JECRC Foundation

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

JAIPUR ENGINEERING COLLEGE AND RESEARCH CENTRE

Year & Sem. – B. Tech I year, Sem.-I

Subject –Engineering Mathematics

Unit – 3

Presented by – Dr. Ruchi Mathur & Dr. Tripati Gupta

Designation - Associate Professor

Department - Mathematics

VISION OF INSTITUTE

To become a renowned centre of outcome based learning, and work towards academic, professional, cultural and social enrichment of the lives of individuals and communities.

MISSION OF INSTITUTE

❖**Focus on evaluation of learning outcomes and motivate students to inculcate research aptitude by project based learning.**

❖**Identify, based on informed perception of Indian, regional and global needs, the areas of focus and provide platform to gain knowledge and solutions.**

❖**Offer opportunities for interaction between academia and industry.**

❖**Develop human potential to its fullest extent so that intellectually capable and imaginatively gifted leaders may emerge in a range of professions.**

Engineering Mathematics: Course Outcomes

Students will be able to:

CO1. Understand fundamental concepts of improper integrals, beta and gamma functions and their properties. Evaluation of Multiple Integrals in finding the areas, volume enclosed by several curves after its tracing and its application in proving certain theorems.

CO2. Interpret the concept of a series as the sum of a sequence, and use the sequence of partial sums to determine convergence of a series. Understand derivatives of power, trigonometric, exponential, hyperbolic, logarithmic series.

CO3. Recognize odd, even and periodic function and express them in Fourier series using Euler's formulae.

CO4. Understand the concept of limits, continuity and differentiability of functions of several variables. Analytical definition of partial derivative. Maxima and minima of functions of several variables Define gradient, divergence and curl of scalar and vector functions.

Change of Interval

Suppose we want to represent the function f(x) defined in the closed interval $[-c, c]$ by a Fourier Series, c being any positive real. We consider this integral as a result of elongating in the interval$[-\pi, \pi]$. The interval $[-\pi, \pi]$ can be obtained from interval $[-c, c]$ by transformation. The interval $-c < x < c$ is transformed into the interval $-\pi < x < \pi$ by the transformation $z = \frac{\pi x}{c}$.

Then $f(z) = f\left(\frac{\pi x}{c}\right)$ say

Let $f(z) = \frac{a_0}{2} + \sum_{n=1}^{\infty}(a_n cos\, nz + b_n \sin nz)$ where

$$a_0 = \frac{1}{\pi}\int_{-\pi}^{\pi} f(z)dz \ldots\ldots..$$

$$a_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(z)Cos\, nz\, dz \ldots\ldots..; \quad n = 1, 2, \ldots \quad \text{and}$$

$$b_n = \frac{1}{\pi}\int_{-\pi}^{\pi} f(z)Sin\, nz\, dz \ldots\ldots..; n = 1,2,3 \ldots\ldots..$$

Applying the transformation $z = \frac{\pi x}{c}$ so that $dz = \frac{\pi dx}{c}$, we get

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n cos\, \frac{n\pi x}{c} + b_n \sin\frac{n\pi x}{c}\right)\ldots..(1) \text{ where}$$

$$a_0 = \frac{1}{c}\int_{-c}^{c} f(x)dx \ldots\ldots(2)$$

$$a_n = \frac{1}{c}\int_{-c}^{c} f(x) Cos\, \frac{n\pi x}{c} dx \ldots\ldots\ldots(3); \;\; n = 1, 2, \ldots \;\; \text{and}$$

$$b_n = \frac{1}{c}\int_{-c}^{c} f(x) Sin\, \frac{n\pi x}{c}\, dx \ldots\ldots\ldots(4); n = 1,2,3 \ldots\ldots\ldots$$

If the function f(x) is defined in the interval 0< x< 2c then (2), (3) and (4) changes to

$$a_0 = \frac{1}{c}\int_{0}^{2c} f(x)dx \ldots\ldots$$

$$a_n = \frac{1}{c}\int_{0}^{2c} f(x) Cos\, \frac{n\pi x}{c} dx \ldots\ldots\ldots; \;\; n = 1, 2, \ldots \;\; \text{and}$$

$$b_n = \frac{1}{c}\int_{0}^{2c} f(x) Sin\, \frac{n\pi x}{c}\, dx \ldots\ldots\ldots; n = 1,2,3 \ldots\ldots\ldots$$

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Example: Find the Fourier series to represent f(x) = x – x^2 from x= -1 to x = 1.

