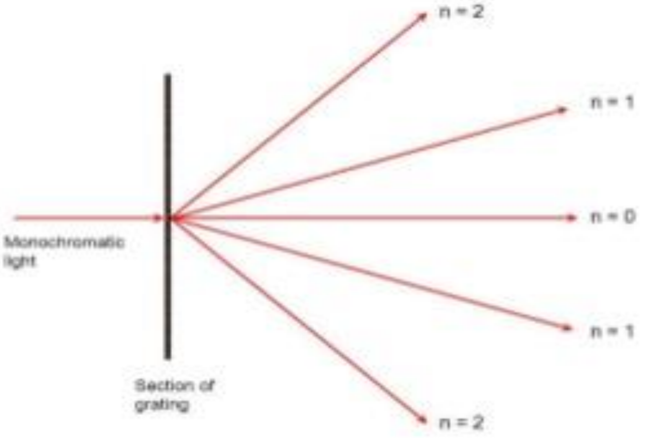
Formation of spectra with Grating

With White Light



With Monochromatic Light





Characteristics of Grating

1. ABSENT SPECTRA

If the angle of diffraction is such that, the minima due to diffraction component in the intensity distribussion falls at the same position of principal maxima due to interference component, then the order of principal maxima then absent. If m^{th} order minima fall on n^{th} order principal maxima then

$$a \sin\theta = m\lambda \quad \text{............... eq 1}$$

$$(a+b)\sin\theta = n\lambda \quad \text{............... eq 2}$$

$$\frac{(a+b)\sin\theta}{a\sin\theta} = \frac{n\lambda}{m\lambda}$$

$$\frac{(a+b)}{a} = \frac{n}{m} \quad \text{............... eq 3}$$

Now we consider some cases

A. If b=a, i.e. width of opaque space in equal to width of slit then from equation 3. n = 2m since m=1, 2, 3 …. Then n = 2nd, 4th , 6th ….spectra will be absent

B. If b=2a, i.e. width of opaque space in equal to width of slit then from equation 3. n = 3m since m=1, 2, 3 …. Then n = 3rd, 6th , 9th ….spectra will be absent

2. Maximum Number of Order Observed by Grating

Principal maximum in grating spectrum is given by

$$(a+b)\sin\theta = n\lambda$$

$$n = \frac{(a+b)\sin\theta}{\lambda} \qquad \text{.................. eq 1}$$

Maximum possible angle of diffraction is 90 degree therefore

$$n_{max} = \frac{(a+b)\sin 90^\circ}{\lambda} \quad \text{So} \quad n_{max} = \frac{(a+b)}{\lambda} \qquad \text{.................. eq 2}$$

Q.1. A plane transmission grating has 6000 lines/cm. Calculate the higher order of spectrum which can be seen with white light of wavelength 4000 angstrom

Sol. Given a+b=1/6000, Wavelength 4000X 10^{-8}cm
As we know that gratings equation written as

$$n = \frac{(a+b)\sin\theta}{\lambda}$$

For maximum order

$$n_{max} = \frac{(a+b)}{\lambda} = \frac{1cm}{6000\times 4000\times 10^{-8}cm} = \frac{100}{24} = 4.16$$

Maximum order will be 4^{th}

3. Width of principal maxima

The angular width of principal maxima of nth order is defined as the angular separation between the first two minima lying adjacent to principal maxima on either side

If θ_n is the position of nth order principal maxima $\theta_n+\delta\theta_n$, $\theta_n-\delta\theta_n$ are positions of first minima adjacent to principal minima then the width of nth principal maxima = $2\delta\theta_n$

