

## UNIT - I

### Introduction to heat transfer

\* Heat Transfer can be defined as the transmission of energy from one region to another region due to temperature difference.

### Modes of Heat Transfer

\* Conduction

\* Convection

\* Radiation

⇒ Heat Conduction is a mechanism of heat transfer from a region of high temperature to a region of low temperature within a medium. [solid, liquid & Gases] or between difference medium in direct physical contact

⇒ In Conduction Energy exchanges takes place by the kinematic motion or direct impact of molecules. Pure Conduction is found out in Solids.



⇒ Convection : Convection is a process of Heat transfer that will occur b/w a solid surface and a fluid medium, when they are at different temperatures.

⇒ Convection is possible only in the presence of fluid medium.

⇒ Radiation : the Heat transfer from one body to another body without any transmitting medium is known as "Radiation"

⇒ It is an "electromagnetic wave phenomena"

### Basic laws of Heat Transfer

\* First law of Thermodynamics : It states that when system undergoes a cyclic process, net heat transfer equal net work transfer

$$\oint Q = \oint W$$

\* Second law of Thermodynamics :

Kelvin's Planck statement : It is impossible to construct an engine working on an cyclic process when converts all the heat energy supplied to it into an equivalent amount of useful work.



Classius Statement : It stated that heat can flow from hot body to Cold body without any external aid, but heat can't be flow from Cold body to hot body without any external aid.

### Law of Conservation of Mass

⇒ This law is used to determine the parameters of flow.

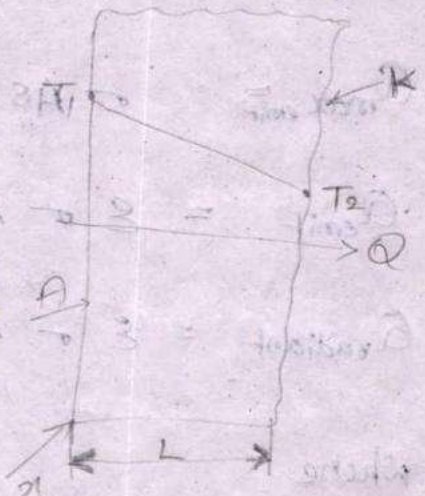
### Fourier law of Equations

⇒ Rate of heat conduction is proportional to the area measured normal to the direction of heat flow and to the temperature gradient in that direction

$$Q \propto -A \frac{dT}{dx}$$

$$Q = -KA \frac{dT}{dx}$$

$$Q = \frac{-KA (T_1 - T_2)}{L}$$



where

$K$  = Thermal conductivity -  $W/mK$

$A$  = Area -  $m^2$

$\frac{dT}{dx}$  = Temperature gradient  $Kel/m$



The negative sign indicates that the heat flow is in a direction along which there is a decrease in temperature.

### Newton's Law of cooling or Convection

⇒ Heat transfer from the moving fluid to a solid surface is given by an equation

$$Q = h A_s (T_{\text{surface}} - T_{\text{surrounding}}) \\ = h A (T_w - T_\infty)$$

where,  $h$  = local heat transfer coefficient -  $\text{W/m}^2\text{K}$

### Radiation

⇒ Maximum rate of radiation <sup>that</sup> can be emitted by the surface.

$$Q_{\text{max emit}} = \sigma A_s T_s^4$$

$$Q_{\text{emit}} = \epsilon \sigma A_s T_s^4$$

$$Q_{\text{radiant}} = \epsilon \sigma A_s (T_s^4 - T_{\text{sur}}^4)$$

where

$\sigma$  = stephen bolzmen constant  $5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$

$T_s$  = surface temperature

$T_{\text{sur}}$  = surrounding temp

$\epsilon$  =  $\epsilon_{\text{psal}}$  (emissivity)



①  
Sol

Given data

$$A = 6 \times 8 = 48 \text{ m}^2$$

$$L = 0.25 \text{ m}$$

$$k = 5.08$$

$$T_1 = 4^\circ\text{C}$$

$$T_2 = 15^\circ\text{C}$$

$$\text{time} = 10 \text{ hrs}$$

$$\text{Amount} = \$ 0.08/\text{kwh.}$$

$$a) Q = -kA \frac{dT}{dx}$$

$$= -kA \frac{T_1 - T_2}{L}$$

$$= \frac{-5.08 \times 48 \times (4 - 15)}{0.25} = 10728.96 \text{ W}$$

$$= 10.728 \text{ kW.}$$

$$b) Q_t = Q \times \text{time}$$

$$= 10.728 \times 10$$

$$= 107.28 \text{ kwh}$$

$$\text{Cost} = \text{Amount} \times Q_t$$

$$= 0.08 \times 107.28 \text{ kwh}$$

$$= \$ 8.58$$

②  
Sol

Given data

$$A = 2 \times 3 = 6 \text{ m}^2$$



2  
801

Given data

$$L = 2\text{m}$$

$$D = 0.003\text{m}$$

$$T_{\infty} = 15^{\circ}\text{C}$$

$$T_w = 152^{\circ}\text{C}$$

$$V = 60\text{V}$$

$$I = 1.5\text{A}$$

$$h = ?$$

$$Q = hA(T_w - T_{\infty})$$

$$Q = V \times I = 60 \times 1.5 = 90\text{W}$$

$$A = \pi DL = \pi \times 0.003 \times 2 = 0.01885$$

$$90 = h \times 0.01885 \times (152 - 15)$$

$$h = 34.9 \text{ W/m}^2\text{K}$$

3

801

Given data

$$\text{Winter } T_{\text{sur}} = 10^{\circ}\text{C} + 273 = 283\text{K}$$

$$\text{Summer } T_{\text{sur}} = 25 + 273 = 298\text{K}$$

$$Q = \epsilon \sigma A_s (T_c^4 - T_{\text{sur}}^4)$$

$$A_s = 1.4\text{m}^2$$

$$T_s = 30^{\circ}\text{C} + 273 = 303\text{K}$$

Emmissivity of a person  $\epsilon = 0.95$

$$\sigma = 5.67 \times 10^{-8} \text{ W/m}^2\text{K}^4$$

$$Q_{\text{rad, summer}} = 0.95 \times 5.67 \times 10^{-8} \times 1.4 \times (303^4 - 298^4)$$



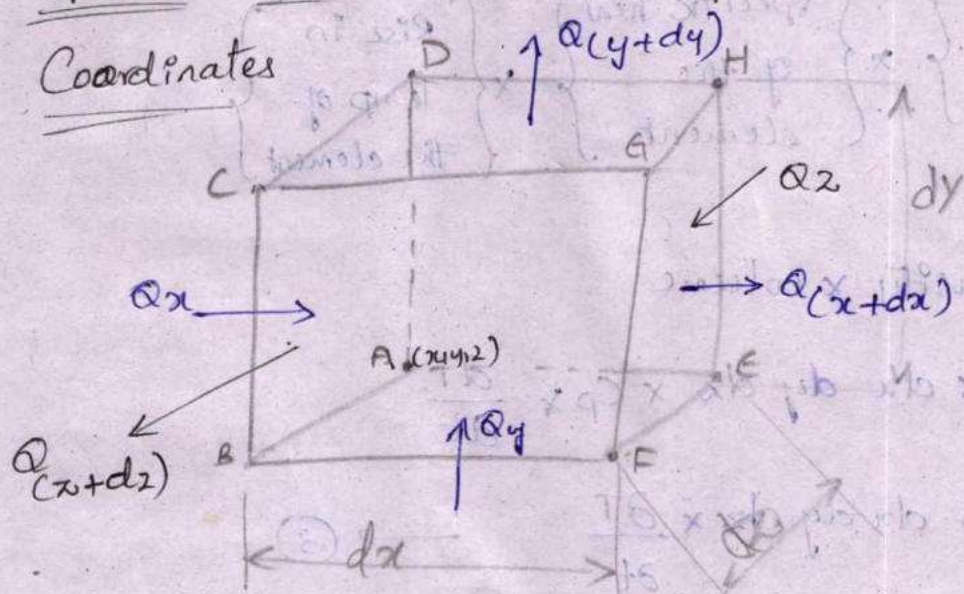
$$= 40.9 \text{ W}$$

$$Q_{\text{rad, winter}} = 0.95 \times 5.67 \times 10^{-8} \times 1.4 (303^4 - 283^4) = 152 \text{ W.}$$

5

## General Heat conduction Equation in Cartesian Coordinates

Anticlock



Net heat conducted into element from

Consider a small rectangular element of sides  $dx, dy, dz$  as shown in fig.

The energy balance of this rectangular element is obtained from first law of Thermodynamics.

Net heat conducted in to element from all the coordinate directions

+

Heat generated with in the element

②

=

Heat stored in the element.

③

①



⇒ Heat generated within the element

$$Q_g = \dot{q} \, dx \, dy \, dz \Rightarrow (2)$$

⇒ Heat stored in the element (3) ⇒

$$\left\{ \begin{array}{l} \text{mass of} \\ \text{the} \\ \text{element} \end{array} \right\} \times \left\{ \begin{array}{l} \text{specific heat} \\ \text{of the} \\ \text{element} \end{array} \right\} \times \left\{ \begin{array}{l} \text{Rise in} \\ \text{Temp of} \\ \text{the element} \end{array} \right\}$$

=  $\rho$  density  $\times$  volume

$$= \rho \times dx \, dy \, dz \times C_p \times \frac{\partial T}{\partial t}$$

$$= \rho C_p \cdot dx \, dy \, dz \times \frac{\partial T}{\partial t} \longrightarrow (3)$$

⇒ Net Heat conducted into the element in X-direction

Let  $q_x$  be the heat flux in the direction of face in

a A, B, C, D and  $q_{x+dx}$  be the heat flux

in a direction of face E, F, G, H.

The rate of heat flow into the element in x-direction through the face ABCD

$$Q_x = q_x \, dy \, dz$$

$$= -k_x \frac{\partial T}{\partial x} \, dy \, dz \longrightarrow (a)$$

The rate of heat flow out of the element in x-direction



through the face EFGH

$$Q_{x+dx} = Q_x + \frac{\partial}{\partial x}(Q_x) dx \longrightarrow (b)$$

Net heat conducted into the element in x-direction

$$Q_x - Q_{x+dx} = Q_x - \left[ Q_x + \frac{d}{dx}(Q_x) dx \right]$$

$$= - \frac{d}{dx} \left( -k_x \frac{dT}{dx} dy dz \right) dx$$

$$= \frac{d^2 T}{dx^2} k_x dx dy dz$$

$$= k_x dx dy dz \frac{d^2 T}{dx^2}$$

Similarly,

$$Q_y - Q_{y+dy} = k_y dx dy dz \frac{d^2 T}{dy^2}$$

$$Q_z - Q_{z+dz} = k_z dx dy dz \frac{d^2 T}{dz^2}$$

Considering the material is isotropic

$$k = k_x = k_y = k_z$$

Net heat conducted into the element, from all the coordinate directions

$$\left( \frac{d^2 T}{dx^2} + \frac{d^2 T}{dy^2} + \frac{d^2 T}{dz^2} \right) k \cdot dx dy dz \longrightarrow (1)$$



$$\left[ \left( \frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} \right) k + \dot{q} \right] dx dy dz = \rho c_p \frac{dT}{dt} dx dy dz \quad (1) \quad (2)$$

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = \frac{\rho c_p}{k} \frac{\partial T}{\partial t}$$

$$= \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

$$\left( \because \alpha = \frac{k}{\rho c_p} = \text{thermal diffusivity} \right)$$

Case (i) : No Heat source

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$

Case (ii) : Steady state Conditions

$$\frac{\partial^2 T}{\partial x^2} + \frac{\partial^2 T}{\partial y^2} + \frac{\partial^2 T}{\partial z^2} + \frac{\dot{q}}{k} = 0$$

$$\nabla^2 T + \frac{\dot{q}}{k} = 0$$

Poisson's equation

Case (iii) : One dimensional steady state heat conduction.

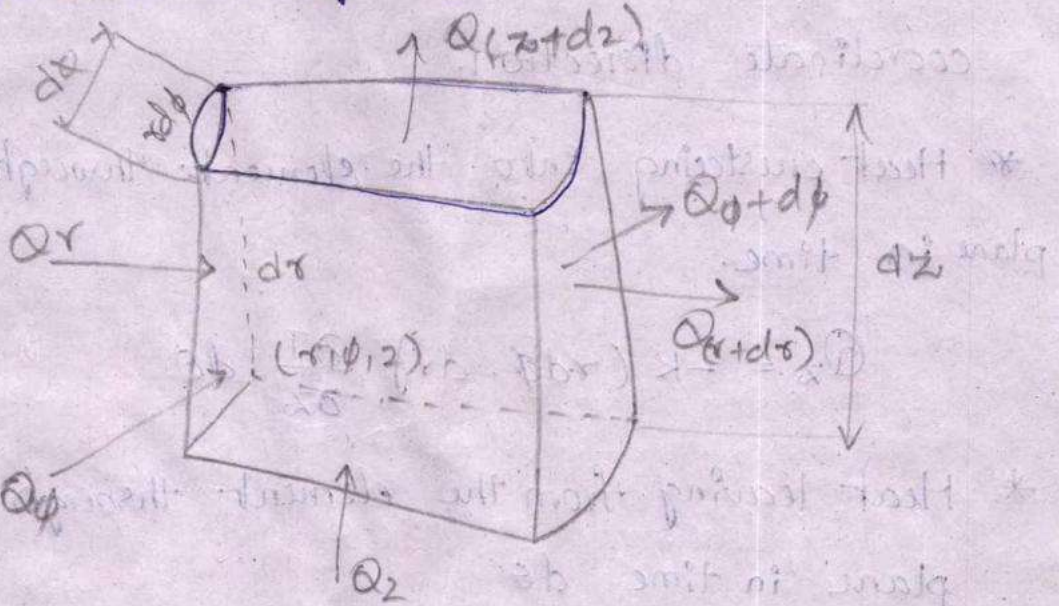
$$\frac{\partial^2 T}{\partial x^2} + \frac{\dot{q}}{k} = 0$$

Case (iv) unsteady one dimensional without Internal Heat generation.  $\frac{\partial^2 T}{\partial x^2} = \frac{1}{\alpha} \frac{\partial T}{\partial t}$



## General Heat Conduction equation in Cylindrical coordinates

The general heat conduction in Cartesian coordinate derived in the previous section is used for solids with rectangular boundaries like square, cube, slab etc. But Cartesian coordinate system is not applicable for, solid, cylinders, cones, spheres etc.



Consider a small cylindrical element of sides  $dr$ ,  $d\phi$ , and  $dz$  as shown in fig.

⇒ The volume of the element  $dv = r d\phi \cdot dr \cdot dz$

⇒ let us assume that thermal conductivity ( $k$ ), specific heat ( $c_p$ ) and density ( $\rho$ ) are constant.

⇒ The energy balance of this cylinder is first law of Thermodynamics.

Net heat conducted into the element from all the coordinate directions  $(\neq)$  Heat generated within the element = Heat stored in the element.



⇒ Heat Generated within the element

$$Q = \dot{q}(r d\phi dr dz) d\theta$$

⇒ Heat stored in the element

$$\int \rho \times r d\phi dr dz \times c_p \cdot \frac{\partial T}{\partial \theta} \cdot d\theta$$

⇒ Net heat conducted into element from all the coordinate direction.

\* Heat entering into the element through  $r$  &  $\phi$  plane in time.

$$Q_z = -k (r d\phi \cdot dr) \frac{\partial T}{\partial z} \cdot d\theta$$

\* Heat leaving from the element through  $r$  &  $\phi$  plane in time  $d\theta$

$$Q_{z+dz} = Q_z + \frac{\partial}{\partial z} (Q_z) dz$$

⇒ Net heat conducted into the element through  $r$  &  $\phi$  plane in time  $d\theta$ .

$$= Q_z - Q_{z+dz}$$

$$= -\frac{\partial}{\partial z} (Q_z) dz$$

$$= -\frac{\partial}{\partial z} \left[ k (r d\phi dr) \left( \frac{\partial T}{\partial z} \right) d\theta \right] dz$$

$$= -k (r \cdot d\phi dr dz) \frac{\partial^2 T}{\partial z^2} \cdot d\theta$$



Similarly,

$$= Q_r - Q_{r+dr}$$

$$= d\theta \cdot K (dr \cdot r d\phi \cdot dz) \left( \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right)$$

Similarly,

$$= Q_\phi - Q_{\phi+d\phi}$$

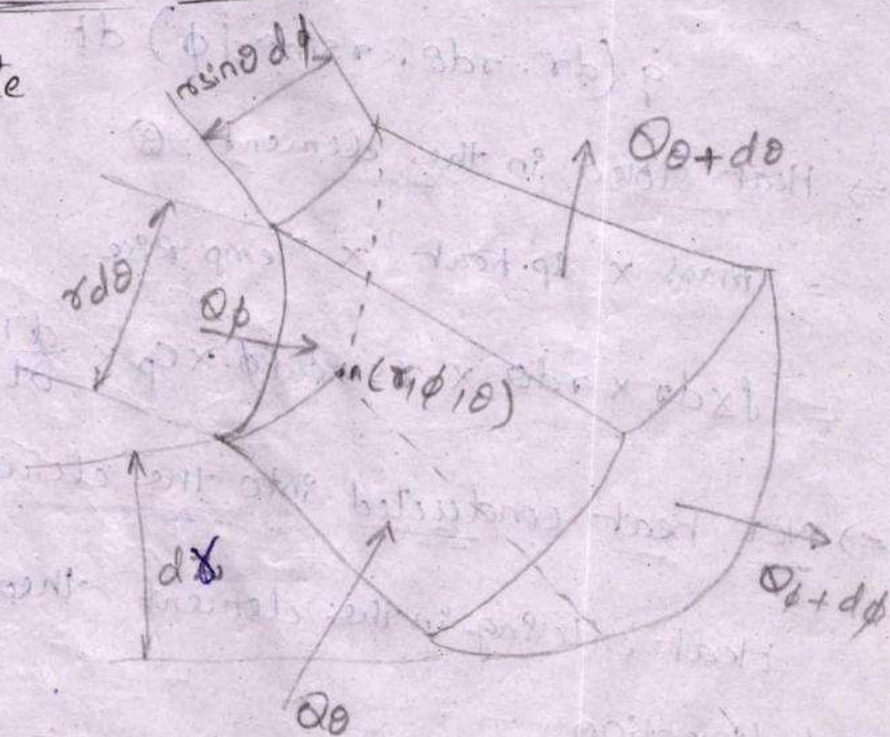
$$= K \left[ \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} \right] (r d\phi \cdot dr \cdot dz) d\phi$$

Adding equation.

$$K (dr \cdot r d\phi \cdot dz)$$

General Heat Conduction Equation Spherical.

Coordinate



Consider a small elemental Volume having the

co-ordinates  $r, \phi, \theta$

$\Rightarrow$  The volume of the element  $dV = dr \cdot r d\theta \cdot r \sin\theta d\phi$



Let us consider Assume that thermal conductivity

$k, \rho, \text{ \& } C_p = \text{constant}$

⇒ The energy balance of this spherical element is obtained from first law of thermodynamics.

Net heat  
Conducted  
into the  
element from  
all co-ordinate  
directions.

①

Heat  
generated  
within  
the  
element

②

Heat  
stored  
in the  
element

③

⇒ Heat Generated within the element = ②

$$\dot{q} (dr \cdot r d\theta \cdot r \sin\theta d\phi) dt$$

⇒ Heat stored in the element = ③

$$= \text{mass} \times \text{sp. heat} \times \text{Temp. Rise.}$$

$$= \int r dr \times r d\theta \times r \sin\theta d\phi \times C_p \times \frac{dT}{dt} dt$$

⇒ Net heat conducted into the element = ①

Heat entering in the element through  $r$  &  $\theta$  plane.

$\phi$  direction

$$Q_\phi = -k dr \cdot r d\theta \frac{\partial T}{\partial \phi} dt \rightarrow \text{a}$$

⇒ Heat leaving from the element

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{\partial \phi} (Q_\phi) r \sin\theta d\phi \rightarrow \text{b}$$



Net heat conducted in  $\phi$  direction

$$d\phi = Q_\phi - Q_{\phi+d\phi}$$

$$= Q_\phi - Q_\phi - \frac{\partial}{\partial \phi} (Q_\phi) r \sin \theta d\phi$$

$$= \frac{\partial}{\partial \phi} \left( k dr \cdot r d\theta \frac{\partial T}{\partial \phi} dt \right) r \sin \theta d\phi$$

$$= k r dr \cdot r d\theta \cdot r \sin \theta d\phi \cdot \frac{1}{r^2 \sin^2 \theta} dt \frac{\partial^2 T}{\partial \phi^2}$$

$\Rightarrow$  Heat entering in the element through  $\theta$  &  $\phi$  plane in  $r$ -direction

$$Q_r = -k r d\theta \cdot r \sin \theta d\phi \frac{\partial T}{\partial r} dt$$

Heat leaving

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} (Q_r) dr$$

Net heat conducted into the element

$$dQ_r = Q_r - Q_{r+dr}$$

$$= Q_r - Q_r - \frac{\partial}{\partial r} (Q_r) dr$$

$$= \frac{\partial}{\partial r} \left( k \cdot r d\theta \cdot r \sin \theta d\phi \frac{\partial T}{\partial r} dt \right) dr$$

$$dQ_r = k dr \cdot r d\theta \cdot r \sin \theta d\phi dt \times \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right)$$

$\Rightarrow$  Heat entering into the element  $\phi$  &  $r$  plane in  $\theta$  direction.



⇒ Heat entering

$$Q_0 = -k \, dr \cdot r \sin \theta \, d\phi \cdot \frac{\partial T}{r \partial \theta} \, dt$$

⇒ Heat leaving

$$Q_{0+d\theta} = Q_0 + \frac{\partial}{\partial \theta} (Q_0) \, r \, d\theta$$

⇒ Net heat conducted

$$dQ_0 = Q_0 - Q_{0+d\theta}$$

$$= \frac{\partial}{\partial \theta} \left( k \, dr \cdot r \sin \theta \, d\phi \cdot \frac{\partial T}{r \partial \theta} \, dt \right) r \, d\theta$$

$$dQ_0 = k \, dr \cdot r \, d\theta \cdot r \sin \theta \, d\phi \, dt \times \frac{1}{r^2 \sin \theta} \times \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right)$$

Net heat conducted into the element from all the co-ordinate direction.

$$= k \, dr \cdot r \, d\theta \cdot r \sin \theta \, d\phi \, dt \left[ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) \right]$$

$$= k \cdot \left[ \frac{1}{r^2 \sin^2 \theta} \frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) \right]$$

$$+ q = \frac{\int_{cp}}{k} \frac{\partial T}{\partial t}$$

$$= \frac{1}{r^2 \sin^2 \theta} \cdot \frac{\partial^2 T}{\partial \phi^2} + \frac{1}{r^2} \frac{\partial}{\partial r} \left( r^2 \frac{\partial T}{\partial r} \right) + \frac{1}{r^2 \sin \theta} \frac{\partial}{\partial \theta} \left( \sin \theta \frac{\partial T}{\partial \theta} \right) = \frac{1}{\alpha} \frac{\partial T}{\partial t}$$



2nd derivation  
Heat Entering in the element

⇒ heat entering into the element through  $\phi$  &  $z$  plane in  $r$ -direction within the time  $d\theta$ .

$$Q_r = -K r d\phi dz \frac{\partial T}{\partial r} d\theta$$

⇒ heat leaving

$$Q_{r+dr} = Q_r + \frac{\partial}{\partial r} (Q_r) \cdot dr$$

⇒ Net heat conducted

$$dQ = Q_r - Q_{r+dr}$$

$$= -\frac{\partial}{\partial r} (Q_r) \cdot dr$$

$$= -\frac{\partial}{\partial r} (K r d\phi dz \frac{\partial T}{\partial r} d\theta) dr$$

$$= K \cdot d\phi dz d\theta dr \left( \frac{\partial}{\partial r} \left( r \frac{\partial T}{\partial r} \right) \right)$$

$$= K \cdot d\phi dz d\theta dr \left[ \frac{\partial T}{\partial r} + r \frac{\partial^2 T}{\partial r^2} \right]$$

$$= K r d\phi dr dz d\theta \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} \right]$$

⇒ Heat Entering into the  $\phi$  direction through  $z$  &  $r$  direction in time  $d\theta$ .

$$Q_\phi = -K dz dr \frac{\partial T}{r \partial \phi} d\theta$$

⇒ Heat leaving

$$Q_{\phi+d\phi} = Q_\phi + \frac{\partial}{\partial \phi} (Q_\phi) r d\phi$$



⇒ Net heat conducted

$$dQ_\phi = Q_\phi - Q_\phi + dQ_\phi$$

$$= -\frac{\partial}{\partial \phi} (Q_\phi) r d\phi$$

$$= \frac{\partial}{\partial \phi} \left( k dz dr \frac{\partial T}{\partial r} d\phi \right) r d\phi$$

$$= k (dz dr r d\phi) d\phi \times \frac{1}{r^2} \times \frac{\partial^2 T}{\partial \phi^2}$$

$$= k (dr \cdot r d\phi \cdot dz) \cdot \frac{1}{r^2} \cdot \frac{\partial^2 T}{\partial \phi^2} \cdot d\phi$$

$$k \left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] r dr d\phi dz$$

$$+ \dot{q} (r dr d\phi dz) d\phi = \int (r dr d\phi dz) \frac{\partial T}{\partial \phi} d\phi$$

$$\left[ \frac{\partial^2 T}{\partial r^2} + \frac{1}{r} \frac{\partial T}{\partial r} + \frac{1}{r^2} \frac{\partial^2 T}{\partial \phi^2} + \frac{\partial^2 T}{\partial z^2} \right] + \frac{\dot{q}}{k} = \frac{1}{k} \frac{\partial T}{\partial \phi}$$

$$= \frac{1}{\alpha} \frac{\partial T}{\partial \phi}$$



If the temperature of a body does not vary with time, it is said to be in a steady state and that type of condition is known as steady state condition.

If the temperature of a body varies with time, it is said to be in a transient state and that type of condition is known as transient condition or steady state condition.

Transient heat condition can be divided into p.

