Weak Students ASSIGNMENT
Year: B. Tech. I Year
Semester: I
Subject: Engineering Mathematics - I
Session: 2019-20

CO1. Understand fundamental concepts of improper integrals, beta and gamma functions and their properties. Evaluation of Multiple Integrals in finding the areas, volume enclosed by several curves after its tracing and its application in proving certain theorems.

Q.1 Find the value of $\int_0^{\frac{\pi}{2}} Sin^6\theta Cos^7\theta\, d\theta$.

Q.2 Prove that $\Gamma n\Gamma(1-n) = \frac{\pi}{\mathrm{Sin} n\pi}$ $0 < n < 1$ **(RTU-2018)**

Q.3 Prove that $B(m,n) = \frac{\Gamma m\Gamma n}{\Gamma(m+n)}$ $, m > 0, n > 0$

Q.4 Show that $B(m,n) = \int_0^\infty \frac{y^{m-1}}{(1+y)^{m+n}}\, dy$

Q.5 Show that $\int_0^2 (8-x^3)^{-\frac{1}{3}} dx = \frac{2\pi}{3\sqrt{3}}$.

Q.6 Evaluate $\int_0^\infty \frac{\mathbf{1}}{\mathbf{1+x^4}} dx$

Q.7 Show that $\int_0^1 \sqrt{1-x^4}\, dx = \frac{\left(\sqrt{\frac{1}{4}}\right)^2}{6\sqrt{2\pi}}$

Q.8 Prove that $\int_0^\infty \frac{x^2}{(1+x^4)^3} dx = \frac{5\pi}{128}$

Q.9 Show that $\int_0^{\frac{\pi}{2}} sin^p\theta cos^q\theta d\theta = \frac{\Gamma\left(\frac{p+1}{2}\right)\Gamma\left(\frac{q+1}{2}\right)}{2\Gamma\left(\frac{p+q+2}{2}\right)}$. **(DTU-17)**

Q.10 Show that $B(m,n) = a^m b^n \int_0^\infty \frac{x^{m-1}}{(ax+b)^{m+n}} = \Gamma m\Gamma n/\Gamma(m+n)$

Q.11 Prove that $B(m,n) = 2\int_0^{\pi/2} sin^{2m-1}\theta cos^{2n-1}\theta\, d\theta$ **(VTU)**

Q.12 The Cardioid $r = a(1+cos\theta)$ revolves about the initial line. Find the volume of solid generated. **(RTU-16)**

Q.13 Find the surface of the solid generated by the Revolution of the ellipse $x^2 + y^2 = 16$ about the major axis.

Q.14 Find the volume bounded by the cylinder $x^2 + y^2 = 4$ and the planes $y + z = 3$ and z=0.

Q.15 Find the surface area of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the x-axis. **(RTU-18)**

Q.16 Find the surface of the solid generated by the Revolution of the ellipse
$x^2 + y^2 = 16$ about the major axis.

Q.17 Evaluate $\int_0^1 \left\{\int_0^1 \frac{(x-y)}{(x+y)^3} dy\right\} dx$.

Q.18 Solve $\int_0^{2a} \int_0^{x^2/4a} xy\,dx\,dy$

Q.19 Find the centre of Gravity of the arc of the curve $x = asin^3\theta, y = acos^3\theta$ lying in the first Quadrant.

Q.20 Evaluate $\int_0^1 \int_0^{\sqrt{2-x^2}} \frac{x}{\sqrt{x^2+y^2}} dx\, dy$ by changing the order of Integration.

Q.21 Evaluate $\int_0^{\pi/2} \int_{r=2sin\theta}^{r=4sin\theta} r^3 d\theta dr$

Q.22 Change into polar form $\int_0^1 \int_x^{\sqrt{2-x^2}} f(x,y)dxdy$

Q.23 Find the area of the region which is enclosed by circle $x^2 + y^2 = a^2$ by double integration.

Q.24 Find by Double Integration the area of the Region enclosed by $x^2 + y^2 = a^2$ and $x + y = a$ (In the First Quadrant)

Q.25 Find the centre of Gravity of the arc of the curve $x = asin^3\theta, y = acos^3\theta$ lying in the first Quadrant.

Q.26 Find the total mass of region in the cube $0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq 1$ with density at any point given by xyz.