From the Grating Equation n[th] order maxima

$$(a+b)\sin\theta_n = n\lambda \quad \text{...............} \quad \text{eq 1}$$

And the position of minima is given by

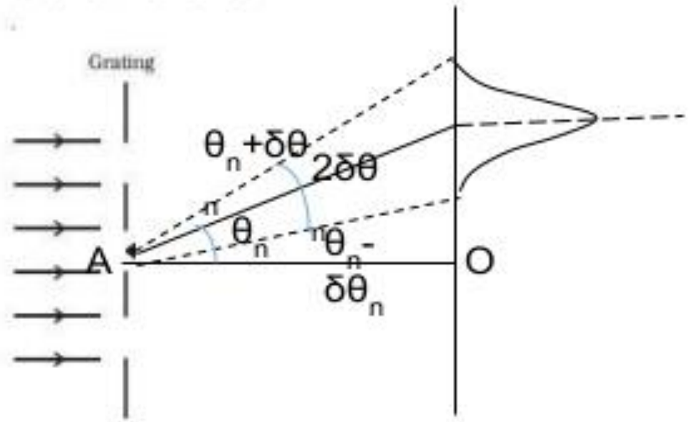
$$(a-b)\sin\theta_n = \frac{p}{N}\lambda, where, p = (nN\pm1)$$

Hence equation rewritten as

$$(a+b)\sin(\theta_n + d\theta_n) = \left(\frac{nN+1}{N}\right)\lambda \quad \text{...eq 2}$$

On dividing eq 2 by eq 1

$$\frac{\sin(\theta_n \pm d\theta_n)}{\sin(\theta_n)} = \left(\frac{nN\pm1}{nN}\right)$$



$$\frac{(\sin\theta_n\cos d\theta_n \pm \cos\theta_n \sin d\theta_n)}{\sin(\theta_n)} = \left(1\pm\frac{1}{nN}\right)$$

$$\cos d\theta_n \pm \frac{\cos\theta_n \sin d\theta_n}{\sin(\theta_n)} = \left(1\pm\frac{1}{nN}\right)$$

If $d\theta_n$ is very small than $\cos d\theta_n = 1$, $\sin d\theta_n = d\theta_n$

$$1\pm\frac{\cos\theta_n d\theta_n}{\sin(\theta_n)} = \left(1\pm\frac{1}{nN}\right)$$

$$\cot\theta_n d\theta_n = \frac{1}{nN}, \quad d\theta_n = \frac{\tan\theta_n}{nN} \qquad 2d\theta_n = \frac{2\tan\theta_n}{nN}$$

4. Dispersive Power of Diffraction Gratings

For a definite order of spectrum, the rate of change of angle of diffraction θ with respect to the wavelength of light ray is called dispersive power of Grating.