- 1) Periodic
- 2) Non Periodic Heat flow.

In Periodic heat flow, the temp varies on a regular basis.

ex: Surface of earth during a period of 24hrs.

In Non-Periodic heat flow, the temp at any point within the system varies non linearly with time.

ex: cooling of bars



# Conduction of heat through a slab or Plain wall

Consider a slab of uniform Thermal Conductivity ( $k$ ), thickness ( $L$ ), with inner Temperature ( $T_1$ ) and outer temperature ( $T_2$ ).

$$Q = -KA \frac{dT}{dx}$$

$$Q \cdot dx = -KA dT$$

Integrating the above eqn

$$Q \cdot \int_0^L dx = -KA \int_{T_1}^{T_2} dT$$

$$Q \cdot [x]_0^L = -KA [T]_{T_1}^{T_2}$$

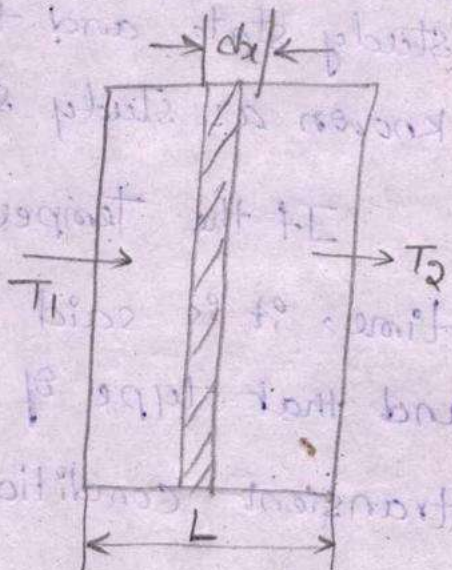
$$Q \cdot L = -KA [T_2 - T_1]$$

$$Q \cdot L = KA [T_1 - T_2]$$

$$Q = \frac{KA [T_1 - T_2]}{L}$$

$$= \frac{T_1 - T_2}{\frac{L}{KA}}$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$



$$\left[ \because \frac{L}{KA} = R \right]$$

thermal resistance



# Conduction of heat through sph Hollow Cylinder

Consider a hollow cylinder of inner radius ( $r_1$ ), outer radius ( $r_2$ ), inner temp ( $T_1$ ), outer temp ( $T_2$ ) and thermal conductivity ( $k$ ).

⇒ let us consider a small elemental area of thickness  $dr$ .

⇒ From Fourier law of conduction, we know that

~~sphere~~ Area =  $2\pi rL$

$$Q = -kA \frac{dT}{dr}$$

$$Q = -k 2\pi rL \frac{dT}{dr}$$

$$Q \cdot \frac{1}{r} dr = -k 2\pi L dT$$

Integrating

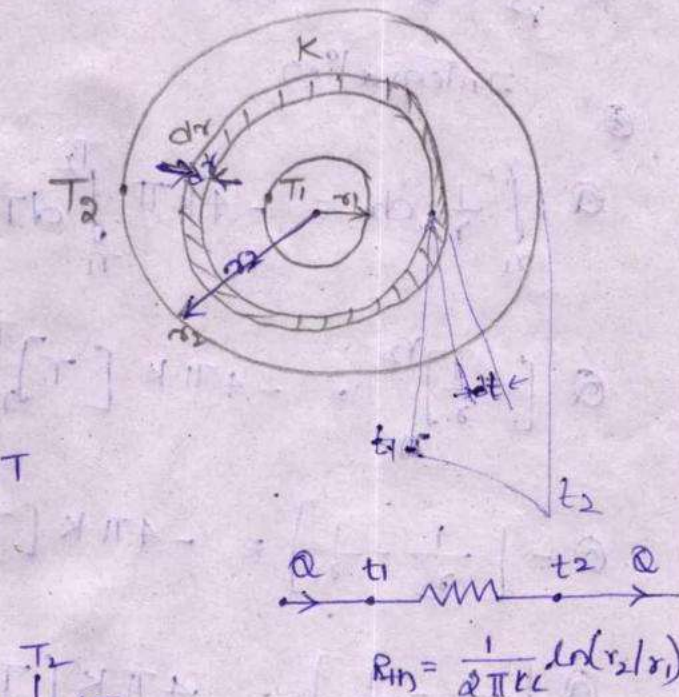
$$Q \cdot \int_{r_1}^{r_2} \frac{1}{r} dr = -k 2\pi L \int_{T_1}^{T_2} dT$$

$$Q \ln[r]_{r_1}^{r_2} = -2\pi KL [T]_{T_1}^{T_2}$$

$$Q \ln[r_2 - r_1] = -2\pi KL (T_2 - T_1)$$

$$Q \ln\left[\frac{r_2}{r_1}\right] = 2\pi KL (T_1 - T_2)$$

$$Q = \frac{T_1 - T_2}{\frac{1}{2\pi KL} \ln\left(\frac{r_2}{r_1}\right)} = \frac{\Delta T_{\text{overall}}}{R}$$



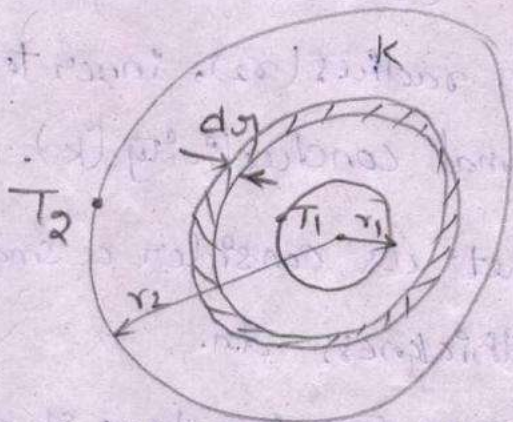


# 128 Conduction of heat through hallow Sphere

$$A = 4\pi r^2$$

$$Q = -KA \frac{dT}{dr}$$

$$Q = -K \cdot 4\pi r^2 \cdot \frac{dT}{dr}$$



$$Q \cdot \frac{1}{r^2} dr = -K 4\pi dT$$

Integrating

$$Q \int_{r_1}^{r_2} \frac{1}{r^2} dr = -4\pi K \int_{T_1}^{T_2} dT$$

$$Q \left[ -\frac{1}{r} \right]_{r_1}^{r_2} = -4\pi K [T]_{T_1}^{T_2}$$

$$Q \left[ \frac{1}{r_2} - \frac{1}{r_1} \right] = -4\pi K [T_2 - T_1]$$

$$Q \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] = 4\pi K [T_1 - T_2]$$

$$Q \left[ \frac{r_2 - r_1}{r_1 r_2} \right] = 4\pi K [T_1 - T_2]$$

$$Q = \frac{(T_1 - T_2) \cdot 4\pi K}{\frac{1}{4\pi K} \left[ \frac{r_2 - r_1}{r_1 r_2} \right]}$$

$$Q = \frac{\Delta T_{\text{overall}}}{R}$$



# Heat transfer through a composite wall with inside and outside convection

$$\rightarrow Q = h_a A [T_a - T_1]$$

$$T_a - T_1 = Q \times \frac{1}{h_a A}$$

$$\rightarrow Q = \frac{k_1 A [T_1 - T_2]}{L_1}$$

$$T_1 - T_2 = Q \times \frac{L_1}{k_1 A}$$

$$\rightarrow Q = \frac{k_2 A [T_2 - T_3]}{L_2}$$

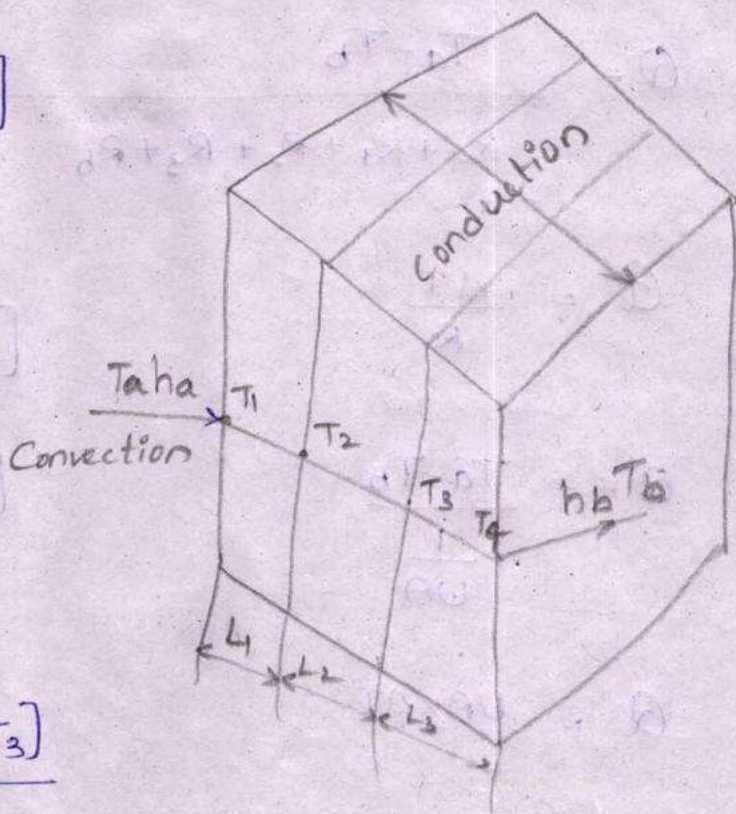
$$T_2 - T_3 = \frac{Q \times L_2}{k_2 A}$$

$$\rightarrow Q = \frac{k_3 A [T_3 - T_4]}{L_3}$$

$$T_3 - T_4 = \frac{Q \times L_3}{k_3 A}$$

$$\rightarrow Q = \frac{h_b A [T_4 - T_b]}{1}$$

$$T_4 - T_b = Q \times \frac{1}{h_b A}$$





$$T_a - T_b = Q \left[ \frac{1}{h_a A} + \frac{L_1}{k_1 A} + \frac{L_2}{k_2 A} + \frac{L_3}{k_3 A} + \frac{1}{h_b A} \right]$$

$$Q = \frac{T_a - T_b}{R_a + R_1 + R_2 + R_3 + R_b}$$

$$Q = \frac{\Delta T}{R}$$

$$Q = \frac{T_a - T_b}{\frac{1}{UA}}$$

$$Q = UA \times \Delta T$$

$$\left[ \because R = \frac{1}{UA} \right]$$

$$\left[ \because \Delta T = T_a - T_b \right]$$

$$\frac{1}{UA} \times Q = T_a - T_b$$

$$\frac{[T_a - T_b] A_s k_s}{\Delta T} = Q$$

$$\frac{\Delta T \times Q}{A_s k_s} = T_a - T_b$$

$$\frac{[T_a - T_b] A_s k_s}{\Delta T} = Q$$

$$\frac{\Delta T \times Q}{A_s k_s} = T_a - T_b$$

$$\frac{[T_a - T_b] A_s k_s}{\Delta T} = Q$$

$$\frac{1}{A_s k_s} \times Q = T_a - T_b$$



①  
Sol

Given data

$$\text{length} = 6\text{m}$$

$$\text{height} = 4\text{m}$$

$$\text{thickness } L = 0.30\text{m}$$

$$\begin{aligned} A &= \text{Length} \times \text{height} \\ &= 6 \times 4 \\ &= 24 \end{aligned}$$

$$T_1 = 100^\circ\text{C} + 273 = 373\text{K}$$

$$T_2 = 40^\circ\text{C} + 273 = 313\text{K}$$

$$K = 0.55 \text{ W/mK}$$

$$Q = \frac{\Delta T}{R} = (T_1 - T_2)$$

$$R = \frac{L}{KA} = \frac{0.3}{0.55 \times 24} = 0.0227$$

$$Q = \frac{(373 - 313)}{0.0227} = 2643 \text{ W}$$

②  
Sol

Given that

$$T_a = -20^\circ\text{C} + 273 = 253\text{K}$$

$$T_b = 25^\circ\text{C} + 273 = 298\text{K}$$

$$L_3 = L_1 = 30\text{cm} = 0.3$$

$$L_2 = 0.2\text{m}$$

$$K_3 = K_1 \text{ for brick} = 2.3 \text{ W/mK}$$

$$\text{Cork} - K_2 = 0.05 \text{ W/mK}$$

{ from data book  
15 Pg.

$$h_b = 55.4 \text{ W/m}^2\text{K}$$

$$h_a = 17 \text{ W/m}^2\text{K}$$

$$Q = \frac{\Delta T}{R} = \frac{298 - 253}{4.337} = 10.37 \text{ W}$$

$$\frac{Q}{A} = \frac{\Delta T}{R}$$

$$R = \left[ \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_b} \right] = 4.337$$



③

801

$$L = 250 \text{ mm} = 0.25 \text{ m}$$

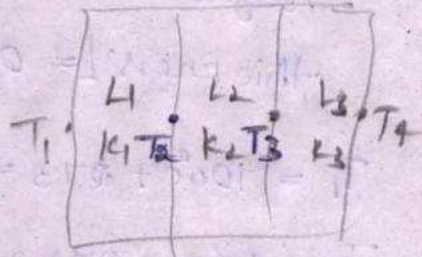
$$K = 1.05 \text{ W/mK}$$

Insulation brick  $L_2 = 0.12 \text{ m}$

$$K_2 = 0.15 \text{ W/mK}$$

Red brick  $L_3 = 200 \text{ mm} = 0.2 \text{ m}$

$$K_3 = 0.85 \text{ W/mK}$$



$$T_1 = 850 + 273 = 1123 \text{ K}$$

$$T_4 = 65 + 273 = 338 \text{ K}$$

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{A} \left[ \frac{1}{h_a} + \frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3} + \frac{1}{h_b} \right]$$

$$\frac{Q}{A} = \frac{T_1 - T_4}{\frac{L_1}{K_1} + \frac{L_2}{K_2} + \frac{L_3}{K_3}} = 616.46 \text{ W/m}^2$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{\frac{L_1}{K_1}} = \frac{T_2 - T_3}{\frac{L_2}{K_2}} = \frac{T_3 - T_4}{\frac{L_3}{K_3}}$$

$$\frac{Q}{A} = \frac{T_1 - T_2}{L_1/K_1} = 976.22 \text{ K} = T_2$$

$$T_3 = 483.05 \text{ K}$$

$$(or) 616.46 = \frac{T_3 - 338}{\frac{0.2}{0.85}}$$

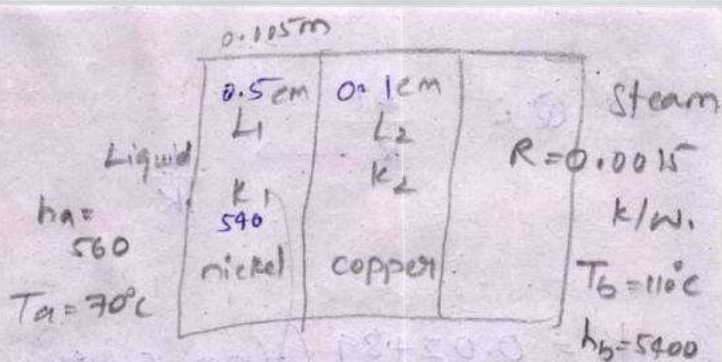
$$T_3 = 483.05 \text{ K}$$



④ Sol

Given data

$$A = 25.2 \text{ m}^2$$



$$R = \frac{1}{UA}$$

$$R = \frac{1}{A} \left[ \frac{1}{h_a} + \frac{L_1}{k_1} + \frac{L_2}{k_2} + \left( \frac{L_3}{k_3} \right) + \frac{1}{h_b} \right]$$

$$= \frac{1}{25.2} \left[ \frac{1}{560} + \frac{0.005}{90} + \frac{0.001}{386} + 0.0015 + \frac{1}{5400} \right]$$

$$R = 1.4 \times 10^{-4} \text{ K/W}$$

$$Q = \frac{\Delta T}{R} = \frac{(343 - 383)}{1.4 \times 10^{-4}} = -28.3 \times 10^4 \text{ W}$$

$$\textcircled{a} Q = UA \Delta T$$

$$-28.3 \times 10^4 = U \times 25.2 \times (343 - 383)$$

$$U = -280.75 \text{ W/m}^2\text{K}$$

~~Calculate~~

$$\textcircled{b} \Rightarrow Q = \frac{\Delta T}{R}$$

$$28.3 \times 10^4 = \frac{\Delta T}{0.0015}$$

$$\Delta T = 424.5 \text{ K}$$

$$(0.5) 151.5^\circ\text{C}$$



$$Q = \frac{\Delta T}{R} \rightarrow \frac{L}{K}$$

$$K = 0.02489 \text{ (from Glass & Kapour Pg. 48) at } 110^\circ\text{C}$$

$$28.3 \times 10^4 = \frac{424.5}{L}$$

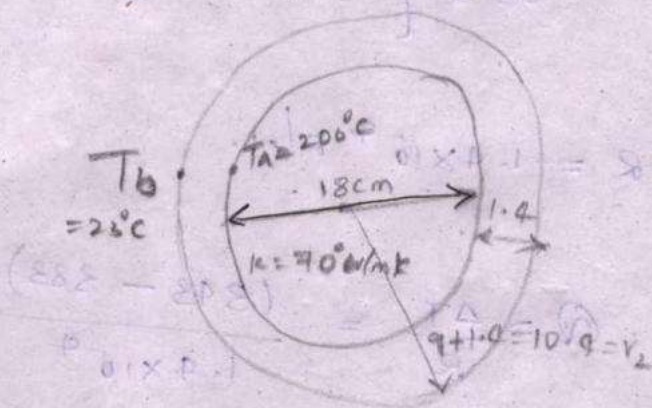
$$\left[ \frac{1}{L} + \frac{1}{0.02489} + \frac{1}{L} + \frac{1}{L} \right] \frac{1}{L} = R$$

$$L = 3.7 \times 10^{-5} \text{ m}$$

⑥

sol

$$Q = \frac{T_a - T_b}{R}$$



$$R = \frac{1}{2\pi k} \left[ \frac{1}{\ln r_2} + \frac{1}{k_1} \ln \left[ \frac{r_2}{r_1} \right] \right] \rightarrow \text{Pg. 54 Composite Cylinder}$$

$$R = \frac{1}{2\pi} \left[ \frac{1}{690 \times 0.9} + \frac{1}{70} \ln \left[ \frac{10.4 \times 10^2}{9 \times 10^2} \right] \right]$$

$$R = 2.8 \times 10^{-3} \text{ K/W}$$

$$Q = \frac{T_a - T_b}{R} = \frac{(200 + 273) - (170 + 273)}{2.8 \times 10^{-3}} = 10.7 \times 10^3 \text{ W}$$



$$Q = UA\Delta T \quad [\because Q = \frac{\Delta T}{R}]$$

$$= U \times 2\pi L r_2 \times \Delta T \quad = \frac{T_a - T_b}{R}$$

$$\frac{Q}{L} = U \times 2\pi \times r_2 \times (T_a - T_b)$$

$$10.7 \times 10^3 = U \times 2\pi \times (10.4 \times 10^{-2}) \times (473 - (23 + 273))$$

$$U = 92.51$$

⑦

80)

$$K_1 = 65 \text{ W/mK}$$

$$r_1 = 60 \text{ mm} = 0.06 \text{ m}$$

$$r_2 = 175 \text{ mm} = 0.175 \text{ m}$$

$$\text{thickness} = 10 \text{ mm} = 0.01$$

$$\text{insulation } K_2 = 10 \text{ W/mK}$$

$$r_3 = 175 + 10 = 185$$

$$T_a = 500 + 273 = 773 \text{ K}$$

$$T_b = 50 + 273 = 323 \text{ K}$$

→ Composite sphere with Convection → Pg. 54

$$Q = \frac{\Delta T}{R}$$

$$R = \frac{1}{4\pi} \left[ \frac{1}{h_i r_1^2} + \frac{1}{K_1} \left[ \frac{1}{r_1} - \frac{1}{r_2} \right] + \frac{1}{K_2} \left[ \frac{1}{r_2} - \frac{1}{r_3} \right] + \frac{1}{h_o r_3^2} \right]$$

→ neglecting  $h_i$  and  $h_o$ ,

$$R = \frac{1}{4\pi} \left[ \frac{1}{65} \left[ \frac{1}{0.06} - \frac{1}{0.175} \right] + \frac{1}{10} \left[ \frac{1}{0.175} - \frac{1}{0.185} \right] \right]$$

$$= 0.01586$$

$$Q = \frac{T_a - T_b}{R} = \frac{773 - 323}{0.01586} = 28373.26 \text{ W (or)} 28.3 \times 10^3 \text{ W}$$



# \* Critical Radius of Insulation

Addition of Insulating material on a surface doesnot reduce the amount of heat transfer rate always.

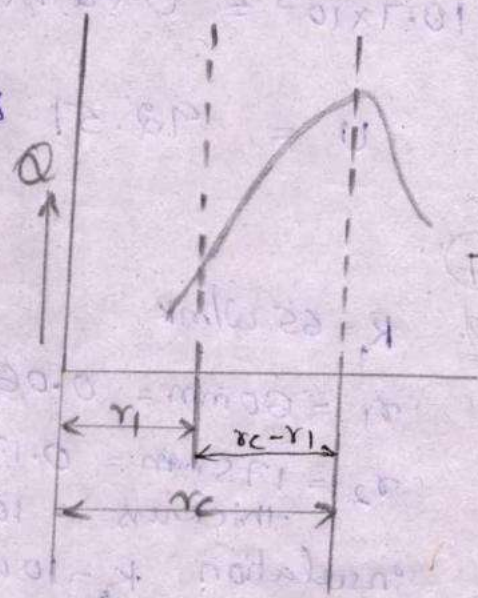
$r_c$  = critical radius

$r_c - r_1$  = critical thickness

$$Q = \frac{T_i - T_{\infty}}{\frac{\ln\left(\frac{r_0}{r_1}\right)}{2\pi KL}}$$

$$Q = \frac{T_i - T_{\infty}}{\frac{\ln\left(\frac{r_0}{r_1}\right)}{2\pi KL} + \frac{1}{A_o h}} \quad \left[ \because A_o = 2\pi r_0 L \right]$$

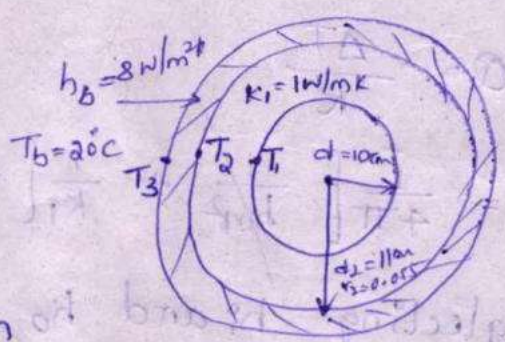
$$r_0 = \frac{k}{h} = r_c$$



8

801

Given data



$$r_c = \frac{k}{h} = \frac{1}{8} = 0.125m$$

$$Q = \frac{\Delta T}{R} = \frac{T_a - T_b}{R}$$

$$R = \frac{1}{2\pi L} \left[ \frac{1}{h_a r_1} + \frac{1}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{k_2} \ln\left(\frac{r_3}{r_2}\right) + \frac{1}{k_3} \ln\left(\frac{r_4}{r_3}\right) + \frac{1}{h_b r_4} \right] \rightarrow Pg. 54$$



$$= \frac{1}{2\pi L} \left[ \frac{1}{k_1} \ln\left(\frac{r_2}{r_1}\right) + \frac{1}{h_b r_c} \right]$$

$$\frac{1}{2\pi L} \times \left[ \frac{1}{1} \ln\left(\frac{0.055}{0.05}\right) + \frac{1}{8 \times 0.125} \right]$$

$$\frac{R}{1} = \frac{0.174}{L}$$

$$Q = \frac{T_a - T_b}{\frac{0.174}{L}}$$

$$\text{Take } L = 1 \text{ m.}$$

$$\frac{Q}{L} = \frac{473 - 293}{0.174} = 1034 \text{ W/m.}$$

⇒ outer surface Temperature.

$$\frac{Q}{L} = \frac{T_a - T_b}{R} = \frac{T_1 - T_2}{R_1} = \frac{T_2 - T_3}{R_L} = \frac{T_3 - T_b}{R_3}$$

$$\frac{Q}{L} = \frac{T_3 - T_b}{R_3} \Rightarrow R_3 = \frac{1}{2\pi L} \times \frac{1}{h_b \times r_c}$$

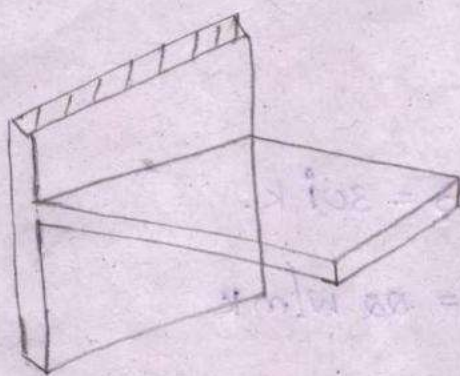
$$1034 = \frac{T_3 - 293}{\frac{1}{2\pi} \times \frac{1}{8 \times 0.125}}$$

$$T_3 = 457.56 \text{ K}$$

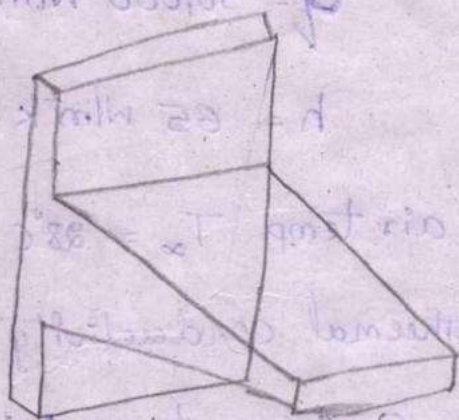


## Fins :

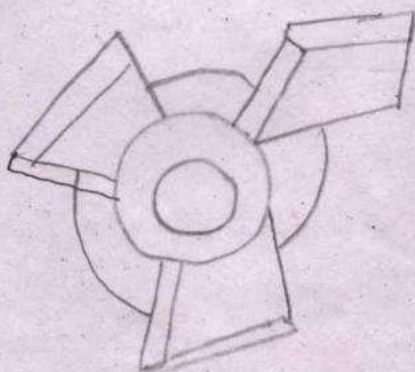
It is possible to increase the heat transfer rate by increasing the surface of heat transfer. The surface used for increasing heat transfer are called extended surfaces are called fins.