Q.27 $\int_0^{log2} \int_0^x \int_0^{x+y} e^{x+y+z}\, dx\, dy\, dz$

Q.28 Evaluate $\iiint \frac{dx\, dy\, dz}{x^2+y^2+z^2}$ throughout the volume of the sphere $x^2+y^2 + z^2 = a^2$.

Q.29 Solve the triple integration $\int_0^1 \int_0^x \int_0^{x+y} (x + y + z)dxdydz$

Q.30 Use Green's Theorem in a Plane to evaluate $\oint_C (2xy - y)dx + (x + y)dy$Where C is the boundary of the circle $x^2 + y^2 = a^2$ in the XY-Plane.

Q.31 Evaluate $\oint_C \vec{F}.d\vec{r}$ by Stoke's Theorem , Where $\vec{F} = y^2\hat{\imath} + x^2\hat{\jmath} - (x + z)\hat{k}$ and C is the boundary of the triangle with vertices at (0,0,0),(1,0,0) and (1,1,0). **(RTU-18)**

Q.32 Verify Divergence Theorem for the function $\vec{F} = y\hat{\imath} + x\hat{\jmath} + z^2\hat{k}$ over the cylindrical region bounded by $x^2 + y^2 = a^2, z = 0\ and\ z = h$.

Q33. Find the volume of the solid of rotation obtained by rotating they are enclosed between parabola $x = y^2$ and the line x=0 and x=1. **(ME 2019, Gate)**

Q34. Find the area of an ellipse represented by the equation $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ **(CE 2020 gate)**

Q35. Find the value of integral $\int_0^{\pi} \int_y^{\pi} \frac{\sin x}{x}\, dx\, dy$. (ECE 2019 gate)

Q36.Solve the integral $\int_0^1 \frac{dx}{\sqrt{1-x}}\, dx$. **(GATE ECE 2016)**

Q37.Find the value of $\int_0^3 \int_0^x (6 - x - y)\, dx\, dy$. (**GATE CSE 2008**)

Q.38 Find the area of an ellipse represented by the equation $\frac{x^2}{a^2} - \frac{y^2}{b^2} = c$.

Q.39 Solve $\int_0^1 \int_0^z \int_0^{x+z} (xy + 4z)dxdydz$.

Q.40 Solve the integral $\int_0^1 \frac{dx}{\sqrt{1+x^2}}\, dx$.

Q. 41 Solve $\int_0^1 \int_0^{\sqrt{2-x^2}} (x^2 + y^2)dxdy$.

Q.42 Solve $\int_0^{2a} \int_0^{y^2/4a} x^2y^2dxdy$.

Q. 43 State and prove Green theorem.
Q.44 State and prove Gauss divergence theorem.
Q.45 State and prove Stock theorem.
Q.46 Evaluate $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{1}{\sqrt{x^2+y^2}} dx\, dy$ by changing the order of Integration.
Q.47 Evaluate $\oint_C (2xy + y^2)dx + (x - y)dy$Where C is the boundary of the circle $x^2 + y^2 = a^2$ in the XY-Plane.
Q.48 Evaluate$\int_0^1 \int_0^1 \frac{(x+y)}{(x-y)^3} dy\, dx$
Q.49 Find the volume of the solid generated by the revolution of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ about the y-axis. **(RTU)**
Q.50 Derive the Duplication formula.

CO2: Interpret the concept of a series as the sum of a sequence, and use the sequence of partial sums to determine convergence of a series. Understand derivatives of power, trigonometric, exponential, hyperbolic, logarithmic series.

Q.1 Test the Convergence of the series whose n^{th} term is $\frac{n^2}{2^n}$
Q.2 Test the Convergence of the series $\sum \frac{4.7.....(3n+1)}{1.2.3.....n} x^n$
Q.3 Test the Convergence of the series $1 - 2x + 3x^2 - 4x^3 + \cdots \infty\ (x < 1)$

Q.4 Test the Convergence of the series $\sum_1^\infty \frac{\sqrt[3]{3n^2+1}}{\sqrt[4]{4n^3+2n+7}}$.
Q.5 Test the Convergence of the series whose n^{th} term is $\frac{2^n}{n^3}$
Q.6 Test the Convergence of the series $\sum_{n=1}^{\infty} \frac{1}{\sqrt{n}+\sqrt{n+1}}$ **(DTU-18)**
Q.7 Test the Convergence of the series $\frac{x}{1.2} + \frac{x^2}{3.4} + \frac{x^3}{5.6} + \frac{x^4}{7.8}$ ....
Q.8 Test the Convergence of the series for positive values of x;

$$1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \cdots + \frac{2^n - 2}{2^n + 1} + \cdots$$