Dispersive Power = $d\theta/d\lambda$ .... eq 1

We know that gratings equation $(a+b)\sin\theta_n = n\lambda$

$$(a+b)\cos\theta\frac{d\theta}{d\lambda} = n$$

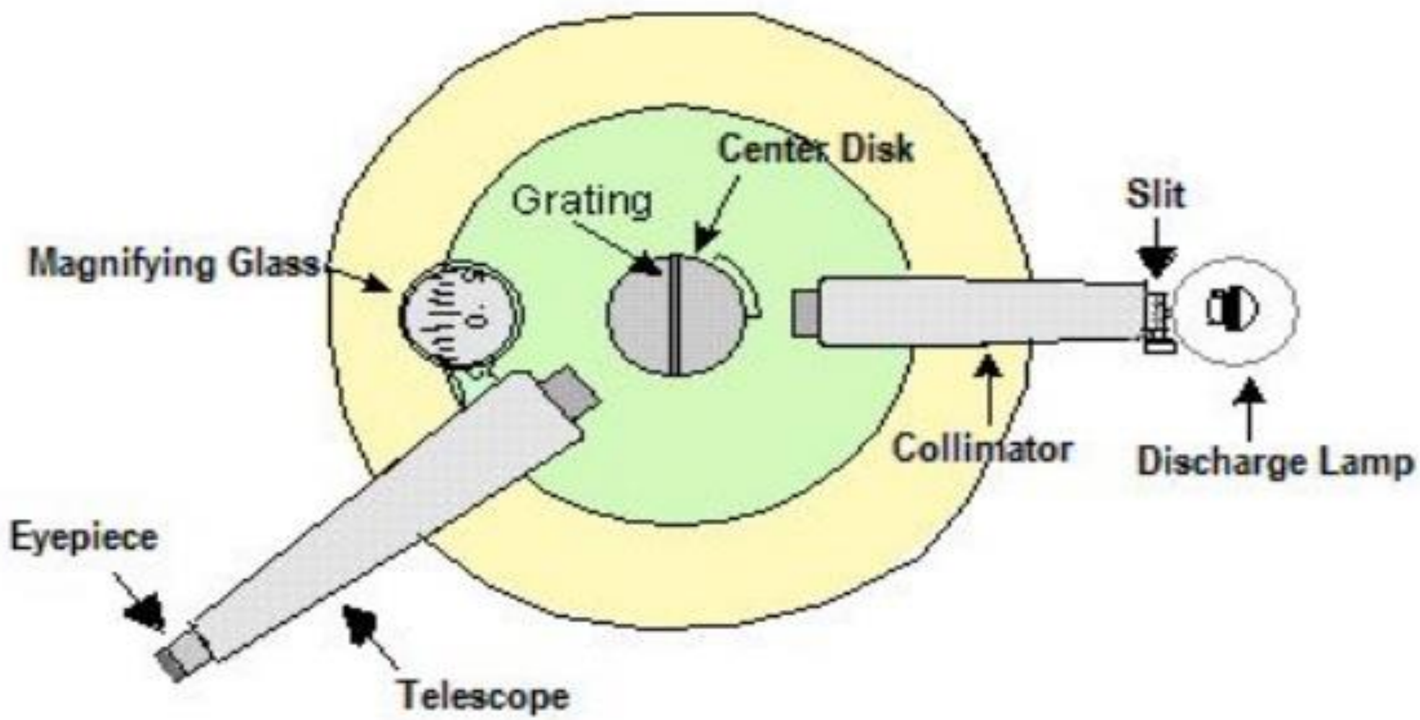
$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\cos\theta}$$ .... eq 2

Also written as

$$\frac{d\theta}{d\lambda} = \frac{n}{(a+b)\sqrt{1-\sin^2\theta}} = \frac{n}{\sqrt{(a+b)^2-(a+b)^2\sin^2\theta}}$$

$$\frac{d\theta}{d\lambda} = \frac{n}{\sqrt{(a+b)^2-n^2\lambda^2}}$$ .... eq 3

5. Experimental demonstration of diffraction grating to determine wavelength



Formula to determine the wavelength of light

- Since $(a+b) \sin\theta = \pm n\lambda$ Or $\boldsymbol{\lambda = \pm (a+b) \sin\theta / n}$, where n=1,2,3……..
- Where (a+b)= Grating element , n= order of spectrum

6. Maximum/highest/largest wavelength obtained by grating :

Since $(a+b)\sin\theta = \pm n\lambda$

Or $\lambda = \pm (a+b)\sin\theta / n$,

Maximum possible angle of diffraction is 90^0,

Therefore $\boldsymbol{\lambda_{max} = (a+b)/n}$ **(since θ=** 90^0)

where (a+b)= Grating element ,
n= order of spectrum

7. Overlapping of Spectral lines :

$$\mathbf{(a+b)\sin\theta = \pm n_1\lambda_1 = n_2\lambda_2}$$

Resolution

To see two close objects just separate is called Resolution.

Types of Resolutions:-

(1) Geometrical Resolution:- When the geometrical position of the two nearby point object are to be resolved then it is called geometrical resolution. For example Microscope, Telescope.