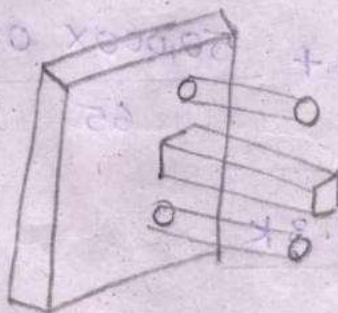
uniform straight fin



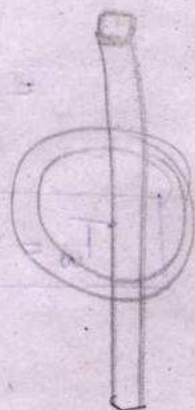
Tapered straight fin



Splines



Pin fin



Annular fin



Commonly there are three types of fins

- 1) Infinitely long fin
- 2) short fin (end is insulated)
- 3) short fin (end is not insulated)

### Temperature distribution and heat Dissipation in Fin

Figure a and b shows the straight fin or longitudinal fin of rectangular section and circular section respectively.

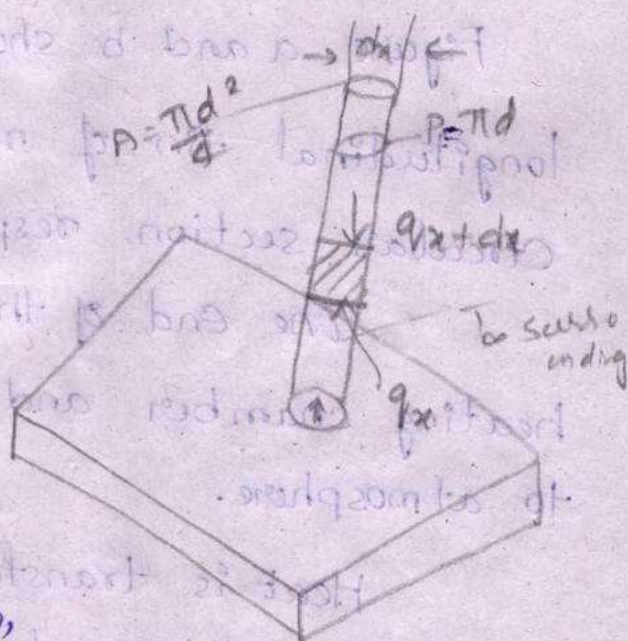
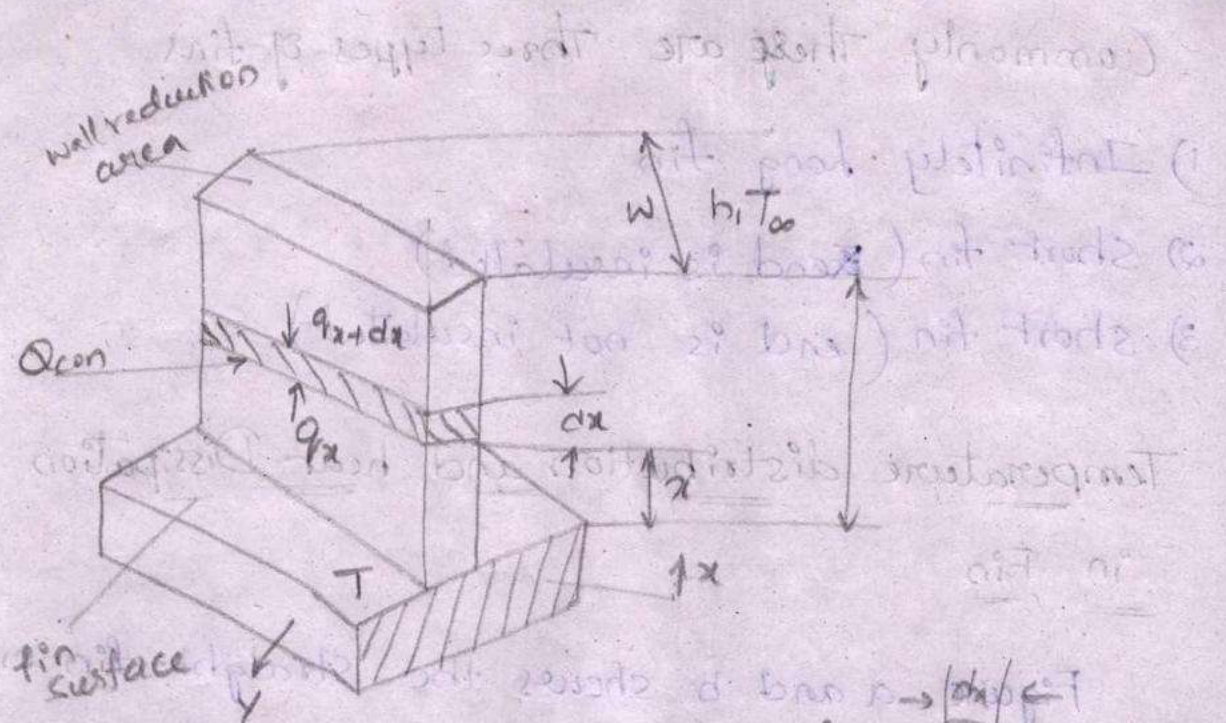
One end of the fin is enclosed in a heating chamber and the other end is exposed to atmosphere.

Heat is transferred across the rectangular fin and circular rod by conduction. from the surface of fins, heat is transferred to a by convection.

Let us consider a small elemental area of thickness  $dx$ , which at a distance of  $x$  from the base.

$$Q_{\text{conduction}} + Q_{\text{convection}} = Q_{\text{out}} - Q_{\text{in}}$$
$$kA \frac{dT}{dx} + hP(T - T_{\infty}) = -kA \frac{dT}{dx}$$
$$(hP(T - T_{\infty}) + kA \frac{dT}{dx}) = -kA \frac{dT}{dx}$$





A study state condition,

Heat balance equation for that element is as follows.

$$\left\{ \begin{array}{l} \text{Heat} \\ \text{conducted} \\ \text{into the} \\ \text{element} \end{array} \right\} = \left\{ \begin{array}{l} \text{Heat} \\ \text{conducted} \\ \text{out of the} \\ \text{element} \end{array} \right\} + \left\{ \begin{array}{l} \text{Heat conducting} \\ \text{to the} \\ \text{surrounding} \\ \text{air} \end{array} \right\}$$

$$-Q_x = Q_{x+dx} + Q_{\text{convection}}$$

$$-KA \frac{dT}{dx} = Q_x + \frac{d}{dx}(Q_x)dx + hA\Delta T + hA(T - T_\infty)$$



$$KA \frac{dT}{dx} = Q_x + \frac{d}{dx}(Q_x) dx + h P dx (T - T_\infty)$$

$$Q_x = Q_x + \frac{d}{dx}(Q_x) dx + h P dx (T - T_\infty)$$

$$\frac{d}{dx} \left( KA \frac{dT}{dx} \right) dx = h P dx (T - T_\infty)$$

$$\frac{d^2 T}{dx^2} KA = h P (T - T_\infty)$$

$$\frac{d^2 T}{dx^2} - \frac{h P}{KA} (T - T_\infty) = 0$$

$$m^2 = \frac{h P}{KA}$$

$$(\theta) \quad \theta = T - T_\infty$$

$$m = \sqrt{\frac{h P}{KA}}$$

$$\theta = C_1 e^{-mx} + C_2 e^{+mx} \rightarrow (1)$$

The above equation shows that  $\theta$  is a function of  $x$  & it is a second order, linear differential equation.  $\frac{d^2 \theta}{dx^2} + m^2 \theta = 0$

The temperature distribution & heat dissipation depends upon the following condition.

Case (i) = Infinitely long fin

In a fin is infinitely long, the temperature at its end is equally to that of its surrounding fluid (air)  
 $\Rightarrow$  at  $x=0$ ,  $T = T_b$



and  $x = \text{infinity}$ ,  $T = T_{\infty}$   
 $x = \infty$ ,  $T = T_{\infty}$

$$0 = C_1 e^{-mx} + C_2 e^{mx}$$

$$0 = T - T_{\infty}$$

Substitute  $x=0$ ,  $T=T_b$

$$T - T_{\infty} = C_1 e^{-mx} + C_2 e^{mx} \rightarrow (2)$$

$$T_b - T_{\infty} = C_1 + C_2 \rightarrow (3)$$

then substitute  $x = \infty$ ,  $T = T_{\infty}$

$$T_{\infty} - T_{\infty} = C_1 e^{-m\infty} + C_2 e^{m\infty}$$

$$0 = C_1 e^{-m\infty} + C_2 e^{m\infty} \rightarrow (4)$$

substitute  $C_2 = 0$  in eq (3), we get

$$T_b - T_{\infty} = C_1$$

Substitute  $C_1$  in eq (2)

$$T - T_{\infty} = (T_b - T_{\infty}) e^{-mx} + 0$$

$$\left[ \because m = \frac{hP}{\sqrt{KA}} \right]$$

$$\frac{T - T_{\infty}}{T_b - T_{\infty}} = e^{-mx} \Rightarrow T - T_{\infty} = e^{-mx} (T_b - T_{\infty})$$

Heat dissipation through the pin is obtained by integrating the heat lost by convection over the entire fin surface, we know that



⇒ Heat lost by convection

$$Q = hA(T - T_{\infty})$$

$$= hP \cdot dx(T - T_{\infty})$$

$$[\because e^{\infty} = -1]$$

$$= \int_0^{\infty} hP \cdot e^{-mx} (T_b - T_{\infty}) dx$$

$$= hP(T_b - T_{\infty}) \int_0^{\infty} e^{-mx} dx$$

$$= hP(T_b - T_{\infty}) \times \frac{-1}{m} [e^{-mx}]_0^{\infty}$$

$$= hP(T_b - T_{\infty}) \times \frac{-1}{m} [e^0 - e^{-\infty}]$$

$$= hP(T_b - T_{\infty}) \times \frac{-1}{m} (-1)$$

$$= hP(T_b - T_{\infty}) \times \frac{1}{m}$$

$$= hP(T_b - T_{\infty}) \times \frac{1}{\sqrt{\frac{hP}{KA}}}$$

$$= hP(T_b - T_{\infty}) \times \sqrt{\frac{KA}{hP}}$$

$$= (hP)^{\frac{1}{2} + \frac{1}{2}} (T_b - T_{\infty}) \times \frac{\sqrt{KA}}{\sqrt{hP}}$$

$$= \sqrt{hPKA}$$

$$Q = \sqrt{hPKA} \times (T_b - T_{\infty})$$



Case (ii) : fin with insulated end (short fin)

The fin has a finite length and the tip of fin is insulated, at  $x=0$  &  $T=T_b \rightarrow (b)$

$$x=L ; \frac{dT}{dx} = 0 \rightarrow (a)$$

$$(T-T_\infty) = C_1 e^{-mx} + C_2 e^{mx} \rightarrow (i)$$

diff. (i)  
w.r.t  $x$

$$\frac{dT}{dx} = C_1 e^{-mx} (-m) + C_2 e^{mx} (m) \rightarrow (c)$$

Apply (a) condition in (c)

$$0 = C_1 e^{-mL} \times -m + C_2 e^{mL} \times m$$

$$m C_1 e^{-mL} = m C_2 e^{mL}$$

$$C_1 = C_2 e^{2mL} \rightarrow (d)$$

Apply (b) conditions in (i)

$$T_b - T_\infty = C_1 e^0 + C_2 e^0 \rightarrow (ii)$$

Substitute  $C_1$  in eq (ii)

$$T_b - T_\infty = C_2 e^{2mL} + C_2$$

$$C_2 = \frac{T_b - T_\infty}{[e^{2mL} + 1]} \rightarrow (e)$$

Apply (e) in (d)

$$C_1 = \frac{T_b - T_\infty}{e^{2mL} + 1} \times e^{2mL}$$

$$e^{-mL} = e^{mL}$$

$$C_1 = \frac{e^{mL}}{e^{-mL}}$$

$$= e^{mL} \times e^{mL}$$

$$= e^{2mL}$$

$$\frac{1}{e^{2mL} + 1} \times e^{2mL}$$

$$\frac{1}{(e^{2mL} + 1)} \times \frac{1}{e^{-2mL}}$$

$$\frac{1}{e^{-2mL} \times (e^{2mL} + 1)}$$

$$\frac{1}{e^{-2mL} + e^{2mL - 2mL}}$$



$$C_1 = \frac{T_b - T_\infty}{1 + e^{-2mL}}$$

Substitute  $C_1$  &  $C_2$  in eq (i)

$$T - T_\infty = \frac{T_b - T_\infty}{1 + e^{-2mL}} \times e^{-mx} + \frac{T_b - T_\infty}{e^{2mL} + 1} \times e^{mx}$$

$$= T_b - T_\infty \left[ \frac{e^{-mx}}{1 + e^{-2mL}} + \frac{e^{mx}}{1 + e^{2mL}} \right]$$

$$\frac{T - T_\infty}{T_b - T_\infty} = \left[ \frac{e^{-mx}}{1 + e^{-2mL}} \times \frac{e^{mL}}{e^{mL}} + \frac{e^{mx}}{1 + e^{2mL}} \times \frac{e^{-mL}}{e^{-mL}} \right]$$

$$= \frac{e^{m(L-x)} + e^{-m(L-x)}}{e^{mL} + e^{-mL}}$$

## Applications of fin

The main applications of fins

- ⇒ Cooling of electrical components
- ⇒ " motor cycle engine
- ⇒ " small capacity compressors.
- ⇒ " transformers.
- ⇒ cooling of radiators & refrigerators etc.



Fin efficiency : It is the ratio of actual heat transfer of fin to <sup>the</sup> maximum possibility heat transfer by the fin.

$$\eta_{fin} = \frac{Q_{fin}}{Q_{max fin}} = \frac{1}{1}$$

for insulated n efficiency =  $\frac{\tanh(mL)}{mL}$

Fin effectiveness: It is defined as the ratio of heat transfer with fin to heat transfer without fin

$$\epsilon = \frac{Q_{with fin}}{Q_{without fin}} \quad [\because \epsilon_{mp} = \epsilon]$$

9

801

Given data

diameter  $d = 3 \text{ mm} = 0.003 \text{ m}$

base temp  $T_b = 140^\circ\text{C} = 413 \text{ K}$

$k = 150 \text{ W/mK}$

$h = 300 \text{ W/m}^2\text{K}$

$T_\infty = 10^\circ\text{C} = 283 \text{ K}$

Heat transfer by fin  $Q = (T_b - T_\infty)(hPKA)^{0.5}$

$\Rightarrow P = \text{Perimeter} = (\pi d) = \pi \times 0.003 = 0.0094$



$$A = \text{area} = \pi/4 \times d^2 = \frac{\pi}{4} \times (0.003)^2 = 7.06 \times 10^{-6}$$

$$Q = (413 - 283) \times (300 \times 0.0094 \times 150 \times 7.06 \times 10^{-6})$$

$$Q = 7.10 \text{ W}$$

(10)

sol

$$\text{dia } d = 5 \text{ cm} = 0.05 \text{ m}$$

$$T_b = 150^\circ\text{C} = 423 \text{ K}$$

$$T_\infty = 20^\circ\text{C} = 293 \text{ K}$$

$$x = 20 \text{ cm} = 20 \times 10^{-2} \text{ m}$$

$$k = 200 \text{ W/mK}$$

$$\text{Intermediate Temp } T = 60^\circ\text{C} = 333 \text{ K}$$

$$m = \sqrt{\frac{hP}{kA}}$$

$$\frac{T - T_\infty}{T_b - T_\infty} = e^{-mx}$$

$$\frac{333 - 293}{423 - 293} = e^{-m \times 20 \times 10^{-2}}$$

$$m = 5.89$$

$$\pi d = P$$

$$\pi \times 0.05 = 0.157$$

$$A = \frac{\pi}{4} d^2$$

$$= \frac{\pi}{4} \times 0.05^2$$

$$= 1.96 \times 10^{-3} \text{ m}^2$$

$$\Rightarrow m = \sqrt{\frac{hP}{kA}}$$

$$5.89 = \sqrt{\frac{h \times 0.157}{200 \times 1.96 \times 10^{-3}}}$$

$$h = 86.6 \text{ W/m}^2\text{K}$$



(12)  
sol

fin dimension  $E = 0.5 \text{ mm}^2 = \frac{5 \times 10^{-7} \text{ m}^2}{0.5 \times 10^{-3} \text{ m}}$  (it is insulated)

long  $l = 12 \text{ mm} = 12 \times 10^{-3} \text{ m}$

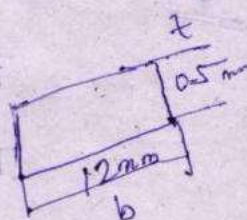
$$T_b = 80^\circ\text{C} = 353 \text{ K}$$

$$Q = 35 \times 10^{-3} \text{ W}$$

$$k = 165 \text{ W/mK}$$

$$h = 10 \text{ W/m}^2\text{K}$$

$$T_\infty = 22 = 295 \text{ K}$$



fin area  $A = b \times t = 5 \times 5 \times 10^{-6} = 2.5 \times 10^{-7} \text{ m}^2$

$$\text{Perimeter } P = 2(b + t)$$

$$= 2(0.012 + 0.5 \times 10^{-3})$$

$$P = 2 \times 10^{-3} \text{ m} \text{ (or) } 0.0025 \text{ m}$$

$$\Rightarrow \text{no. of fins required} = \frac{\text{heat generated}}{\text{heat transfer per fin}}$$

$$\Rightarrow \text{short fin (end insulated)} Q = (h P k A)^{0.5} (T_b - T_\infty) \tanh(mL)$$

$$\Rightarrow m = \sqrt{\frac{hP}{kA}} = 22 \text{ m}^{-1}$$

$$\Rightarrow Q = 0.0135 \text{ per fin}$$

$$\eta = \frac{35 \times 10^{-3}}{0.0135} = 2.59$$



(14)

So)

$$\text{wide } l = 140 \text{ mm} = 0.14$$

$$t = 5 \text{ mm} = 0.005 \text{ m}$$

$$\text{dia } d_i = 200 \text{ mm} = 0.2 \text{ m}$$

$$r_i = 100 \text{ mm} = 0.1 \text{ m}$$

$$\text{fin base temp } T_b = 170 = 443 \text{ K}$$

$$T_\infty = 25^\circ\text{C} = 298 \text{ K}$$

$$K = 220 \text{ W/mK}$$

$$h = 140 \text{ W/m}^2\text{K}$$

$$L_c = L + \frac{t}{2} = 0.014 + \frac{0.005}{2}$$

$$= 0.1425 \text{ m}$$

$$\Rightarrow r_{2c} = r_i + L_c = 0.2425 \text{ m}$$

$$\Rightarrow A_m = t(r_{2c} - r_i) = 0.005(0.2425 - 0.1) = 7.125 \times 10^{-4} \text{ m}^2$$

$$\Rightarrow A_s = 2\pi(r_{2c}^2 - r_i^2) = 2\pi(0.2425^2 - 0.1^2) = 0.3065 \text{ m}^2$$

$$x\text{-axis} = L_c^{1.5} \left( \frac{h}{KA_m} \right)^{0.5}$$

$$= 1.6$$

$$\text{Curve} = \frac{r_{2c}}{r_i} = \frac{0.2425}{0.1} = 2.425$$

$$\eta = 35\%$$

$$Q = \eta A_s h (T_b - T_\infty)$$

$$Q = 2712 \text{ W}$$



Transient heat condition occurs in cooling of IC engines, Automobile engines, boiler tubes, heating & cooling of metal pellets, Rocket nozzles, electric iron etc.

Transient heat condition can be divided into

1) Periodic heat flow: Temperature varies on a regular basis.

ex: cylinder of an IC engine.

2) Non-Periodic heat flow: Temperature at any point within the system varies non linearly with time.

ex: cooling of bars with atmospheric temperature and Ingote

Biot number: The ratio of Internal conduction resistance to the surface convection resistance.  $\rightarrow$  120 Pg.

$$Bi = \frac{hL}{k_s}$$

Significance

$\Rightarrow$  lumped parameter System  $= \frac{hL}{k_s} < 0.1 \rightarrow$  66 pg.

$Bi = \text{infinity}$  (semi infinity solid)



⇒ Infinite Solid  $0.1 < B_i < 100$

⇒ Volume

Characteristic length

$$L = \frac{V}{A}$$

$V = \text{area} \times \text{thickness of the slab}$

$A = 2 \times \text{area}$

$$L = \frac{\cancel{A} \times \text{thickness } (x)}{2 \times \cancel{A}}$$

$$L = x/2$$

for Cylinder

$$L = \frac{V}{A} = \frac{\pi r^2 x}{2\pi r x} = \frac{r}{2} = \text{Radius of cylinder.}$$

for Sphere

$$L = \frac{V}{A} = \frac{\frac{4}{3}\pi r^3}{4\pi r^2} = \frac{r}{3}$$

Significance of Fourier Number: → 120pg

The ratio of characteristic body dimension to the Temperature wave penetration depth in time,  $\tau$

Lumped Heat Analysis: (Negligible Internal Resistance)

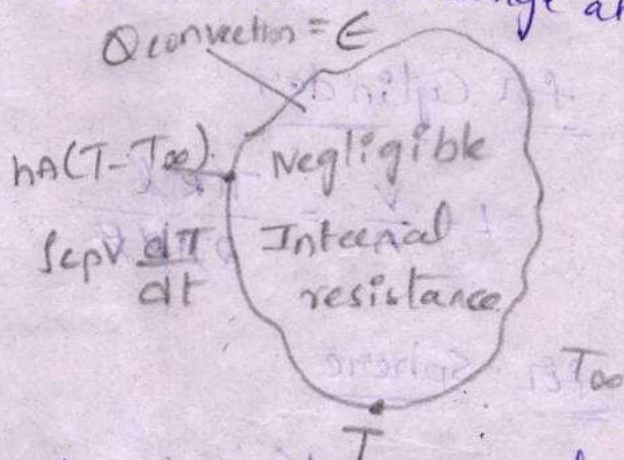
The process in which the internal resistance is negligible in comparing with its surface resistance in its Newtonian



heating or cooling process.

In a Newton's Newtonian heating or cooling process the temperature is considered to be uniform at a given time. Such that Analysis is called as lumped parameter Analysis.

Let us consider a solid whose initial temperature is  $T_0$  and it is placed suddenly in ambient air temperature  $T_\infty$ . The transient response of the body can be determined by relating its rates of change of internal energy with convective exchange at the surface.



Convective Heat transfer = Rate of change of Internal Energy.

$$-hA(T - T_\infty) = \rho \times C_p \times V \times \frac{dT}{dt}$$

$$\frac{dT}{T - T_\infty} = \frac{-hA}{\rho C_p V} \times dt$$

$$\int \frac{dT}{T - T_\infty} = \frac{-hA}{\rho C_p V} \int dt$$

$$\ln[T - T_\infty] = \frac{-hA}{\rho C_p V} t + C_1 \rightarrow (1)$$



Apply boundary conditions to eq (1), we get  
At  $T = T_0$  ;  $t = 0$

$$\ln [T_0 - T_\infty] = 0 + C_1$$

$$C_1 = \ln [T_0 - T_\infty]$$

Substitute  $C_1$  in eq (1)

$$\ln [T - T_\infty] = \frac{-hA}{\rho C_p V} t + \ln [T_0 - T_\infty]$$

$$\ln [T - T_\infty] - \ln [T_0 - T_\infty] = \frac{-hA}{\rho C_p V} t$$

$$\ln \left[ \frac{T - T_\infty}{T_0 - T_\infty} \right] = \frac{-hA}{\rho C_p V} t$$

$$\frac{T - T_\infty}{T_0 - T_\infty} = e^{-\left[ \frac{hAt}{\rho C_p V} \right]}$$

3) A  $50 \times 50 \text{ cm}^2$  aluminium slab of thickness

6mm. is at  $400^\circ\text{C}$  initially and it is

suddenly immersed in water. So its surface temp is lowered to  $250^\circ\text{C}$ . determine the time

required for the slab to reach  $120^\circ\text{C}$

Take  $h = 100 \text{ W/m}^2\text{K}$ .

Sol

Given data

$$\text{Dimension} = 50 \times 50 \text{ cm}^2$$

$$= 5 \times 50 \times 10^{-4} \text{ m}^2$$

$$L = 6 \text{ mm} = 6 \times 10^{-3} \text{ m}$$



$$T_0 = 400^\circ\text{C} = 673\text{K}$$

$$T_\infty = 50^\circ\text{C} = 323\text{K}$$

$$T = 120^\circ\text{C} = 393\text{K}$$

$$\text{Biot number } Bi = \frac{hL}{k_s} \rightarrow \text{Gppg.}$$

$$L = \text{significant length } L_c = \frac{\text{thickness}}{2} = \frac{6 \times 10^{-3}}{2}$$

$$L_c = 3 \times 10^{-3}$$

⇒ from data book (Pure alloy in Properties) → Pg. 1

$$\rho = 2707 \text{ kg/m}^3$$

$$c_p = 896 \text{ J/kgK}$$

$$k = 204.2 \text{ W/mK}$$

$$Bi = \frac{hL}{k_s} = \frac{100 \times 3 \times 10^{-3}}{204.2} = 1.46 \times 10^{-3}$$

$Bi < 0.1$ , so it is lumped parameter system.

$$\text{slab} = \frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\frac{hAs}{c_p V \rho} t\right] \quad \boxed{\tau = t, \text{ time}}$$

$$\tau = 117.1 \text{ sec}$$



1) A mildsteel sphere of 15mm dia is planned to be cooled by an air flow at  $20^\circ\text{C}$  the convective heat transfer coefficient is  $110 \text{ W/m}^2\text{K}$ . Calculate the following. (i) Time required to cool the sphere  $700 - 150^\circ\text{C}$

(ii) Instantaneous heat transfer rate at  $150^\circ\text{C}$

(iii) total energy transfer upto  $150^\circ\text{C}$

Take for mildsteel  $\rho = 7850 \text{ kg/m}^3$ ,  $c_p = 474 \text{ J/kgK}$  and thermal conductivity  $k = 43 \text{ W/mK}$ ,

$\alpha = 0.044 \text{ m}^2/\text{hr}$

Sol Given  $d = 15 \text{ mm} = 0.015 \text{ m}$ .

$$T_\infty = 20^\circ\text{C} = 293 \text{ K}$$

$$h = 110 \text{ W/m}^2\text{K}$$

$$T_0 = 700^\circ\text{C} = 973 \text{ K}$$

$$T = 150^\circ\text{C} = 423 \text{ K}$$

$$\alpha = \frac{0.044}{60 \times 60} = 1.22 \times 10^{-5} \text{ m}^2/\text{sec}$$

$$Bi = \frac{hL}{k_s} = \frac{110 \times 2.5 \times 10^{-3}}{43} = 6.39 \times 10^{-3}$$

$$L = \frac{\text{char. } R}{3} = \frac{7.5 \times 10^{-3}}{3} = 2.5 \times 10^{-3} \text{ m}$$

$Bi < 0.1$ , Lumped.