Q.9 Test the Convergence of the series $\frac{x^2}{2log2} + \frac{x^3}{3log3} + \frac{x^4}{4log4} + \cdots$
Q.10 Test the Convergence of the series $2x + \frac{3}{8}x^2 + \frac{4}{27}x^3 + \cdots + \frac{n+1}{n^3}x^n + \cdots \infty$
Q.11Test the Convergence of the series $x + \frac{1}{2} \times \frac{x^3}{3} + \frac{1\times3}{2\times4} \times \frac{x^5}{5} + \frac{1\times3\times5}{2\times4\times6} \times \frac{x^7}{7} + \cdots$
Q.12 Test the Convergence of the series $\sum \frac{1}{\left(1+\frac{1}{n}\right)^{n^2}}$
Q.13 Discuss the Convergence of the series $\frac{1}{2} + \frac{2}{3}x + \left(\frac{3}{4}\right)^2 x^2 + \left(\frac{4}{5}\right)^3 x^3 + \cdots \infty$

Q.14 Test the Convergence of the series $x + \frac{2^2.x^2}{!2} + \frac{3^3.x^3}{!3} + \frac{4^4.x^4}{!4} + \cdots \infty$

Q.15 Test the Convergence of the series $1^p + \left(\frac{1}{2}\right)^p + \left(\frac{1\times3}{2\times4}\right)^p + \left(\frac{1\times3\times5}{2\times4\times6}\right)^p + \cdots$ **(DTU-17)**

Q.16 Test the Convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n+10}$

Q.17 Test the Convergence of the series $\sum_{n=1}^{\infty} ne^{-n^2}$

Q.18 Test the Convergence the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots\infty$

Q.19 Test the Convergence the series $1 + \frac{1}{2^2} + \frac{2^2}{3^3} + \frac{3^3}{4^4} + \cdots$

Q.20 Test the Convergence the series $1^{logx} + 2^{logx} + 3^{logx} + 4^{logx} + \cdots\infty$

Q.21 Test the Convergence the series $\frac{1}{2}x + x^2 + \frac{9}{8}x^3 + x^4 + \frac{25}{32}x^5 + \cdots\infty$

Q.22 Test the Convergence the series $1 + \frac{3}{7}x + \frac{3.6}{7.10}x^2 + \frac{3.6.9}{7.10.13}x^3 + \frac{3.6.9.12}{7.10.13.16}x^4 + \cdots$

Q.23 Test the Convergence the series $1 + \frac{2x}{2!} + \frac{3^2x^2}{3!} + \frac{4^3x^3}{4!} + \cdots$

Q.24 Test the Convergence the series $1 + \frac{1}{2(log2)^p} + \frac{1}{3(log3)^p} + \cdots \frac{1}{n(logn)^p} + \cdots$

Q.25 Test the Convergence the series $1 - \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{3}} - \frac{1}{\sqrt{4}} + \cdots\infty$

Q.26 Find whether series is $\sum \frac{n}{n+10}$ convergent or not. **(RTU-18)**

Q.27 Give an example of two divergent series whose sum is convergent. **(RTU-18)**

Q.28 Find Taylor series expansion of $f(x) = cos5x^2$ about the point $x = \pi$. **(RTU-18)**

Q.29 Discuss the convergence of the series $\sum \frac{\sqrt{n}}{\sqrt{n^2+1}}x^n$. **(RTU-18)**

Q.30 Write the condition for p series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ to be convergent and divergent. **(RTU-19)**

Q.31 Determine the radii of convergence of series $\sum_{n=1}^{\infty} \frac{(2n)!}{(n!)^2} z^n$ **(RTU-19)**

Q.32 Use Taylor's series show that

$\sin(x + h) = \sin x + h\cos x - \frac{h^2}{2!}\sin x \ldots.$ **(RTU-19)**

Q.33 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1^2 5^2 9^2 \ldots\ldots (4n-3)^2}{4^2 8^2 12^2 \ldots\ldots 4n^2}$ **(RTU-19)**

Q.34 What is Monotonic sequence?

Q.35 Define Oscillatory sequence.

Q.36 Write the properties of a Convergent sequence.