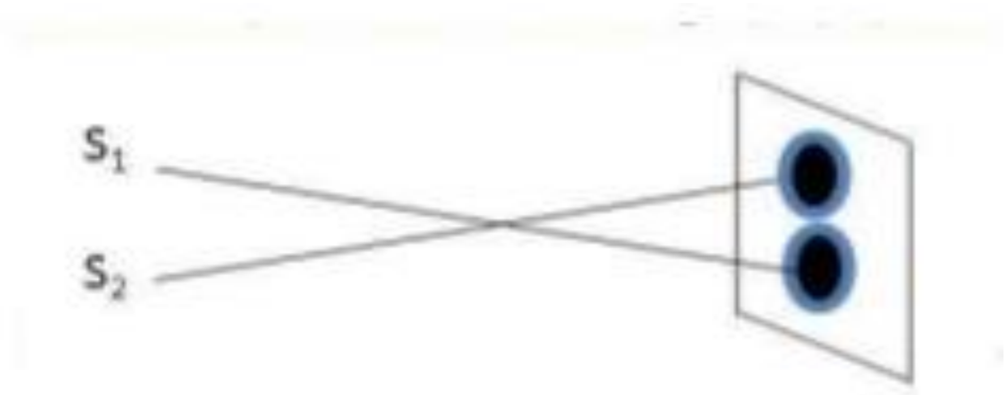
(2) Spectral Resolution:- When spectral lines corresponding to wavelength having small distances are to be just resolved then it is called spectral resolution. For example Diffraction Grating.

Resolution Limit:- The smallest distance between the two close objects whose images can be seen just as separate is called resolution limit.

Resolving Power:- The ability or the power of an optical instruments to see to close objects just as separate is called resolving power.

Relation between resolving power and resolution limit:-

Resolution limit α 1/ Resolving power



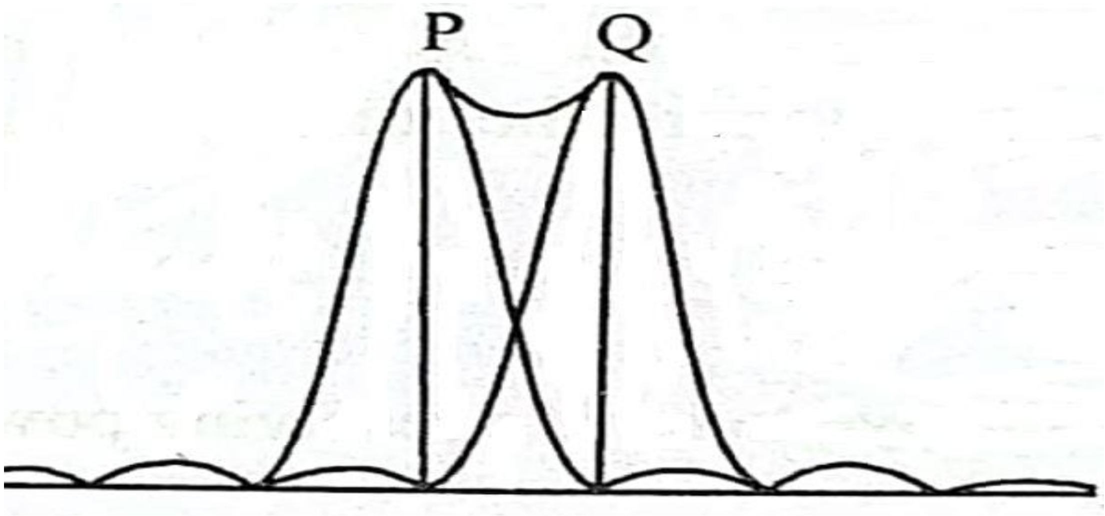
Lord Rayleigh Criterion for Resolution

According to this method resolving power of any optical instruments like grating can be determined.