Lumped

$$\frac{T - T_\infty}{T_0 - T_\infty} = \exp\left[-\frac{hA_s}{\rho V c_p} \tau\right]$$



$$L = \frac{V}{A_s}$$

$$\frac{A_s}{V} = \frac{1}{L}$$

$$\text{then, } \frac{423 - 293}{973 - 293} = \exp \left[ \frac{-110 \times 0.0706 \times t}{474 \times 2.5 \times 10^{-3} \times 7850} \right]$$

$$t = 139 \text{ sec}$$

Instantaneous heat flow

$$Q = h A_s (T - T_\infty) = 110 \times 5.30 \times (923 - 293)$$

$$A = 4\pi R^2$$

$$= 1086.8 \text{ KW}$$

$$= 4\pi \times (7.5 \times 10^{-3})^2$$

$$= 10.8 \text{ W}$$

$$= 5.30 \text{ m}^2$$

$$= 0.0706$$

$$\Rightarrow \text{Volume} = \frac{4}{3} \pi R^3 = \frac{4}{3} \times \pi \times (0.075)^3 = 1.76 \times 10^{-3} \text{ m}^3$$

$$\Rightarrow q_t = \rho c_p V (T - T_0)$$

$$= 7850 \times 474 \times 1.76 \times 10^{-3} (923 - 973)$$

$$= -36.16 \text{ W}$$

The negative sign shows that the heat is coming out of the sphere



### Infinite solid:

A solid which extends its self infinitely in all direction of space is a infinite solid.

The heat transfer coefficient b/w the surface of the plate and the fluid on both sides is assumed to be constant. The center of the plate is selected as the origin.

⇒ The governing differential eqn

$$\frac{d^2 T}{dx^2} = \frac{1}{\alpha} \frac{dT}{dt}$$

① At  $t=0$ ,  $T_0 = T_i$       &     $x=0$ ,  $\frac{dT}{dx} = 0$

$$x = \pm L, \quad kA \frac{dT}{dx} = hA(T_0 - T_\infty)$$

$$\frac{T_0 - T_x}{T_i - T_\infty} = f \left[ \frac{x}{L}, \frac{hL}{k}, \frac{\alpha t}{L^2} \right]$$

① A Aluminium slab of 5cm thick initially at a temperature of  $400^\circ\text{C}$ , It is suddenly immersed in a water at  $90^\circ\text{C}$ . Calculate the midplane temperature after 1min calculate temp inside the plate at a distance 10mm from



the mid plane. Take  $h = 1800 \text{ W/m}^2\text{K}$

sol Given data

thickness  $L = 5 \text{ cm} = 5 \times 10^{-2} \text{ m}$

$$T_i = 400^\circ\text{C} + 273 = 673 \text{ K}$$

$$T_\infty = 90^\circ\text{C} + 273 = 363 \text{ K}$$

$$t = 1 \text{ min} = 60 \text{ sec}$$

$$x = 10 \text{ mm} = 10 \times 10^{-3} \text{ m}$$

$$h = 1800 \text{ W/m}^2\text{K}$$

(i)  $T_x = ?$

$$\frac{T(x,t) - T_\infty}{T_0 - T_\infty}$$

$$\text{curve} = \frac{x}{L} \Rightarrow L = \frac{5}{2} = \frac{0.01}{0.025} = 4$$

x-axis

$$= \frac{hL}{k} = 0.225$$

$$, k = 0.22$$

$$= \frac{10 \times 10^{-3} \times 1800}{0.22}$$

$$\text{curve} = 81.8$$

$$\underline{\text{y-axis}} = 0.98$$

$$= \frac{hL}{k} = \frac{1800 \times 16 \times 10^{-3}}{0.98} = 18.36$$

$$= \frac{T_{(x/L)} - T_\infty}{T_0 - T_\infty}$$

$$T_0 = 421.9$$

$$= \frac{\left(\frac{0.01}{0.05}\right) - 363}{-363}$$



## Unit-II Convection



## Reynolds Number (Re)

It is defined as the ratio of inertia force to viscous force.

$$Re = \frac{\text{Inertia force}}{\text{Viscous force}} = \frac{\rho U^2 L^2}{\mu UL} = \frac{\rho UL}{\mu}$$

$$Re = \frac{UL}{\nu}$$

$U$  = velocity, m/s

$L$  = length, m,

$\nu = \frac{\mu}{\rho}$  = kinematic viscosity, m<sup>2</sup>/s

## Prandtl Number (Pr)

It is the ratio of the momentum diffusivity to the thermal diffusivity.

$$Pr = \frac{\text{Momentum diffusivity}}{\text{Thermal diffusivity}} = \frac{\mu c_p}{k} = \frac{\nu}{\alpha}$$

$\nu$  = kinematic viscosity, m<sup>2</sup>/s,

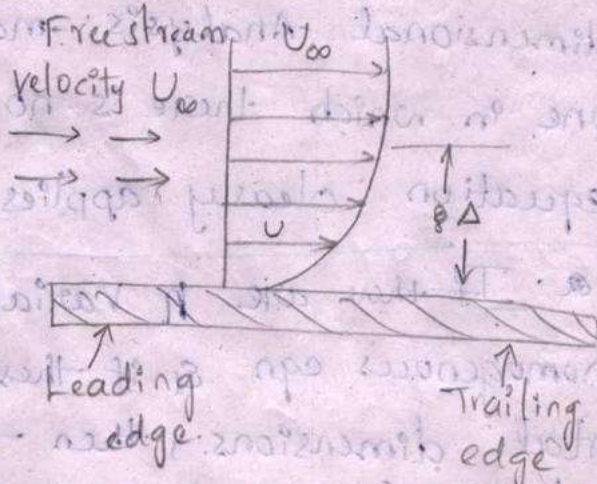
$\alpha$  = thermal diffusivity, m<sup>2</sup>/s.

## Nusselt number (Nu)

It is defined as the ratio of the heat flow by convection process under an unit temperature gradient.

$$Nu = \frac{hL}{k}$$

## Newtonian Non Newtonian, laminar Turbulant Boundary layer Concept



Boundary layer on flat plate



## Types of Boundary Layer

1. Hydrodynamic boundary layer  
(or)

velocity boundary layer

2) thermal boundary layer.

### ① Hydrodynamic Boundary Layer

In hydrodynamic boundary layer, velocity of the fluid is less than 99% of free stream velocity.

Thermal boundary layer

In thermal boundary layer, temperature of the fluid is less than 99% of free stream temperature.

## Types of Convection:

1) Free Convection      2) Forced Convection

Free (or) Natural Convection

If the fluid motion is produced due to change in density resulting from temperature gradients, the mode of heat transfer is said to be free or natural convection.

### Forced Convection

If the fluid motion is artificially created by means of an external force like a blower or fan that type of heat transfer is known as forced convection.



11/05/2023

External Flow - 121

- i) Air at  $20^\circ\text{C}$ ; at a pressure of 1 bar flowing over a flat plate at a velocity of 3 m/sec. If the plate is maintain at  $60^\circ\text{C}$ , calculate the heat transfer per unit width of the plate. Assuming the length of the plate along the flow of air is 2 m.

Sol

$$T_\infty = 20^\circ\text{C}$$

$$P = 1 \text{ bar.}$$

$$u = 3 \text{ m/sec}$$

$$T_w = 60^\circ\text{C}$$

$$\text{width } w = 1 \text{ m}$$

$$L = 2 \text{ m}$$

$$T_f = \frac{T_w + T_\infty}{2}$$

$$= \frac{60 + 20}{2} = 40^\circ\text{C}$$

from data book Properties of Gas & Vapour (42)

$$\rho = 1.128 \text{ kg/m}^3$$

$$k = 0.02756 \text{ W/mK.}$$

$$Pr = 0.699$$

$$\nu = 16.96 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$Re = \frac{uL}{\nu} = \frac{3 \times 2}{16.96 \times 10^{-6}} = 353 \times 10^3 = 3 \times 10^5$$

$\therefore$  it is less than  $5 \times 10^5 < 3 \times 10^5$

it is a laminar flow.



Nusselt number  $Nu = \frac{hL}{k} =$

$$Nu = 0.332 Re_x^{0.5} Pr^{0.333}$$

$$= 0.332 \times (3 \times 10^5)^{0.5} \times (0.699)^{0.333}$$

$$= 156.54$$

$$\Rightarrow h = \frac{Nu \cdot k}{L} = \frac{156.5 \times 0.02756}{2} = 2.157$$

$$\Rightarrow Q = hA(T_w - T_\infty)$$

$$Q = 2.157 \times (2 \times 1) (60 - 20)$$

$$= 172.52 \text{ W} \times 2$$

$$= 345.05 \text{ W}$$

$$h = 2 \times h_x$$

$$= 2 \times 172.5$$

$$=$$

② Air at  $20^\circ\text{C}$  at atmospheric pressure flows over a plate at a velocity of  $30 \text{ m/sec}$ . If the plate is  $1 \text{ m}$  wide and  $80^\circ\text{C}$ . Calculate the following at  $x = 300 \text{ mm}$  =

- i) hydrodynamic boundary layer thickness.
- ii) Thermal boundary layer thickness.
- iii) Local friction coefficient
- iv) avg. friction coefficient
- v) Local heat transfer coefficient
- vi) Avg. heat transfer coefficient
- vii) Heat transfer.



801

Given data

$$T_{\infty} = 20^{\circ}\text{C}$$

$$u = 30 \text{ m/sec}$$

$$W = 1 \text{ m}$$

$$T_w = 80^{\circ}\text{C}$$

$$x = 0.3 \text{ m} = \text{Length.}$$

$$T_f = \frac{T_w + T_{\infty}}{2} = \frac{20 + 80}{2} = 50^{\circ}\text{C}$$

(i) hydrodynamic boundary layer thickness.

taking values from data book: at  $50^{\circ}\text{C}$  (properties)

$$\rho = 1.093 \text{ kg/m}^3$$

$$\nu = 17.95 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$\text{Pr} = 0.698$$

$$k = 0.02826 \text{ W/mK}$$

$$\Delta x = 5x \text{Re}^{-0.5}$$

$$= 5 \times 0.3 \times (5 \times 10^4)^{-0.5}$$

$$= 6.7 \times 10^{-3} \text{ m}$$

$$\text{Nu} = \frac{hL}{k}$$

$$\text{Re} = \frac{uL}{\nu} = \frac{30 \times 0.3}{17.95 \times 10^{-6}} = 5 \times 10^4$$

it is laminar flow.



(ii) Thermal boundary layer thickness

$$\begin{aligned}\Delta_{Tx} &= \Delta_{hx} \cdot Pr^{-0.333} \\ &= 6.7 \times 10^{-3} \times (0.698)^{-0.333} \\ &= 7.56 \times 10^{-3} \text{ m}\end{aligned}$$

(iii) Local friction coefficient

$$\begin{aligned}C_{fx} &= 0.664 Re_x^{-0.5} \\ &= 0.664 \times (5 \times 10^4)^{-0.5} \\ &= 2.96 \times 10^{-3}\end{aligned}$$

(iv) Avg. friction coefficient

$$\begin{aligned}C_{fL} &= 1.328 Re_L^{-0.5} \\ &= 1.328 \times (5 \times 10^4)^{-0.5} \\ &= 5.9 \times 10^{-3}\end{aligned}$$

(v) Local heat transfer coefficient

$$Nu = \frac{hL}{k}$$

$$\begin{aligned}Nu &= 0.332 \times Re^{0.5} \times Pr^{0.333} \\ &= 65.86\end{aligned}$$

$$h_x = \frac{65.86 \times 0.02826}{0.3} = 6.2 \text{ W/m}^2\text{K}$$



⑥ Avg. heat transfer coefficient (h)

$$h = 2 \times h_x = 12.4 \text{ W/m}^2\text{K}$$

$$h = 6.2 \times 2 = 12.4 \text{ W/m}^2\text{K}$$

⑦ Heat transfer (Q)

$$Q = h \times A (T_w - T_\infty)$$

$$A = W \times L = 1 \times 0.3 = 0.3$$

$$Q = 223.38 \text{ W}$$

$$= 12.4 \times 0.3 (80 - 20)$$

$$Q = 223.2 \text{ W}$$

③ Air at  $30^\circ\text{C}$  flows over a flat plate at a velocity of  $4 \text{ m/sec}$ . The plate is maintained at  $90^\circ\text{C}$ . The plate dimension  $90 \times 30 \text{ cm}^2$  calculate the heat transfer for following condition.

(i) ~~first~~ half half the plate

(ii) full plate

(iii) Next half half the plate

Sol  $T_\infty = 30^\circ\text{C}$

$$T_w = 90^\circ\text{C}$$

$$u = 4 \text{ m/sec}$$

$$\text{Plate} = 90 \times 30 \text{ cm}^2$$

$$= 0.9 \times 0.3 \text{ m}^2$$

$$T_f = \frac{T_w + T_\infty}{2} = \frac{30 + 90}{2} = \frac{120}{2} = 60^\circ\text{C}$$



$$\rho = 1.060 \text{ kg/m}^3$$

$$k = 0.02896 \text{ W/mK}$$

$$V = 18.97 \times 10^{-6} \text{ m}^3/\text{sec}$$

$$Pr = 0.696$$

$$\textcircled{1} \quad x = L = 0.9 \text{ m}$$

$$x = \frac{L}{2} = \frac{0.9}{2} = 0.45 \text{ m.}$$

$$Re = \frac{UL}{V} = \frac{4 \times 0.45}{18.97 \times 10^{-6}} = 0.9 \times 10^5$$

$$\begin{aligned} Nu &= 0.332 \times Re^{0.5} \times Pr^{0.333} \\ &= 0.332 \times (0.96 \times 10^5)^{0.5} \times (0.696)^{0.333} \\ &= 91.17 \end{aligned}$$

$$Nu = \frac{hL}{k}$$

$$h_x = \frac{91.17 \times 0.02896}{0.45} = 5.86 \text{ W/m}^2\text{K.}$$

$$h = 2 \times 5.86 = 11.73 \text{ W/m}^2\text{K.}$$

$$\begin{aligned} Q_1 &= hA (T_w - T_\infty) \\ &= 11.73 \times 0.135 (90 - 30) \end{aligned}$$

$$Q_1 = 95.013 \text{ W}$$

$$\begin{aligned} A &= 0.45 \times 0.3 \\ &= 0.135 \end{aligned}$$



(ii) - full plate

$$L = 0.9$$

$$Re = \frac{UL}{\nu} = \frac{4 \times 0.9}{18.97 \times 10^{-6}} = 1.89 \times 10^5$$

$$\begin{aligned} Nu &= 0.332 \times Re^{0.5} \times Pr^{0.333} \\ &= 0.332 \times (1.89 \times 10^5)^{0.5} \times (0.696)^{0.333} \\ &= 127.92 \end{aligned}$$

$$\Rightarrow Nu = \frac{hL}{k}$$

$$h_x = \frac{Nu k}{L} = \frac{127.05 \times 0.02396}{0.9} = 4.11$$

$$h = 4.11 \times 2 = 8.22 \text{ W/m}^2\text{K}.$$

$$Q_2 = hA(T_w - T_b)$$

$$= 8.22 \times 0.27(90 - 30)$$

$$Q_2 = 133.16 \text{ W}$$

$$\begin{aligned} A &= 0.9 \times 0.3 \\ &= 0.27 \text{ m}^2 \end{aligned}$$

iii

$$Q = Q_2 - Q_1$$

$$= 133.16 - 95.013$$

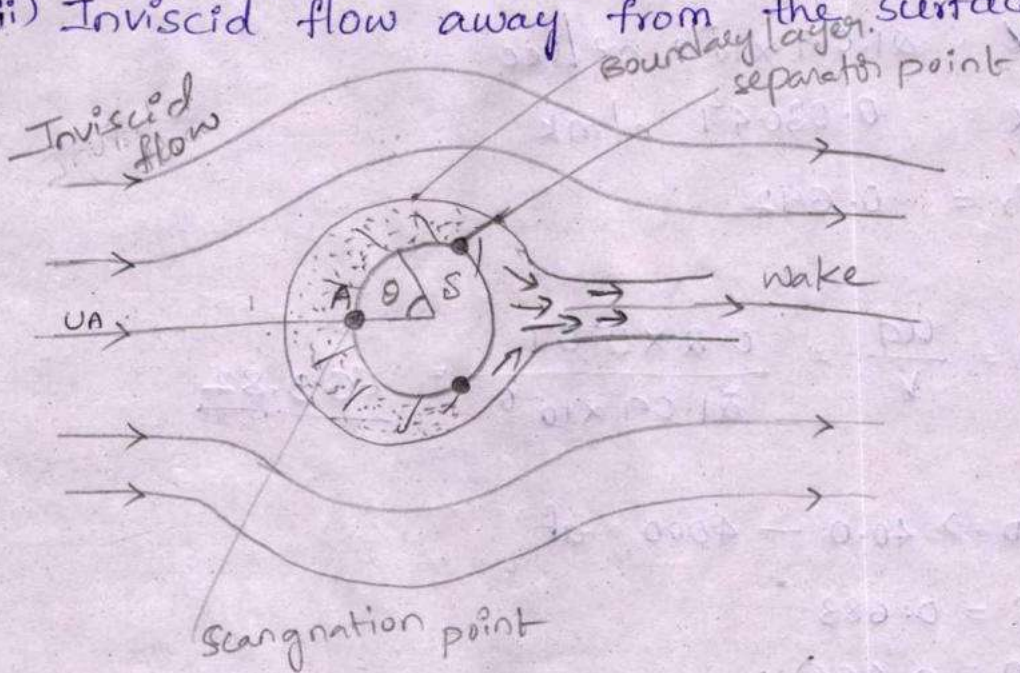
$$Q = 38.15 \text{ W}$$



# Flows over a Cylinder & Sphere

The flow field can be divided into two regions. They are

- (i) Boundary layer region near the surface
- (ii) Inviscid flow away from the surface



① Air at  $30^\circ$ ,  $0.2 \text{ m/sec}$  flows across a  $120 \text{ W}$  electric bulb at  $130^\circ \text{C}$  find heat transfer & Power lost due to convection. if bulb diameter is  $70 \text{ mm}$ .

sol Given

fluid temp  $T_\infty = 30^\circ \text{C}$

velocity  $U = 0.2 \text{ m/sec}$

heat energy  $Q_i = 120 \text{ W}$

Surface temp  $T_w = 130^\circ \text{C}$

dia  $D = 70 \text{ mm} = 0.07 \text{ m}$ .



$$T_f = \frac{T_w + T_o}{2} = \frac{30 + 130}{2} = \frac{160}{2} = 80^\circ\text{C}$$

properties of air

$$\rho = 1.000 \text{ kg/m}^3$$

$$\mu = 21.09 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$k = 0.03047 \text{ W/mK}$$

$$Pr = 0.692$$

$$Re = \frac{u_d}{\nu} = \frac{0.2 \times 0.07}{21.09 \times 10^{-6}} = \underline{663.82}$$

$$Re \Rightarrow 40.0 - 4000 \text{ at}$$

$$C = 0.683$$

$$m = 0.466$$

$$Nu = C \cdot Re^m \cdot Pr^{0.333}$$

$$= 0.683 \times (663.82)^{0.466} \times (0.692)^{0.333}$$

$$\Rightarrow Nu = 0.37 \cdot Re^{0.6}$$

$$= 0.37 \times (663.8)^{0.6}$$

$$= 18.25$$

$$Nu = \frac{hD}{k}$$

$$h = \frac{18.25 \times 0.03047}{0.07} = 7.94$$

$$Q = 7.94 \times 0.0153 \times (130 - 30)$$

$$Q_2 = 12.14 \text{ W}$$

$$A = 4\pi r^2$$

$$= 4\pi \times 0.035^2$$

$$= 0.0153$$



(ii) % of heat lost due to convection.

$$= \frac{Q_2}{Q_1} \times 100$$

$$= \frac{12.14}{120} \times 100 = 10.11$$

Q2) Air at  $40^\circ\text{C}$  flows over a tube with a velocity of  $30 \text{ m/sec}$ . The tube surface temp is  $120^\circ\text{C}$ . Calculate the heat transfer coefficient for the following cases.

(i) tube is considered to be square of side  $6 \text{ cm}$

(ii) tube is circular cylinder of diameter  $6 \text{ cm}$ .

80)  $T_\infty = 40^\circ\text{C}$

$$T_w = 120^\circ\text{C}$$

$$U = 30 \text{ m/sec}$$

$$T_f = \frac{T_w + T_\infty}{2} = \frac{40 + 120}{2} = \frac{160}{2} = 80^\circ\text{C}$$

air =

$$\rho = 1.000 \text{ kg/m}^3$$

$$V = 21.09 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$K = 0.03047 \text{ W/mK}$$

$$Pr = 0.692$$

(i)  $h = ?$

$$L = 6 \text{ cm} = 0.06 \text{ m}$$

$$Re = \frac{UL}{V} = \frac{30 \times 0.06}{21.09 \times 10^{-6}} = 85 \times 10^3$$

$$(i) Nu = 173.3$$

$$h = 88 \text{ (127, 110)}$$

$$(ii) Nu = 219.3$$

$$(124) \cdot h = 111.3$$

$$Re = 0.85 \times 10^5$$



$$\Rightarrow N = C Re^n = 0.092 (0.85 \times 10^5)^{0.675}$$

$$= 195.4$$

$$= 1.13 Pr^{0.33} = 1.13 \times (0.692)^{0.33}$$

$$\Rightarrow h = \frac{Nu k}{L} = 195.54$$

(ii) flow over cylinder (124 Pa)

$$Nu = C Re^m \cdot Pr^{0.33}$$

$$C = 0.0266$$

$$0.805 = m$$

$$= 0.0266 \times (0.85 \times 10^5)^{0.805} \times (0.692)^{0.33}$$

$$= 219.3$$

$$Nu = \frac{hD}{k} = 111.3 \text{ W/m}^2\text{K}$$

## Flows over Banks of Tubes

① In a surface condenser, water flows through inline Tubes while the air is passed in cross-flow over the tubes. The temp and velocity of air are  $30^\circ\text{C}$  &  $8 \text{ m/sec}$  respectively. The longitudinal and Transverse pitch are  $22 \text{ mm}$  and  $20 \text{ mm}$  respectively. The tube outside diameter is  $18 \text{ mm}$  & Tube surface temp is  $90^\circ\text{C}$ .

Given

$$T_\infty = 30^\circ\text{C}$$

$$u = 8 \text{ m/sec}$$

$$S_L = 22 \text{ mm} = 0.022 \text{ m}$$

$$S_T = 20 \text{ mm} = 20 \times 10^{-3} \text{ m}$$

$$D = 18 \text{ mm} = 0.018 \text{ m}$$

$$T_w = 90^\circ\text{C}$$

Sol



$$T_f = \frac{T_w + T_o}{2} = 40^\circ\text{C} \rightarrow \rho, k, \nu, \Pr$$

$$Nu = \frac{h d}{k} = h = ? \quad 426.6 \text{ W/m}^2\text{K}$$

$$Nu = 0.33 \times 1.13 \times Pr^{0.33} \times C \cdot Re^n \quad [131 \text{ pg}]$$

$$\Rightarrow Re = \frac{u d}{\nu} = 7.6 \times 10^4 = \frac{80 \times 0.018}{12.97 \times 10^{-6}}$$

Inline [129 pg]

$$U_{\max} = \left[ \frac{St}{St - D} \right] U_\infty$$

$$= \left[ \frac{20 \times 10^3}{20 \times 10^3 - 0.018} \right] \times 8$$

$$= 80 \text{ m/sec}$$

$$\frac{St}{D} = 1.11$$

$$\frac{St}{D} = 1.22$$

$$C = 0.348$$

$$n = 0.592$$



## Internal Flow: Flow through a Cylinder

- ① Lubricating oil at a temperature of  $60^\circ\text{C}$  enters 1 cm diameter tube with a velocity of 3 m/sec. The tube surface is maintained at  $40^\circ\text{C}$ . Assuming that the oil has the following average properties, calculate the tube length required to cool the oil at  $45^\circ\text{C}$ .

$$\rho = 865 \text{ kg/m}^3, \quad k = 0.140 \text{ W/mK}$$

$$c_p = 1.78 \text{ kJ/kg}^\circ\text{C}$$

$$1.78 \times 10^3 \text{ J/kg}^\circ\text{C}$$

sol)  $T_{m_i} = 60^\circ\text{C}$

$$D = 1 \text{ cm} = 0.01 \text{ m}$$

$$u_m = 3 \text{ m/sec}$$

$$T_w = 40^\circ\text{C}$$

$$T_{m_o} = 45^\circ\text{C}$$

$$\Rightarrow Q = hA \Delta T$$

$$\Rightarrow Q = m c_p \Delta T$$

$$\Rightarrow Nu = 3.66 = \frac{h d}{k}$$

$$h = 51.24 \text{ W/m}^2\text{K}$$

$$m = \rho \times A \times u$$

$$= \frac{\pi d^2}{4}$$

$$= 0.204 \text{ m/sec}$$



$$Q = hA \Delta T$$

$$\left( \frac{T_m + T_i}{2} \right) - T_w$$

$$A = \pi \times d \times L \rightarrow 270.6 \text{ m}$$

$$Q = m_{cp} (T_m - T_{Ti})$$

$$= 56.2 \times 4178 (50 - 30)$$

$$= 4.69 \times 10^6 \text{ W}$$

$$Q = hA (T_w - T_m)$$

$$A = \pi DL$$

$$L = ?$$



## Free Convection

due to temp difference, density changed in the fluid flow.