Q.37 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{(4n-3)^2}{4n^2}$

Q.38 Discuss the convergence of the series $\sum \frac{2x_n+3}{4}$

Q.39 Discuss the convergence of the series $\sum \frac{1}{n(n+1)}$

Q.40 Write an example of oscillating series.

Q.41 What is necessary condition for the convergence of a series.

Q.42 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{n^2}{n+1}$

Q.43 Explain the comparison test.

Q.44 What is D'Alembert ration test.

Q.45 Discuss the convergence of the series $\sum \frac{n^n}{n!}$

Q.46 Define Raabe's test.

Q.47 Test the convergence of the series $\sum \frac{1}{\sqrt{n+1}-1}$.

Q.48 What is Gauss test.

Q.49 Test the convergence of the series $\sum_{n=1}^{\infty} \frac{2^2 4^2 6^2 \ldots\ldots (2n-2)^2}{3^2 5^2 7^2 \ldots\ldots (2n-1)^2}$.

Q.50 Test the series for convergence $\sum \frac{n+1}{n^p}$.

CO3. Recognize odd, even and periodic function and express them in Fourier series using Euler's formulae.

Q.1 Define Fourier series f(x) in the interval $[-\pi, \pi]$. State Dirichlet's conditions for convergence of Fourier series f(x). **(RTU 2018, 17)**

Q.2 Write Dirichlet's conditions for Fourier expansion of a function. **(RTU 2019, 18)**

Q.3 Define (Write about) even and odd function with examples.

Q.4 The value of integral $\int_{\alpha}^{\alpha+2\pi} cos^2 nx\, dx = \cdots$

Q.5 Explain Dirichlet's condition for any function f(x) developed as a Fourier series.

Q.6 Define a Fourier series.

Q.7 State Euler's formulae. **(RTU 2019, 17, 16 ,14)**

Q.8 Fourier expansion of an odd function has only…………… terms.

Q.9 The function $f(x) = \begin{Bmatrix} 1 - x\ in -\pi < x < 0 \\ 1 + x\ in \quad 0 < x < \pi \end{Bmatrix}$ is an odd function. Is the above function is true or false?

Q.10 Using sine series for $f(x) = 1\ \ in\ 0 < x < \pi$, find the value of $1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots \infty = \ldots$

Q.11 Determine the Fourier coefficient of a_0 in the Fourier series expansion.

Q.12 Find a series of sines and cosines of multiples of x which will represent the function $f(x) = x + x^2$ in the interval $-\pi < x < \pi$. Hence show that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$ **(RTU 2019, 2020, 17)**

Q.13 Find the Fourier series of the function $f(x)$, where $f(x) = \begin{Bmatrix} -1, -1 < x < 0 \\ 2x,\ 0 < x < 1 \end{Bmatrix}$ and $f(x + 2) = f(x)$.

Q.14 Expand the function $f(x) = x \sin x$ as a Fourier series in $-\pi \le x \le \pi$. Deduce that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots \infty = \frac{\pi - 2}{4}$ **(RTU 2019, 18)**

Q.15 Obtain a Fourier series for $(x) = \begin{cases} \pi x, & 0 < x < 1 \\ \pi(2-x), & 1 < x < 2 \end{cases}$. Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$

Q.16 Obtain Fourier's series in the interval $(-\pi, \pi)$ for the function $f(x) = x \cos x$

Q.17 A sinusoidal voltage $E \sin \omega t$ is passed through a half-wave rectifier which clips the negative portion of the wave. Develop the resulting periodic function

$$V(t) = \begin{cases} 0, & \frac{-T}{2} < t < 0 \\ E \sin wt, & 0 < t < \frac{T}{2} \end{cases}$$ and $T = \frac{2\pi}{w}$, in a Fourier series.

Q.18 If $f(x) = \begin{cases} x, & 0 < x < \frac{\pi}{2} \\ \pi - x, & \frac{\pi}{2} < x < \pi \end{cases}$

show that $f(x) = \frac{\pi}{4} - \frac{2}{\pi}\left\{\frac{1}{1^2}\cos 2x + \frac{1}{3^2}\cos 6x + \frac{1}{5^2}\cos 10x + \cdots\right\}$

Q.19 If $f(x) = \begin{cases} 0, & -\pi \le x \le 0 \\ \sin x, & 0 \le x \le \pi \end{cases}$ then prove that

$f(x) = \frac{1}{\pi} + \frac{\sin x}{2} - \frac{2}{\pi}\sum_{n=1}^{\infty} \frac{\cos 2nx}{4n^2-1}$.