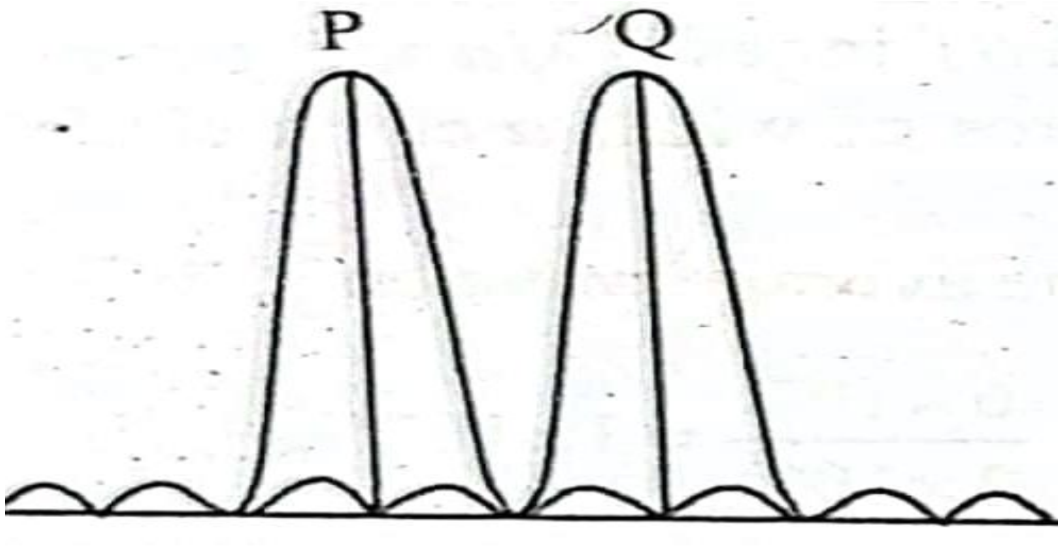
In this two closed sources or images of nearly equal intensity are said to be just resolved when the central maxima of diffraction pattern of one source falls at the first minima of diffraction pattern of second source or vice versa.

Cases :-

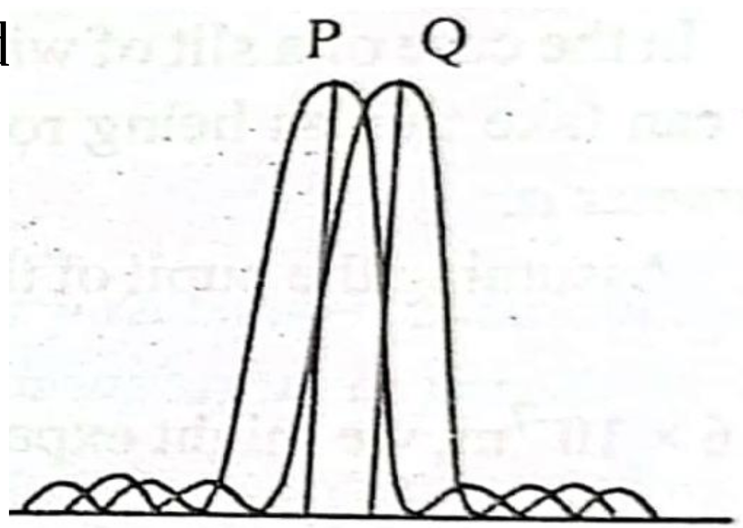
(1) Just resolved:- In case of just resolution the distances between two closed sources is called resolution limit.



(2) Completely separated:- If the distance between two closed sources is greater than the resolution limit than the will appear as completely resolved.