- ① A vertical plate of 0.75m height is at 170°C and is exposed to air at a temp of 105°C and 1 atmospheric pressure. Calculate
- Mean heat transfer coefficient
  - rate of heat transfer per unit width

So)  $L = 0.75 \text{ m} = x$

$$T_w = 170^\circ\text{C}$$

$$T_\infty = 105^\circ\text{C}$$

$$T_f = \frac{T_w + T_\infty}{2} = 137.5 \approx 140^\circ\text{C}$$

$$\rho = 0.854 \text{ kg/m}^3$$

$$V = 27.80 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$Pr = 0.684$$

$$k = 0.03489 \text{ W/mK}$$

$$Gr \cdot Pr$$

$$Gr = \frac{\rho \times \beta \times x^3 \Delta T}{\nu^2} = \frac{2.5 \times 10^9 \times 835 \times 10^{-6}}{1}$$

$$\beta = \frac{1}{T_f} = \frac{1}{(137.5 + 273)} = \frac{1}{410.5} = 2.4 \times 10^{-3}$$

$$Gr \cdot Pr = 2.5 \times 10^9 \times 0.684$$



$$Gr \cdot Pr = 8.3 \times 10^8 \times 0.684$$

$$Gr \cdot Pr = 5.7 \times 10^8$$

$\left( \begin{array}{l} Gr \cdot Pr < 10^9 \\ \Rightarrow \text{it is laminar flow.} \end{array} \right.$

$$\begin{aligned} \Rightarrow Nu &= 0.59 (Gr \cdot Pr)^{0.25} \longrightarrow 194 \text{ Pg.} \\ &= 0.59 (5.7 \times 10^8)^{0.25} \quad \text{higher values.} \\ &= 91.16 \end{aligned}$$

$$\Rightarrow Nu = \frac{hL}{K}$$

$$\begin{aligned} h &= \frac{91.16 \times 0.03489}{0.75} \\ &= 4.24 \text{ W/m}^2\text{K} \end{aligned}$$

$$\begin{aligned} \Rightarrow Q &= hA(T_w - T_\infty) & \left| \begin{array}{l} A = W \times L \\ = 1 \times 0.75 \\ = 0.75 \text{ m}^2 \end{array} \right. \\ &= 4.24 \times 0.75 (170 - 105) \\ &= 206.7 \text{ W} \end{aligned}$$

② A thin 100cm long and 10cm wide horizontal plate is maintained at a uniform temperature of  $150^\circ\text{C}$  in a large tank of water at  $75^\circ\text{C}$ . Estimate the rate of heat to be supplied to the plate maintain constant plate temp as heat is dissipated either side of plate



Ex) Given data  $\rho = 961 \times 8.08$   $x = w$  while cal at Gr

$$L = 100 \text{ cm} = 1 \text{ m}$$

$$W = 10 \text{ cm} = 0.1 \text{ m} = \text{horizontal position.}$$

$$T_w = 150^\circ\text{C}$$

$$T_\infty = 75^\circ\text{C}$$

$$T_f = \frac{T_w + T_\infty}{2} = \frac{150 + 75}{2} = \frac{225}{2} = 112.5^\circ\text{C}$$

at  $100^\circ\text{C}$

$$\rho = 961 \text{ kg/m}^3$$

$$v = 0.293 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$Pr = 1.740$$

$$k = 0.6804 \text{ W/mK}$$

$$\beta = \frac{1}{(T_f + 273)} = \frac{1}{(112.5 + 273)} = 2.5 \times 10^{-3}$$

$$Gr = \frac{\rho \cdot \beta \cdot L^3 \Delta T}{v^2} = \frac{961 \times 2.5 \times 10^{-3} \times (0.1)^3 \times (25)}{(0.293 \times 10^{-6})^2}$$

$$x = \frac{w}{2} \text{ only for finding Gr value.} = \frac{0.1}{2} = 0.05$$

$$Gr = 2.67 \times 10^9$$

$$Gr Pr = 2.67 \times 10^9 \times 1.740$$

$$= 4.6 \times 10^9$$



$$\Rightarrow Nu = 0.15 (GrPr)^{0.333}$$

$$= 247.62$$

$$\Rightarrow Nu = \frac{hL}{k}$$

$$h = \frac{247.62 \times 0.6804}{0.05}$$

$$\Rightarrow h_F = 3369.6 \text{ W/m}^2\text{K}$$

$$\Rightarrow Nu = 0.27 \times (GrPr)^{0.25}$$

=

$$= 70.31$$

$$\Rightarrow h_2 = \frac{70.31 \times 0.6804}{0.05} = 956.77 \text{ W/m}^2\text{K}$$

$$Q = hA(\Delta T)$$

$$= 3439.9 \times 0.1 \times (75)$$

$$Q = 25799.25 \text{ W}$$

$$h = h_1 + h_2$$

$$= 3439.91$$

$$A = 0.1 \times 1$$

$$= 0.1 \text{ m}^2$$



### Unit-III

## Heat transfer with phase change.

Boiling : liquid - vapour

Condensing : Vapour - liquid

Pool boiling heat transfer phenomena

Boiling is a convection process involving a change of phase from liquid to Vapour state.

⇒ According to Convection law

$$Q = hA (\Delta T)$$

$\Delta T = \text{excess temperature} = T_w - T_{\text{sat}}$

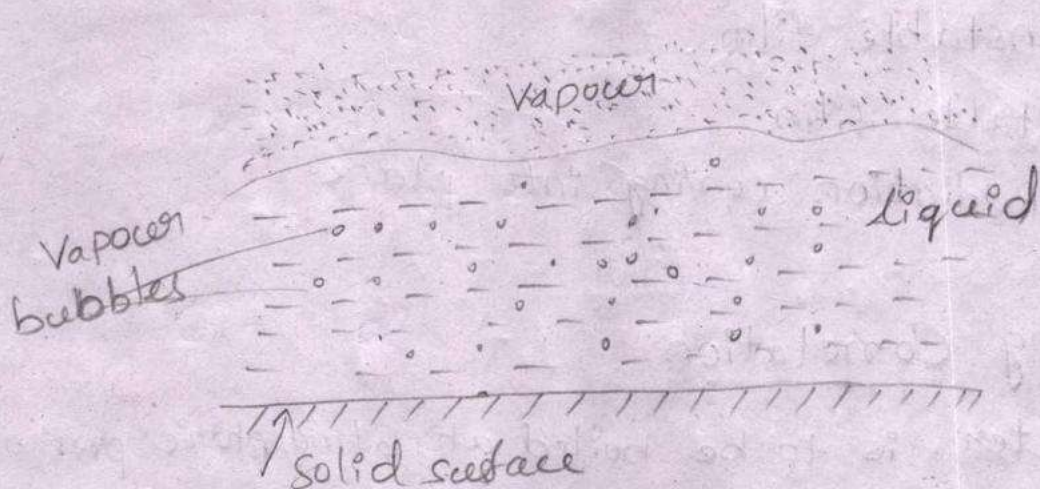


fig: Pool boiling



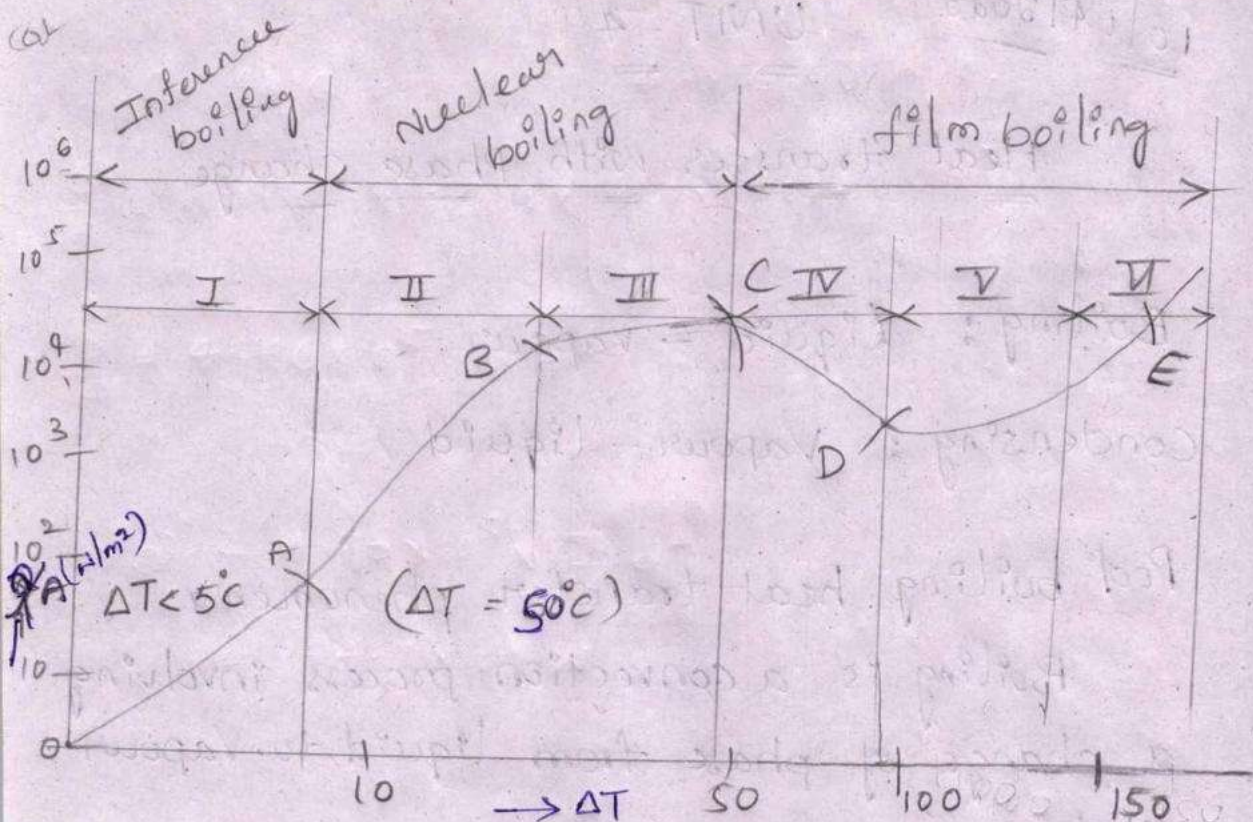


fig: Pool Boiling curve for water

1) free convection

2) bubble condensed in superheated liquid

3) bubble rise to surface

4) unstable film

5) stable film

6) Radiation coming into plane

## Boiling Correlation

① Water is to be boiled at atmospheric pressure in a polished copper pan by means of an electric heater. The diameter of the pan is 0.38m and is kept at 115°C. Calculate the following.

i) Power required to boil the water

ii) rate of evaporation.

iii) Critical heat flux.



Sol Given data

$$P = 1.03 \text{ bar} = T_{\text{sat}} = 100^\circ\text{C}$$

$$\text{dia} = d = 0.38 \text{ m}$$

$$T_w = 115^\circ\text{C}$$

$$\Delta T = T_w - T_{\text{sat}}$$

$$= 115 - 100$$

$$= 15^\circ\text{C}$$

$\therefore$  Nucleate pool boiling.

$$\frac{Q}{A} = \mu_f h_{fg} \left[ g \cdot \frac{(\rho_l - \rho_v)}{\sigma} \right]^{0.5} \left[ \frac{C_l \Delta T}{C_{sf} h_{fg} Pr^n} \right]^{3.0}$$

$\Rightarrow$  Property values of water at  $100^\circ\text{C}$

$$\rho_l = 961 \text{ kg/m}^3$$

$$v = 0.293 \times 10^{-6} \text{ m}^3/\text{kg}$$

$$Pr = 1.740$$

$$\mu = \rho \times v = 961 \times 0.293 \times 10^{-6}$$

$$\mu = 2.81573 \times 10^{-4} \text{ Ns/m}^2$$

$\Rightarrow$  from steam tables

$$h_{fg} = 2266.9 \text{ kJ/kg}$$

$$v_g = 1.693 \text{ m}^3/\text{kg}$$

$$\rho_v = \frac{1}{v_g} = \frac{1}{1.693} = 0.597$$

$$\sigma = 0.0588 \text{ N/m.} \rightarrow$$

$$C_{fs} = 0.013$$



$n=1$  for water, other for 0.17

$$C_p = 4216 \text{ J/kgK} = 4.216 \text{ kJ/kgK}$$

$$\frac{Q}{A} = 2.81 \times 10^{-4} \times 2266.9 \times \left[ 9.81 \times \frac{(961 - 0.597)}{0.0588} \right]^{0.5} \left[ \frac{9.216 \times 15}{0.013 \times 2266.9} \right]^{3.1}$$
$$\frac{Q}{A} = 4.76 \times 10^5 \text{ W/m}^2$$

$$\Rightarrow Q = 4.76 \times 10^5 \times \frac{\pi}{4} d^2$$

$$= 54.7 \times 10^3 \text{ W}$$

$$\Rightarrow Q = \dot{m} \times h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} \text{ — J/kgK}$$

$$= 0.024 \text{ kg/sec}$$

$\Rightarrow$  critical or maximum heat flux.  $1.52 \times 10^6 \text{ W/m}^2$

$$\frac{Q}{A} = 0.18 h_{fg} \sqrt{\left[ \frac{\sigma \cdot g \cdot (s_l - s_v)}{s_v^2} \right]^{0.25}}$$

$$= 0.18 \times$$

$$= 1.52 \times 10^6 \text{ W/m}^2$$



② A Nickel wire carrying electric current of 1.5 mm diameter and 50 cm long is submerged in a water bath which is open to atmospheric pressure. Calculate the voltage at the burn out point, if at this point, the wire carries a current of 200 amperes.

Sol Given data

$$d = 1.5 \text{ mm} = 1.5 \times 10^{-3} \text{ m}$$

$$l = 50 \text{ cm} = 50 \times 10^{-2} \text{ m}$$

$$I = 200 \text{ A}$$

$$V = ?$$

$$Q = V \times I$$

Assume that  $T_{\text{sat}} = 100^\circ \text{C}$  ✓

from problem ①  $\frac{Q}{A} = \text{Critical heat flux}$

$$= 1.52 \times 10^6 \text{ W/m}^2$$

"multiply & divide with 'A'"

$$\frac{Q}{A} = \frac{V \times I}{A}$$

$$1.52 \times 10^6 = \frac{V \times 200}{\pi d l}$$

$$V = 17.9 \text{ volts}$$

$$Q = V \times I$$

$$54.7 \times 10^3 = V \times 200$$

$$V = \frac{54.7 \times 10^3}{200}$$

$$V = 273.5$$



③ A heating element clad with a metal is 8mm diameter and half emissivity is 0.92. The element is horizontally immersed in a water bath. The surface temp of the metal is 260°C under steady state boiling conditions. Calculate the power dissipation per unit length of the heater.

sol  $d = 8 \text{ mm} = 8 \times 10^{-3} \text{ m}$

$\epsilon = 0.92$

$T_w = 260^\circ\text{C}$

$Q = h A \Delta T$

$h = ?$

$\Delta T = T_w - T_{\text{sat}}$   
 $= 260 - 100$   
 $= 160^\circ\text{C}$

film boiling

$$h_c = 0.62 \left[ \frac{K_f^3 \rho_f (\rho_f - \rho_v) \cdot g (h_{fg} + 0.68 C_{pf} \Delta T)}{\mu_f D \Delta T} \right]^{0.25}$$

$$h_r = \epsilon \left[ \frac{T_w^4 - T_{\text{sat}}^4}{T_w - T_{\text{sat}}} \right]$$

$h = h_c + 0.75 h_r$

$h = 436.02 \text{ W/m}^2\text{K}$



$$Q = hA \Delta T$$

$$Q = h \pi D L \Delta T$$

$$\frac{Q}{L} = h \pi D \Delta T$$

$$= 436.02 \times \pi \times 8 \times 10^{-3} \times 160$$

$$\frac{Q}{L} = 1753.6 \text{ W/m}$$

## Condensation

Changing of phase from Vapour to Liquid

### Film wise Condensation

The liquid condensate wets the solid surface spreads out and forms a film over the entire surface.

⇒ Film Condensation occurs when a Vapour free from impurities.

### Drop wise Condensation

The vapour condenses into small ~~var~~ liquid droplets of various sizes. which fall down the surface in a random fraction

⇒ Dropwise Condensation heat transfer rate is 10-times higher than film wise Condensation.



## Nusselt's Theory for film Condensation

The following Assumptions are made

- 1) The plate is maintain at the uniform temperature
- 2) Fluid properties are constant
- 3) The shear stress of the liquid vapour interface is negligible
- 4) The heat transfer across the condensate layer is by pure convection & the temp distribution is linear
- 5) The condensing vapour is entirely clean and free from gases, air and non-condensing impurities.

① Dry saturated steam at a pressure of 3 bar, Condenses on the surface of a vertical tube of height 1m. The tube surface temp is kept at  $110^{\circ}\text{C}$ . Calculate thickness of the condensate film. (ii) local heat transfer coefficient at a distance of 0.25m

Sol

$$P = 3 \text{ bar}$$

$$h = 1 \text{ m}$$

$$T_{\text{ms}} = 110^{\circ}\text{C} \text{ (surface)}$$

at 3 bar from steam tables.



$$T = T_v = 133.5^\circ\text{C}$$

$$h_{fg} = 2163.2 \text{ kJ/kg} = 2163.2 \times 10^3 \text{ J/kg}$$

$$T_f = \frac{(T_v + T_s)}{2} = \frac{133.5 + 110}{2} = 121.75^\circ\text{C}$$

$$\text{density } \rho = 945 \text{ kg/m}^3$$

$$v = 0.247 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$k = 0.6850 \text{ W/mK}$$

$$\text{dynamic viscosity } \mu = \rho \times v$$

$$= 945 \times 0.247 \times 10^{-6}$$

$$= 2.3 \times 10^{-4} \text{ N-s/m}^2$$

$$\text{thickness } \Delta x = \left[ \frac{4\mu, k \cdot x (T_v - T_s)}{g h_{fg} \cdot \rho^2} \right]^{0.25}$$

$$x = 0.25 \text{ m}$$

$$\Delta x = \left[ \frac{4 \times 2.3 \times 10^{-4} \times 0.6850 \times 0.25 (133.5 - 110)}{9.81 \times 2163.2 \times 945^2 \times 10^3} \right]^{0.25}$$

$$= 1.18 \times 10^{-4} \text{ m}$$

(ii) Local heat transfer coefficient ( $h_x$ )

$$h_x = \frac{k}{\Delta x} = \frac{0.6850}{1.18 \times 10^{-4}} = 5805. \text{ W/m}^2\text{K}$$



- ① A Vertical tube of 65 mm outside diameter and 1.5 m long is exposed to steam at atmospheric pressure. The outer surface of the tube is maintained at a temperature of  $60^\circ$  by circulating cold water through the tube. Calculate the following
- the rate of heat transfer to the coolant
  - the rate of condensation of steam.
  - which type of flow it is?

Sol Given data

$$d = 65 \text{ mm} = 65 \times 10^{-3} \text{ m}$$

$$L = 1.5 \text{ m}$$

$$P = \text{atmospheric pressure @ } T = T_v = 100^\circ\text{C}$$

$$T_s = 60^\circ\text{C}$$

$$T_f = \frac{(T_v + T_s)}{2} = \frac{(100 + 60)}{2} = 80^\circ\text{C}$$

properties at  $100^\circ\text{C}$  from steam tables.

$$h_{fg} = 2256.9 \text{ kJ/kg} = 2256.9 \times 10^3 \text{ J/kg}$$

$$\text{at } 80^\circ\text{C} \rightarrow \nu = 0.364 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$\rho = 974 \text{ kg/m}^3$$

$$k = 0.6687 \text{ W/mK}$$



$$\mu = \int V = 974 \times 0.369 \times 10^{-6} = \frac{3.54 \times 10^{-4}}{\text{Ns/m}^2}$$

$$Q = hA(T_v - T_s)$$

$$\Rightarrow h = 0.943 \left[ \frac{k^3 \rho^2 g h_{fg}}{\mu L (T_v - T_s)} \right]^{0.25}$$

The factor 0.943 may be replaced by 1.13 for more accurate result as suggested by Mc. Adams

$$h = 1.13 \left[ \frac{0.6687^3 \times 974^2 \times 9.81 \times 2256.9 \times 10^3}{3.54 \times 10^{-4} \times 1.5 (100 - 60)} \right]^{0.25}$$

$$\boxed{h = 4685.84 \text{ W/m}^2\text{K}}$$

$$\Rightarrow A = \pi d L = \pi \times 65 \times 10^{-3} \times 1.5 = 0.3063 \text{ m}^2$$

$$\begin{aligned} \Rightarrow Q &= hA(T_v - T_s) \\ &= 4685.8 \times 0.3063 (100 - 60) \end{aligned}$$

$$\boxed{Q = 57410.42 \text{ W}}$$

(ii) The rate of Condensation of steam

$$Q = \dot{m} \times h_{fg}$$

$$57410 = \dot{m} \times 2256.9 \times 10^3$$

$$\boxed{\dot{m} = 0.025 \text{ kg/sec.}}$$



(iii) Type of flow.

$$Re = \frac{4\dot{m}}{P\mu} = \frac{4 \times 0.025}{\pi \times (0.065) \times 3.54 \times 10^{-4}}$$

$$P = \pi d$$

$$Re = 1383.35$$

$$= \pi \times 0.065$$

$$= \pi \times 0.065$$

It is less than  $Re < 1800$ , Laminar flow.

② Steam at 0.08 bar is arranged to condense over a 50cm square vertical plate. The surface temperature is maintained at  $20^\circ\text{C}$ . Calculate the following.

(i) Film thickness at a distance of 25cm from the top plate

(ii) Local heat transfer coefficient at a distance of 25cm from the top of the plate.

(iii) Avg. heat transfer coefficient.

(iv) Total heat transfer? (Q)

(v) Total steam condensation rate (m)

(vi) What would be the heat transfer coefficient, if the plate is inclined  $\alpha$  with horizontal plane.



## Heat exchangers

A heat exchanger is defined as an equipment which transfers the heat from a hot fluid to a cold fluid.

### Classification of heat exchangers

There are several types of heat exchangers which may be classified on the basis of:

1. Nature of heat exchange process
2. Relative direction of fluid motion
3. Design & constructional features
4. Physical state of fluids

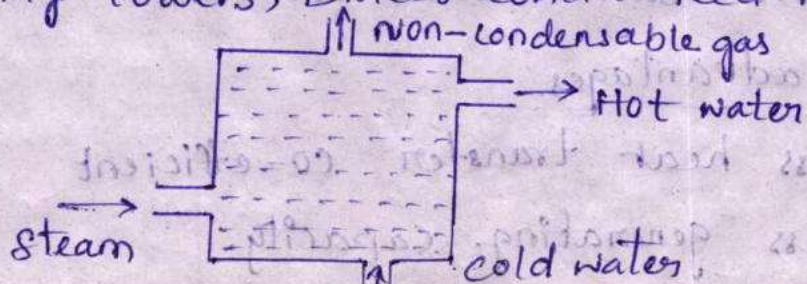
#### I. Nature of heat exchange Process

on the basis of the nature of heat exchange process, heat exchangers are classified as:

- (a) Direct Contact heat exchanger (or)
- (b) Open heat exchangers;

In direct contact heat exchanger, the heat exchange takes place by direct mixing of hot & cold fluids. This heat transfer is usually accompanied by mass transfer.

Eg: cooling towers, Direct contact feed heater





## ⑥ Indirect Contact heat exchangers:

In this type of heat exchangers, the transfer of heat b/w two fluids could be carried out by transmission through a wall, which separates the two fluids.

It may be classified as

(i) Regenerators

(ii) Recuperators (or) Surface heat exchangers.

### (i) Regenerators:

In this type of heat exchangers, hot & cold fluids flow alternately through the same space.

Eg: IC engines, gas turbines.

### (ii) Surface heat exchangers:

This is the most common type of heat exchanger in which the hot & cold fluids don't come into direct contact with each other but are separated by a tube wall or a surface.

Eg: Automobile radiators, Economisers etc.,

### Advantages:

- 1) Easy construction.
- 2) More economical.
- 3) More surface area for heat transfer.

### Dis-advantages

- 1) Less heat transfer co-efficient
- 2) Less generating capacity.



## II. Relative direction of fluid motion:

This type of heat exchangers are classified as follows:

- (a) Parallel flow heat exchanger
- (b) Counter flow heat exchanger
- (c) Cross flow heat exchanger.

### (a) Parallel flow heat exchanger.

In this type, hot & cold fluids move in the same direction.

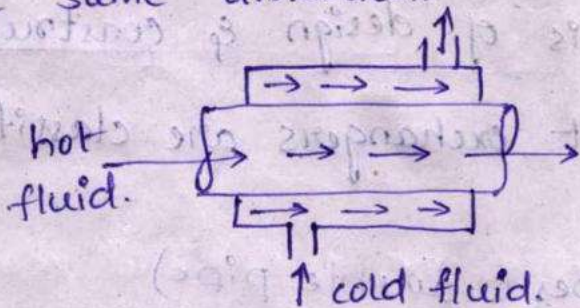


fig: Parallel flow heat exchangers.

### (b) Counter flow heat exchanger:

In this types, hot & cold fluids move in Parallel but opposite directions.