Hence show that $\frac{1}{1.3} - \frac{1}{3.5} + \frac{1}{5.7} - \cdots - \infty = \frac{\pi - 2}{4}$ **(RTU 2019, 17,16)**

Q.20 Obtain the Fourier series for $f(x) = e^x$ in the interval $0 < x < 2\pi$

Q.21 Find a Fourier series to represent $x - x^2$ from $x = -\pi$ to $x = \pi$

Q.22 Find the Fourier series expansion for $f(x)$if $(x) = \begin{cases} -\pi, & -\pi < x < 0 \\ x, & 0 < x < \pi \end{cases}$. Deduce that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \cdots \infty = \frac{\pi^2}{8}$ **(RTU 16, 18, 19, 11 , 13)**

Q.23 Expand $f(x) = x \sin x$ in the range $0 < x < 2\pi$ as a Fourier series

Q.24 Expand $f(x) = \sqrt{1 - \cos x}, 0 < x < 2\pi$ in a Fourier series. Hence evaluate $\frac{1}{1.3} + \frac{1}{3.5} + \frac{1}{5.7} + \cdots$

Q.25 Find the Fourier series for $f(x) = |x|, -\pi < x < \pi$.
Hence show that $\frac{1}{1^2} + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots = \frac{\pi^2}{8}$ **(RTU 2010, 12, 18)**

Q.26 Expand $f(x) = x \sin x$ in the range $0 < x < 2\pi$ as a Fourier series.

Q.27 Prove Parseval's formula for f(x) in the range $-c < x < c$

Q.28 Find the Fourier series to represent $f(x)$, where $f(x) = \begin{cases} x & for\ 0 < x < 1 \\ 0 & for\ 1 < x < 2 \end{cases}$

Q.29 Obtain the Fourier series to represent the function $f(x) = x^2, -\pi < x < \pi$

Hence show that $\frac{1}{1^4} + \frac{1}{2^4} + \frac{1}{3^4} + \cdots .. = \frac{\pi^4}{90}$ using Parseval's theorem.

Q.30 Find the half range cosine series for the function $f(x) = 2x - 1$ in the interval $0 < x < 1$.

Q.31 Find the half range sine series for the function $f(x) = x$ in the interval $0 < x < 2$.

Q.32 Find the Fourier series to represent the function $f(x) = x^2, -a < x < a$

Q.33 Find half range cosine series for the function

$$f(x) = \begin{cases} kx, & 0 \le x \le l/2 \\ k(l - x), & l/2 \le x \le l \end{cases}$$

Q.34 Find the half range cosine series for $f(x) = 2x - 1, \quad 0 < x < 1$

Q.35 Find the half range sine series for $f(x) = x\, sinx, \quad 0 < x < \pi$

Q.36 Find the Fourier series to represent $f(x) = x - x^2 \ \ from\ x = -1\ to\ x = 1$.

Q.37 If the function $f(x)$ is defined by $f(x) = |\sin x|, -\pi < x < \pi$.

Q.38 If the function $f(x)$ is defined by $f(x) = |\cos x|, -\pi < x < \pi$. **(RTU 2019)**

Q.39 Find the Fourier series to represent $f(x) = x^3 \ , \ in \ \ 0 < x < 4$.

Q.40 Find the Fourier series to represent $f(x) = x - x^2 \ \ from\ x = -1\ to\ x = 1$.

Q.41 Find the half range sine series for the function $(x) = x(\pi - x) \ , \quad 0 < x < \pi$ **(RTU 2017)**

Q.42 Find the half range sine series for the function $(x) = \frac{l}{2} - x \ , \quad 0 < x < l$.

Q.43 Find the Fourier series for the function $f(x) = \pi x \ , \ -l < x < l$.

Q.44 Find half range sine series for the function $f(x) = 1\ in\ 0 < x < \pi$. Hence show that $\frac{\pi^2}{8} = 1 + \frac{1}{3^2} + \frac{1}{5^2} + \frac{1}{7^2} + \cdots$

Q.45 Find a series of sines and cosines of multiples of x which will represent the function $(x) = x + x^2 \ , -\pi < x < \pi$.