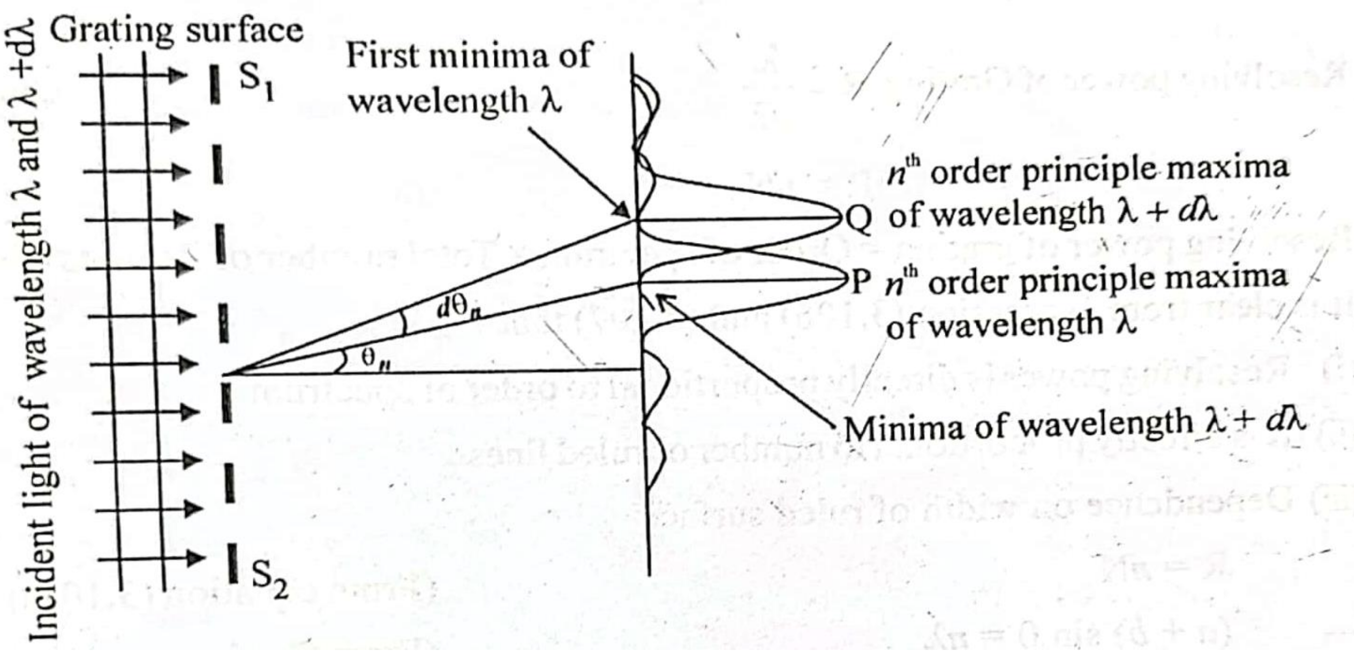
(3) **Mixed or unresolved**:- If the distance between two closed sources is less than the resolution limit than the will appear as mixed or unresolved



Resolving Power of Grating

The resolving power of diffraction grating is the ability of grating to resolve two nearby spectral lines that is, to see these two spectra lines just as separate. It is measured in terms of the ratio λ / dλ. Here λ is the wavelength of any spectral line (or their mean) and dλ is the difference in the two wavelengths.

Derivation:-



Let a parallel beam of light of wavelength λ and $\lambda + d\lambda$ is incident normally on the grating surface.

Let n^{th} order principal maxima of wavelength λ is form the direction θ_n , than

$$(a + b) \sin \theta_n = n \lambda$$

For a principal maxima of wavelength $\lambda + d\lambda$ formed in the direction $\theta_n + d\theta_n$

$$(a + b) \sin (\theta_n + d\theta_n) = n (\lambda + d\lambda) \qquad \ldots\ldots\ldots\ldots\ldots 1$$

Let the position of first minima adjacent to the n^{th} order principal maxima of wavelength λ is formed in the direction $\theta_n + d\theta_n$, is given by

$$(a+b) \sin (\theta_n + d\theta_n) = m \lambda / N \qquad \ldots\ldots\ldots\ldots\ldots 2$$