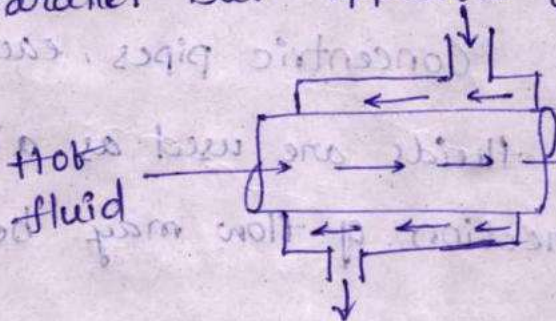
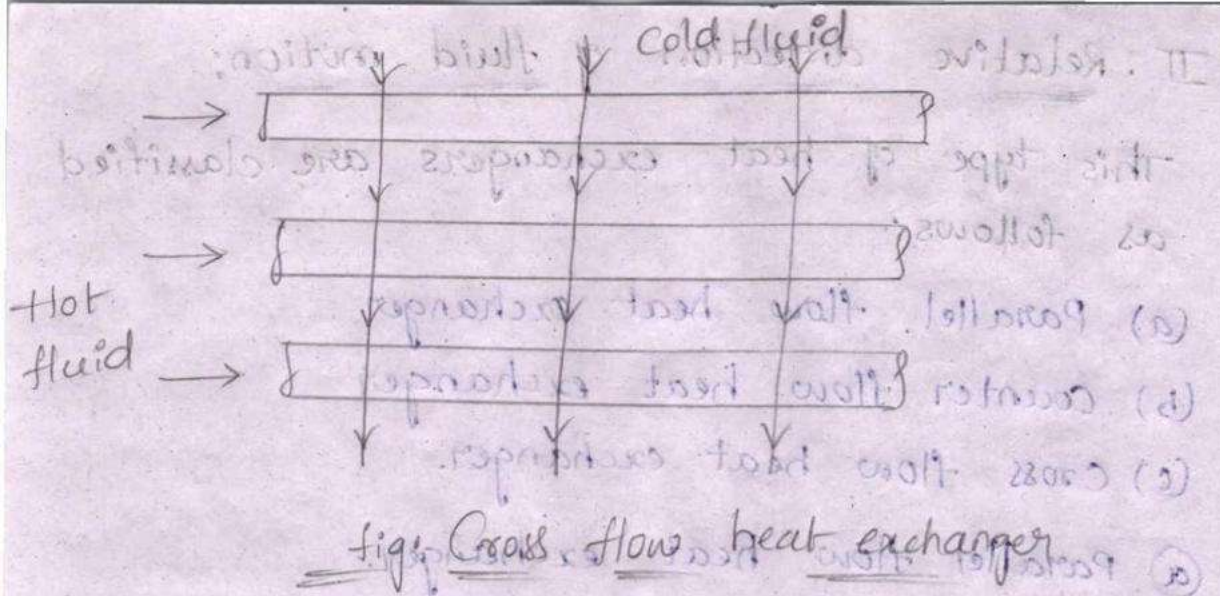


fig: counter flow heat exchanger.

### (c) Cross flow heat exchanger:

In this type, the hot & cold fluids move at right angles to each other.





### III. Design & Constructional features:

on the basis of design & construction features, the Heat exchangers are classified as follows:

(a) Concentric tubes (double pipe)

(b) Shell & tube

(c) multiple shell & tube passes

(d) compact heat exchangers.

(a) Concentric tubes:

In this type, Concentric pipes, each carrying one of the fluids are used as a heat exchangers. The direction of flow may be parallel or counter

(b) Shell & Tube:

In this type of heat exchangers, one of the fluids move through a bundle of tubes enclosed by a shell. The other fluid is forced through the shell and it moves over the outside surface of the tubes.



31/03/2023

In a double pipe heat exchanger, hot fluid with a specific heat of  $2300 \text{ J/kg K}$  enters at  $380^\circ\text{C}$  and leaves at  $300^\circ\text{C}$ . Cold fluid enters at  $25^\circ\text{C}$  and leaves at  $210^\circ\text{C}$ . Calculate the heat exchanger area required for Counter flow and what would be the percentage increase in area. If the fluid flows parallel. And find mass flow rate of water.

Take overall heat transfer coefficient is  $750 \text{ W/m}^2\text{K}$  and mass flow rate of hot fluid is  $1 \text{ kg/sec}$ .  $C_{PC} = 4186 \text{ J/kg K}$ ,  $Q = 184 \text{ kW}$

Sol Given data

$$C_{PH} = 2300 \text{ J/kg K}$$

$$T_1 = 380^\circ\text{C}$$

$$T_2 = 300^\circ\text{C}$$

$$t_1 = 25^\circ\text{C}$$

$$t_2 = 210^\circ\text{C}$$

Counter flow.  $(\Delta T)_{lm} = \frac{1 \cdot (T_1 - t_2) - (T_2 - t_1)}{\ln \left[ \frac{T_1 - t_2}{T_2 - t_1} \right]}$

$$= \frac{(380 - 210) - (300 - 25)}{\ln \left[ \frac{380 - 210}{300 - 25} \right]} = 218.3$$



Heat exchanger  $Q = UA (\Delta T)_{lm}$

Where  $U = 750 \text{ W/m}^2\text{K}$

$A = ?$

$184 \times 10^3 = 750 \times A (218.3)$

$\frac{184 \times 10^3}{750} = 218.3 A$

$A = \frac{184 \times 10^3}{218.3 \times 750}$

$A = 1.12 \text{ m}^2$

(ii) Parallel flow.  $(\Delta T)_{lm} = \frac{(T_1 - t_1) + (T_2 - t_2)}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$

$= \frac{(380 - 25) + (300 - 210)}{\ln \left[ \frac{380 - 25}{300 - 210} \right]}$

$(\Delta T)_{lm} = 193.1$

$\Rightarrow 184 \times 10^3 = 750 \times A \times 193.1$

$A = 1.27 \text{ m}^2$



$$(iii) \text{ Percentage of Area.} = \frac{1.27 - 1.12}{1.27}$$

$$= 11.8 \%$$

Case (iii)

$$Q = m_c C_{Pc} (t_2 - t_1) =$$

$$184 \times 10^3 = m_c \times 4186 (210 - 25)$$

$$m_c = 0.237 \text{ kg/s}$$

Length of the tube

② Saturated steam at  $126^\circ\text{C}$  is Condensing on the outer tube surface of a single pass heat exchanger. The heat exchanger heats 1050 kg/hr of water from  $20^\circ\text{C} - 45^\circ\text{C}$ . The overall heat transfer coefficient is  $1800 \text{ W/m}^2\text{K}$ . Calculate the following

- (i) Area of heat exchanger
- (ii) Length of the heat exchanger
- (iii) Rate of condensation of steam.

Take  $h_{fg} = 2185 \text{ kJ/kg}$ ,  $\theta_1 = 0.4 \text{ m}$ ,  $D_2 = 0.75 \text{ m}$



Sol Given data =  $1 \text{ hr} = 3600 \text{ sec}$

$$T_1 = T_2 = 126^\circ\text{C}$$

$$m_c = 1050 \text{ kg/hr} = \frac{1050}{3600} = 0.29 \text{ kg/sec}$$

$$t_1 = 20^\circ\text{C}$$

$$t_2 = 95^\circ\text{C}$$

(iii) 2650

$T_2 > t_2$ , then it is parallel flow.

$$U = 1800 \text{ W/m}^2\text{K}$$

$$h_{fg} = 2185 \text{ kJ/kg}$$

$$\left[ \because \text{cp of water} \right. \\ \left. \frac{2}{\text{p2}} \text{ F.E.S.O} = 4186 \text{ J/kgK} \right]$$

$$D_1 = 0.4 \text{ m}$$

$$D_2 = 0.45 \text{ m}$$

$$Q = m_h C_{ph} (T_2 - T_1) = m_c C_{pc} (t_2 - t_1)$$

$$Q = 0.29 \times 4186 \times (95 - 20)$$

$$Q = 91045.5 \text{ W (or)} 91 \times 10^3 \text{ W}$$

$$\Rightarrow Q = UA (\Delta T)_{\text{lm}}$$

$$(\Delta T)_{\text{lm}} \text{ Parallel flow} = \frac{(T_1 - t_1)(T_2 - t_2)}{\ln \left[ \frac{T_1 - t_1}{T_2 - t_2} \right]}$$

$$= \frac{(126 - 20) - (126 - 95)}{\ln \left[ \frac{126 - 20}{126 - 95} \right]}$$

$$(\Delta T)_{\text{lm}} = 61^\circ\text{C}$$



$$\Rightarrow 91 \times 10^3 = (1800 \times A \times 61)$$

$$\boxed{A = 0.82 \text{ m}^2}$$

(ii) Length of heat exchanger.

$$\text{Area} = \pi d L = \pi \times D_1 \times L$$

$$0.82 = \pi \times 0.4 \times L$$

$$\boxed{L = 0.65 \text{ m}}$$

(iii) Rate of condensation of steam.

$$Q = m_h \times h_{fg}$$

$$m_h = \frac{Q}{h_{fg}} = \frac{91 \times 10^3}{2185 \times 10^3} = 0.0416 \text{ kg/sec}$$

③ In a Cross flow heat exchanger both fluids ~~and~~ <sup>are</sup> mixed hot fluid with a specific heat of  $2300 \text{ J/kg K}$  enters at  $380^\circ\text{C}$  and leaves at  $300^\circ\text{C}$ . Cold water enters at  $25^\circ\text{C}$  and leaves at  $210^\circ\text{C}$ . Calculate the (required) surface area of heat exchanger. Take  $U = 750 \text{ W/m}^2\text{K}$  mass flow rate of hot fluid is  $1 \text{ kg/sec}$ .

So)

Given data:

$$C_h = 2300 \text{ J/kg K}$$

$$T_1 = 380^\circ\text{C}$$

$$T_2 = 300^\circ\text{C}$$

$$t_2 = 210^\circ\text{C}$$

$$U = 750 \text{ W/m}^2\text{K}$$

$$m_h = 1 \text{ kg/sec}$$



$$\text{Counter flow } (\Delta T)_{lm} = \frac{(T_1 - t_2) - (T_2 - t_1)}{\ln \left[ \frac{T_1 - t_2}{T_2 - t_1} \right]}$$

$$= \frac{(380 - 210) - (300 - 25)}{\ln \left[ \frac{380 - 210}{300 - 25} \right]}$$

$$(\Delta T)_{lm} = 218.30^\circ\text{C}$$

F = cross flow, Both fluids are unmixed

$$x = P = \frac{t_2 - t_1}{T_1 - t_1} = \frac{210 - 25}{380 - 25} = 0.52$$

$$\text{Curren} = R = \frac{T_1 - T_2}{t_2 - t_1} = \frac{380 - 300}{210 - 25} = 0.43$$

$$F = 0.95$$

$$Q = FUA (\Delta T)_{lm}$$

$$\Rightarrow Q = m_h c_h (T_1 - T_2)$$

$$= 1 \times 2300 (380 - 300)$$

$$Q = 334880 \text{ W } 184 \times 10^3 \text{ W}$$

$$\Rightarrow 334880 = 0.95 \times 750 \times A (218.30)$$

$$A = 2.18$$



④ A parallel flow heat exchanger. has hot and cold water stream running through it. The flow rates are 10 & 25 kg/min. respectively. Inlet temperatures are 75°C & 25°C on hot & cold sides. The exit temperature on the hot side should not exceed 50°C. Assume  $h_i = h_o = 600 \text{ W/m}^2\text{K}$ . Calculate the area of heat exchanger using effectiveness-NTU approach

Sol Given data

$$m_h = 10 \text{ kg/min} = 0.16 \text{ kg/sec}$$

$$m_c = 25 \text{ kg/min} =$$

$$T_1 = 75^\circ\text{C}$$

$$t_1 = 25^\circ\text{C}$$

$$T_2 = 50^\circ\text{C}$$

$$h_i = h_o = 600 \text{ W/m}^2\text{K}$$

$$C_h = m_h C_p = 694.87 \text{ J/kgK} = C_{\min}$$

$$C_c = m_c C_p = 0.416 \times 4186 = 1741 \text{ W/K} = C_{\max}$$

$$\text{effectiveness} = \varepsilon = \frac{m_h C_p}{C_{\min}} \left[ \frac{T_1 - T_2}{T_1 - t_1} \right]$$

$$= 0.5 = 50\%$$



4) A parallel flow heat exchanger has

$$Y_{axis} = 0.5 = 50\%$$

$$C_{ratio} = \frac{C_{min}}{C_{max}} = \frac{694}{1741} = 0.399$$

$$x_{axis} = 2 \text{ (temperature rise of cold side)}$$

$$NTU = 0.84$$

$$NTU = \frac{UA}{C_{min}} \quad (10)$$

$$\frac{1}{U} = \frac{1}{U_o} = \frac{1}{U_i}$$

$$= \frac{1}{U}$$

$$U = 300 \text{ W/m}^2\text{K}$$

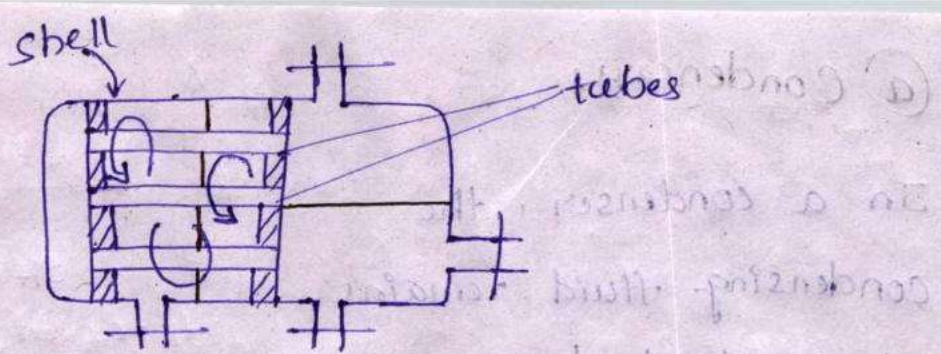
$$NTU = \frac{UA}{C_{min}} = 1.94$$

$$A = \frac{NTU \times C_{min}}{U}$$

$$\text{Effectiveness} = \frac{C_{min} [T_1 - T_2]}{C_{max} [T_1 - T_2]} = 3$$

$$0.2 = 2.0$$





### c) Multiple shell & tube passes

In order to increase the overall heat transfer multiple shell & tube passes are used. In this type, the 2 fluids traverse the exchanger more than one time. This type of exchanger is preferred due to its low cost of manufacture & easy to repair.

### (d) Compact heat exchangers

There are many special purpose heat exchanger called compact heat exchangers. They are generally employed when convective heat transfer co-efficient associated with one of the fluids is much smaller than that associated with the other fluid.

### (iv) Physical state of fluids:

Based on the physical state of fluids inside the exchangers, heat exchangers are classified as:

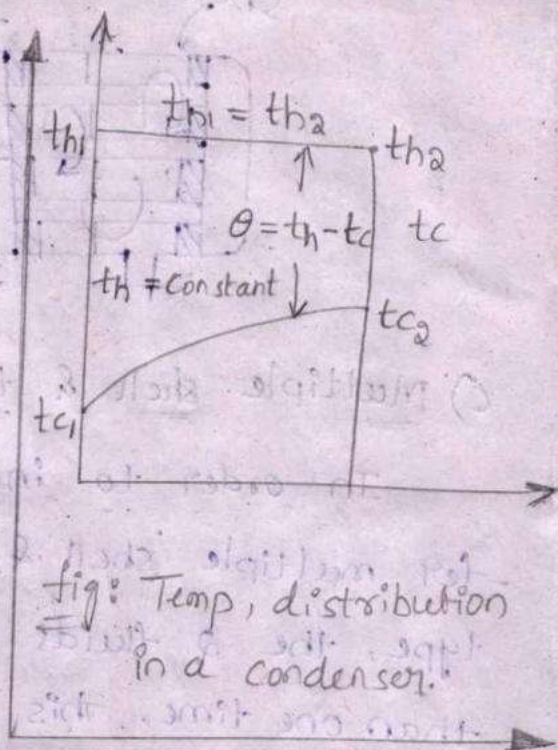
(a) Condensers

(b) evaporators



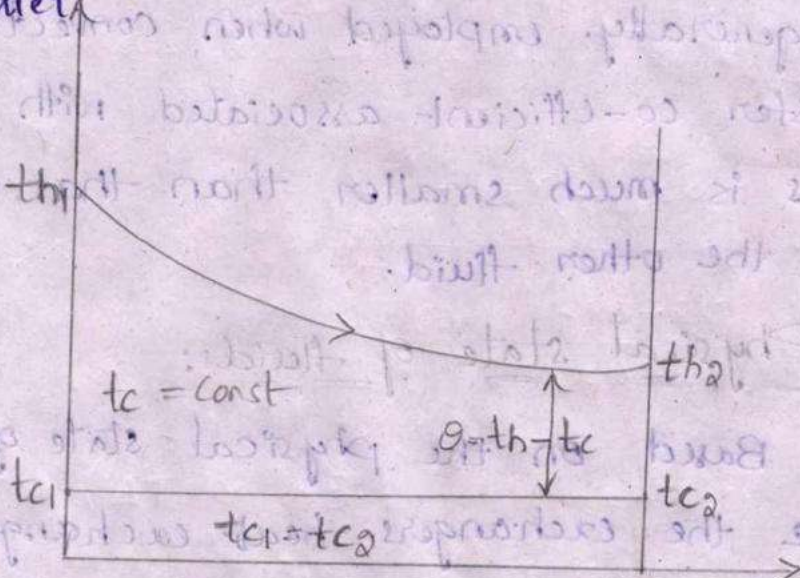
## (a) Condensers:

In a condenser, the condensing fluid remains at constant temp throughout the exchanger while the temp. of the colder fluid gradually increased from inlet to outlet.



## (b) Evaporators:

In a evaporators, the cold fluid remains at constant temperature while the temperature of hot fluid gradually decreases from inlet to outlet.





## ⇒ Logarithmic mean temp difference (LMTD)

The temp diff b/w the hot & cold fluids in the heat exchanger varies from point to point. In addition various modes of heat transfer are involved. Therefore, based on the concept of appropriate mean temp difference also called LMTD, the total heat transfer rate in the heat exchanger is expressed as

$$Q = UA (\Delta T)_{lm}$$

$U$  = overall heat transfer co-efficient,  $W/m^2K$ .

$A$  = Area,  $m^2$

$(\Delta T)_m$  = LMTD.

### Assumptions:

- 1) flow is steady
- 2) the overall heat transfer co-efficient is constant.
- 3) The specific heats of both fluids are const.
- 4) The mass flow rate of both fluids are constant
- 5) Axial conduction along the tube is negligible
- 6) The change in kinetic & potential energies



Let  $m_h$  = mass flow rate of hot fluid

$m_c$  = mass flow rate of cold fluid

$C_{ph}$  = specific heat of hot fluid

$C_{pc}$  = " " " " cold " " " "

$T_1$  = Entry temp. of hot fluid

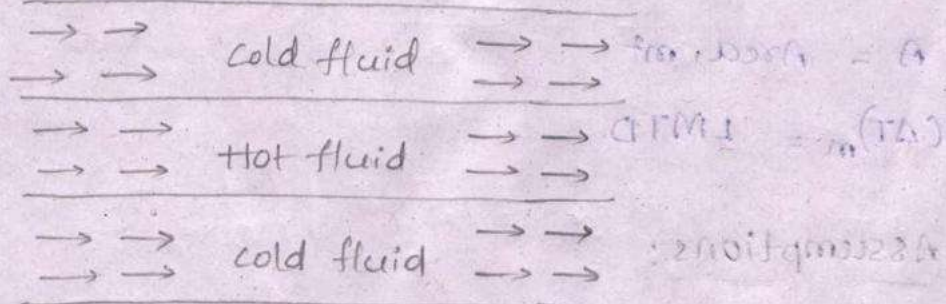
$t_1$  = " " " " cold " " " "

$T_2$  = Exit temp. of hot fluid

$t_2$  = " " " " cold " " " "

$$m_1(T_1 - T_2) = m_2(t_2 - t_1)$$

(i) LMTD for Parallel flow.



Let us consider an elemental area  $dA$  of the heat exchanger. The heat flow rate

is given:  $dQ = U dA (T - t) \rightarrow (1)$

W.K.T

$$dQ = -m_h C_{ph} dT \rightarrow (2)$$

$$dQ = +m_c C_{pc} dt$$

$$dT = \frac{-dQ}{m_h C_{ph}}$$

$$dT = \frac{-dQ}{C_h} \rightarrow (3)$$



$$\therefore ch = m_h \times c_{ph}$$

$$dt = \frac{d\theta}{mc_{pc}}$$

$$= \frac{d\theta}{cc} \rightarrow (4)$$

$$\therefore cc = m_c \times c_{pc}$$

$$dT - dt = \frac{-d\theta}{ch} - \frac{d\theta}{cc}$$

$$d\theta = -d\theta \left[ \frac{1}{ch} + \frac{1}{cc} \right] \rightarrow (5)$$

$$\therefore d\theta = dT - dt$$

Substituting  $d\theta$  value from eq (1) in eq (5)

$$d\theta = -UdA (T-t) \left[ \frac{1}{ch} + \frac{1}{cc} \right]$$

$$\therefore \theta = T - t$$

$$d\theta = -UdA \theta \left[ \frac{1}{ch} + \frac{1}{cc} \right]$$

$$\frac{d\theta}{\theta} = -UdA \left[ \frac{1}{ch} + \frac{1}{cc} \right]$$

Integrating

$$\int \frac{d\theta}{\theta} = -U \left[ \frac{1}{ch} + \frac{1}{cc} \right] \int dA$$

$$[\ln \theta]_1^2 = -U \left[ \frac{1}{ch} + \frac{1}{cc} \right] A$$

$$\ln \theta_2 - \ln \theta_1 = -UA \left[ \frac{1}{ch} + \frac{1}{cc} \right]$$

$$\ln \left( \frac{\theta_2}{\theta_1} \right) = -UA \left[ \frac{1}{ch} + \frac{1}{cc} \right] \rightarrow (6)$$



W. K. T

$$Q = m_h c_{ph} (T_1 - T_2) = m_c c_{pc} (t_2 - t_1)$$

$$Q = C_h (T_1 - T_2) = C_c (t_2 - t_1) \rightarrow (7)$$

$$Q = C_h (T_1 - T_2)$$

$$\frac{1}{C_h} = \frac{T_1 - T_2}{Q} \rightarrow (8)$$

$$\begin{aligned} \therefore C &= m \times c_p \\ Q &= C_c (t_2 - t_1) \\ \frac{1}{C_c} &= \frac{t_2 - t_1}{Q} \rightarrow (9) \end{aligned}$$

Substitute  $\frac{1}{C_h}$  &  $\frac{1}{C_c}$  values in eq (6)

$$\ln \left( \frac{\theta_2}{\theta_1} \right) = -UA \left[ \frac{T_1 - T_2}{Q} + \frac{t_2 - t_1}{Q} \right]$$

$$= -\frac{UA}{Q} [T_1 - T_2 + t_2 - t_1]$$

$$= -\frac{UA [T_1 - T_2 + t_2 - t_1]}{\ln \left( \frac{\theta_2}{\theta_1} \right)}$$

$$Q = \frac{+UA [T_2 - T_1 + t_1 - t_2]}{\ln \left( \frac{T_2 - t_2}{T_1 - t_1} \right)}$$

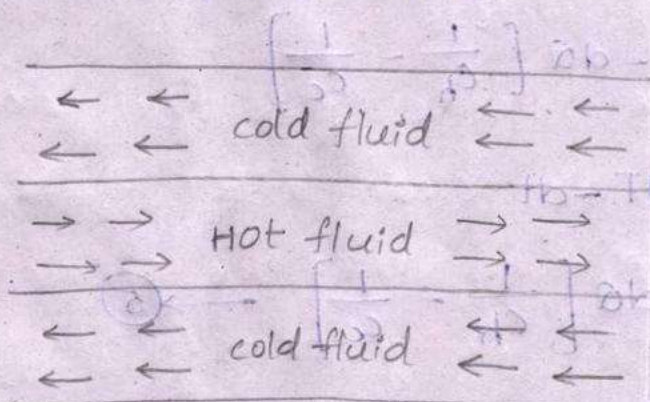
$$\theta = T - t = \frac{UA [(T_2 - t_2) - (T_1 - t_1)]}{\ln \left[ \frac{T_2 - t_2}{T_1 - t_1} \right]}$$



$$= \frac{UA \left[ (T_1 - t_1) - (T_2 - t_2) \right]}{\ln \left( \frac{T_1 - t_1}{T_2 - t_2} \right)}$$

$$Q = UA (\Delta T)_m$$

② LMTD for counter flow.



Let us consider, on elemental area  $dA$  of the heat exchanger.