Solution : Here c = 1, therefore the fourier series is

$$x - x^2 = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{c} + b_n \sin\frac{n\pi x}{c}\right)$$

where $a_0 = \frac{1}{c}\int_{-c}^{c} f(x)dx$

$$a_0 = \frac{1}{1}\int_{-1}^{1}(x - x^2)dx = \left[\frac{x^2}{2} - \frac{x^3}{3}\right]_{-1}^{1} = -\frac{2}{3}$$

$$a_n = \frac{1}{c}\int_{-c}^{c} f(x)\cos\frac{n\pi x}{c}dx$$

$$a_n = \frac{1}{1}\int_{-1}^{1}(x - x^2)\cos n\pi x\, dx = \int_{-1}^{1} x^2 \cos n\pi x\, dx \qquad \{ \text{ as } \int_{-1}^{1} x \cos n\pi x\, dx = 0; being\ odd$$

$$= -2\left[\frac{x^2}{n\pi}\sin n\pi x + \frac{2x}{n^2\pi^2}\cos n\pi x - \frac{2}{n^3\pi^3}\sin n\pi x\right]_0^1 = -2\left[\frac{2}{n^2\pi^2}\cos n\pi\right] = -\frac{4}{n^2\pi^2}(-1)^n$$

$$b_n = \frac{1}{1}\int_{-1}^{1}(x - x^2)\sin n\pi x\, dx = \int_{-1}^{1} x\ \sin n\pi x\, dx \qquad \{ \text{ as } \int_{-1}^{1} x^2 \sin n\pi x\, dx = 0; being\ odd$$

$$= 2\left[\frac{-x}{n\pi}\cos n\pi x + \frac{1}{n^2\pi^2}\sin n\pi x\right]_0^1 = -2\left[\frac{\cos n\pi}{n\pi}\right] = -\frac{2}{n\pi}(-1)^n$$

Hence the required fourier series is

$$x - x^2 = -\frac{1}{3} + \frac{4}{\pi^2}\left[\frac{\cos \pi x}{1^2} - \frac{\cos 2\pi x}{2^2} + \frac{\cos 3\pi x}{3^2} + \cdots .\right] + \frac{2}{\pi}\left[\frac{\sin \pi x}{1} - \frac{\sin 2\pi x}{2} + \frac{\sin 3\pi x}{3} + \cdots\right]$$

Example : In the range (0, 2l) , f(x) is defined as $f(x) = \begin{Bmatrix} \mathbf{0}, & \mathbf{0 \le x \le l} \\ \mathbf{a}, & \mathbf{l \le x \le 2l} \end{Bmatrix}$. Expand f(x) in fourier series.

Solution : Here 2c = 2l and c= l

Therefore , the fourier series is

$$f(x) = \frac{a_0}{2} + \sum_{n=1}^{\infty}\left(a_n \cos\frac{n\pi x}{l} + b_n \sin\frac{n\pi x}{l}\right), \quad 0 < x < 2l$$

Where

$$a_0 = \frac{1}{l}\int_0^{2l} f(x)dx$$

$$\frac{1}{l}\left[\int_0^{l} 0dx + \int_l^{2l} a\, dx\right] = \frac{a}{l}[x]_l^{2l} = a$$

$$a_n = \frac{1}{l}\int_0^{2l} f(x)\cos\frac{n\pi x}{l}dx$$

$$\frac{1}{l}\left[\int_0^{l} 0\cos\frac{n\pi x}{l}dx + \int_l^{2l} a\cos\frac{n\pi x}{l}\, dx\right] = \frac{a}{l}\frac{l}{n\pi}\left[\frac{\sin n\pi x}{l}\right]_l^{2l} = 0$$

$$b_n = \frac{1}{l}\int_0^{2l} f(x)\sin\frac{n\pi x}{l}dx$$

$$\frac{1}{l}\left[\int_0^{l} 0\sin\frac{n\pi x}{l}dx + \int_l^{2l} a\sin\frac{n\pi x}{l}\ dx\right] = \frac{a}{l}\frac{l}{n\pi}\left[\frac{-cos\ n\pi x}{l}\right]_l^{2l} = \frac{a}{n\pi}[-cos2n\pi + \cos n\pi]$$

$$= \frac{a}{n\pi}[-1 + (-1)^n]$$

Therefore Fourier series is

$$f(x)\ = \frac{a}{2} + \sum_{n=1}^{\infty} b_n \sin\frac{n\pi x}{l}\ = \frac{a}{2} + \sum_{n=1}^{\infty}\frac{a}{n\pi}[-1 + (-1)^n]\sin\frac{n\pi x}{l}$$

$$f(x) = \frac{a}{2} + \frac{a}{\pi}\left[\frac{-2}{1}\sin\frac{\pi x}{l} - \frac{2}{3}\sin\frac{3\pi x}{l} - \frac{2}{5}\sin\frac{5\pi x}{l} - \ \dots\dots\dots.\right]$$

Suggested links from NPTEL & other Platforms:

- Advanced Engineering Mathematics: Erwin Kreyszig, Wiley plus publication
- https://www.youtube.com/watch?v=LGxE_yZYigI (NPTEL-NOC IITM)
- https://www.youtube.com/watch?v=SHx32HD8vDI

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Thank you!