Where 'm' is the order of spectrum for minima and

'N' is the no. of lines on the grating

Since first minima corresponds to

$$m = nN \pm 1 \qquad \ldots\ldots\ldots\ldots\ldots.3$$

By putting the value of 'm' from equation no. 3 into 2, we get

$$(a + b) \sin (\theta_n + d\,\theta_n) = (nN \pm 1)\,\lambda / N \qquad \ldots\ldots\ldots\ldots\ldots.4$$

By comparing equation 2 and equation 4, we get

$$m\,\lambda / N = (nN \pm 1)\,\lambda / N$$

On solving we get - $\boldsymbol{\lambda / d\lambda = nN}$

Or resolution limit $\boldsymbol{= d\lambda / \lambda = 1 / nN}$

Where n= order of spectrum

N = number of lines on the grating .

Bragg's Law

X-ray diffraction by Crystals & Bragg's Law

Crystal :

A solid composed of atoms, ions or molecules arranged in a pattern that is repeated in thee dimensions are known as crystal. In crystals, atoms are arranged in various systematic planes. Normally, interplanner spacing of crystal is order of 1A to 5A^o.

X-rays :

X-rays are invisible electromagnetic radiations in the range of 1A^o – 10 A^o

Soft X-rays ; 10 A^o

Hard X-rays : 1A^o

Why X-rays useful for diffraction in crystals?

- For X-rays, crystals act as a three dimensional gratings, because interplaner spacing of crystal is same order of wave length of X-rays. When X-rays fall on crystal surface, they get reflected (or diffracted) from different layers of atoms.

Statement of Bragg's Law

This deals with X-ray diffraction, it gives rise mathematical condition for constructive interference for X-rays which occurs at glancing angle (θ)

$$2d \, \text{Sin} \, \theta = n \lambda$$

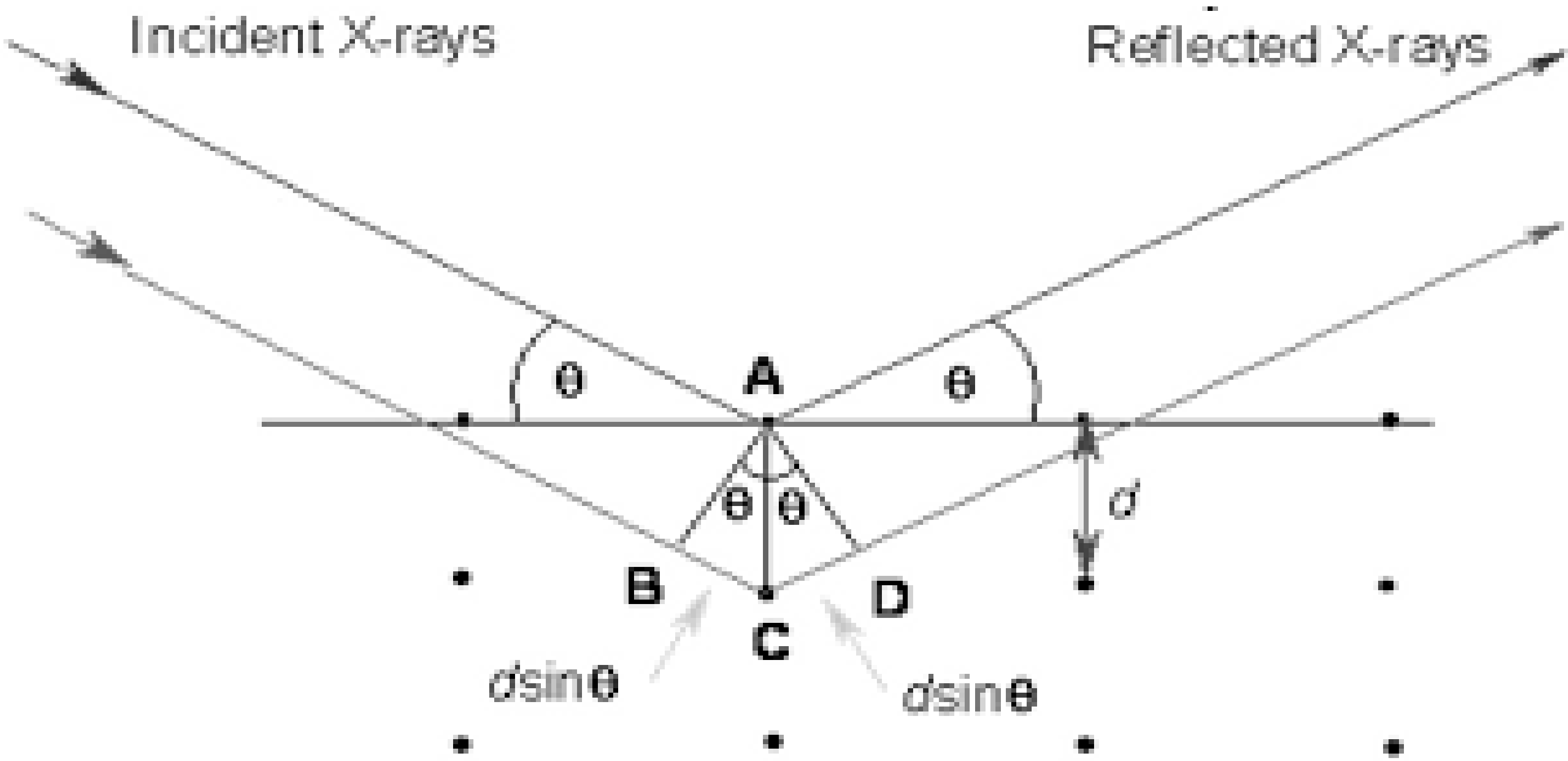
d → interplaner spacing of crystal.