The heat flow rate is given by

$$dQ = U dA (T - t) \quad \text{--- (1)}$$

W.K.T

$$dQ = -m_h c_{ph} (dT) = -m_c c_{pc} (dt) \quad \text{--- (2)}$$

$$dQ = -m_h c_{ph} dT$$

$$dT = -\frac{dQ}{m_h c_{ph}}$$

$$dT = -\frac{dQ}{c_h} \quad \text{--- (3)} \quad \because c_h = m_h \times c_{ph}$$

$$\left[ \frac{1}{c_h} - \frac{1}{c_c} \right] dQ = -dT$$



$$d\theta = -\frac{\dot{m}_c c_{pc}}{(\dot{m}_h - \dot{m}_c)} dt \quad (1)$$

$$dt = \frac{-d\theta}{\frac{\dot{m}_c c_{pc}}{(\dot{m}_h - \dot{m}_c)}} \quad (2)$$

$$dt = \frac{-d\theta}{C_c} \quad (3)$$

$$\therefore C_c = \dot{m}_c c_{pc}$$

$$(\dot{m}_h - \dot{m}_c) \Delta T = 0$$

$$dT - dt = \frac{-d\theta}{C_h} + \frac{d\theta}{C_c}$$

$$= -d\theta \left[ \frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$\therefore d\theta = dT - dt$$

$$d\theta = -d\theta \left[ \frac{1}{C_h} - \frac{1}{C_c} \right] \quad (5)$$

Substituting  $d\theta$  value from eq (2) in eq (5)

$$d\theta = -U dA (T - t) \left[ \frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$= -U dA \theta \left[ \frac{1}{C_h} - \frac{1}{C_c} \right]$$

$$\therefore \theta = T - t$$

$$\frac{d\theta}{\theta} = -U dA \left[ \frac{1}{C_h} - \frac{1}{C_c} \right]$$

Integrating

$$\int_1^2 \frac{d\theta}{\theta} = -U \left[ \frac{1}{C_h} - \frac{1}{C_c} \right] \int_{A_1}^{A_2} dA$$

$$[\ln \theta]_1^2 = -UA \left[ \frac{1}{C_h} - \frac{1}{C_c} \right] \frac{\theta_2 - \theta_1}{\theta_1 \theta_2}$$

$$\ln \theta_2 - \ln \theta_1 = -UA \left[ \frac{1}{C_h} - \frac{1}{C_c} \right]$$



$$\ln \left( \frac{\theta_2}{\theta_1} \right) = -UA \left[ \frac{1}{C_h} - \frac{1}{C_c} \right] \rightarrow (6)$$

$$Q = m_h C_{ph} (T_1 - T_2) = m_c C_{pc} (t_2 - t_1)$$

$$Q = C_h (T_1 - T_2) = C_c (t_2 - t_1) \rightarrow (7)$$

$$\therefore C = m \times C_p$$

$$Q = C_h (T_1 - T_2)$$

$$\frac{1}{C_h} = \frac{T_1 - T_2}{Q} \rightarrow (8)$$

$$Q = C_c (t_2 - t_1) \Rightarrow \frac{1}{C_c} = \frac{t_2 - t_1}{Q} \rightarrow (9)$$

substitute  $\frac{1}{C_h}$  &  $\frac{1}{C_c}$  values in eq (6)

$$\ln \left( \frac{\theta_2}{\theta_1} \right) = -UA \left[ \frac{(T_1 - T_2)}{Q} - \frac{(t_2 - t_1)}{Q} \right]$$

$$= \frac{-UA}{Q} [(T_1 - T_2) - (t_2 - t_1)]$$

$$= \frac{-UA}{Q} [(T_1 - t_2) - (T_2 - t_1)]$$

$$\ln \left( \frac{\theta_2}{\theta_1} \right) = \frac{UA}{Q} [(T_2 - t_1) - (T_1 - t_2)]$$

$$Q = \frac{UA [(T_2 - t_1) - (T_1 - t_2)]}{\ln \left( \frac{T_2 - t_1}{T_1 - t_2} \right)}$$

$$Q = UA (\Delta T)_m$$



$$Q_2 = T_2 - t_1 \left[ \frac{1}{20} - \frac{1}{10} \right] AU = \left( \frac{20}{10} \right) Q_1$$

$$Q_1 = T_1 - t_2$$

$$(1 - 0.5) Q_1 = (0.5 - 1) Q_2 \quad Q_1 = Q_2$$

$$Q = \frac{UA [(T_1 - t_2) - (T_2 - t_1)]}{\ln \left( \frac{T_1 - t_2}{T_2 - t_1} \right)}$$

$$\Rightarrow \text{fouling factors} \quad \boxed{\frac{1}{U_{foul}} = R_f + \frac{1}{U_{clean}}}$$

The surfaces of a heat exchangers do not remain clean after it has been in use for some time. The surface become fouled with scaling or deposits. The effect of these deposits affecting the value of overall heat transfer co-efficient ( $U$ ). This effect is take care of by introducing an additional thermal resistance called the fouling resistance ( $R_f$ ) which is gives by as follows.

$$U_{outer} = \frac{1}{\frac{1}{h_o} + R_{fo} + \frac{r_o}{K} \ln \left( \frac{r_o}{r_i} \right) + \left( \frac{r_o}{r_i} \right) R_{fi} + \left( \frac{r_o}{r_i} \right) \frac{1}{h_i}}$$

$$U_{inner} = \frac{1}{\frac{1}{h_i} + R_{fi} + \frac{r_i}{K} \ln \left( \frac{r_o}{r_i} \right) + \left( \frac{r_i}{r_o} \right) R_{fo} + \left( \frac{r_i}{r_o} \right) \frac{1}{h_o}}$$



Effectiveness by using number of Trans  
-fer units (NTU):

A heat exchanger can be designed by the LMTD when inlet & outlet condition are specified. ~~But when inlet & outlet condition are specified.~~ But when the problem is to determine the inlet (or) exit temp of heat exchanger, effectiveness method is used.

The heat exchanger effectiveness is defined as the ratio of actual heat transfer to the max. possible heat transfer.

$$\text{Effectiveness } \epsilon = \frac{\text{Actual heat transfer}}{\text{max. possible heat transfer}}$$

$$= \frac{Q}{Q_{\max.}}$$



Effectiveness by using number of hours  
for unit (unit):

A heat exchanger can be classified by  
the limit when inlet & outlet condition  
are specified. But when inlet & outlet  
condition are specified but when the  
problem is to determine the inlet (or)  
outlet temp of heat exchanger, effectiveness  
method is used

The heat exchanger effectiveness is  
define as the ratio of actual heat  
transfer to the max possible heat  
transfer

Effectiveness =  $\frac{\text{actual heat transfer}}{\text{max. possible heat transfer}}$

$\frac{Q}{Q_{max}}$



Sol

$$P = 0.08 \text{ bar} = @ T = 41.54^\circ\text{C} = T_v$$

$$A = 50 \times 50 = 2500 \text{ cm}^2 = 2500 \times (10^{-2})^2$$

$$T_s = 20^\circ\text{C}$$

$$\Rightarrow h_{fg} = 2403.2 \text{ kJ/kg} = 2403.2 \times 10^3 \text{ J/kg}$$

$$\Rightarrow T_f = \frac{(T_v + T_s)}{2} = \frac{41.54 + 20}{2} = 30.77 =$$

$$\Rightarrow \rho = 997 \text{ kg/m}^3$$

$$\Rightarrow v = 0.8 \times 10^{-6} \text{ m}^2/\text{sec}$$

$$\Rightarrow k = 0.61280 \text{ W/mK}$$

$$\Rightarrow \mu = \rho v$$

$$\mu = 995 \times 0.657 \times 10^{-6} = 6.53 \times 10^{-4} \text{ Ns/m}^2$$
$$\boxed{\mu = 827 \times 10^{-6} \text{ Ns/m}^2}$$

film thickness

$$(i) \Delta x = \left[ \frac{4 \mu_1 k x (T_v - T_s)}{g \cdot h_{fg} \cdot \rho^2} \right]^{0.25}$$
$$= \left[ \frac{4 \times 827 \times 10^{-6} \times 0.612 \times (25 \times 10^2) (41.54 - 20)}{9.81 \times 2403.2 \times 10^3 \times 997^2} \right]^{0.25}$$

$$\boxed{\Delta x = 1.46 \times 10^{-4} \text{ m.}}$$

(ii)

$$\Rightarrow h_x = \frac{k}{\Delta x} = \frac{0.612}{1.46 \times 10^{-4}} = 4191.7 \text{ W/m}^2\text{K}$$



(iii) Avg. heat transfer coefficient.

$$h = 0.943 \left[ \frac{k^3 \rho^2 g h_{fg}}{\mu_1 L (T_v - T_s)} \right]^{0.25}$$
$$= 1.13 \left[ \frac{0.612^3 \times 997^2 \times 9.81 \times 2403.2 \times 10^3}{827 \times 10^{-6} \times 50 \times 10^{-2} (41.54 - 20)} \right]^{0.25}$$

$$h = 5599.8 \text{ W/m}^2\text{K}.$$

(iv) Total heat transfer

$$Q = hA [T_v - T_s]$$

$$= 5599.8$$

$$= 30139.8 \text{ W}$$

(v) steam condensation

$$Q = \dot{m} h_{fg}$$

$$\dot{m} = \frac{Q}{h_{fg}} = 0.0125 \text{ kg/sec}$$

(vi)  $h_{\text{horizontal}}$   $\theta = 30^\circ$

$$h = 0.728 \left[ \frac{k^3 \rho^2 (g \cdot \cos \theta) h_{fg}}{\mu_1 D (T_v - T_s)} \right]^{0.25}$$

$$h = 0.728 \left[ \frac{0.612^3 \times 997^2 (\cos 30^\circ) \times 2403.2 \times 10^3}{827 \times 10^{-6} \times 50 \times 10^{-2} (41.54 - 20)} \right] = 3480 \text{ W/m}^2\text{K}$$



# Radiation

## Unit-V

The heat transfer from one body to another body without any transmitting medium is known as Radiation

- \* It is an electromagnetic wave phenomena
- \* These waves are classified in terms of wavelength and are propagated at the speed of light i.e.,  $3 \times 10^8$  m/sec

### Emission Properties:

- \* The rate of emission of radiation by a body depends upon the following factors.
  - a) the wavelength or frequency of radiation
  - b) The temp of the surface
  - c) The nature of the surface

Emissive Power  $[E_p] = W/m^2$

The emissive power is defined as the total amount of radiation emitted by a body per unit time & unit area.

Mono chromatic emissive power  $[E_{b\lambda}]$

The energy emitted by the surface at a given length per unit time per unit area in all directions.

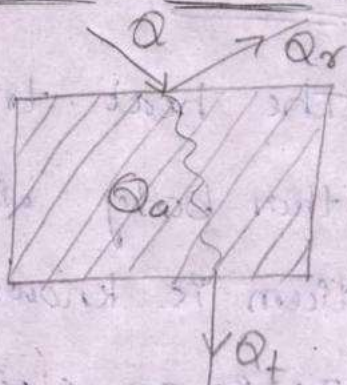


## Absorption, Reflection & Transmission:

$$Q = Q_a + Q_r + Q_t$$

$$\frac{Q}{Q} = Q_a + Q_r + Q_t$$

$$\boxed{1 = \alpha + \rho + \tau}$$



absorptivity  $\alpha = \frac{\text{Radiation absorbed}}{\text{Incident radiation}}$

Reflectivity  $\rho = \frac{\text{Radiation reflected}}{\text{Incident radiation}}$

Transmissivity  $\tau = \frac{\text{Radiation Transmitted}}{\text{Incident Radiation}}$

⇒ There are two types of reflection phenomena they are:

a) Specular Reflection 

b) Diffused Reflection 

## Concept of Black Body

Black body is an ideal surface having the following properties.

⇒ A black body absorbs all incident radiation, regardless of wavelength & direction.



⇒ For a prescribed temperature & Wavelength,  
NO surface can emit more energy than  
black body.

An interesting point note here

↳ black body continuous to emit radiation  
even when it is in thermal equilibrium with its  
surroundings.

### Planck's Distribution law

The relationship between the monochromatic  
emissive power of a black body & wavelength  
of a radiation at a particular temperature.

$$E_{bx} = \frac{C_1 \lambda^{-5}}{e^{\left[\frac{C_2}{\lambda T}\right]} - 1}$$

$$C_1 = 0.374 \times 10^{-15} \text{ W m}^2$$

$$C_2 = 144 \times 10^{-3} \text{ mK}$$

### Wien's Displacement law

The relationship between the temperature  
and wavelength corresponding to the maximum  
emissive power at a black body

$$\lambda_{\max} T = 2.9 \times 10^{-3} \text{ mK} \quad \left[ \because 1 \mu = 10^{-6} \text{ m} \right]$$

micro

$$\lambda_{\max} T = 2898 \text{ } \mu\text{mK}$$



## Stefen Boltzman law:

The emissive power of a black body is proportional to the fourth power of absolute temperature.

$$E_b \propto T^4$$

$$E_b = \sigma T^4$$

where  $\sigma = 5.67 \times 10^{-8} \text{ W/m}^2 \text{ K}^4$

## Maximum Emissive power

A combination of plancks law and Wien's displacement law yield's the condition for the max. monochromatic emission power for a black body.

$$(E_{bx})_{\text{max.}} = C_4 T^5$$

$$C_4 = 1.307 \times 10^{-5}$$

where  $C_4 = (\text{Radiation constant})$

Emissivity: ratio of nonblack body to the black body of emissive power.

$$e = \frac{E}{E_b}$$



Grey body:

Kirchoff's law of Radiation:

$$\frac{E_1}{\alpha_1} = \frac{E_2}{\alpha_2}$$

$\alpha$  = absorptivity.

$$E_1 = \alpha_1$$

Intensity of radiation

$$I_n = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

① A black body at  $3000\text{K}$  emits radiation  
calculate the following

- (i) Monochromatic emissive power at  $1\mu\text{m}$
- (ii) wavelength at which emissive is max.
- (iii) Max. emissive power
- (iv) Total emissive power.
- (v) calculate the total emissive of the furnace  
if it is assumed as a real surface  
having emissivity equal to  $0.85$

Sol

Surface temp  $T = 3000\text{K}$

wavelength  $\lambda = 1\mu\text{m}$ .

$$\lambda = 1 \times 10^{-6}\text{m}.$$

Γ



$$(i) E_{bx} = \frac{C_1 \lambda^{-5}}{e^{\left[ \frac{C_2}{\lambda T} \right]} - 1}$$

$$= \frac{0.374 \times 10^{-15} \times (1 \times 10^{-6})^{-5}}{e^{\left[ \frac{0.01438752}{1 \times 10^{-6} \times 3000} \right]} - 1}$$

$$E_{bx} = 3.11 \times 10^{12} \text{ W/m}^2$$

$$(ii) \lambda_{max} T = 2898 \times 10^{-6} \text{ mK}$$

$$\lambda_{max} = \frac{2898 \times 10^{-6}}{3000}$$

$$= 9.6 \times 10^{-7}$$

$$(iii) E_b = \epsilon \sigma T^4$$

$$(E_{bx})_{max} = C_4 \times T^5$$

$$= 1.307 \times 10^{-5} \times T^5$$

$$= 1.307 \times 10^{-5} \times 3000^5$$

$$(E_{bx})_{max} = 3.17 \times 10^{12}$$



$$\begin{aligned}
 \text{(iv)} \quad E_b &= \sigma T^4 \\
 &= 5.67 \times 10^{-8} \times 3000^4 \\
 E_b &= 4.59 \times 10^6 \text{ W/m}^2
 \end{aligned}$$

(v) Total emissive power

$$(E_b)_{\text{rad}} = \epsilon \sigma T^4$$

$$(\because \epsilon = 0.85)$$

$$(E_b)_{\text{rad}} = 0.85 \times 4.59 \times 10^6$$

$$(E_b)_{\text{rad}} = 3.9 \times 10^6 \text{ W/m}^2$$

② A Black body of  $1200 \text{ cm}^2$  emits radiation at  $1000 \text{ K}$ . calculate the following

- (i) Total rate of energy emission
- (ii) intensity of normal radiation
- (iii) wave length of max. Monochromatic emissive power.
- (iv) Intensity of radiation along a direction at  $60^\circ$  to the normal.

Sol Given data

$$\begin{aligned}
 A &= 1200 \text{ cm}^2 \\
 &= (1200 \times 10^{-2})^2
 \end{aligned}$$

$$T = 1000 \text{ K}$$



(i) Total rate of energy emission.

$$E_b = \sigma T^4 \times A$$

$$= 5.67 \times 10^{-8} \times 1000^4 \times (1200 \times 10^{-2})^2$$

$$= 6804 \text{ W}$$

(ii) Intensity of normal radiation.

$$I_n = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

$$= \frac{5.67 \times 10^{-8} \times 1000^4}{\pi}$$

$$= 18048 \text{ W/m}^2$$

(iii) Wavelength of max. monochromatic emission power

$$\lambda_{\max} T = 2898 \text{ } \mu\text{mK}$$

$$\lambda_{\max} = \frac{2898}{1000}$$

$$\lambda_{\max} = 2.89 \text{ } \mu\text{m}$$

(iv)

$$I_n = I_0$$

$$= 18.048 \text{ W/m}^2$$



③ Assuming sun to be black body emitting radiation at  $6000\text{K}$  at a mean distance of  $12 \times 10^{10}\text{m}$  for the earth. the diameter of sun is  $1.5 \times 10^9\text{m}$ . and that of the earth is  $13.2 \times 10^6\text{m}$ . calculate the following.

- (i) Total energy emitted by the sun
- (ii) The emission received per  $\text{m}^2$  just outside the earth atmosphere.
- (iii) The total energy received by the earth if no radiation is blocked by the earth atmosphere
- (iv) The energy received by a  $2 \times 2\text{m}$  solar collector whose normal is inclined at  $45^\circ$  to the sun. the energy loss through the atmosphere is  $50\%$  and the diffused radiation is  $20\%$  of direct radiation.

Sol Given data

$$T = 6000\text{K}$$

$$R = 12 \times 10^{10}\text{m}$$

$$D_{\text{sun}} = 1.5 \times 10^9\text{m}$$

$$D_{\text{earth}} = 13.2 \times 10^6\text{m}.$$

$$\begin{aligned}
 (i) E_b &= \sigma T^4 \cdot A \\
 &= \sigma T^4 \cdot 4\pi (R_{\text{sun}}^2) \\
 &= 5.67 \times 10^{-8} \times 6000^4 \times 4\pi \left( \frac{1.5 \times 10^9}{2} \right)^2 \\
 &= 5.19 \times 10^{26} \text{ W}
 \end{aligned}$$



$$\begin{aligned}
 \text{ii) } E_{b(\text{atm})} &= \frac{E_b}{A} \\
 &= \frac{E_b}{4\pi R_{\text{distance}}^2} \\
 &= \frac{5.19 \times 10^{26}}{4\pi \times (12 \times 10^{10})^2} \\
 &= 2868.10 \text{ W/m}^2
 \end{aligned}$$

$$\begin{aligned}
 \text{iii) } E_b(\text{earth atm}) \times A \\
 &= 2868.10 \times \frac{\pi}{4} (d^2) \\
 &= 2868.10 \times \frac{\pi}{4} (13.2 \times 10^6)^2 \\
 &= 3.92 \times 10^{17} \text{ W}
 \end{aligned}$$

iv) Energy loss through atmosphere is 50%.

$$E_1 = \frac{50}{100} \times 2868$$

$$E_1 = 1434 \text{ W/m}^2$$

The diffused radiation is 20% of direct radiation

$$E_2 = 1434 \times \frac{20}{100}$$

$$= 286.8 \text{ W/m}^2$$

Total radiation reaching the collector

$$E_t = E_1 + E_2$$

$$= 286.8 + 1434$$

$$E = 1720.8 \text{ W/m}^2$$



Energy received by the solar panel

$$E \times A \times \cos \theta$$

$$= 1720 \times 2 \times 2 \times \cos(45^\circ)$$

$$= 48648.9 \text{ W}$$

④  $800 \text{ W/m}^2$  of radiant energy is Incident upon a surface of out of which  $300 \text{ W/m}^2$  is absorbed,  $100 \text{ W/m}^2$  is reflected & the remainder is Transmitted. Calculate the following. (i) Absorptivity (ii) Reflectivity (iii) Transmittivity.

sol  $Q = Q_a + Q_r + Q_t$

$$800 = 300 + 100 + Q_t$$

$$Q_t = 400 \text{ W/m}^2$$

$$(i) \text{ Absorptivity} = \frac{Q_a}{Q} = \frac{300}{800} = 0.375$$

$$(ii) \text{ Reflectivity} = \frac{Q_r}{Q} = \frac{100}{800} = 0.125$$

$$(iii) \text{ Transmittivity} = \frac{Q_t}{Q} = \frac{400}{800} = 0.5$$

$$\boxed{\alpha + r + \tau = 1}$$



## ⑤ Radiation exchange b/w surfaces

Radiant energy exchange between surfaces depends not only on the emission, absorption and reflection characteristics of the surfaces but also on their geometrical arrangements.

\* This Heat exchange will be effect further due to the presence partially emitting & absorbing medium between the surfaces.

\* To account this radiation exchange following assumptions are made.

⇒ All the surfaces are considered to be either black or grey.

⇒ Radiation and reflection process are assumed to diffusion.

⇒ The absorptivity of surface is taken equals to its emissivity & independent of temperature of the source of the Incident radiation.

Radiation Exchange Between two black surfaces separated by a non-absorbing medium



Let us consider two black bodies separated by a non-absorbing medium.

⇒ The problem is to determine the Net radiation exchange between them.

⇒ Consider Area

elements  $dA_1$  &  $dA_2$

are the two surfaces

the distance b/w them is 'r'

⇒ The angle normals two area elements make with the line joining them are  $\phi_1$  &  $\phi_2$

⇒ The rate of radiative energy  $dQ$ , leaving  $dA_1$  that strikes  $dA_2$  is given by

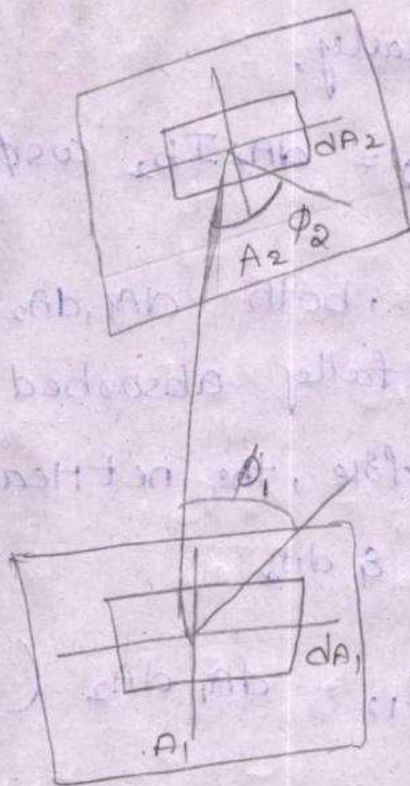
$$dQ_1 = dA_1 I_{b1} \cos \phi_1 d\omega_{12}$$

where solid angle

$$d\omega_{12} = \frac{dA_2 \cos \phi_2}{r^2}$$

$$dQ_1 = dA_1 I_{b1} \cos \phi_1 \frac{dA_2 \cos \phi_2}{r^2}$$

$$dQ_1 = dA_1 I_{b1} \cos \phi_1 \frac{dA_2 \cos \phi_2}{r^2}$$





Similarly,

$$dQ_2 = dA_2 I_{b2} \cos \phi_2 \cdot \frac{dA_1 \cos \phi_1}{r^2}$$

Since, both  $dA_1, dA_2$  black surface,  $dQ_1$  &  $dQ_2$  are fully absorbed by  $dA_1$  &  $dA_2$  respectively. Therefore, the net Heat transfer energy between  $dA_1$  &  $dA_2$

$$dQ_{12} = dA_1 dA_2 (I_{b1} - I_{b2}) \frac{\cos \phi_2 \cos \phi_1}{r^2}$$

$$\therefore I_b = \frac{E_b}{\pi} = \frac{\sigma T^4}{\pi}$$

$$dQ_{12} = \left( \frac{E_{b1}}{\pi} - \frac{E_{b2}}{\pi} \right) (dA_1 dA_2) \left( \frac{\cos \phi_2 \cos \phi_1}{r^2} \right)$$

The net energy exchange between  $A_1$  &  $A_2$  is obtained by integrating

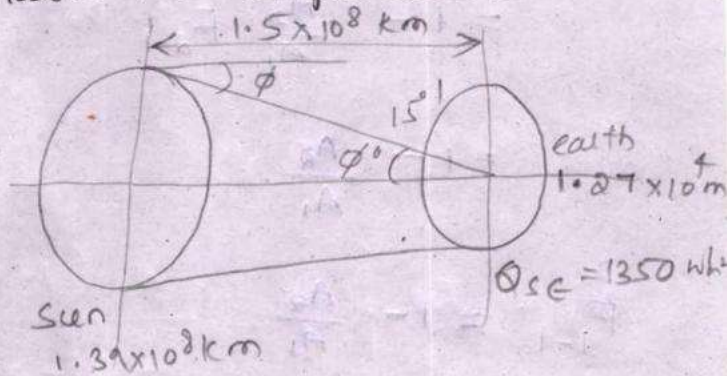
$$Q_{12} = \iint_{A_1 A_2} \frac{\sigma}{\pi r^2} (T_1^4 - T_2^4) dA_1 dA_2 \cos \phi_1 \cos \phi_2$$

Shape Factor :

① 800 W/m<sup>2</sup> radiant energy is Incident upon a surface, out of it which 300 W/m<sup>2</sup> is observed,



① The sun is a spherical mass of extremely hot gas continuously generating heat by thermo nuclear fusion reaction. This energy is radiated from the sun in all directions and a small fraction of it reaches the earth. The orientation of sun and earth is shown in fig. On a clear day, the solar radiation on the earth has been found to be  $1350 \text{ W/m}^2$ . Assuming sun to be a black body, estimate its temperature.