Hence show that $1 + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{4^2} + \cdots = \frac{\pi^2}{6}$

Q.46 Find the Fourier series expansion for $f(x) = \begin{cases} -\pi, -\pi < x < 0 \\ x, \ \ 0 < x < \pi \end{cases}$ **(RTU 2019)**

Q.47 Explain the expression of Fourier series for Even And Odd function.

Q.48 Find the half range cosine series for $f(x) = 2x - 4, \quad 0 < x < 4$

Q.49 Find the Fourier series for the function $f(x) = \pi x\ ,\ 0 < x < 4$

Q.50 Find the cosine series for the function

$$f(x) = e^x \quad in\ \ 0 < x < 2\pi.$$

Co 4: Understand the concept of limits, continuity and differentiability of functions of several variables. Analytical definition of partial derivative. Maxima and minima of functions of several variables Define gradient, divergence and curl of scalar and vector functions.

Q.1 Find the first and second partial derivatives of $z = x^3 + y^3 - 3axy$

Q.2 If $u = \log(x^3 + y^3 + z^3 - 3xyz)$, show that $\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)^2 u = -\frac{9}{(x+y+z)^2}$

Q.3 If $\theta = t^n e^{\frac{-r^2}{4t}}$, What value of n will make $\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2\frac{\partial\theta}{\partial r}\right) = \frac{\partial\theta}{\partial t}$

Q.4 If $u = f(r)$and $x = r\cos\theta\ , y = r\sin\theta$,Prove that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = f''(r) + \frac{1}{r}f'(r)$ **(RTU 2016, 17)**

Q.5 If $x^x y^y z^z = c$, Show that at $x = y = z, \frac{\partial^2 z}{\partial x \partial y} = -(x\log ex)^{-1}$ **(RTU 2009, 12, 15)**

Q.6 If $u = x^2 tan^{-1}\left(\frac{y}{x}\right) - y^2 tan^{-1}\left(\frac{x}{y}\right)$, Show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{x^2 - y^2}{x^2 + y^2}$ and $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$

Q.7 If $u = x^y$, Show that $\frac{\partial^3 u}{\partial x^2 \partial y} = \frac{\partial^3 u}{\partial x \partial y \partial x}$

Q.8 If $u = e^{xyz}$ then find the value of $\frac{\partial^3 u}{\partial x \partial y \partial z}$ **(RTU 2020, 2016)**

Q.9 If $z(x + y) = x^2 + y^2$, Show that $\left(\frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)^2 = 4\left(1 - \frac{\partial z}{\partial x} - \frac{\partial z}{\partial y}\right)$

Q.10 Find the Curl of the vector $xyz\hat{\imath} + 3x^2y\hat{\jmath} + (xz^2 - y^2z)\hat{k}$

Q.11 Find the Tangent plane and normal line to the surface $f(x, y, z) = x^2 + y^2 + z = 0$ at the point $P(1,2,4)$. **(RTU 2018)**

Q.12 Find the Tangent plane and normal line to the surface $f(x, y, z) = x^2 + 2y^2 + 3z^2 = 12$ at the point $P(1,2,-1)$.

Q.13 If $u = f(y - z, z - x, x - y)$, prove that $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} + \frac{\partial u}{\partial z} = 0$ **(RTU 2018, 16)**

Q.14 Find the directional derivative of $f(x, y)$ at the given point P in the direction indicated by the angle $\theta\ for\ f(x, y) = x^2y^3 - y^4. P = (2,1), \theta = \frac{\pi}{4}$

Q.15 If $f(x, y) = sinx + e^{xy}$, then find $\nabla f(x, y)$.

Q.16 Prove that (i) $\nabla^2\left(\frac{1}{r}\right) = 0$ (ii) $\nabla.\left[r\nabla\left(\frac{1}{r^3}\right)\right] = 3r^{-4}$

Q.17 Determine the maximum and minimum values of the function

$$f(x) = 12x^5 - 45x^4 + 40x^3 - 20.$$

Q.18 Discuss the maxima or minima of $f = (x^2 + y^2)e^{6x+2y}$

Q.19 Mention the condition for the vector field A to be solenoidal. **(EC 2020 gate)**

Q.20 Find the partial derivative of the function $f(x, y, z) = e^{1-x\cos z} + xze^{-\frac{1}{1+y}}$ **(EC 2020 gate)**

Q.21 Discuss the maximum or minimum of the following function $u = x^4 + 2x^2y - x^2 + 3y^2$ **(RTU 2009, 13, 17 , 18)**

Q.22 Discuss the maximum or minimum of the following function $u = x^3y^2 - x^4y^2 - x^3y^3$

Q.23 Examine the following function $f(x, y)$ for extreme : $\sin x + \sin y + \sin(x + y)$ **(GATE 2005)**

Q.24 Divide 24 into three parts such that the continued product of the first, square of second and the cube of third may be maximum.