q → glancing angle

n → order of reflection = 1,2,3,4 ......

λ → wavelength of X-ray beams.

Derivation of Bragg's law



When X rays incident on a certain crystal each atom scatters waves in all directions and the secondary waves coming from these atoms interfere constructively in one direction and destructive interferly in other directions .

Consider two X-rays incident in a set of crystal planes and they got reflected by the set of atoms . Now the total path difference between the reflected rays will be

Δ = BC+CD ,

From figure , BC= d sinθ , CD= d sinθ

Δ = 2d sinθ ………………………………………………… 1

For constructive interference, Δ = n λ ………………………... 2

From 1 & 2

n λ =2d sinθ …. This is Bragg's law

Numericals

- A diffraction grating has 5000 lines per cm and the total rules width is 5 cm. Calculate for a wavelength of 5000Å in second order (i) the resolving power (ii) the smallest difference in wavelength resolved (iii) what happens if half the ruling width is covered.
- A grating has 9600 lines uniformly spaced over a width of 3.0 cm and is illuminated by light from mercury vapour lamp. Find resolving power of grating in fifth order.
- What must be minimum number of lines per cm in half inch width grating to resolve the D1 and D2 lines sodium in the first order (λ_{D1}=5896Å, λ_{D2}=5896Å).
- Find the maximum inter-atomic spacing of the light rays scattered by rock salt crystal in the second order spectrum . Given that wavelength is 1Å.

Lecture contents with a blend of NPTEL contents and other platforms

- http://nptel.iitm.ac.in by Prof. S. Bhardwaj, IIT Kharagpur
- http://nptel.ac.in by Dr. S.Sankaran , IIT Madras
- https://www.youtube.com/watch?v=F8Cn6jAMa-A by Prof. G. S. Raghuvanshi , JIET Jodhpur.
- https://www.youtube.com/watch?v=n65gZGwiZtk by Prof. M. K. Srivastava, IIT Roorkee.

References and Bibliography

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- Fundamental of Optics by Jetkins and White, Tata McGraw Hill, New Delhi
- Engineering Physics by Prof. Y. C. Bhatt, Ashirwad Publications
- Optics by Subhramanium and Brij lal, S. Chand Publications.

Thank You