Sol

$$E_b = \sigma T_s^4 = \frac{dQ_{se}}{dA_e}$$

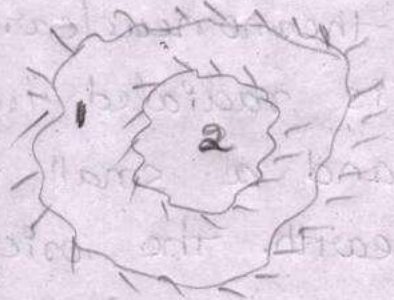
$$= \frac{\pi r^2}{\frac{\pi}{4} r^2}$$

$$= \frac{\pi (1.5 \times 10^{11})^2 \times 1350}{\frac{\pi}{4} (1.39 \times 10^9)^2}$$

$$T_s = 5400^\circ \text{C}$$



② Calculate the shape factor for the configuration shows in the fig.



$$F_{1 \rightarrow 1} + F_{1 \rightarrow 2} = 1$$

$$A_1 F_{1 \rightarrow 2} = A_2 F_{2 \rightarrow 1}$$

$$F_{1 \rightarrow 2} = \frac{A_2 F_{2 \rightarrow 1}}{A_1}$$

$$F_{1 \rightarrow 1} = 1 - F_{1 \rightarrow 2}$$

$$= 1 - \frac{A_2}{A_1} F_{2 \rightarrow 1}$$

$$= 1 - \frac{A_2}{A_1}$$

$$F_{1 \rightarrow 1} = 1 - \frac{A_2}{A_1}$$

$$F_{2 \rightarrow 1} = 1$$

$$F_{1 \rightarrow 1} = 1 - \frac{A_2}{A_1}$$

$$F_{1 \rightarrow 2} = \frac{A_2 F_{2 \rightarrow 1}}{A_1}$$

$$F_{2 \rightarrow 1} = 1$$

③

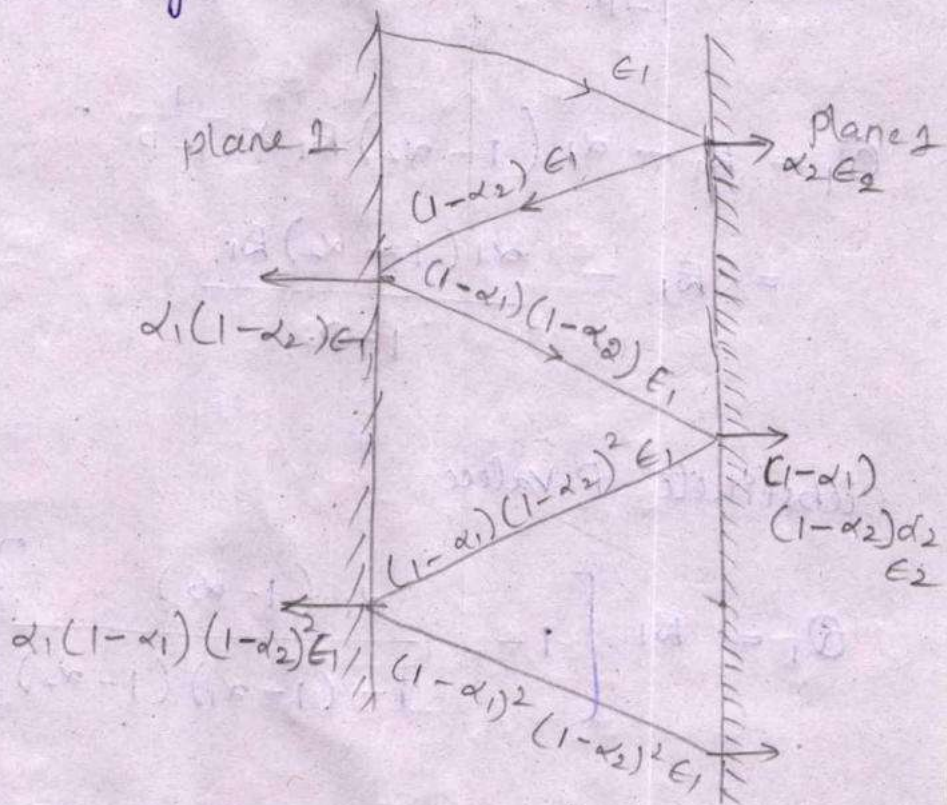
$$\frac{2 \pi r_1^2 \times 2 \pi r_2^2}{\pi}$$

$$= \frac{2 \pi r_1^2 \times 2 \pi r_2^2}{\pi}$$



## Radiation shield (442 Pg)

Consider two very large parallel gray surfaces of areas  $A_1$  and  $A_2$ , at a small distance apart, and exchanging radiation as shown in Fig.



the rate of radiant energy leaving surface 1 is

$$Q_1 = E_1 - [\alpha_1(1-\alpha_2)E_1 + \alpha_1(1-\alpha_1)(1-\alpha_2)^2 E_1 + \alpha_1(1-\alpha_1)^2(1-\alpha_2)^3 E_1 + \dots]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1 [1 + (1-\alpha_1)(1-\alpha_2) + (1-\alpha_1)^2(1-\alpha_2)^2 + \dots]$$

$$= E_1 - \alpha_1(1-\alpha_2)E_1 [1 + P + P^2 + \dots]$$



where  $P = (1 - \alpha_1)(1 - \alpha_2)$

Since  $\alpha_1$  &  $\alpha_2$  are less than 1, as  $P < 1$

$1 + P + P^2 + \dots$  when extended to infinity

$$= \frac{1}{1-P}$$

$$\begin{aligned} Q_1 &= E_1 - \alpha_1 (1 - \alpha_2) E_1 \times \frac{1}{1-P} \\ &= E_1 - \frac{\alpha_1 (1 - \alpha_2) E_1}{1-P} \end{aligned}$$

Substitute  $P$  value

$$Q_1 = E_1 \left[ 1 - \frac{\alpha_1 (1 - \alpha_2)}{1 - (1 - \alpha_1)(1 - \alpha_2)} \right]$$

from Kirchhoff's law, emissivity = absorptivity

$$\alpha_1 = \epsilon_1 \quad \& \quad \alpha_2 = \epsilon_2$$

$$\Rightarrow Q_1 = E_1 \left[ 1 - \frac{\epsilon_1 (1 - \epsilon_2)}{1 - (1 - \epsilon_1)(1 - \epsilon_2)} \right]$$

$$= E_1 \left[ \frac{1 - (1 - \epsilon_1)(1 - \epsilon_2) - \epsilon_1(1 - \epsilon_2)}{1 - (1 - \epsilon_1)(1 - \epsilon_2)} \right]$$



$$= E_1 \left[ \frac{1 - (1 - \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2) - \epsilon_1 (1 - \epsilon_2)}{1 - (1 - \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2)} \right]$$

$$= E_1 \left[ \frac{1 - 1 + \epsilon_2 + \epsilon_1 - \epsilon_1 \epsilon_2 - \epsilon_1 + \epsilon_1 \epsilon_2}{1 - 1 + \epsilon_2 + \epsilon_1 - \epsilon_1 \epsilon_2} \right]$$

$$= \frac{E_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$Q_1 = \frac{E_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

the net radiative heat exchange from surface 1-2 is given by

$$Q_{12} = Q_1 - Q_2$$

$$= \frac{E_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} - \frac{E_2 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$Q_{12} = \frac{E_1 \epsilon_2 - E_2 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$E_p = \sigma T^4$$

$$E_1 = \sigma T_1^4$$

$$E_1 = \epsilon_1 \sigma T_1^4$$

$$E_2 = \epsilon_2 \sigma T_2^4$$



Substitute  $\epsilon_1$  &  $\epsilon_2$  values in eq

$$Q_{12} = \frac{\epsilon_1 \sigma T_1^4 \epsilon_2 - \epsilon_2 \sigma T_2^4 \epsilon_1}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$= \frac{\epsilon_1 \epsilon_2 \sigma [T_1^4 - T_2^4]}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$= \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2} \times \sigma [T_1^4 - T_2^4]$$

$$Q_{12} = \bar{\epsilon} \sigma [T_1^4 - T_2^4]$$

where  $\Rightarrow \bar{\epsilon} = \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$

Divide by  $\epsilon_1 \epsilon_2$ ,

$$\Rightarrow \bar{\epsilon} = \frac{\epsilon_1 \epsilon_2}{\epsilon_1 + \epsilon_2 - \epsilon_1 \epsilon_2}$$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_2} + \frac{1}{\epsilon_1} - 1}$$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$



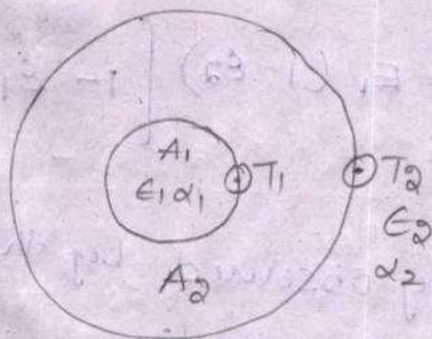
Heat exchange between two parallel surfaces is given by (considering area)

$$Q_{12} = \bar{e} \sigma A [T_1^4 - T_2^4]$$

$$\bar{e} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$

Heat exchange between two large Concentric cylinders or spheres (427)

Consider two large Concentric cylinders of area  $A_1$  &  $A_2$



$$F_{12} A_1 = F_{21} A_2$$

$$F_{12} = 1$$

$$F_{21} = \frac{A_1}{A_2}$$

Considering the energy emitted by the inner cylinder.

① inner cylinder emits the energy =  $E_1$

② outer cylinder absorbs energy =  $\alpha_2 E_1$   
 $= \epsilon_2 E_1$



③ Outer cylinder reflects energy  $= E_1 (1 - \epsilon_2)$

④ Inner cylinder absorbs energy  $= E_1 (1 - \epsilon_2) F_{21} \alpha_1$

$$= E_1 (1 - \epsilon_2) \frac{A_1}{A_2} \epsilon_1$$

$$\left[ \because F_{21} = \frac{A_1}{A_2}, \alpha_1 = \epsilon_1 \right]$$

⑤ Inner cylinder reflects energy

$$= E_1 (1 - \epsilon_2) - E_1 (1 - \epsilon_2) \epsilon_1 \frac{A_1}{A_2}$$

$$= E_1 (1 - \epsilon_2) \left[ 1 - \epsilon_1 \frac{A_1}{A_2} \right]$$

⑥ Energy observed by the inner cylinder on the second reflection.

$$= E_1 (1 - \epsilon_2)^2 \epsilon_1 \frac{A_1}{A_2} \left[ 1 - \frac{A_2}{A_1} \epsilon_1 \right] + \dots$$

$$Q_1 = E_1 - E_1 (1 - \epsilon_2) \epsilon_1 \frac{A_1}{A_2} -$$

$$E_2 (1 - \epsilon_2)^2 \epsilon_1 \frac{A_1}{A_2} \left[ 1 - \frac{A_1}{A_2} \epsilon_1 \right] + \dots$$

$$= E_1 \left[ 1 - \frac{A_1}{A_2} \epsilon_1 (1 - \epsilon_2) \left\{ 1 + (1 - \epsilon_2) \left( 1 - \frac{A_1}{A_2} \epsilon_1 \right) + (1 - \epsilon_2)^2 \times \left( 1 - \frac{A_1}{A_2} \epsilon_1 \right)^2 + \dots \right\} \right]$$



$$F_1 \left[ 1 - \frac{\frac{A_1}{A_2} \epsilon_1 (1 - \epsilon_2)}{1 - (1 - \epsilon_2) \left[ 1 - \frac{A_1}{A_2} \epsilon_1 \right]} \right]$$

$$Q_1 = \frac{\cancel{F_1} F_1 \epsilon_2}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2}$$

Similarly

$$Q_2 = \frac{F_2 \epsilon_1 \frac{A_1}{A_2}}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2}$$

The net radiation heat transfer between the inner & outer concentric cylinders is given by

$$Q_{12} = Q_1 - Q_2$$

$$Q_{12} = \frac{F_1 \epsilon_2}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2} - \frac{F_2 \epsilon_1 \frac{A_1}{A_2}}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2}$$

Considering area  $A_1$  &  $A_2$

$$\Rightarrow Q_{12} = \frac{A_1 F_1 \epsilon_2}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2} - \frac{A_2 F_2 \epsilon_1 \frac{A_1}{A_2}}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2}$$

$$\Rightarrow Q_{12} = \frac{A_1 F_1 \epsilon_2 - A_1 F_2 \epsilon_1}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2} \rightarrow (a)$$



from Stefan Boltzman Law, we know that,

$$E_b = \epsilon \sigma T^4$$

$$E_1 = \epsilon_1 \sigma T_1^4$$

$$E_2 = \epsilon_2 \sigma T_2^4$$

Substituting  $E_1$  &  $E_2$  in eq

$$Q_{12} = \frac{A \epsilon_1 \sigma T_1^4 \epsilon_2 - A_1 \epsilon_2 \sigma T_2^4 \epsilon_1}{\frac{A_1}{A_2} \epsilon_1 + \epsilon_2 - \frac{A_1}{A_2} \epsilon_1 \epsilon_2}$$

$$= \frac{A_1 \sigma \epsilon_1 \epsilon_2 [T_1^4 - T_2^4]}{\left[ \frac{A_1}{A_2} \epsilon_1 \epsilon_2 \left( \frac{1}{\epsilon_2} - 1 \right) + \epsilon_2 \right]}$$

Dividing by  $\epsilon_1 \epsilon_2$

$$Q_{12} = \frac{A_1 \sigma [T_1^4 - T_2^4]}{\frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right) + \frac{1}{\epsilon_1}}$$

$$Q_{12} = \frac{A_1 \sigma (T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

$$Q_{12} = \bar{\epsilon} A_1 \sigma (T_1^4 - T_2^4)$$



$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)}$$

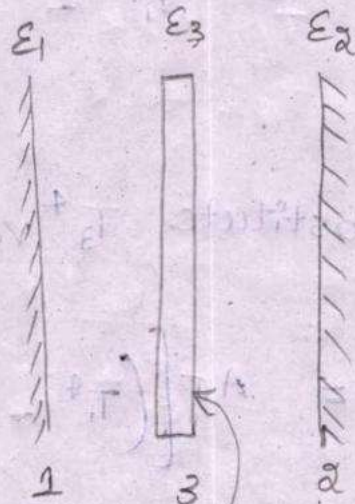
for cylinder, Area  $A = 2\pi rL$

for sphere  $A = 4\pi r^2$

### Radiation shield (442)

The net heat exchange between parallel plates without radiation shield

$$Q_{12} = \frac{A\sigma(T_1^4 - T_2^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1}$$



radiation shield.

Heat exchange between 1-3

$$Q_{13} = \frac{A\sigma(T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} \rightarrow \textcircled{a}$$

Heat exchange between 3-2

$$Q_{32} = \frac{A\sigma(T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1} \rightarrow \textcircled{b}$$



from (a)

$$T_1^4 - T_3^4 = \frac{Q_{13} \left[ \frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right]}{A\sigma}$$

Let's find the heat exchange between the two spheres

Let's find the heat exchange between the two spheres

$$\Rightarrow T_3^4 = T_1^4 - \frac{Q_{13} \left[ \frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right]}{A\sigma}$$

Let's find the heat exchange between the two spheres

Substitute  $T_3^4$  value in (b)

$$Q_{32} = A\sigma \left[ \left( T_1^4 - \frac{Q_{13} \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1 \right)}{A\sigma} \right) - T_2^4 \right]$$
$$\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1$$

$$\Rightarrow Q_{32} \left( \frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1 \right) = A\sigma T_1^4 - Q_{13} \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right)$$
$$Q_{32} \left( \frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1 \right) = A\sigma T_1^4 - Q_{13} \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right)$$

under equilibrium condition

$$Q_{13} = Q_{32}$$

$$Q_{13} \left[ \left( \frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1 \right) + \left( \frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1 \right) \right] = A\sigma (T_1^4 - T_2^4)$$



$$Q_{13} = \frac{A\sigma(T_1^4 - T_2^4)}{\left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right)} \rightarrow \textcircled{c}$$

Dividing the equation  $\textcircled{c}$  in eq  $\textcircled{a}$

$$\frac{Q_{13}}{Q_{12}} = \frac{\left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1\right)}{\left(\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1\right) + \left(\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1\right)}$$

If  $\epsilon_1 = \epsilon_2 = \epsilon_3$

$$\frac{Q_{13}}{Q_{12}} = \frac{1}{2}$$

$$Q_{13} = \frac{1}{2} Q_{12} \quad (\text{or}) \quad Q_{32} = \frac{1}{2} Q_{12}$$

Thus by inserting one shield between two parallel surfaces the direct radiation heat transfer between them is halved.



① Calculate net radiant interchange for square meter. for two large planes at a temperature of 900K and 400K respectively. Assume that the emissivity of hot plane is 0.9 and that of cold plane is 0.7.

Sol

$$T_1 = 900\text{K}$$

$$\epsilon_1 = 0.9$$

$$T_2 = 400\text{K}$$

$$\epsilon_2 = 0.7$$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{\frac{1}{0.9} + \frac{1}{0.7} - 1} = 0.649$$

$$\frac{Q_{12}}{A} = \bar{\epsilon} \sigma [T_1^4 - T_2^4]$$

$$\frac{Q_{12}}{A} = 0.649 \times 5.67 \times 10^{-8} [900^4 - 400^4]$$

$$= 23.20 \text{ kW/m}^2$$

② Two large parallel plates are maintained at a temperature of 900K & 500K respectively. Each plate has an area of 6m<sup>2</sup>. Compare the net heat exchange between the plates for the following cases. (i) both plates are black  
ii) plates of an emissivity of 0.5.



50)  $T_1 = 900 \text{ K}$

$T_2 = 500 \text{ K}$

$A = 6 \text{ m}^2$

(i) both plates are black

$\epsilon = 1$

$$\begin{aligned} Q_{12} &= \epsilon \sigma A (T_1^4 - T_2^4) \\ &= 1 \times 5.67 \times 10^{-8} \times 6 \times (900^4 - 500^4) \\ &= 201942.7 \text{ W/m}^2 \text{ W} \end{aligned}$$

(ii) plates of an emissivity

$\epsilon_1 = \epsilon_2 = 0.5$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{\frac{1}{0.5} + \frac{1}{0.5} - 1} = 0.33$$

$$\begin{aligned} Q_{12} &= \bar{\epsilon} \sigma A (T_1^4 - T_2^4) \\ &= 0.33 \times 5.67 \times 10^{-8} \times 6 \times (900^4 - 500^4) \\ &= 66641.09 \text{ W} \end{aligned}$$

③ Calculate the heat exchange by radiation between the surfaces of two long cylinders having radii 120 mm & 60 mm respectively. The axis of the cylinder are parallel to each other. The inner cylinder is maintain at



130°C and emissivity of 0.6. The outer cylinder is maintain at 30°C and emissivity of 0.5

$$\text{in-out} = A_1$$

$$\text{out-in} = A_2$$

Sol  $r_1 = 60 \times 10^{-3} \text{ m}$

$$r_2 = 120 \times 10^{-3} \text{ m}$$

$$T_1 = 403 \text{ K}$$

$$\epsilon_1 = 0.6$$

$$\epsilon_2 = 0.5$$

$$T_2 = 303 \text{ K}$$

$$L_1 = L_2 = 1$$

$$\epsilon = \frac{1}{\frac{1}{\epsilon_1} + \frac{A_1}{A_2} \left( \frac{1}{\epsilon_2} - 1 \right)} = \frac{1}{\frac{1}{0.6} + \frac{0.376}{0.753} \left[ \frac{1}{0.5} - 1 \right]}$$

$$= 0.4616$$

$$A_1 = \pi D_1 L = \pi \times 120 \times 10^{-3} \text{ m} \times 1 = 0.376$$

$$A_2 = \pi D_2 L = \pi \times 240 \times 10^{-3} \times 1 = 0.753$$

$$Q_{12} = \epsilon \sigma A_1 (T_1^4 - T_2^4)$$

$$= 0.4616 \times 5.67 \times 10^{-8} \times 0.376 (403^4 - 303^4)$$

$$= 176.6 \text{ W}$$

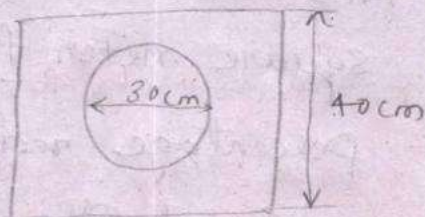
3(b) rate of evaporation of liquid oxygen if its rate of evaporation of latent heat

is  $200 \text{ kJ/kg}$

$$\frac{Q}{\text{Latent heat}} = \text{Rate of evaporation} = \frac{176}{200 \times 10^3} = 8.8 \times 10^{-4} \text{ kg/sec}$$



- ④ A pipe of outer diameter is 30cm having emissivity 0.6 and at a temperature of 600K runs centrally in a brick duct of 40cm side square section having emissivity 0.8 and at a temperature of 300K. Calculate the following (i) Heat exchange per meter length (ii) Calculate the Convective heat transfer Co-efficient when surrounding of duct is 280K.



Sol  $D_1 = 30 \times 10^{-2} \text{ m}$

$\epsilon_1 = 0.6$

$T_1 = 600 \text{ K}$

Side =  $40 \times 10^{-2} \text{ m}$

$T_2 = 300 \text{ K}$

$\epsilon_2 = 0.8$

$$\bar{\epsilon} = \frac{1}{\frac{1}{\epsilon_1} + \left( \frac{1}{\epsilon_2} - 1 \right) \frac{A_1}{A_2}} = \frac{1}{\frac{1}{0.6} + \left( \frac{1}{0.8} - 1 \right) \frac{0.942}{1.6}} = 0.5513$$

$A_1 = \pi D L = \pi \times 30 \times 10^{-2} \times 1 = 0.942$

$A_2 = (0.4 \times 4) \times 1 \times 1 = 1.6 \text{ m}^2$

$\frac{Q_{12}}{A} = \bar{\epsilon} \sigma \times A_1 (T_1^4 - T_2^4) = 0.55 \times 5.67 \times 10^{-8} \times 0.942 \times (600^4 - 300^4) = 3569.2 \text{ W/m.}$



(ii) Convective heat transfer co-efficient

$$Q = hA(T_w - T_s)$$

$$3569.2 = h \times 1 (300 - 280)$$

$$h = 178.46 \text{ W/m}^2\text{K}$$

⑤ Emissivity of two large parallel plates maintain at  $800^\circ\text{C}$ ,  $300^\circ\text{C}$  are 0.3 & 0.5 respectively. Find the net radiant heat exchange per square meter for these plates. Find the percentage reduction in heat transfer with a polished Aluminium radiation shield of emissivity 0.06. Find is placed between them. and also Find the temperature of the shield.

Sol Given  $\epsilon_1 = 0.3$   $\epsilon_3 = 0.06$   
 $\epsilon_2 = 0.5$   $T_1 = 1073 \text{ K}$   
 $T_2 = 573 \text{ K}$

$$Q_{12} = \epsilon \sigma A (T_1^4 - T_2^4)$$

$$\epsilon = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_2} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.5} - 1} = 0.23$$

$$\frac{Q_{12}}{A} = 0.23 \times 5.67 \times 10^{-8} \left( 1073^4 - (300+273) 573^4 \right)$$
$$= 15.8 \text{ kW/m}^2$$



$$Q_{13} = Q_{32}$$

$$\bar{\epsilon} A \sigma (T_1^4 - T_3^4) = \bar{\epsilon} A \sigma (T_3^4 - T_2^4)$$

$$\frac{A \sigma (T_1^4 - T_3^4)}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{A \sigma (T_3^4 - T_2^4)}{\frac{1}{\epsilon_3} + \frac{1}{\epsilon_2} - 1}$$

$$\frac{(1073^4 - T_3^4)}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = \frac{(T_3^4 - 573^4)}{\frac{1}{0.06} + \frac{1}{0.5} - 1}$$

$$T_3 = 911.5 \text{ K}$$

$$Q_{13} = \epsilon \sigma A (T_1^4 - T_3^4)$$

$$\frac{Q_{13}}{A} = \epsilon \sigma (T_1^4 - T_3^4)$$

$$= 0.05 \times 5.67 \times 10^{-8} (1073^4 - 911.4^4)$$

$$= 1081.86 \text{ W/m}^2$$

$$\epsilon_{13} = \frac{1}{\frac{1}{\epsilon_1} + \frac{1}{\epsilon_3} - 1} = \frac{1}{\frac{1}{0.3} + \frac{1}{0.06} - 1} = 0.05$$



$$(iii) \text{ Percentage reduction} = \frac{Q_{12} - Q_{13}}{Q_{12}}$$

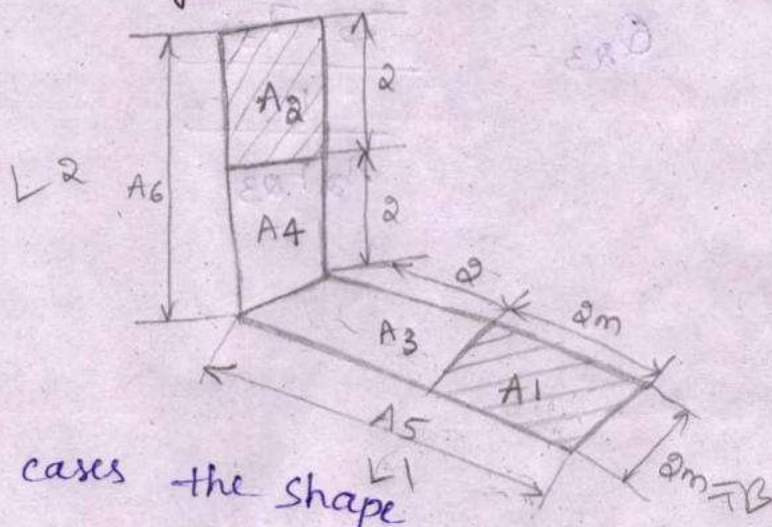
$$= \frac{15880 - 1895.7}{15880}$$

$$= 1 - \frac{1}{8.8} = 88\%$$



① Find the shape factor  $F_{12}$  for the figure shown below. In the figure the areas  $A_1$  and  $A_2$  are perpendicular but don't share the common edge.

Sol



⇒ for such cases the shape factor is evaluated by introducing hypothetical area. So, that the arrangement of perpendicular surface has a common edge.

Now  $A_5 = A_1 + A_3$

$A_6 = A_4 + A_2$

$$A_5 F_{5-6} = A_1 F_{1-6} + A_3 F_{3-6}$$

$$= (A_1 F_{1-4} + A_1 F_{1-2}) + A_3 F_{3-6}$$

$$= [(A_5 F_{5-4} - A_3 F_{3-4}) + A_1 F_{1-2}] + A_3 F_{3-6} = A_1 F_{1-2}$$

$$\Rightarrow A_1 F_{12} = A_5 F_{5-6} - A_3 F_{3-6} - A_5 F_{5-4} + A_3 F_{3-4}$$

$$\Rightarrow A_1 F_{12} = (A_5 F_{5-6} + A_3 F_{3-4}) - (A_5 F_{5-4} + A_3 F_{3-6})$$



surface  $z/a$   $y/a$  shape factor

$A_{5-6}$   $4/2 = 2$   $4/2 = 2$   $0.14930$

$A_{3-4}$   $2/2 = 1$   $2/2 = 1$   $0.2004$

$A_{5-4}$   $2/2 = 1$   $4/2 = 2$   $0.11643$

$A_{3-6}$   $4/2 = 2$   $2/2 = 1$   $0.23285$

$Z = \frac{L}{B}$

$A_1 = (2 \times 2) F_{1-2} = 4x$

$V = \frac{L}{B}$

$A_5 = (4 \times 2) F_{5-6} = 8x 0.14930 =$

$A_3 = (2 \times 2) F_{3-4} = 4x 0.2004 =$

$A_6$

$(F_{1-2}) = (2 \times 2) x F_{1-2} = (4 \times 2) \times 0.149 + (2 \times 2) \times 0.2004$   
 $- (4 \times 2) \times 0.11 + (2 \times 2) \times 0.23$

$F_{1-2} = 0.05$