Q.25 Divide an iron rod of length 48cm into three parts such that the continued product of the first, square of second and the cube of third may be maximum.

Q.26 Find the values of x, y, z for which $f(x, y, z) = \frac{5xyz}{x+2y+4z}$ is a maximum subject to the condition $xyz = 8$

Q.27 Find the maxima and minima of $u = x^2 + y^2 + z^2$ subject the conditions $ax^2 + by^2 + cz^2 = 1$ and $lx + my + nz = 0$

Q.28 A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimensions of the box requiring least material for its construction. **(RTU 2010)**

Q.29 Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq.cm

Q.30 A rectangular box, open at the top, is to have given capacity. Find the dimensions of the box requiring least material for its construction. **(RTU 2011, 2012)**

Q.31 Prove that of all rectangular parallelepiped of given surface, cube has the maximum volume

OR

Show that the rectangular parallelepiped of maximum volume that can be inscribed in a sphere is a cube.

OR

Prove that the rectangular parallelepiped of maximum volume that can be inscribed in a sphere is a cube.

Q.32 Show that the rectangular solid of maximum volume that can be inscribed in a sphere is a cube. **(RTU 2008, 2010, 2015, 2017)**

Q.33 Find the volume of the greatest rectangular parallelepiped that can be inscribed in the ellipsoid whose equation is $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ **(RTU 2016, 17, 19)**

Q.34 Find the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid of revolution $4x^2 + 4y^2 + 9z^2 = 36$

Q.35 Find the maximum and minimum distances of the point (3, 4, 12) from the sphere $x^2 + y^2 + z^2 = 1$

Q.36 Obtain the extreme points of $f = x_1^3 + x_2^3 + 2x_1^2 + 4x_2^2 + 6$

Q.37 Obtain the extreme points of $f = 20x_1 + 26x_2 + 4x_1x_2 - 4x_1^2 - 3x_2^2$.

Q.38 Find the value of line integral $\int_1^2 2xy^2dx + 2yx^2dy + dz$ along a path joining the origin (0,0,0) and the point (1,1,1). **(GATE EE 2016)**

Q.39 Find the line integral of function F=yzi in the counterclockwise direction, along the circle $x^2 + y^2 = 1$ at z=1, (**GATE EE 2014)**

Q.40 If $f = 2x^3 + 3y^2 + 4z, find \int gradf.dr$ along the path $(-3,-3,2)\ to\ (2,-3,2)\ to\ (2,6,2)\ to\ (2,6,-1)$.(EE 2019 gate)

Q.41 Obtain the extreme points of $f(x_1, x_2, x_3) = x_1 + x_2 + x_3$; sub. to $x_1^2 + x_2^2 + x_3^2 = 1$.Find whether the points is maxima or minima.

Q.42 Solve the following Min $f(X) = x_1x_2x_3$ sub to g≡ $\{x_1 + x_2 + x_3 - 1 = 0\}$

Q.43 Find gradient of $f(x,y) = 5xy^2 - 4x^3y$ t the point P(1,2) (**RTU-2018)**

Q.44 Discuss the necessary and sufficient condition of function of two variable$f(x,y)$ to be maximization or minimization.

Q.45 Define Divergence and curl of a vector point function.

Q.46 Write the statement of following

(a) Green theorem in a plane (**RTU 2008)**
(b) Stokes theorem
(c) Gauss divergence theorem

Q.47 Show that the vector field $\vec{F} = (y+z)i + (z+x)j + (x+y)k$ is irrotational. If so find scalar potential. (**RTU 2020)**

Q.48 Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2,-1,2)$

Q.49 Find the directional derivative of $\emptyset(x,y,z) = xy + yz + zx$ in the direction of the vector $2i + 3j + 6k$ at the point (3,1,2). (**RTU 2019,10)**

Q.50 Define Gradient of scalar point function. Also explain its physical interpretation.