JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem – I Year & II Sem

Subject –Engineering Mathematics-II

Unit – I

Presented by – (Dr.Vishal Saxena, Associate Professor)

VISION AND MISSION OF INSTITUTE

VISION OF INSTITUTE

To became a renowned centre of outcome based learning and work towards academic professional, cultural and social enrichment of the lives of individuals and communities .

MISSION OF INSTITUTE

- Focus on evaluation of learning, outcomes and motivate students to research aptitude by project based learning.
- Identify based on informed perception of Indian, regional and global needs, the area of focus and provide platform to gain knowledge and solutions.
- Offer opportunities for interaction between academic and industry .
- Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge.

CONTENTS (TO BE COVERED)

MATRICES

A set of mn numbers arranged in a rectangular array of m horizontal lines (rows) & n vertical lines (Columns) is known as matrix of order $m \times n$. These numbers are called elements being enclosed in brackets [ ] or || ||. In compact form the matrix is represented as $A = [a_{ij}]_{m \times n}$ where $i = 1, 2, \ldots, m$ and $j = 1, 2, \ldots n$.

It is usually written as

$$A = [a_{ij}] = \begin{bmatrix} a_{11} & a_{12} & \cdots & a_{1n} \\ a_{21} & a_{22} & \cdots & a_{2n} \\ a_{31} & a_{32} & \cdots & a_{3n} \\ \vdots & \vdots & & \\ a_{m1} & a_{m2} & \cdots & a_{mn} \end{bmatrix}_{m \times n}$$

Inverse of a matrix

Let A be a non singular matrix of order n. If there exists another non singular matrix B of same order as A such that $AB = BA = I_n$ then B is called inverse of A and is denoted by A^{-1}. Here I_n is an identity matrix of order n. We find the inverse of a matrix by elementary transformation

Q. Find the inverse by elementary row transformation of a matrix when

$$A = \begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix}$$

Sol: we have $A = IA$

So. $$\begin{bmatrix} 0 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 3 & 1 & 1 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$R_3 \rightarrow R_3 - 3R_1$$

$$\begin{bmatrix} 1 & 2 & 3 \\ 0 & 1 & 2 \\ 0 & -5 & -8 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & -3 & 1 \end{bmatrix} \cdot A$$

$R_1 \to R_1 - 2R_2 \quad , \quad R_3 \to R_3 + 5R_2$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 2 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5 & -3 & 1 \end{bmatrix} \cdot A$$

$R_3 \to \frac{R_3}{2}$

$$\begin{bmatrix} 1 & 0 & -1 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} -2 & 1 & 0 \\ 1 & 0 & 0 \\ 5/2 & -3/2 & +1/2 \end{bmatrix} \cdot A$$

$R_1 \to R_1 + R_3 \quad , \quad R_2 \to R_2 - 2R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix} \cdot A$$

So $A^{-1} = \begin{bmatrix} 1/2 & -1/2 & 1/2 \\ -4 & 3 & -1 \\ 5/2 & -3/2 & 1/2 \end{bmatrix}$

Q. Find the Inverse by elementary row transformation of a matrix when

$$A = \begin{bmatrix} 3 & 1 & 5 \\ 1 & 3 & 2 \\ 5 & -2 & 4 \end{bmatrix}$$

Sol: we have $A = IA$

$$\begin{bmatrix} 3 & 1 & 5 \\ 1 & 3 & 2 \\ 5 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$R_1 \leftrightarrow R_2$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 3 & 1 & 5 \\ 5 & -2 & 4 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \cdot A$$

$$R_2 \to R_2 - 3R_1, \quad R_3 \to R_3 - 5R_1$$

$$\begin{bmatrix} 1 & 3 & 2 \\ 0 & -8 & -1 \\ 0 & -17 & -6 \end{bmatrix} = \begin{bmatrix} 0 & 1 & 0 \\ 1 & -3 & 0 \\ 0 & -5 & 1 \end{bmatrix} \cdot A$$

$$R_1 \to R_1 + 3/8\, R_2 \quad , \quad R_3 \to R_3 + \frac{17}{8} R_2$$

$$\begin{bmatrix} 1 & 0 & \frac{13}{8} \\ 0 & -8 & -1 \\ 0 & 0 & \frac{-31}{8} \end{bmatrix} = \begin{bmatrix} \frac{3}{8} & -1/8 & 0 \\ 1 & -3 & 0 \\ -17/8 & 11/8 & 1 \end{bmatrix} \cdot A$$

$$R_2 \to -\frac{R_2}{8} \quad , \quad R_3 \to -\frac{8}{31}$$

$$\begin{bmatrix} 1 & 0 & 13/8 \\ 0 & 1 & 1/8 \\ 0 & 0 & 1 \end{bmatrix} = \begin{bmatrix} 3/8 & -1/8 & 0 \\ -1/8 & 3/8 & 0 \\ 17/31 & -11/31 & \frac{-8}{31} \end{bmatrix} \cdot A$$

$R_1 \to R_1 - \frac{13}{8} R_3 , \quad R_2 \to R_2 - \frac{1}{8} R_3$

$$\begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{bmatrix} = \begin{bmatrix} -16/31 & 14/31 & 13/31 \\ -6/31 & 13/31 & 1/31 \\ +17/31 & -11/31 & \frac{-8}{31} \end{bmatrix} \cdot A$$

So $$A^{-1} = \frac{1}{31} \begin{bmatrix} -16 & 14 & 13 \\ -6 & 13 & 1 \\ 17 & -11 & -8 \end{bmatrix}$$

Ex:1. If $A = \begin{bmatrix} 1 & 3 & 1 \\ 1 & 4 & 2 \\ -1 & -2 & 3 \end{bmatrix}$ find A^{-1}.

Ex.2. If $A = \begin{bmatrix} -3 & 6 & -11 \\ 3 & -4 & 6 \\ 4 & -8 & 13 \end{bmatrix}$ find A^{-1}.

Ans: (1) $A^{-1} = \frac{1}{3}\begin{bmatrix} 16 & -11 & 2 \\ -5 & 4 & -1 \\ 2 & -1 & 1 \end{bmatrix}$ (2) $A^{-1} = \frac{1}{10}\begin{bmatrix} -4 & 10 & -8 \\ -15 & 5 & -15 \\ -8 & 0 & -6 \end{bmatrix}$

Refrences

1.Advanced Engineering Mathematics by Prof.ERWIN KREYSZIG (Ch.10,page no.557-580)
2. Advanced Engineering Mathematics by Prof.H.K Dass (Ch.14,page no.851-875)
3.Advanced Engineering Mathematics by B.V RAMANA (Ch.20,pageno.20.1.20.5)
4.NPTEL Lectures available on

http://www.infocobuild.com/education/audio-video-courses/mathematics/TransformTechniquesForEngineers-IIT-Madras/lecture-47.html

JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Thank you!